

## The Green Solow Model

William A. Brock\*  
M. Scott Taylor\*\*

**Abstract:** We demonstrate that a key empirical finding in environmental economics - The Environmental Kuznets Curve - and the core model of modern macroeconomics - the Solow model - are intimately related. Once we amend the Solow model to incorporate technological progress in abatement, the EKC is a necessary by product of convergence to a sustainable growth path. Our amended model, which we dub the "Green Solow", generates an EKC relationship between both the flow of pollution emissions and income per capita, and the stock of environmental quality and income per capita. The resulting EKC may be humped shaped or strictly declining. We explain why current methods for estimating an EKC are likely to fail whenever they fail to account for cross-country heterogeneity in either initial conditions or deep parameters. We then develop an alternative empirical method closely related to tests of income convergence employed in the macro literature. Preliminary tests of the model's predictions are investigated using data from OECD countries.

\*Brock: Vilas Professor of Economics, Department of Economics, University of Wisconsin ([wbrock@ssc.wisc.edu](mailto:wbrock@ssc.wisc.edu)); \*\*Taylor, Professor of Economics, Department of Economics, University of Wisconsin, Faculty Research Associate National Bureau of Economic Research, Cambridge, MA. ([staylor@ssc.wisc.edu](mailto:staylor@ssc.wisc.edu) and web: [www.ssc.wisc.edu/~staylor](http://www.ssc.wisc.edu/~staylor)). We thank Henning Bohn, Brian Copeland, Steven Durlauf, John List, and conference participants at the UNCTEE 2004 conference at UC-Santa Barbara for helpful comments on an earlier draft. Excellent research assistance was provided by Pablo Pincheira. Research funding was generously provided by the Vilas Trust, University of Wisconsin.

# 1 Introduction

The goal of this paper is to provide a cohesive theoretical explanation for three puzzling features of the pollution and income per capita data. To do so we introduce the reader to a very simple growth model closely related to the one-sector Solow model. We show how this amended model generates predictions closely in line with evidence on emissions, emission intensities and pollution abatement costs. We then use this model to derive a simple estimating equation linking a measure of emissions growth to initial emission levels and other controls drawn from theory. Preliminary tests of the model are encouraging and well in accord with the theoretical predictions of the Green Solow model.

Our work is related to recent attempts to explain the Environmental Kuznets Curve (thereafter EKC) but differs from other contributions in two important ways. First, we attempt to fit more features of the data than just the EKC and employ data on both pollution abatement costs and emission intensities to identify key features of the data that are largely inconsistent with existing theories. Second, we derive an estimating equation directly from our theory. By doing so we provide the first rigorously developed link between theory and empirical work in this area.

The EKC has captured the attention of policymakers, theorists and empirical researchers alike since its discovery in the early 1990s. The theory literature has from the start focussed on developing models that replicate the inverted U shaped relationship. Prominent explanations are threshold effects in abatement that delay the onset of policy, income driven policy changes that get stronger with income growth, structural change towards a service based economy, and increasing returns to abatement that drive down costs of pollution control.<sup>1</sup>

While each of these explanations succeeds in predicting a EKC, they are typically less successful at matching other features of the income and pollution data. One key feature of this data concerns the timing of pollution reductions. Models of threshold effects predict no pollution policy at all over some initial period followed by a period of active regulation.<sup>2</sup> When policy is inactive, emissions are produced lock step with output. When policy is active emissions per unit of output fall as do aggregate emissions. As a result, the decline in the

---

<sup>1</sup>See for example Stokey (1998), Andreoni et al. (2001), and Lopez (1994) for original contributions. A review of the competing explanations appears in Chapter 2 of Copeland and Taylor (2003).

<sup>2</sup>This is for example the exact prediction of both Stokey (1998) and Brock and Taylor (2003a).

emissions to output ratio occurs simultaneously with the reduction in aggregate pollution levels. This temporal correlation is however strongly contradicted by the data.

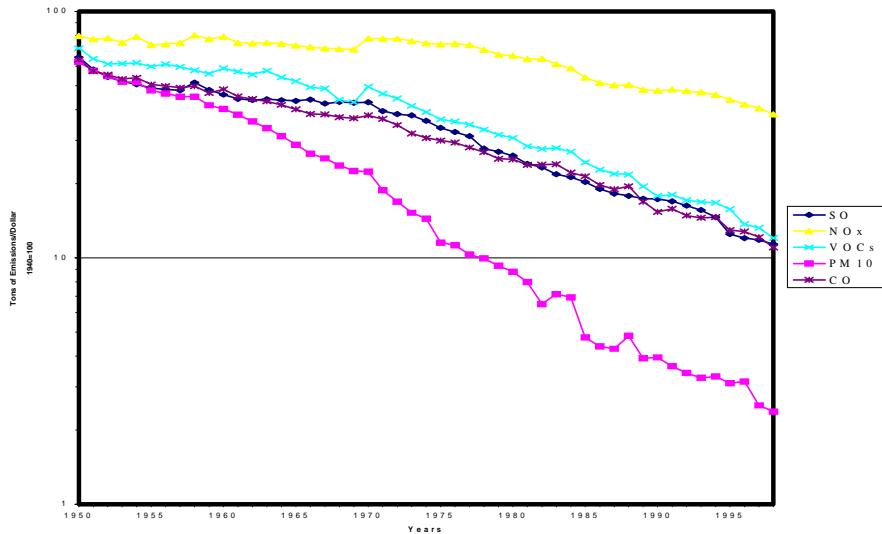


Figure 1: Emission Intensities

In Figure 1 we plot US data giving emissions per dollar of (real) GDP over the 1950 to 2001 period.<sup>3</sup> For ease of reading we have adopted a log scale. We plot emission intensities for sulfur dioxide, nitrogen dioxide, particulate matter, carbon monoxide, and volatile organic compounds. There are two features of note in the figure. The first is simply that the emission to output ratio is in decline from the start of the period in 1950. The second is that (given the log scale for emissions per dollar of output) the percentage rate of decline has been roughly constant over the fifty-year period (although it does vary across pollutants).

