DECISION SURFACE (HYPERSURFACE) FOR CLASSIFICATION

- THE GOAL OF ANY CLASSIFIER IS TO PARTITION THE FEATURE SPACE INTO REGIONS. THE PARTITION IS ACHIEVED BY:
  - POINTS IN $\mathbb{R}$
  - ~~BOTH~~ CURVES IN $\mathbb{R}^2$
  - SURFACES IN $\mathbb{R}^3$
  - HYPERSURFACES IN $\mathbb{R}^l$
- A HYPERSURFACE S IS EXPRESSED AS:

$$g: \mathbb{R}^l \rightarrow \mathbb{R}$$

$$S = \{x \in \mathbb{R}^l \mid g(x) = 0\}$$

ALL POINTS LYING ON ONE-SIDE OF THE HYPERSURFACE SCORE $g(x) > 0$; AND ALL POINTS ON THE OTHER SIDE SCORE $g(x) < 0$

THE RESULTING (HYPER) SURFACES ARE KNOWN AS DECISION (HYPER) SURFACES (OR DISCRIMINANT).

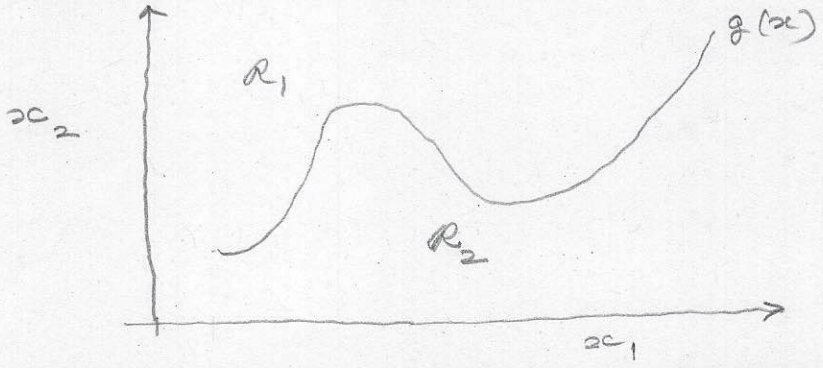
EXAMPLE: IN CASE OF TWO-CLASS BAYESIAN CLASSIFIER,

$$g(x) = P(\omega_1 | x) - P(\omega_2 | x) = 0$$

WE DECIDE IN FAVOR OF CLASS ω_1 (REGION R_1) IF $g(x) > 0$.

" " " " ω_2 (REGION R_2) IF $g(x) < 0$

POINTS WHERE NO DECISION IS TAKEN ARE THOSE THAT LIE CLOSE TO THE DECISION HYPERSURFACE.



DISCRIMINANT FUNCTIONS

LET THE K NUMBER OF CLASSES BE : $C_1, C_2, \ldots, C_K$

CLASSIFICATION CAN BE DONE VIA A SET OF DISCRIMINATING FUNCTIONS : $g_i(x)$; $i = 1, 2, \ldots, K$

CHOOSE C_i IF $g_i(x) = \underset{1 \le k \le K}{\text{MAX}} \; g_k(x)$

EXAMPLES

1. $g_i(x) = w_i^T x + w_{i0}$; $1 \le i \le K$

   THESE FUNCTIONS ARE LINEAR IN x.

   THEREFORE THESE ARE CALLED LINEAR DISCRIMINANTS.

2. $g_i(x) = x^T W_i x + w_i^T x + w_{i0}$; $1 \le i \le K$

   THESE FUNCTIONS ARE QUADRATIC IN x. THEREFORE THESE ARE CALLED QUADRATIC DISCRIMINANTS.

LINEAR DISCRIMINANT ANALYSIS (LDA)

- GIVEN A SET OF LABELED d- DIMENSIONAL POINTS x_i, AND ITS CLASS y_i; THE GOAL OF LDA IS TO FIND A VECTOR w THAT MAXIMIZES THE SEPARATION BETWEEN CLASSES, AFTER PROJECTION ON TO w.
- IN PRINCIPAL COMPONENT ANALYSIS (PCA), PRINCIPAL COMPONENT IS THE VECTOR THAT MAXIMIZES THE PROJECTED VARIANCE OF THE POINTS.
- THE KEY DIFFERENCE BETWEEN PCA AND LDA IS ~~THAT~~ THAT, THE FORMER DEALS WITH UNLABELED DATA AND TRIES TO MAXIMIZE VARIANCE; WHEREAS THE LATTER DEALS WITH LABELED DATA AND TRIES TO MAXIMIZE THE DISCRIMINATION BETWEEN THE CLASSES.