

# Programming Assignment #6. Problem #1

$$X = \begin{bmatrix} 1 & 1 & 2 \\ 1 & -3 & 2 \\ 1 & 2 & -1 \end{bmatrix}; Y = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

$$X^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -3 & 2 \\ 2 & 2 & -1 \end{bmatrix}; X^T X = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 14 & -6 \\ 3 & -6 & 9 \end{bmatrix}$$

$$(X^T X)^{-1} = \begin{bmatrix} 5/8 & -1/8 & -7/24 \\ -1/8 & 1/8 & 1/8 \\ -7/24 & 1/8 & 7/24 \end{bmatrix}; X^T Y = \begin{bmatrix} 2 \\ -7 \\ 7 \end{bmatrix}$$

$$\hat{\beta} = (X^T X)^{-1} (X^T Y) = \begin{bmatrix} 1/2 \\ -1/4 \\ 1/12 \end{bmatrix}$$

## Problem 2 (a)

Model Matrix:  $X = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 4 \\ 1 & -3 & -6 \end{bmatrix}$  Observations  $Y = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

Suppose error  $\epsilon = y - X\beta$ , where  $\beta$  is param vector.

Then Sum of Sq  $L = \sum_{i=1}^n \epsilon_i^2 = (y - X\beta)^T (y - X\beta)$

The least squares estimator  $\hat{\beta}$  is the solution for  $\beta$  in

$\frac{\partial L}{\partial \beta} = 0$ . Minimizing  $\frac{\partial L}{\partial \beta}$ , we have normal equations.

$X^T X \hat{\beta} = X^T Y$ . Solving  $\hat{\beta} = (X^T X)^{-1} X^T Y$

$$X^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & 4 & -6 \end{bmatrix}$$

$$X^T X = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 14 & 28 \\ 0 & 28 & 56 \end{bmatrix}$$

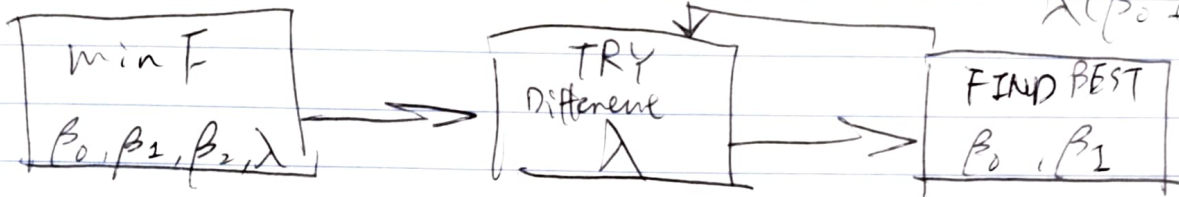
$$X^T Y = \begin{bmatrix} 0 \\ 14 \\ 28 \end{bmatrix}$$

$(X^T X)^{-1}$  is singular,  $(X^T X)$  inverse doesn't exist!!

Problem 2 (b) Ridge Reg. Goal  $\min_{\vec{\beta}} \|\vec{y} - A\vec{\beta}\|_2^2$  s.t.  $\|\vec{\beta}\|_2 \leq C^2$

$$\min_{\vec{\beta}} \sum_i (y_i - x_i \vec{\beta})^2 \text{ s.t. } \beta_0^2 + \beta_1^2 + \beta_2^2 \leq C^2$$

Lagrange Function  $F(\beta_0, \beta_1, \lambda) = \sum_i (y_i - x_i \vec{\beta})^2 + \lambda(\beta_0^2 + \beta_1^2 + \beta_2^2 - C^2)$



$$\min_{\vec{\beta}} \|\vec{y} - A\vec{\beta}\|_2^2 + \lambda \|\vec{\beta}\|_2^2$$

Ridge Solution  $\vec{\beta}^R = (X^T X + \lambda I)^{-1} X^T \vec{y}$

$\lambda$   
 $10^{-5}$

$\vec{\beta}^R$   
 $\vec{\beta}$   
 $\begin{bmatrix} 0 \\ 0.2 \\ 0.4 \end{bmatrix}$

$-10^{-5}$

$\begin{bmatrix} 0 \\ 0.2 \\ 0.4 \end{bmatrix}$

See second PDF