In Figure 2 we plot the corresponding emission levels for these same pollutants over the same time period. Figure 2 shows a general tendency for

---

<sup>3</sup>Data on US emissions of the criteria pollutants graphed in Figures 1 and 2 come from the US E.P.A. The long series of historical data presented in the figures is taken from the EPA's 1998 report National Pollution Emission Trends, at <http://www.epa.gov/ttn/chief/trends/trends98>. Because prior to 1985 fugitive dust sources and other miscellaneous emissions were not included in PM10 we have removed these components to make the data comparable over time.

emissions to at first rise and then fall over time. Since the US exhibited trend growth in real income per capita of approximately 2% a year over this period, the time scale in the figure could just as well be replaced by income per capita, and hence it offers a strong confirmation of the EKC as found, for example, by Grossman and Krueger (1994,1995). The EKC pattern is visible in the data for all pollutants except nitrogen oxides that may at present be approaching a peak, and particulates which peaked before the sample period.

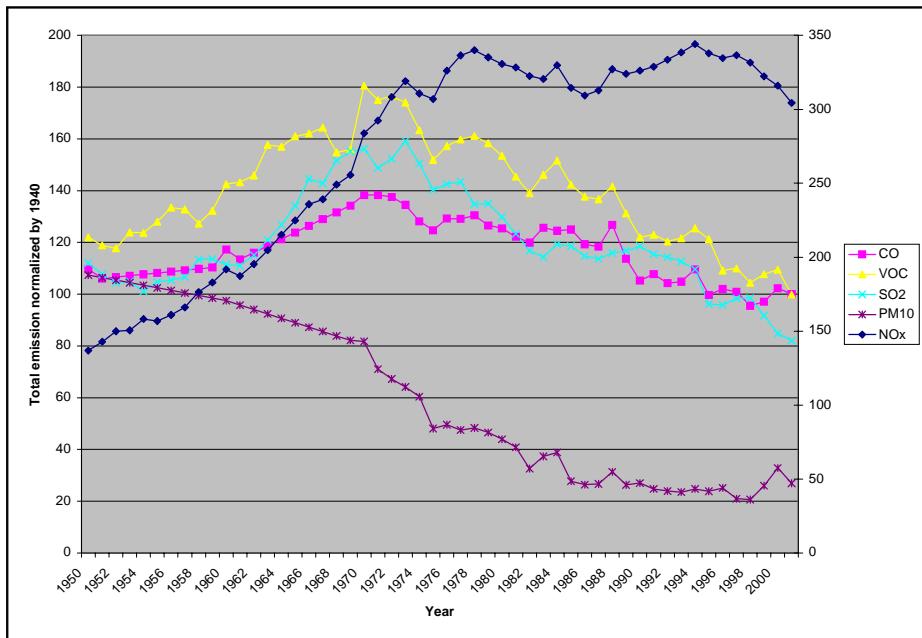


Figure 2: Emission Levels

It is clear however that the reduction in emission intensities shown in Figure 1 precede the peak level of pollutants in Figure 2 by 25 years for sulfur dioxide, carbon monoxide, and volatile organic compounds. Particulates have however been falling throughout, but the peak for nitrogen oxides occurs approximately 50 years after their emissions per unit output started to decline.

If we take the early 1970's as the start of serious pollution regulation, then threshold theories predict an unchanged and therefore horizontal line for the emissions to output ratio until the mid 1970s, and then a steep decline that forces aggregate emissions downward. This is not what Figure 1 and 2 show. The peaks in these pollution profiles – to the extent that they have peaked at all – occur much too late relative to the decline in emission intensities.

A second feature of the data that is difficult to reconcile with many theories is the magnitude of pollution abatement costs. Theories that rely on rising incomes driving down emissions via tighter pollution policy must square very large reductions in emissions with very small pollution abatement costs. For example in Figure 2, sulfur dioxide emissions peaked in 1973 at approximately 32,000 tons and fell almost in half to approximately 17,000 tons in 2001. Correspondingly large changes in emissions per unit output also occurred. But over much of this period, pollution abatement costs as a fraction of GDP or manufacturing value-added, remained both small and without much of a positive trend. Theories that rely on tightening environmental policy predict ever increasing costs of abatement, since emissions per unit of output must fall faster than aggregate output to hold pollution in check. In a world without technological progress in abatement, this requires larger and larger investments in pollution control.<sup>4</sup>

In Figure 3 we plot business expenditures on pollution abatement costs per dollar of GDP over the period 1972-1994. These twenty-two years are the only time period where data is available. As shown, pollution abatement costs as a fraction of GDP rise quite rapidly until 1975 and then remain relatively constant. As a fraction of overall output, these costs are small. Generating a similar plot for costs as a fraction of manufacturing value-added produces similar results. Alternatively, if we consider pollution abatement costs specifically directed to the six criteria air pollutants and scale this by real US output, the ratio is then incredibly small – approximately one half of one percent of GDP - and has remained so for over twenty years (See Vogan (1996)). There is of course considerable controversy over whether these figures represent the full cost of environmental regulation, and they necessarily ignore the significant abatement done prior to the 1970s by cities, utilities, and businesses.<sup>5</sup> Nevertheless, data

---

<sup>4</sup>Stokey (1998), Aghion and Howitt (1998) and others adopt an abatement function relating emissions per unit final output,  $E/Y$ , to the share of productive factors used in abatement  $\theta$ , as follows:  $E/Y = (1 + \theta)^\beta$ ,  $\beta > 0$ . Copeland and Taylor (2003, Chapter 2) show this relationship arises from an assumption on joint production and constant returns to scale in abatement. For emissions to decline while final output  $Y$  grows,  $E/Y$  must fall and this implies  $\theta$  must approach 1. That is, the share of the economy's resources dedicated to abatement must rise along the model's balanced growth path and approach one in the limit. The interested reader can verify this by making the translation into Stokey's notation by setting  $1 + \theta = z$ , and interpreting the gap between Stokey's potential and actual output as the output used in abatement.

