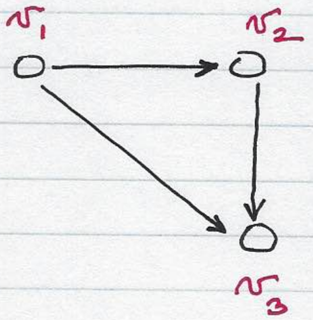
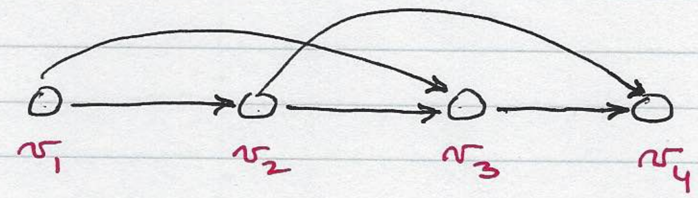
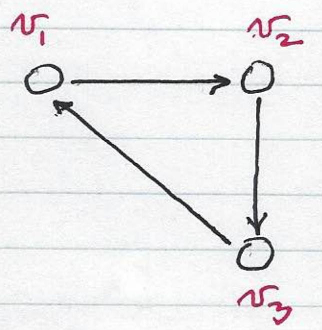
EXAMPLES

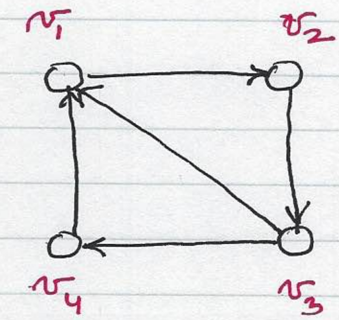




NO CYCLES

NO CYCLES





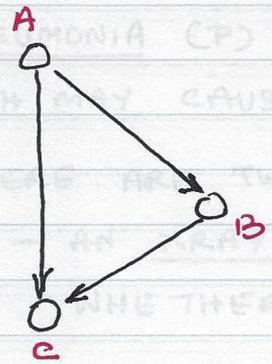
CYCLE

(CLOSED LOOP)

CYCLES

DIRECTED GRAPHICAL MODEL

TERMINOLOGY

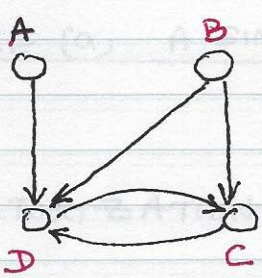


- NODE ~~OF~~ A IS PARENT OF NODE B
- NODE B IS CHILD OF NODE A
- A NODE REPRESENTS AN EVENT / RANDOM VARIABLE
- FOR EACH CONDITIONAL DISTRIBUTION ADD LINKS (ARROWS)
- FOR $P(C|A,B)$ HAVE LINKS (A,C) AND (B,C)

$$P(A,B,C) = P(A)\,P(B|A)\,P(C|A,B)$$

- $P(A,B,C)$ IS SPECIFIED ~~AS~~ BY PRODUCT OF THREE TERMS.
- THERE ARE THREE TERMS, AND EACH TERM IS A DISTRIBUTION. IT COULD BE SPECIFIED BY A TABLE.

EXAMPLE



$$P(A,B,C,D)$$
$$= P(A)\,P(B)\,P(C|B,D)\,P(D|A,B,C)$$

□

EXAMPLE A SIMPLE YET SERIOUS EXAMPLE OF BAYESIAN NW.

