MATHEMATICAL STATISTICS

1. GIVEN A SET OF DATA POINTS:

$X = \{x_1, x_2, \dots, x_n\}$

i) SAMPLE MEAN OF THE DATA POINTS IN THE SET X IS $\bar{x}$

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

ii) SAMPLE VARIANCE OF THE DATA POINTS IN THE SET X IS s^2

$$s^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} (x_i - \bar{x})^2 \quad ; \quad n > 1$$

EXAMPLE 1: $X = \{-3, -4, +5, +3\}$

FIND SAMPLE MEAN AND SAMPLE VARIANCE OF X

SOLUTION: $|X| = 4 = n$

SAMPLE MEAN $= \bar{x} = \frac{1}{4}(-3-4+5+3) = \boxed{\frac{1}{4}} = \bar{x}$

SAMPLE VARIANCE $= s^2 = \frac{1}{3} \sum_{i=1}^{4} (x_i - \bar{x})^2$

$= \frac{1}{3}\left[(-3-0.25)^2 + (-4-0.25)^2 + (5-0.25)^2 + (3-0.25)^2\right]$

$\approx \boxed{19.58} = s^2$

$$\boxed{\bar{x} = 0.25 \; ; \; s^2 \approx 19.58}$$

$\square$

EXAMPLE 2 : $X = \{-2, 7, -3, 2\}$

a) FIND SAMPLE MEAN OF POINTS IN THE SET X
b) " " VARIANCE " " " " " " X
c) SHIFT EACH DATA POINT IN THE SET BY 3 UNITS
 i) FIND THE NEW SAMPLE MEAN
 ii) " " " " VARIANCE.

SOLUTION :

a) $\bar{x} = \frac{1}{4}(-2+7-3+2) = \boxed{1}$

b) $s_x^2 = \frac{1}{3}\left[(-2-1)^2 + (7-1)^2 + (-3-1)^2 + (2-1)^2\right]$
$= \frac{1}{3}\left[9+36+16+1\right] = \frac{62}{3} = \boxed{20\frac{2}{3}}$

c) NEW SET OF DATA POINTS:
$Y = \{1, 10, 0, 5\}$

 i) SAMPLE MEAN $= \bar{y} = \frac{1}{4}(1+10+0+5) = \boxed{4} = \bar{x}+3$

 ii) $s_y^2 = \frac{1}{3}\left[(1-4)^2 + (10-4)^2 + (0-4)^2 + (5-4)^2\right]$
$= \frac{1}{3}\left[9+36+16+1\right] = \boxed{20\frac{2}{3}} = s_x^2$ □

LESSON IF EACH DATA POINT IS SHIFTED BY THE SAME VALUE, THEN:

1. THE SAMPLE MEAN SHIFTS BY THE SAME VALUE.

2. THE SAMPLE VARIANCE DOES NOT CHANGE. □

EXAMPLE 3 : $X = \{-1, 2, -3, -2\}$

a) FIND SAMPLE MEAN OF THE POINTS IN THE SET X

b) FIND " VARIANCE " " " " " " X

c) DILATE EACH DATA POINT IN THE SET BY -2

i) FIND THE NEW SAMPLE MEAN

ii) " " " " VARIANCE.

SOLUTION :

a) $\bar{x} = \frac{1}{4}(-1+2-3-2) = \boxed{-1}$

b) $s_x^2 = \frac{1}{3}\left[(-1+1)^2 + (2+1)^2 + (-3+1)^2 + (-2+1)^2\right]$

$= \frac{1}{3}\left[9+4+1\right] = \boxed{4\frac{2}{3}}$

c) NEW SET OF DATA POINTS :

$Y = \{2, -4, 6, 4\}$

i) SAMPLE MEAN $= \bar{y} = \frac{1}{4}(2-4+6+4) = \boxed{2} = (-2)\,\bar{x}$

ii) $s_y^2 = \frac{1}{3}\left[(2-2)^2 + (-4-2)^2 + (6-2)^2 + (4-2)^2\right]$

$= \frac{1}{3}\left[0+36+16+4\right] = \frac{56}{3} = \boxed{18\frac{2}{3}} = (-2)^2 s_x^2$

□

LESSON : IF EACH DATA POINT IS DILATED BY THE SAME AMOUNT, THEN :

1. THE SAMPLE MEAN IS ALSO DILATED BY THE SAME AMOUNT.
2. THE SAMPLE VARIANCE IS MULTIPLIED BY THE SQUARE OF THE DILATED VALUE.

□