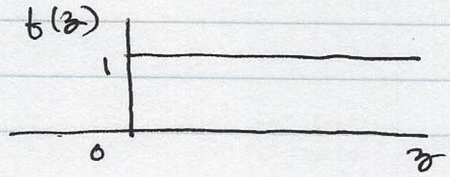
IMPLEMENTATION OF BOOLEAN FUNCTIONS VIA ANNS

THE ACTIVATION FUNCTION IN THE FOLLOWING IMPLEMENTATIONS IS HEAVISIDE-FUNCTION.



$y = f(z)$

$$y = \begin{cases} 1 & \text{IF } z \geq 0 \\ 0 & \text{IF } z < 0 \end{cases}$$

BOOLEAN FUNCTIONS OF INTEREST ARE:

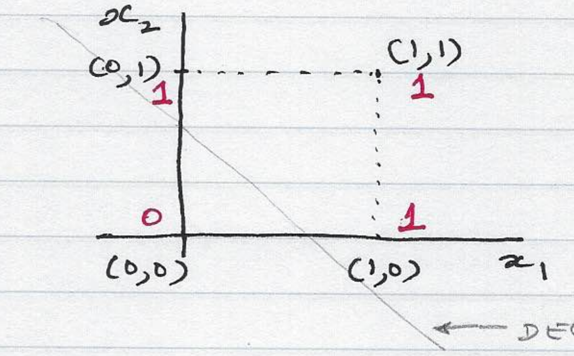
1. OR OPERATION DENOTED BY $\vee$ OR SIMPLY $+$
2. AND " " " $\wedge$
3. NOT " " " $-$ COMPLEMENTATION (BAR)
4. XOR " $\oplus$ EXCLUSIVE OR

BOOLEAN VARIABLES TAKE VALUES FROM THE SET $\{0, 1\}$.

x, x_1, x_2 ARE BOOLEAN VARIABLES.

NOTE THAT ALL BOOLEAN FUNCTIONS CAN BE IMPLEMENTED BY: (AND, NOT) PAIR
(OR, NOT) PAIR

OR OPERATION



← DECISION BOUNDARY (LINE)

x_1	x_2	$x_1 \vee x_2$
0	0	0
0	1	1
1	0	1
1	1	1

$w_1 = w_2 = 1 \; ; b = -0.5$

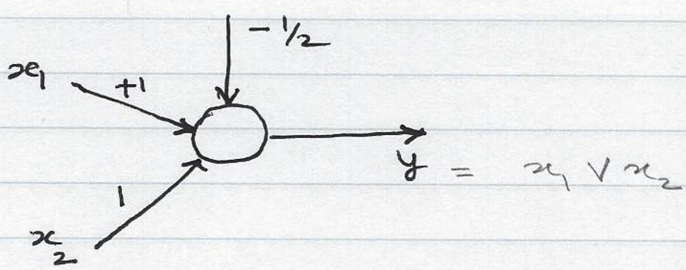
$z = w_1 x_1 + w_2 x_2 + b$

$\quad = x_1 + x_2 - 0.5$

$y = f(z)$ (HEAVISIDE FUNCTION)

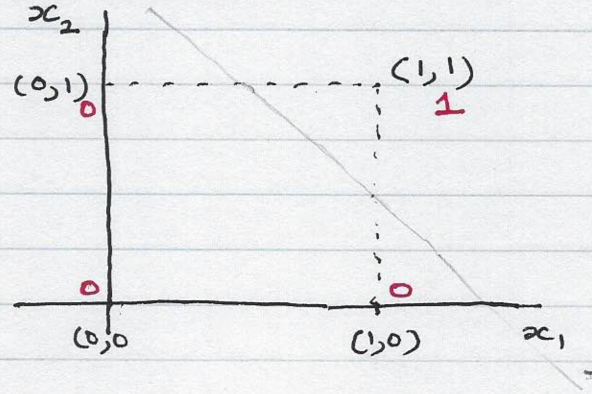
x_1	x_2	z	y
0	0	-0.5	0
0	1	0.5	1
1	0	0.5	1
1	1	1.5	1