<sup>5</sup>For an illuminating historical account of pollution regulation in the US from 1940 to 1970 see Dewey (2000). Dewey details the efforts at pollution control in major US cities such as New York, St. Louis, Pittsburgh and Los Angeles. The analysis shows serious pollution regulation is not a post 1970s phenomena.

from other countries supports our general conclusion that pollution abatement costs are a small fraction of GDP and show at best a slight upward trend.<sup>6</sup>

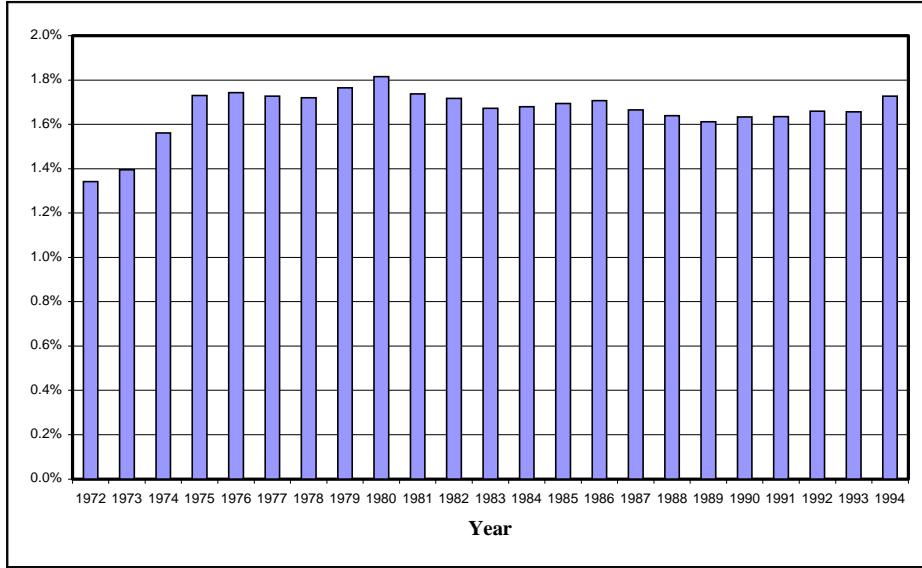


Figure 3: US Abatement Costs/GDP

While income effect theories often point to the creation of the EPA in the 1970s and more activist environmental policy, we should again return to Figure 1 and note that the trend in emissions to output was already declining and strongly so prior to 1970. Therefore the advent of more activist federal policy in the 1970s can only be a contributor to processes already at play in the 1950s.

Theories relying on strong compositional shifts or increasing returns also have difficulty matching these data. Changes in the composition of output towards less pollution intensive goods can lower emissions in the medium term, but in the long term reductions can only occur if emissions per unit of output in the cleanest of goods falls. This of course places us back where we started, asking how to lower emissions per unit output without ever rising costs. Moreover, empirical work has found a changing composition of output plays at most a bit part in the reductions we have observed (Selden et al. (1997), Bruvoll et al. (2003)).

And while increasing returns to abatement may be important in some in-

---

<sup>6</sup>US Data shown in Figure 3 is taken from Vogan (1996). See Table 3, section 4 for International data on pollution abatement costs, and our data appendix for a summary of the measures used in our empirical work.

dustries and for some processes, a large portion of emissions come from small diffuse sources such as cars, houses and individual consumptive activity. In each of these cases, increasing returns to abatement seems unlikely. Increasing returns also presents strong incentives for mergers and natural monopoly and unless we bound the strength of increasing returns carefully, IRS models predict negative pollution emissions at large levels of output.<sup>7</sup>

To us the pollution data and the related empirical work on the EKC present three puzzles that need to be resolved by any successful theory.

The first puzzle is how do we square the very large reductions in emission intensities shown in Figure 1 with the relatively small pollution abatement costs shown in Figure 3?

The second puzzle is the EKC: what is responsible for the shape of the pollution profiles in Figure 2?

The third puzzle comes from the empirical literature itself. What explains the current disconnect between the evidence for the EKC present in plots of raw data like Figure 2, and the difficulty empirical researchers have in estimating EKC relationships? It is now well known that empirical estimates from EKC style regressions can vary greatly with the sample used and estimation procedure. How do we make sense of the finding of an EKC in raw country level data as shown in Figure 2, and the fragile cross-country empirical results that are now commonplace to the literature?

In this paper we show that the Green Solow model provides a very simple explanation for all three puzzles. Our explanation starts with the observations in Figure 1 and 3. We square the rapidly declining emission intensities shown in Figure 1 with the constant pollution abatement costs in Figure 3 by assuming ongoing technological progress in abatement. To capture this possibility we introduce exogenous technological progress into a standard abatement function and then couple this abatement function with a standard fixed savings rate Solow model. The resulting "Green Solow model" then generates a pattern of incomes per capita and pollution consistent with Figure 2; i.e. it generates an EKC.

