Probability Distributions
of Continuous Random Variables
by
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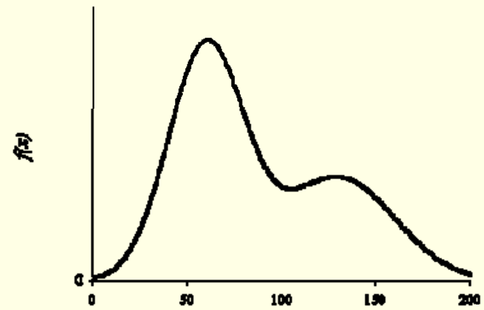
(modified)

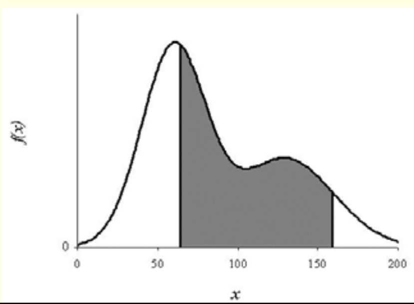
Probability Density Function

The **probability distribution** of a ***continuous*** random variable is describe by ***probability density curve f(x)***.

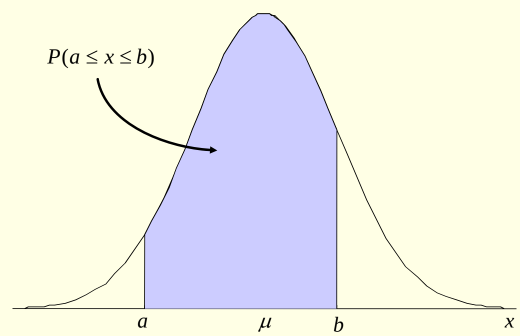


Notes:

- The Total Area under the probability density curve is 1.
- The Area under the probability density curve is from a to b is *P[a < X < b]*.



Normal Probability Distributions
(Bell shaped curve)



Mean, Variance, and Standard Deviation of a Continuous Probability Distribution

- Describe the center and spread of a probability distribution
- The mean (denoted by greek letter μ (mu)), measures the centre of the distribution.
- The variance (σ^2) and the standard deviation (σ) measure the spread of the distribution.

  σ is the greek letter for s.

Mean of a Continuous Random Variable (uses calculus)

- The mean, μ, of a discrete random variable *x*

$$\mu = \int_{-\infty}^{\infty} x f(x)\,dx$$

Notes:

- The mean is a ***weighted average*** of the values of *X*.
- The mean is the ***long-run average*** value of the random variable.
- The mean is ***centre of gravity*** of the probability distribution of the random variable

Variance and Standard Deviation

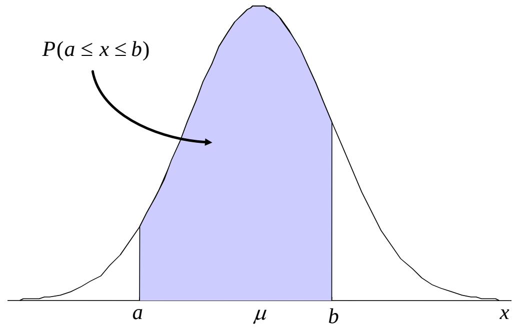
Variance of a Continuous Random Variable: Variance, σ^2, of a discrete random variable x

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 \, f(x)\, dx$$

Standard Deviation of a Discrete Random Variable: The positive square root of the variance:

$$\sigma = \sqrt{\sigma^2}$$

Normal Probability Distributions

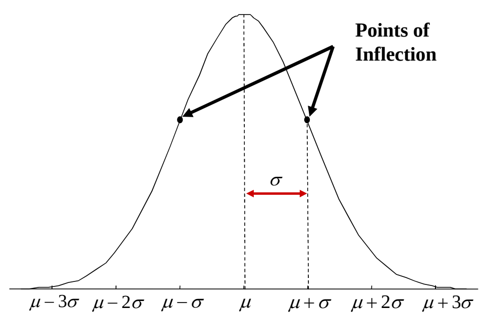


Normal Probability Distributions

- The normal probability distribution is the most important distribution in all of statistics

- Many continuous random variables have normal or approximately normal distributions

