EXPECTED MAXIMIZATION ALGORITHM

1. FINDING MAXIMUM LIKELIHOOD ESTIMATES REQUIRE A NUMERICAL METHOD.
   - CLASSICAL TECHNIQUES LIKE NEWTON-RAPHSON AND GAUSS-NEWTON HAVE INSPIRATION FROM CALCULUS.
   - EXPECTED MAXIMIZATION (EM) ALGORITHM IS A STATISTICALLY MOTIVATED TECHNIQUE.

2. THIS IS USEFUL WHERE
   - SOME PARTS OF THE DATA ARE MISSING, AND ANALYSIS OF INCOMPLETE DATA IS SOMEWHAT COMPLICATED OR NONLINEAR.
   - IT IS POSSIBLE TO 'FILL IN' THE MISSING DATA, AND THEN ANALYSIS OF THE COMPLETE DATA IS RELATIVELY SIMPLE.

3. THE PHRASE EM WAS COINED BY:
   A. P. DEMPSTER, N.M. LAIRD, AND D. B. RUBIN (1977).

MAIN IDEA

$x = \{x_1, x_2, \ldots, x_n\}$ = SET OF AVAILABLE DATA POINTS

β = VECTOR OF PARAMETERS TO BE ESTIMATED

$L_x(\beta)$ = LOG-LIKELIHOOD FUNCTION

$\beta^{(\tau)}$ = INITIAL ESTIMATE OF β

τ = ITERATING INDEX

E = EXPECTATION OPERATOR

z = HIDDEN RANDOM VECTOR

EM ALGORITHM OBTAINS THE MLE $\hat{\beta}$ BY THE FOLLOWING ITERATION.:

- E-STEP : COMPUTE THE CONDITIONAL EXPECTATION

$$Q(\beta, \beta^{(\tau)}) = E_{z|x, \beta^{(\tau)}} \{ L_x(\beta) \mid x, \beta^{(\tau)} \}$$

- M-STEP : MAXIMIZE $Q(\beta, \beta^{(\tau)})$ TO OBTAIN AN UPDATED VALUE $\beta^{(\tau+1)}$, THEN GO TO E-STEP, USING THE UPDATED VALUE, AND ITERATE TILL CONVERGENCE.

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