The logic is simple: Ongoing technological progress in abatement drives emissions per unit of output downward at a constant rate both in and out of steady state (as in Figure 1). Initially the Solow model's fast initial growth

---

<sup>7</sup>The simplest version of Andreoni and Levinson's theory of increasing returns to abatement has the property that pollution becomes negative for some large, but finite level of output. This feature poses problems in dynamic models where output grows exponentially.

overwhelms progress in abatement to produce a period of initially rising emission levels. Aggregate emissions rise even though emissions per unit of output are falling (recall Figure 2). Technological progress in abatement however eventually overwhelms the slowing growth of output as the economy approaches its balanced growth path. Aggregate emissions start to decline while emissions per unit of output continue their fall. Throughout the model's measure of pollution abatement costs as a fraction of GDP is constant (recall Figure 3). We offer these features of the model as potential explanations for the first two puzzles in the data.

Another model prediction is that the path for emissions, peak level of emissions, and income per capita at peak emissions will typically be country specific. Even countries that share identical parameter values will exhibit different EKC patterns if they differ in initial conditions. Additional cross-country heterogeneity is introduced by differences in savings rates, population growth rates or abatement intensities. Failing to account for this heterogeneity could be responsible for the failed empirical tests, and the sensitivity of estimates to the sample. We take this feature of the model as a potential explanation for the third puzzle - the current disconnect between the plots of raw data showing an EKC within countries, and the fragility of cross country empirical results. While much of current empirical work on the EKC includes controls for cross-country heterogeneity these controls are typically level variables such as population density, openness to trade, or measures of democracy and not the rates of change variables suggested by our analysis.

Finally to complete our argument, we provide empirical evidence in support of our approach from sources outside the dataset we sought to explain. Since our theoretical work shows that EKC profiles are not unique we focus our attention on a model prediction that holds more generally: convergence in a measure of emissions per capita. By borrowing from techniques used in the macro literature on income convergence we derive a simple linear estimating equation linking growth in emissions per capita over a fixed time period to emissions per capita in an initial period and a limited set of controls. These controls include typical Solow type regressors such as population growth and the savings rate, but also include a measure of pollution abatement costs and a proxy for technological progress in abatement. To demonstrate the potential usefulness of our approach we estimate our specification on OECD data. The results are encouraging.

Not surprisingly, the Green Solow model bears a family resemblance to many

other contributions in the literature given its close connection to Solow (1956). It is similar in purpose to that of Stokey (1998) but differs because Stokey does not consider technological progress in abatement. It is related to the new growth theory model of Bovenberg and Smulders (1995) because these authors allow for "pollution augmenting technological progress", which is, under certain circumstances, equivalent to our technological progress in abatement. The focus of their work is however very different from ours. It is perhaps most closely related to our own earlier work (Brock and Taylor (2003a)) where we tried to match data on pollution abatement costs, the EKC, and emission intensities within a modified AK model with ongoing technological progress in both goods and abatement production. While our earlier work was successful in some respects, like other models with threshold effects it failed to predict the steady fall in emission to output ratios prior to peak pollution levels. And while this earlier work contained a prediction regarding convergence in emission levels, this prediction did not follow from the neoclassical forces we highlight here. This paper grew out of our earlier attempts to match key features of the pollution and income per capita data within the simplest model possible. Our work also owes much to previous work in macroeconomics on conditional and absolute convergence; in particular Barro (1991) and Barro and Sala-i-Martin (1992).

The rest of the paper proceeds as follows. Section 2 sets up the basic model and develops three propositions concerning its behavior. In section 3 we derive an estimating equation from the model and present a preliminary empirical implementation using CO<sub>2</sub> data from the OECD. Section 4 contains a discussion of our assumptions and offers some international evidence. To make our points clear we develop the model under the assumption that both savings rates and abatement intensities are fixed over time. The appendix contains all proofs and lengthy calculations.

## 2 The Model

We develop an augmented Solow model where exogenous technological progress in both goods production and abatement leads to continual growth with rising environmental quality. We present the simplest specification where both savings and abatement choices are exogenously set. The fixed savings rate assumption is commonly used in the Solow model and is often innocuous; the assumption of a fixed abatement intensity helps us demonstrate how changes in the intensity

of abatement need not play any role in generating an Environmental Kuznets Curve. Together they render the model simple and tractable.

Consider the standard one sector Solow model with a fixed savings rate  $s$ . Output is produced via a constant returns to scale and strictly concave production function taking effective labor and capital to produce output,  $Y$ . Capital accumulates via savings and depreciates at rate  $\delta$ . We assume the rate of labor augmenting technological progress is given by  $g$ . All this implies:

$$\begin{aligned} Y &= F(K, BL), \dot{K} = sY - \delta K \\ L &= nL, \dot{B} = gB \end{aligned} \tag{1}$$

where  $B$  represents labor augmenting technological progress and  $n$  is population growth.

To model the impact of pollution we follow Copeland and Taylor (1994) by assuming every unit of economic activity,  $F$ , generates  $\Omega$  units of pollution as a joint product of output.<sup>8</sup> The amount of pollution released into the atmosphere may differ from the amount produced if there is abatement. We assume abatement is a constant returns to scale activity and write the amount of pollution abated as an increasing and strictly concave function of the total scale of economic activity,  $F$ , and the economy's efforts at abatement,  $F^A$ . If abatement at level  $A$ , removes the  $\Omega A$  units of pollution from the total created, we have:

$$\begin{aligned} \text{pollution emitted} &= \text{pollution created} - \text{pollution abated} \\ E &= \Omega F - \Omega A(F, F^A) \\ E &= \Omega F - A(1, F^A/F) \\ E &= \Omega Fa(\theta), \\ \text{where } a(\theta) &= 1 - A(1, F^A/F) \quad \text{and } \theta = F^A/F \end{aligned} \tag{2}$$

where the third line follows from the linear homogeneity of  $A$ , and the fourth