- SHOWS INTERACTION BETWEEN TWO POTENTIAL DISEASES: PNEUMONIA (P) ; AND TUBERCULOSIS (T).
  BOTH MAY CAUSE A PATIENT TO HAVE LUNG INFILTRATES (I)
- THERE ARE TWO SEPARATE TESTS THAT CAN BE PERFORMED.
  - AN XRAY (X) CAN BE TAKEN, WHICH MAY INDICATE WHETHER THE PATIENT HAS LUNG INFILTRATES.
  - THERE IS A SEPARATE SPUTUM SMEAR (S) TEST FOR TUBERCULOSIS.
- FIGURE (a) SHOWS DEPENDABILITY STRUCTURE.
  - ALL VARIABLES P, T, I, X, S ARE BOOLEAN

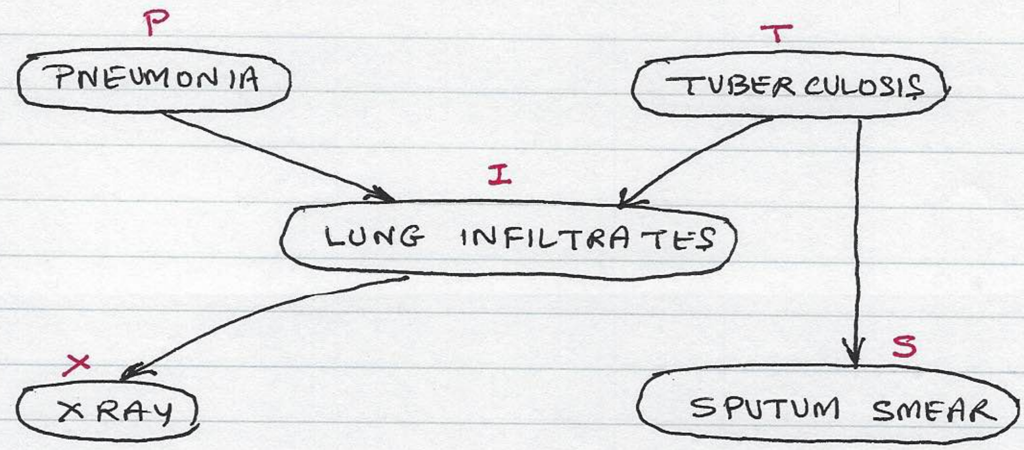


FIGURE (a) A SIMPLE BAYESIAN NETWORK FOR TWO POTENTIAL DISEASES.

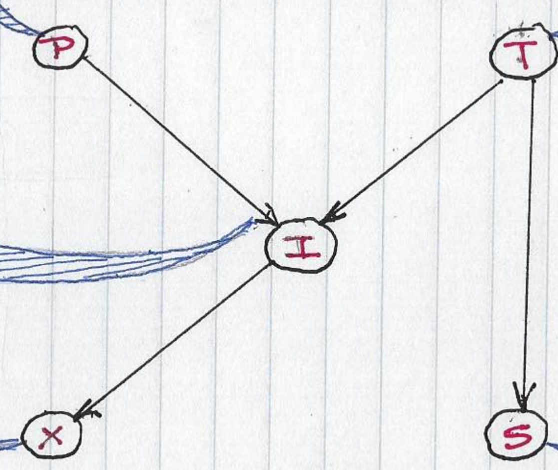
- FACTORIZATION FOR $P(P, T, I, X, S)$

$$P(P, T, I, X, S) = P(P)\,P(T)\,P(I \mid P, T)\,P(X \mid I)\,P(S \mid T)$$

FIGURE (b) SHOWS CONDITIONAL PROBABILITY DISTRIBUTIONS FOR EACH RANDOM VARIABLE.

