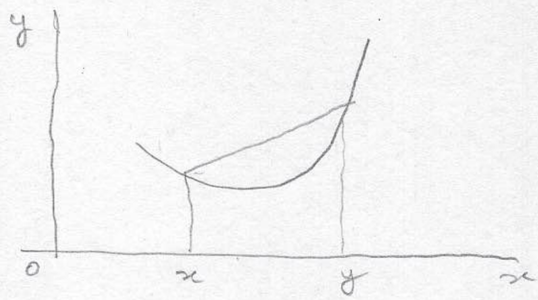
ASSIGNMENT #3 CONTD.

CONVEX FUNCTION - SINGLE VARIABLE.

DEF 1: A FUNCTION f IS CONVEX FOR ALL x, y AND ALL $\alpha \in (0,1)$ IF

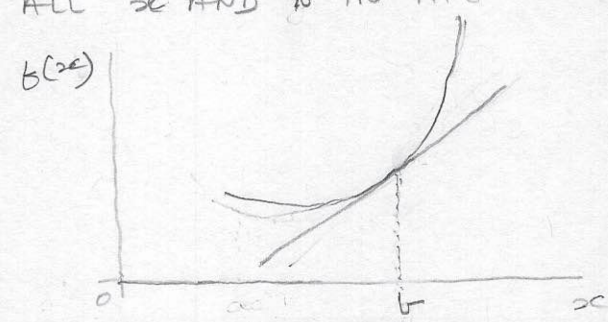
$$f(\alpha x + (1-\alpha)y) \leq \alpha f(x) + (1-\alpha) f(y)$$



DEF 2: A CONTINUOUSLY DIFFERENTIABLE FUNCTION OF ONE VARIABLE IS CONVEX ON AN ITERVAL I IF AND ONLY IF THE FUNCTION LIES ABOVE ALL OF ITS TANGENTS. THAT IS,

$$f(x) \geq f(b) + f'(b)(x-b)$$

FOR ALL x AND b IN THE INTERVAL I



EXAMPLE OF CONVEX FUNCTIONS

x^2, x^4 ; $|x|^p$; $p \geq 1$

e^x

ASSIGNMENT #3 (CONTD.)

#3 (a) $f(x) = x^3 - 12x + 1$

$f'(x) = 3x^2 - 12 = 0 \Rightarrow 3(x^2-4) = 0 \Rightarrow x = \pm 2$

$f''(x) = 6x.$

$f''(2) = 12 > 0 \Rightarrow x = 2$ IS A LOCAL MINIMA ;

$f''(-2) = -12 < 0 \Rightarrow x = -2$ IS " " MAXIMA

(b) $f(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$

$f'(x) = 4x^3 - 12x^2 + 12x - 4 = 4(x^3 - 3x^2 + 3x - 1) = 4(x-1)^3$

$= 0 \Rightarrow x = 1$

$f''(x) = 12(x-1)^2$; $f''(1) = 0 \Rightarrow x = 1$ MAY OR MAY NOT

BE A LOCAL EXTREMUM.

$f(x) = (x-1)^4$:

$f(1) = 0$; $\left.\begin{array}{l} f(1-\epsilon) = \epsilon^4 > 0 \\ f(1+\epsilon) = \epsilon^4 > 0 \end{array}\right\} \forall \epsilon > 0$

$x = 1$ IS A LOCAL MINIMA

(c) $f(x) = x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$

$f'(x) = 5x^4 - 20x^3 + 30x^2 - 20x + 5$

$= 5(x^4 - 4x^3 + 6x^2 - 4x + 1) = 5(x-1)^4 = 0 \Rightarrow x = 1$

$f''(x) = 20(x-1)^3$; $f''(1) = 0$

$f(1-\epsilon) < 0$; $f(1+\epsilon) > 0 \quad \forall \epsilon > 0$

$x = 1$ IS NEITHER A MAXIMA NOR MINIMA

(d) $f(x) = (4-2x)e^{x^2}$

$f'(x) = (4-2x)2xe^{x^2} + (-2)e^{x^2} = e^{x^2}(-4x^2 + 8x - 2)$

$f'(x) = 0 \Rightarrow (4x^2 - 8x + 2) = 0 \Rightarrow (2x^2 - 4x + 1) = 0$

$\Rightarrow x = \frac{+4 \pm \sqrt{16-8}}{4} = \frac{4 \pm 2\sqrt{2}}{4} = \frac{2 \pm \sqrt{2}}{2} = 1 \pm \frac{1}{\sqrt{2}} = x_1, x_2$

$f''(x) = e^{x^2}(-8x + 8) + 2xe^{x^2}(-4x^2 + 8x - 2)$

$f''(x_i) = e^{x_i^2} 8(1 - x_i) = e^{x_i^2} 8\left(\mp \frac{1}{\sqrt{2}}\right)$; $i = 1, 2$

i) $f''(x_1) < 0 \Rightarrow f(x)$ IS MAXIMUM AT $x = \left(1 + \frac{1}{\sqrt{2}}\right)$

ii) $f''(x_2) > 0 \Rightarrow f(x)$ IS MINIMUM A $x = \left(1 - \frac{1}{\sqrt{2}}\right)$