---

<sup>8</sup>This approach has been subsequently employed by many authors (Stokey (1998), Aghion and Howitt (1998), etc.). In these other papers,  $\Omega$  is taken as constant over time and by choice of units set to one. Some authors who adopt this approach refer to the firms or planners problem as one of choosing across dirty or clean technologies rather than less or more abatement. Copeland and Taylor (2003, chapter 2) provides background and shows the two approaches are identical.

by the definition of  $\theta$  as the fraction of economic activity dedicated to abatement. We assume the intensive abatement function satisfies  $a(0) = 1$  and note  $a'(0) < 0$  and  $a''(0) > 0$  by concavity. Abatement has a positive but diminishing marginal impact on pollution reduction. In some cases we will adopt the specific form  $a(\theta) = (1 + \theta)^\epsilon$  where  $\epsilon > 1$ .

The relationship in 2 requires several comments. The first is simply that 2 shows emissions are determined by the scale of economic activity  $F$ , and the techniques of production as captured by  $\Omega a(\theta)$ . Techniques can be influenced by changes in the intensity of abatement,  $\theta$ , or by technological progress that lowers the parameter  $\Omega$  over time. Since  $F^A$  is included in  $F$ , even the activity of abatement itself pollutes. Second, abatement uses factors in the same proportion as does final output hence we can think of the fraction  $\theta$  of capital and effective labor being allocated directly to abatement with the remaining fraction  $(1 + \theta)$  available for production of consumption or investment goods. Finally, it is important to note that a fixed abatement intensity,  $\theta$ , does not correspond to a situation of static or non-existent environmental policy. We show in the appendix that  $\theta$  remains constant over time if governments raise technology standards slowly over time. Our reading of environmental history suggests this may be a reasonably accurate characterization of slowly evolving technology standards imposed via command and control.

To combine our assumptions on pollution in 2 with the Solow model, we note that once we take abatement into account, output available for consumption or investment  $Y$ , then becomes  $Y = [1 + \theta]F$ .

Since we wish to generate predictions on both environmental quality and emissions we must adopt some assumption concerning natural regeneration. The simplest form has exponential dissipation of pollution so that the stock of pollution  $X$  is related to the flow of emissions  $E$  according to:

$$\overset{\circ}{X} = E + \eta X \quad (3)$$

where  $\eta > 0$  is the natural rate of regeneration and  $X = 0$  represents a pristine environment with a zero pollution stock.

Finally, to match the Solow model's exogenous technological progress in goods production raising effective labor at rate  $g$ , we assume exogenous technological progress in abatement lowering  $\Omega$  at rate  $g^A > 0$ . Putting these assumptions together and transforming our measures of output, capital and pollution into intensive units, the Green Solow model becomes:

$$y = f(k)[1 - \theta] \quad (4)$$

$$\dot{k} = sf(k)[1 - \theta] + [\delta + n + g]k \quad (5)$$

$$e = f(k)\Omega a(\theta) \quad (6)$$

where  $k = K/BL$ ,  $y = Y/BL$ ,  $e = E/BL$  and  $f(k) = F(k, 1)$ .

## 2.1 Balanced growth path

Assume the Inada conditions hold for  $F$ , then with  $\theta$  fixed it is immediate that starting from any  $k(0) > 0$ , the economy converges to a unique  $k^*$  just as in the Solow model. As the economy approaches its balanced growth path aggregate output, consumption and capital all grow at rate  $g + n$  while their corresponding per capita magnitudes grow at rate  $g$ . Using standard notation for growth in per capita magnitudes, along the balanced growth path we must have  $g_y = g_k = g_e = g > 0$ . A potentially worsening environment however threatens this happy existence. Since  $k$  approaches the constant  $k^*$  along the balanced growth path we can infer from 6 that the growth rate of aggregate emissions along the balanced growth path,  $G_E$ , can be positive or negative:

$$G_E = g + n + g_A \quad (7)$$

The first two terms in 7 represent the scale effect of growth on emissions since aggregate output grows at rate  $g + n$  along the balanced growth path. The second term is a technique effect created by technological progress in abatement. Using this information and referring to 3 it is easy to see that constant growth in  $X$  along the balanced growth path occurs when  $G_X = G_E$ .

Define sustainable growth as a balanced growth path generating rising consumption per capita and an improving environment. Sustainable growth is guaranteed by:

$$g > 0 \text{ and } g_A > g + n \quad (8)$$

Technological progress in goods production is necessary to generate per capita income growth. Technological progress in abatement must exceed growth in aggregate output in order for pollution to fall and the environment to improve.

## 2.2 Green Solow and the EKC

The Green Solow model, although simple, generates a very suggestive explanation for much of the empirical evidence relating income levels to environmental quality. Despite the fact that the intensity of abatement is fixed, there are no composition effects in our one good framework, and no political economy or intergenerational conflicts to resolve, the Green Solow model produces a path for income per capita and environmental quality that traces out an Environmental Kuznets Curve. This is true whether we measure environmental quality via our stock variable  $X$  or the flow of emissions  $E$ . This result is shown in Figure 4.<sup>9</sup>

In Figure 4 we present the trajectories for two economies that are identical in all respects except for their allocation to abatement  $\theta$ . We plot both emissions  $E$  and the pollution stock  $X$ . Each economy starts from an initially pristine environment and a small initial capital stock,  $k(0) > 0$ . One economy allocates 5% of its output to abatement which we refer to as the strong abatement case; the other economy is the weak abatement case as it allocates only .5% of its output to abatement. Parameters were chosen for the purposes of illustration. We have taken  $f(k)$  to be Cobb-Douglas with a capital share of .35. Per capita income grows at 1.5% along the balanced growth path, the population grows at 1% and the abatement technology improves at 3%. These parameters ensure sustainable growth is possible. The savings rate is 25%, depreciation is 3.5%, regeneration is set at .03 implying a 3% rate of dissipation of  $X$  per unit time.