The Normal Probability Distribution

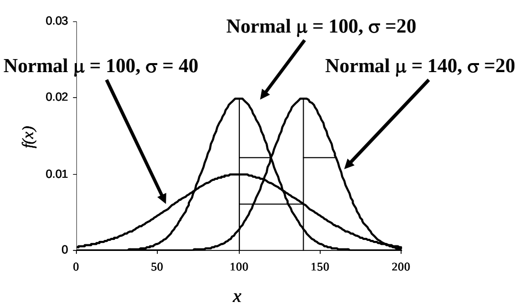


Main characteristics of the Normal Distribution

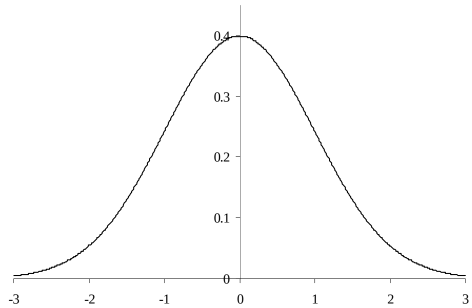
- Bell Shaped, symmetric
- Points of inflection on the bell shaped curve are at $\mu - \sigma$ and $\mu + \sigma$. That is one standard deviation from the mean
- Area under the bell shaped curve between $\mu - \sigma$ and $\mu + \sigma$ is approximately 2/3.
- Area under the bell shaped curve between $\mu - 2\sigma$ and $\mu + 2\sigma$ is approximately 0.95.

There are many Normal distributions

depending on by μ and σ



The Standard Normal Distribution
$\mu = 0, \sigma = 1$



- There are infinitely many normal probability distributions (differing in μ and σ)
- Area under the Normal distribution with mean μ and standard deviation σ can be converted to area under the **standard normal distribution**
- If X has a Normal distribution with mean μ and standard deviation σ than

$$z = \frac{X - \mu}{\sigma}$$

has a **standard normal distribution.**

Converting Area
under the Normal distribution with mean μ and standard deviation σ
to
Area under the standard normal distribution

Perform the z-transformation

$$z = \frac{X - \mu}{\sigma}$$

Area under the Normal distribution with mean μ and standard deviation σ

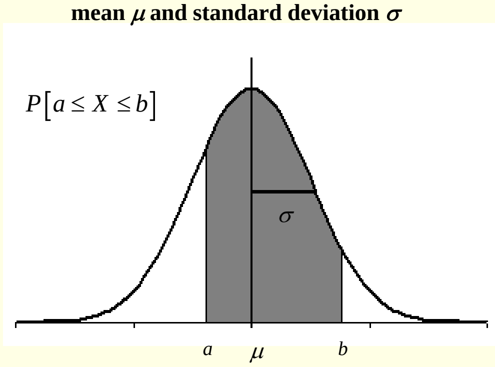
then $P\left[a \le X \le b\right]$

$$= P\left[\frac{a-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{b-\mu}{\sigma}\right]$$

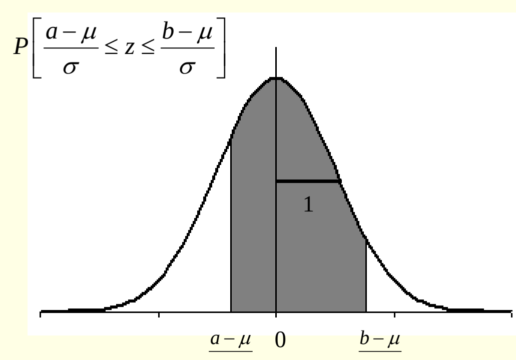
$$= P\left[\frac{a-\mu}{\sigma} \le z \le \frac{b-\mu}{\sigma}\right]$$

Area under the **standard normal distribution**

Area under the Normal distribution with mean μ and standard deviation σ



Area under the **standard normal distribution**



Example 2

A bottling machine is adjusted to fill bottles with a mean of 32.0 oz of soda and standard deviation of 0.02. Assume the amount of fill is normally distributed and a bottle is selected at random:

1) Find the probability the bottle contains between 32.00 oz and 32.025 oz
2) Find the probability the bottle contains more than 31.97 oz

Solution part 1)

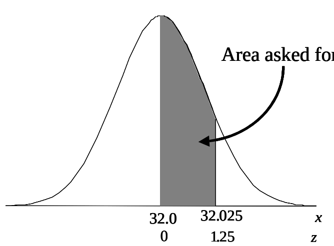
When x = 32.00

$$z = \frac{32.00 - \mu}{\sigma} = \frac{32.00 - 32}{0.02} = 0.00$$

When x = 32.025

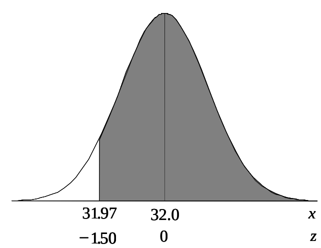
$$z = \frac{32.025 - \mu}{\sigma} = \frac{32.025 - 32}{0.02} = 1.25$$

Graphical Illustration:



$$P(32.0 < X < 32.025) = P\left(\frac{32.0 - 32.0}{0.02} < \frac{X - 32.0}{0.02} < \frac{32.025 - 32.0}{0.02}\right)$$
$$= P(0 < z < 1.25) = 0.3944$$

Example 2, Part 2



$$P(x > 31.97) = P\left(\frac{x - 32.0}{0.02} > \frac{31.97 - 32.0}{0.02}\right) = P(z > -1.50)$$
$$= 1.0000 - 0.0668 = 0.9332$$