LINEAR COMBINATIONS

1. A VECTOR b OF SIZE m ($=$ # OF COMPONENTS OF b) IS A LINEAR COMBINATION OF THE k VECTORS $v_1, v_2, \ldots, v_k$, EACH OF SIZE m, IF IT IS POSSIBLE TO FIND k REAL NUMBERS $x_1, x_2, \ldots, x_k$ SATISFYING.

$$x_1 v_1 + x_2 v_2 + \cdots + x_k v_k = b$$

TO FIND $\underbrace{\text{THESE NUMBERS}}_{x_1, x_2, \ldots, x_k}$ VIEW THE ABOVE EQUATION AS A SYSTEM OF LINEAR EQUATIONS, AND SOLVE BY THE GAUSS-JORDAN METHOD.

2. A SET OF VECTORS (ALL HAVING m COMPONENTS) IS LINEARLY DEPENDENT, IF AT LEAST ONE VECTOR IS A LINEAR COMBINATION OF THE OTHERS. OTHERWISE THEY ARE LINEARLY INDEPENDENT. $\square$

EXAMPLE 1

$(2,3)^T$ IS A LINEAR COMBINATION OF $(1,3)^T$ AND $(1,1)^T$,

WHERE

$$x_1\begin{bmatrix}1\\3\end{bmatrix}+x_2\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}2\\3\end{bmatrix}$$

$$\Rightarrow \left.\begin{aligned} x_1+x_2&=2\\ 3x_1+x_2&=3\end{aligned}\right\} \Longrightarrow x_1=1/2 \;;\; x_2=3/2$$

□

EXAMPLE 2

$(1,0)^T$ IS A LINEAR COMBINATION OF $(1,3)^T$ AND $(1,1)^T$;

WHERE

$$x_1\begin{bmatrix}1\\3\end{bmatrix}+x_2\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$$

$$\Rightarrow \left.\begin{aligned} x_1+x_2&=1\\ 3x_1+x_2&=0\end{aligned}\right\} \Longrightarrow x_1=-1/2 \;;\; x_2=3/2$$

□