As shown, the environment at first worsens with both  $X$  and  $E$  rising. After approximately 40 years emissions start to fall. After approximately 90 years the pollution stock  $X$ , starts to fall and the economy converges on its balanced growth path. Using 7 we know that along the balanced growth path emissions fall at .5% per year, which is close to what the simulation delivers in its last periods. Outside of the balanced growth path, emissions growth is of course positive for a long period of time.

---

<sup>9</sup>During the final writing of this paper we discovered that Xepapadeas (2003) also notes that technological progress in abatement can generate an EKC pattern. His discussion is brief and appears in a review article as does our first discussion of Green Solow in Brock and Taylor (2003b).

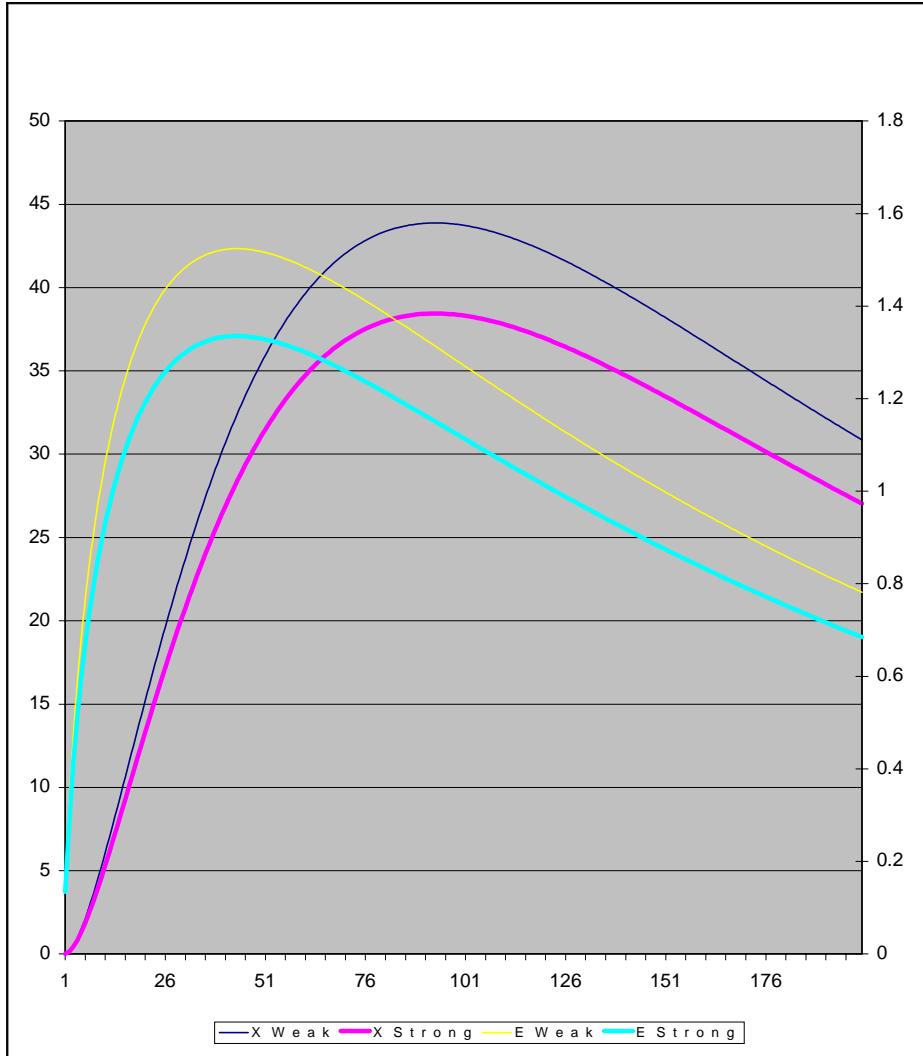


Figure 4: The EKC

The result shown in Figure 4 follows for very simple reasons. The convergence properties of the Solow model imply that output growth is at first very rapid but slows as  $k$  approaches its balanced growth path level  $k^*$ . Pollution emissions grow quickly at first but slower later. Both during the transition phase and beyond, emissions per unit output are falling at the constant rate  $g_A$  because of technological progress in abatement. This works to drive emissions downward. Finally, we have assumed growth is sustainable in the long run so that  $g_A > g+n$ . It is then immediate that the typical convergence properties of the Solow model

ensure that rapid growth in output will first overwhelm falling emissions per unit output when the economy is far from its balanced growth path, but growth in output will in turn be overwhelmed by technological progress in abatement sometime before the economy enters its balanced growth path. The interplay of technological progress and diminishing returns generates an EKC.

Outside observers may interpret the correlation between emission reduction and income growth in a variety of ways. One interpretation could be that environmental policy has finally come of age and is now aggressive enough to cause emission levels to fall. Another is that the slowdown in output growth is caused by the tightening environmental policy that is also driving emissions downward. Both of these interpretations are wrong in the context of the Green Solow model. The decline in emissions is not reflective of a new and invigorated environmental policy since  $\theta$  is constant over time. And the slowdown in growth is caused by diminishing returns not environmental policy. In fact, the slowdown in growth is the cause of emission decline - not the reverse. While it is quite natural to link a turning point in emissions with a discrete change in circumstances, the model shows that the turning point may instead reflect a more subtle weighing of various forces long at work in the economy.

