LAPLACE PROBABILITY DENSITY FUNCTION

LAPLACE pdf DECAYS EXPONENTIALLY.

THAT IS:

$$f_X(x) = c\, e^{-\alpha|x|} \quad ; \quad x \in \mathbb{R} \; ; \; \alpha > 0$$

i) IT CAN BE SHOWN THAT:

$$f_X(x) = \frac{\alpha}{2} e^{-\alpha|x|} \quad ; \quad x \in \mathbb{R}$$

THAT IS: $c = \frac{\alpha}{2}$

ii) $P(|X| < r) = (1 - e^{-\alpha r}) \quad ; \quad r \geq 0$

LONG ROPE TO CATCH HORSES & CATTLE

LASSO REGRESSION

LASSO REGRESSION ASSUMES THAT THE PRIOR PROBABILITY IS DRAWN FROM LAPLACE DISTRIBUTION.

LET $\mathcal{D}$ = DATA SET

β = PARAMETER SPACE

$\|\cdot\|_2$ = EUCLIDEAN NORM

$\|\cdot\|_1$ = ABSOLUTE NORM

LET $\beta = (\beta_0, \beta_1)$; THEN $\|\beta\|_1 = |\beta_0| + |\beta_1|$

PARAMETER OF THE LAPLACE DISTRIBUTION $= \alpha$

$\alpha = \|\beta\|_1 = |\beta_0| + |\beta_1| \triangleq \alpha(\beta)$

$P(\beta) = \frac{\lambda}{2} e^{-\lambda\alpha(\beta)}$; $\lambda > 0$

LIKELIHOOD FUNCTION $= P(\mathcal{D}|\beta)$

$$= \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2}\left(y_i - (\beta_0 + \beta_1 x_i)\right)^2\right]$$

WE MAXIMIZE:

$$\max_{\beta} \ln\{P(\mathcal{D}|\beta)P(\beta)\}$$

$\uparrow_\beta$

$$= \max_{\beta}\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{n}\{y_i - (\beta_0 + \beta_1 x_i)\}^2 - \lambda\alpha(\beta)\right]$$

$$\Leftrightarrow \min_{\beta}\left\{\frac{1}{2\sigma^2}\|y - X\beta\|^2 + \lambda\|\beta\|_1\right\}$$

DEFINED AS IN THE LINEAR REGRESSION MODEL

THE ABOVE FUNCTION IS NOT SMOOTH, AND THE FUNCTION $\|\beta\|_1$ IS NOT DIFFERENTIABLE.

MURPHY (2012) DISCUSSES AN APPROACH TO ADDRESS THIS ISSUE.

RECALL THAT THE OBJECTIVE FUNCTION TO BE MINIMIZED IN THE ANALYSIS OF RIDGE REGRESSION USES ONLY EUCLIDEAN NORM.