$x_1 + x_2 - 0.5 = 0$ ← DECISION BOUNDARY



ARTIFICIAL NEURAL OR

AND OPERATION



DECISION BOUNDARY (LINE)

x_1	x_2	$x_1 \wedge x_2$
0	0	0
0	1	0
1	0	0
1	1	1

$w_1 = w_2 = 1 \; ; \; b = -1.5$

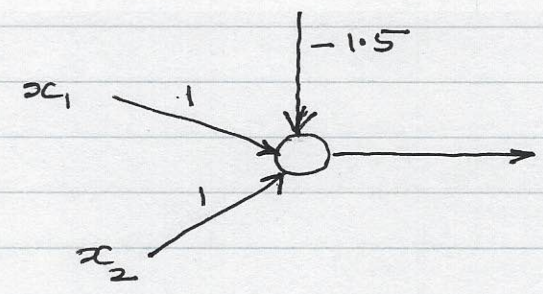
$z = w_1 x_1 + w_2 x_2 + b$

$\quad = x_1 + x_2 - 1.5$

$y = f(z)$ (HEAVISIDE FUNCTION)

x_1	x_2	z	y
0	0	$-\cancel{0}1.5$	0
0	1	-0.5	0
1	0	-0.5	0
1	1	0.5	1

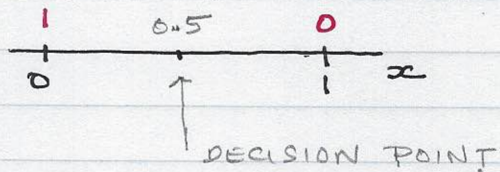
$x_1 + x_2 - 1.5 = 0$ ← DECISION BOUNDARY



ARTIFICIAL NEURON AND

NOT OPERATION

x	$\bar{x}$
0	1
1	0



$w = -1$; $b = +0.5$

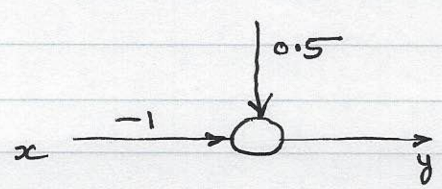
$\overset{z}{\cancel{y}} = wx + b$

$= -x + 0.5$

x	z	y
0	0.5	1
1	-0.5	0

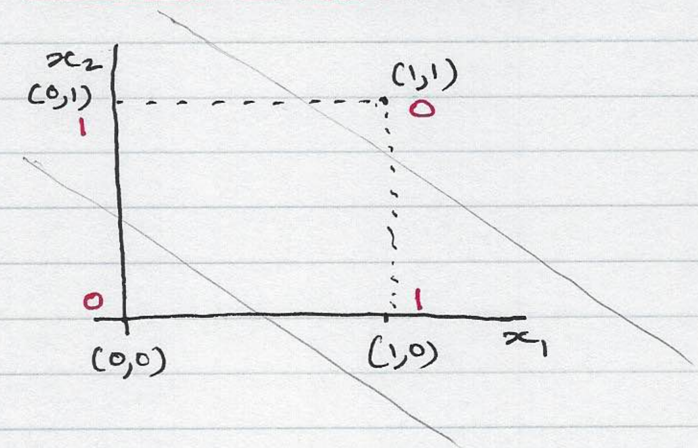
$y = f(z)$ (HEAVISIDE FUNCTION)

$-x + 0.5 = 0$ ← DECISION POINT



ARTIFICIAL NEURAL NOT

EXCLUSIVE OR OPERATION



x_1	x_2	$x_1 \oplus x_2$
0	0	0
0	1	1
1	0	1
1	1	0

TWO STRAIGHT LINES ARE REQUIRED, IN ORDER TO SPECIFY THE DECISION BOUNDARY

THEREFORE EXCLUSIVE OR OPERATION CANNOT BE IMPLEMENTED BY A SINGLE NEURON. (ARTIFICIAL)

HOWEVER IT CAN BE IMPLEMENTED BY MORE THAN A SINGLE ARTIFICIAL NEURON.

