Machine Learning

Assignment 4

85 points

1. (10 points) Consider the 3×3 symmetric matrix

$$A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 3 & 0 \\ 2 & 0 & 5 \end{bmatrix}$$

If $x^T = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$, find the quadratic form $x^T A x$ associated with the matrix A.

2. (20 points) Let

$$f(y) = y^T B y = y_1^2 - y_2^2 + 4y_3^2 - 2y_1 y_2 + 4y_2 y_3$$

where $y^T = \begin{bmatrix} y_1 & y_2 & y_3 \end{bmatrix}$, and B is a 3×3 symmetric matrix, find B.

3. (15 points) Minimize the function

$$f(x_1, x_2) = x_1^2 + x_2^2 - x_1 x_2$$

Hint: Equate the gradient to zero and find the critical point. Prove that the Hessian is positive definite at the critical point.

4. (20 points) Find the global minimizer of

$$f(x, y) = e^{x-y} + e^{y-x} + e^{x^2}$$

Hint: Equate the gradient to zero and find the critical point. Prove that the Hessian is positive definite at the critical point.

5. (20 points)
$$\min \left(10x + 2y^2 + z^2 + 8z \right)$$
$$\text{subject to } (x + y + z) = 100$$