In generating this result we have of course assumed the fraction of aggregate resources allocated to abatement is roughly constant - recall Figure 3 - and we have assumed technological progress in abatement works to lower emissions per unit output continuously - recall Figure 1. In fact, Green Solow equates the slope of  $\log E/Y$  shown in Figure 1 to  $g_A$  which we have assumed is constant over time. Since the model predicts that emissions per unit of output fall at a constant rate both during the transition period *and* along the balanced growth path, emissions per unit of output are falling long before emissions or the pollution stock peaks. It is tempting therefore to construct the model's analog to the emission intensities graphed in Figure 1 and pollution levels graphed in Figure 2. We construct such a graph for the strong abatement case and present it as Figure 5. The match with the earlier figures is striking.

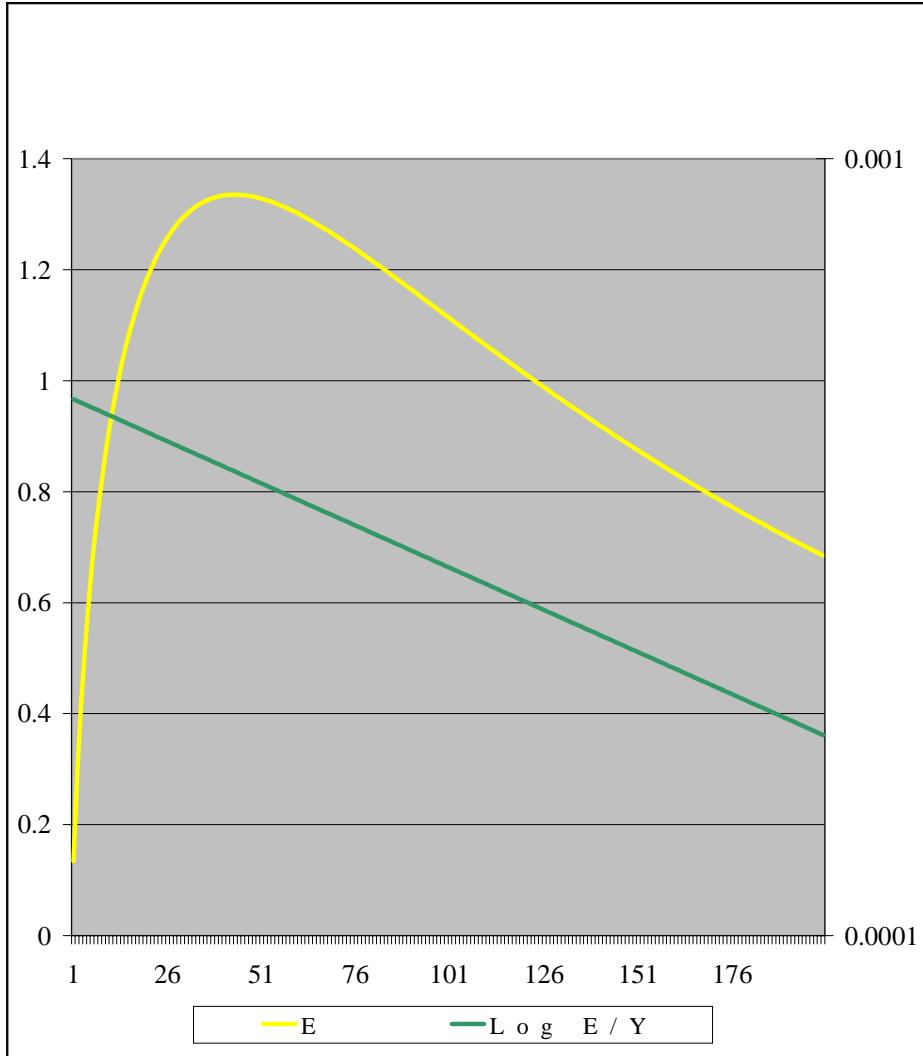


Figure 5: Matching  $E$  and  $E/Y$

Thus far we have illustrated the properties of Green Solow by simulation. To investigate how general these results are we need to solve for the transition dynamics explicitly.

### 2.3 Diminishing Returns and the Dynamics of Transition

We examine the transitional dynamics with the aid of two diagrams. The first plots the growth rate of emissions and capital against capital per effective labor and is very similar to graphical representations of the Solow model. The second

follows from the first and plots the level of emissions as a function of capital per effective worker and is very similar to representations of the EKC. To start we need to develop a differential equation for emissions. To do so write emissions at any time  $t$  as:

$$E = B(0)L(0)\Omega(0)a(\theta) \exp[G_E t]k^\alpha \quad (9)$$

where  $B(0)$ ,  $L(0)$ , and  $\Omega(0)$  are initial conditions, and  $G_E$  was given earlier. Differentiate with respect to time to obtain the growth rate of emissions:

$$\frac{\dot{E}}{E} = G_E + \alpha \frac{\dot{k}}{k} \quad (10)$$

where we note the rate of change of capital per effective worker is simply:

$$\frac{\dot{k}}{k} = sk^{\alpha-1}(1-\theta) - (\delta + n + g) \quad (11)$$

Using these two expressions we now depict the dynamics in the two panels of Figure 6.

