## OVERDETERMINED SYSTEM

- IN MATHEMATICS, A SYSTEM OF EQUATIONS IS CONSIDERED OVERDETERMINED, IF THERE ARE MORE EQUATIONS THAN UNKNOWNS.
- AN OVERDETERMINED SYSTEM IS ALMOST ALWAYS INCONSISTENT (IT HAS NO SOLUTIONS).
- METHOD OF ORDINARY LEAST SQUARES CAN BE USED TO FIND AN APPROXIMATE SOLUTION TO THE OVERDETERMINED SYSTEMS
- FOR THE SYSTEM $Ax = b$, THE LEAST-SQUARES FORMULA IS OBTAINED FROM THE PROBLEM

$\min_x \|Ax - b\|$, WHERE $\|\cdot\|$ IS THE EUCLIDEAN NORM.

THE SOLUTION OF WHICH CAN BE WRITTEN AS

$$x = (A^T A)^{-1} (A^T b)$$

## UNDETERMINED SYSTEM

IN MATHEMATICS, A SYSTEM OF LINEAR EQUATIONS IS CONSIDERED UNDETERMINED, IF THERE ARE FEWER EQUATIONS THAN UNKNOWNS.

## WELL-POSED PROBLEM

ACCORDING TO THE FRENCH MATHEMATICIAN JACQUES HADAMARD, MATHEMATICAL MODELS OF PHYSICAL PHENOMENON SHOULD HAVE THE FOLLOWING PROPERTIES:

1. A SOLUTION EXISTS
2. SOLUTION IS UNIQUE.
3. SOLUTION'S BEHAVIOR CHANGES CONTINUOUSLY WITH INITAL CONDITIONS.

- PROBLEMS THAT ARE NOT WELL-POSED IN THE SENSE OF HADAMARD ARE CALLED ILL-POSED.

- EVEN IF A PROBLEM IS WELL-POSED, IT MAY STILL BE ILL-CONDITIONED, MEANING THAT A SMALL ERROR IN THE INITIAL DATA CAN RESULT IN MUCH LARGER ERRORS IN THE ANSWERS. AN ILL-CONDITIONED PROBLEM IS INDICATED BY LARGE CONDITION NUMBER.

- IF THE PROBLEM IS NOT WELL-POSED, IT NEED TO BE RE-FORMULATED FOR NUMERICAL TREATMENT. TYPICALLY THIS INVOLVES INCLUDING ADDITIONAL ASSUMPTIONS, SUCH AS SMOOTHNESS OF SOLUTION. THIS PROCESS IS KNOWN AS REGULARIZATION.

TIKHONOV REGULARIZATION IS ONE OF THE MOST COMMONLY USED, FOR REGULARIZATION OF LINEAR ILL-POSED PROBLEMS IN STATISTICS, THE METHOD IS KNOWN AS RIDGE REGRESSION.