P(P = T)	P(P = F)
0.05	0.95

P(T = T)	P(T = F)
0.02	0.98



P	T	P(I = F | P,T)	P(I = T | P,T)
F	F	0.99	0.01
T	F	0.40	0.60
F	T	0.80	0.20
T	T	0.20	0.80

NODE	TABLE-SIZE
P	2
T	2
I	8
X	4
S	4

I	P(X = F | I)	P(X = T | I)
F	0.4	0.6
T	0.2	0.8

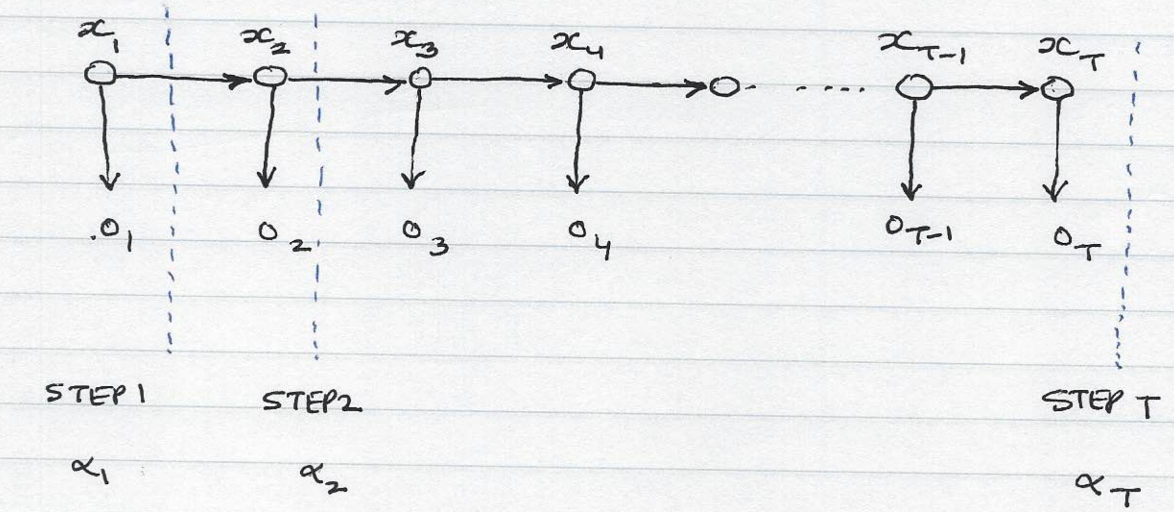
T	P(S = F | T)	P(S = T | T)
F	0.4	0.6
T	0.2	0.8

FIG (6) A SIMPLE BAYESIAN NW FOR TWO POTENTIAL DISEASES & CORRESPONDING PROBABILITY TABLES

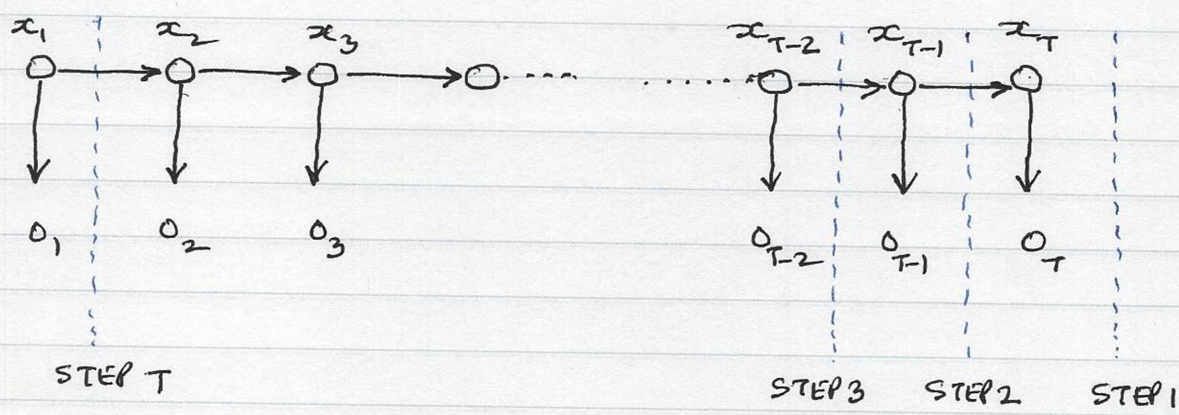
TOTAL TABLE ENTRIES = 20 ; TOTAL POSSIBILITIES = 2^5 = 32.