NOTE THAT:

$$x_1 \oplus x_2 = \underbrace{x_1\bar{x}_2 \overbrace{+}^{\text{OR}} \bar{x}_1 x_2}_{}$$

OR, AND

$$= (x_1 + x_2)(\bar{x}_1 + \bar{x}_2)$$

OR AND OR

i) $(x_1 + x_2)$ REQUIRES OR OPERATION (KNOWN)

ii) 'AND' OPERATION KNOWN.

iii) $(\bar{x}_1 + \bar{x}_2)$ IS ON NEXT PAGE

$w_1 = -1$; $w_2 = -1$; $b = \frac{3}{2}$

$$z = w_1 x_1 + w_2 x_2 + b$$

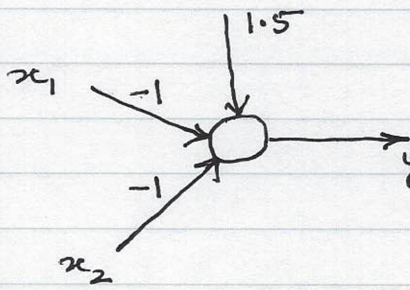
$$= -x_1 - x_2 + \frac{3}{2}$$

$y = f(z)$ (HEAVISIDE FUNCTION)

x_1	x_2	$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_1 + \bar{x}_2$
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	0

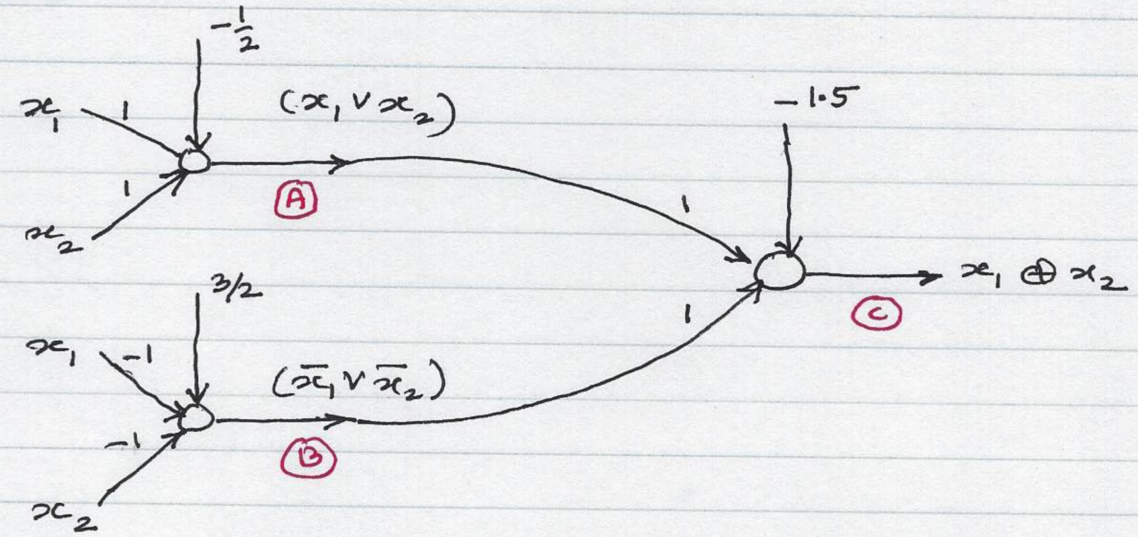
$-x_1 - x_2 + \frac{3}{2} = 0 \leftarrow$ DECISION BOUNDARY

x_1	x_2	z	y
0	0	1.5	1
0	1	0.5	1
1	0	0.5	1
1	1	-0.5	0



$y = (\bar{x}_1 + \bar{x}_2) = \bar{x}_1 \vee \bar{x}_2$

ARTIFICIAL NEURON IMPLEMENTS $\bar{x}_1 \vee \bar{x}_2$



x_1	x_2	A	B	C
0	0	0	1	0
0	1	1	1	1
1	0	1	1	1
1	1	1	0	0

$x_1 \vee x_2$ $\qquad$ $\bar{x}_1 \vee \bar{x}_2$ $\qquad$ $x_1 \oplus x_2$

$\therefore$ $\oplus$ IS IMPLEMENTED BY THREE ARTIFICIAL NEURONS.