Multi Dimensional Callatz Sequence Generalization

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The Collatz Conjecture

The collatz conjecture goes as follows
start with and arbitrary number in the positive integers
$n \in \mathbb{Z}^{\geq 1}$
then follow the following update rule

$$n_{i+1} = \begin{cases} \frac{n_i}{2} & \text{if n is even} \\ 3 \times n_i + 1 & \text{if n is odd} \end{cases}$$

All numbers tested so far have all converged to the same cycle which is $4 \to 2 \to 1 \to 4$
The question remains if all numbers converge to this cycle or if there is a starting point
that does not converge to any number or converges to a different cycle.

The Multi Dimensional Generalization of the Collatz Conjecture

We define the multi dimensional generalization of the Collatz conjecture to be as follows :
We start with an arbitrary vector $\vec{v} \in \mathbb{Z}^n$

We disambiguate the n in this statement from the n in the definition of the original Collatz conjecture. The n in the original reffered the number that was being iterated on, and in the current context refers to the dimension of the vector $\vec{v}$. Ie if $n = 5$, the vector $\vec{v}$ has 5 components $\vec{v} = (v_0, v_1, v_2, v_3, v_4)$

Now we define the update rule as follows

$$\vec{v}^{\,(i+1)} = \begin{cases} \frac{\vec{v}^{\,(i)}}{2} & \text{if all values of } \vec{v}^{\,(i)} \text{ are even} \\ \\ 3 \times \vec{v}^{\,(i)} + \vec{v}^{\,(i)} \ \% \ 2 & \text{if there is at least one component of} \\ & \vec{v}^{\,(i)} \text{ that is odd} \end{cases}$$

The super script (i) and $(i+1)$ represent the current iteration and the next iteration of the vector respectively.
This update rule becomes the normal Collatz conjecture when the dimension of the vector is 1, and we will see that the diagonal vectors ie (1, 1), (2, 2), (3, 3) all converge and are the same as the original Collatz conjecture.

lets go through a brief example starting with the vector (4, 2, 6)

(4, 2, 6)
all values are even so we follow the even values rule
and divide all values by 2
$\frac{4}{2} = 2$

$\frac{2}{2} = 1$

$\frac{6}{2} = 3$

(2, 1, 3)
there are odd values so we follow the odd values rule
3x2 + 2 % 2 = 6+0 = 6,
3x1 + 1 % 2 = 3 + 1 = 4,
3x3 + 3 % 2 = 9 + 1 = 10

(6, 4, 10)
even rule

(3, 2, 5)
odd rule

(10, 6, 16)
even rule

(5, 3, 8)
odd rule

(16, 10, 24)
even rule

(8, 5, 12)
odd rule

(24, 16, 36)*
even rule

(12, 8, 18)
even rule

(6, 4, 9)
odd rule

(18, 12, 28)
even rule

(9, 6, 14)
odd rule

(28, 18, 42)
even rule

(14, 9, 21)
odd rule

(42, 28, 64)
even rule

(21, 14, 32)
odd rule

(64, 42, 96)
even rule

(32, 21, 48)
odd rule

(96, 64, 144)
even rule

(48, 32, 72)
even rule

(24, 16, 36)*
We stop here becuase we have reached a vector that we have seen before, meaning that we have entered into a cycle.

Here are some images that the algorithm makes. We have to cap the number of iterations to search for a cycle because it appears that many of the starting vectors do not converge to a cycle. The colored blocks in the images correspond to the starting vectors where the algorithm has converged to a cycle that has been found within a specified number of steps. This update rule can also find cycles for the negative integers. This assumes that the modulo operator returns a value with the same sign as the divisor or returns zero.

The behaviour of the Modulo operator in this context is $a, b \in \mathbb{Z}$, $a \,\%\, b = a - (b \times \lfloor \frac{a}{b} \rfloor)$

Figure 1: General Collatz update with dimension 2

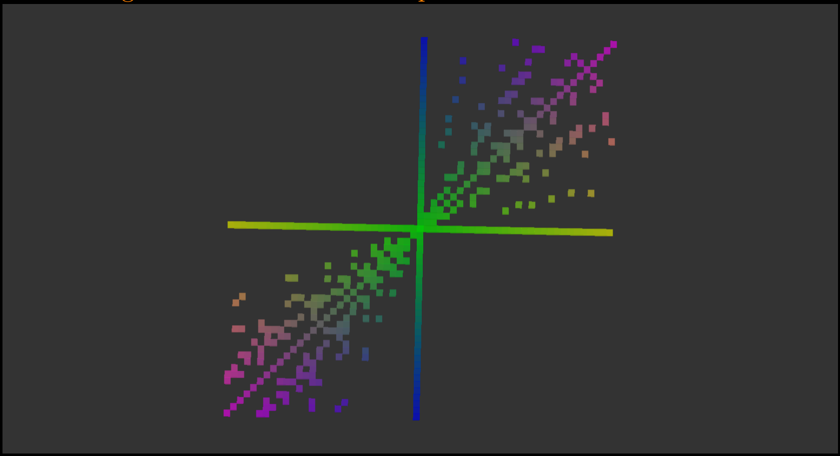
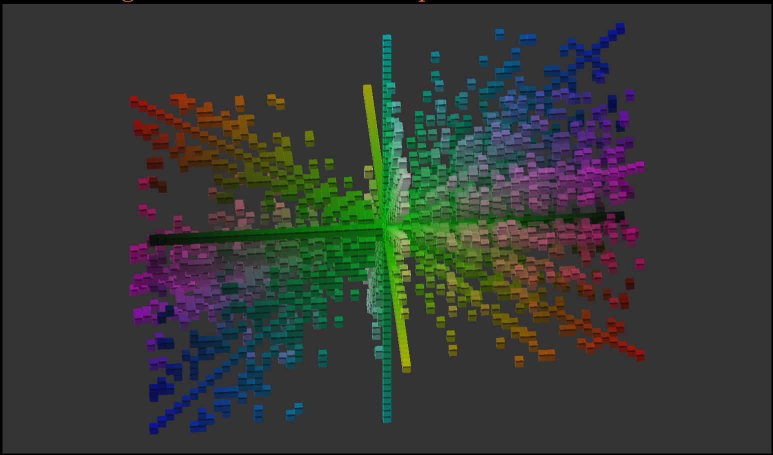


Figure 2: General Collatz update with dimension 3



Appendix

i

For the computation, only the quadrants where all values are positive or zero and all values are negative or zero were considered. This is due to the observation that the other quadrants do no have starting vectors that converge to cycles in any reasonable amount of steps.

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The previous vector generalization also gives rise to the numeric generalization as follows.
$\vec{v} \in \mathbb{Z}^n$
$a, b \in \mathbb{Z}$

$$\vec{v}^{\,(i+1)} = \begin{cases} \frac{\vec{v}^{\,(i)}}{b} & \text{if all values of } \vec{v}^{\,(i)} \text{ are evenly divisible by } b \\ a \times \vec{v}^{\,(i)} + \vec{v}^{\,(i)} \,\%\, b & \text{if there is at least one component of } \vec{v}^{\,(i)} \text{ that is not evenly divisible by } b \end{cases}$$