EXAMPLE HIDDEN MARKOV MODEL (HMM)

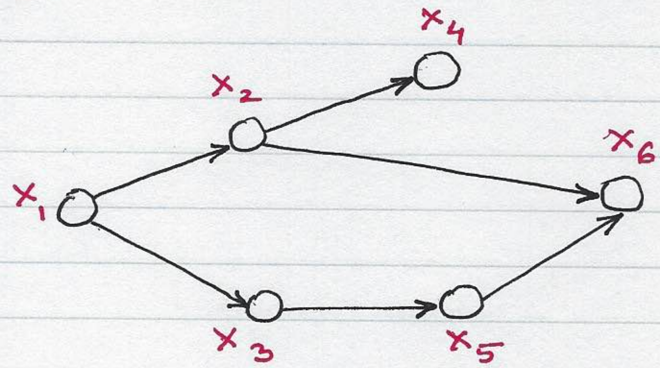
FORWARD ALGORITHM:



BACKWARD ALGORITHM



EXAMPLE



RANDOM VARIABLES ARE: $\{x_1, x_2, x_3, x_4, x_5, x_6\}$

$\Theta_1 = \{\cancel{x_1}\}$ ϕ NULL SET ; $\Theta_2 = \{x_1\}$; $\Theta_3 = \{x_1\}$; $\Theta_4 = \{x_2\}$

$\Theta_5 = \{x_3\}$; $\Theta_6 = \{x_2, x_5\}$

THE GRAPH DEFINES A FACTORIZATION OF THE JOINT DISTRIBUTION IN TERMS OF CONDITIONAL DISTRIBUTIONS $P(x_i \mid \Theta_i)$

$$P(x_1, x_2, \ldots, x_6) = P(x_1) P(x_2 \mid x_1) P(x_3 \mid x_1) P(x_4 \mid x_2) \cdot P(x_5 \mid x_3) P(x_6 \mid x_2, x_5)$$

$\square$

EXAMPLES ON GRAPHICAL MODELING

EXAMPLE EVENT B IS CONDITIONALLY INDEPENDENT OF EVENT A, GIVEN EVENT C. THAT IS,

$$P(B|A,C) = P(B|C)$$

$$\begin{tikzcd} A \arrow[r] & C \arrow[r] & B \end{tikzcd}$$

JOINT PROBABILITY $P(A,B,C) = ?$

SOLUTION:

i) VIA CONDITIONAL DISTRIBUTIONS (USE PARENT RULE)

$$P(A,B,C) = P(A)\,P(\overset{C}{\cancel{B}}|A)\,P(B|C)$$

ii) VIA PRODUCT RULE:

$$P(A,B,C) = P(A)\,P(C|A)\,P(B|A,C)$$
$$= P(A)\,P(C|A)\,P(B|C) \qquad \square$$

EXAMPLE

EVENT A INDEPENDENT OF EVENT B. $\Rightarrow P(B|A) = P(B)$

$$\begin{tikzcd} A \arrow[dr] & & B \arrow[dl] \\ & C & \end{tikzcd}$$

JOINT $P(A,B,C) = ?$

SOLUTION:

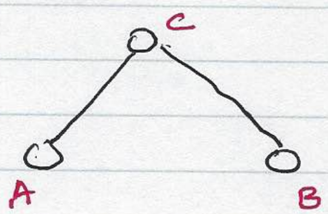
i) VIA PARENT RULE: $P(A,B,C) = P(A)\,P(B)\,P(C|A,B)$

ii) VIA PRODUCT RULE :

$$P(A,B,C) = P(A)\,P(B|A)\,P(C|A,B)$$
$$= P(A)\,P(B)\,P(C|A,B) \qquad \square$$

EXAMPLE

EVENT B IS CONDITIONALLY INDEPENDENT OF EVENT A, GIVEN EVENT C. $\Rightarrow$ $P(B|A,C) = P(B|C)$



JOINT $P(A,B,C) = ?$

SOLUTION :

i) VIA PARENT RULE : $P(A,B,C) = P(C)\,P(A|C)\,P(B|C)$

ii) VIA PRODUCT RULE :

$$P(A,B,C) = P(C)\,P(A|C)\,P(B|A,C)$$
$$= P(C)\,P(A|C)\,P(B|C) \qquad \square$$