In the top panel of Figure 6 we plot the rates of change of ( $\alpha$  times) capital per effective worker  $\alpha \dot{k}/k$  and aggregate emissions  $\dot{E}/E$  on the vertical axis against capital per effective worker  $k$  on the horizontal. In drawing the figure we have implicitly assumed growth is sustainable. We refer to the negatively sloped line as the savings locus since it is given by  $\alpha sk^{\alpha-1}[1-\theta]$  and shifts with the savings rate  $s$ . The savings locus starts at plus infinity and approaches zero as  $k$  grows large; therefore, it must intersect the two horizontal lines at points T and B as shown. From 11 it is clear that the vertical distance between  $\alpha sk^{\alpha-1}[1-\theta]$  and the horizontal line with height  $\alpha[\delta + n + g]$  is just  $\alpha$  times the growth rate of capital per effective worker or  $\alpha \dot{k}/k$ . Capital per effective worker is rising at all points to the left of B and falling at all points to the right. As is well known, the intersection at point B gives us the steady state capital per effective worker  $k^*$ . Growth is most rapid for small  $k$  and falls as  $k$  approaches  $k^*$ . When the economy enters its balanced growth path,  $k$  is zero and the economy's aggregate output and capital grow at rate  $g + n$ .

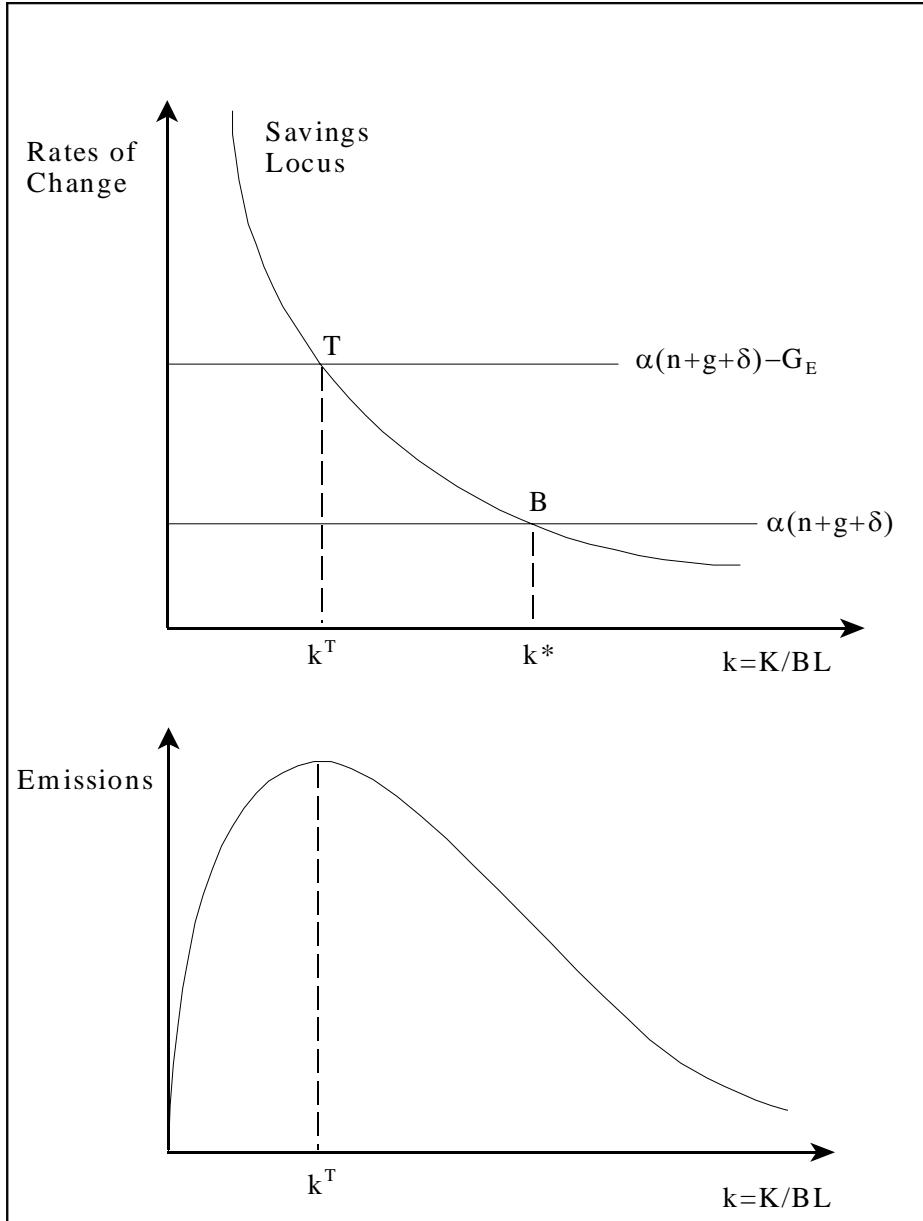


Figure 6: The Green Solow Model

To determine the time path for emissions recall that  $G_E$  is constant, and therefore from 10 we conclude that the growth rate of emissions inherits most of the properties of the growth rate of capital. Most importantly, the growth rate of emissions is very rapid for small  $k$  and falls monotonically as the economy

approaches its balanced growth path. We will exploit this property later when we derive an estimating equation predicting the convergence in emissions across countries. But for now it is important to recognize that the growth rate of emissions falls regardless of whether growth is sustainable or not.

To determine the peak level of emissions we use 10 and 11. By construction the vertical distance between the savings locus  $\alpha s k^{\alpha i} [1 + \theta]$  and the horizontal line with height  $\alpha [\delta + n + g]$  in  $G_E$  equals the percentage rate of change of emissions or  $E/E$ . Therefore, at point T the growth rate of emissions is zero:  $E = 0$ . Point T represents the turning point in emissions as shown in the bottom panel of Figure 6. Under the assumption that growth is sustainable,  $G_E < 0$ , and point T lies to the left of B; when growth is not sustainable  $G_E > 0$  and T lies to the right of B.

The figure illustrates several features of the model. It shows that if an economy's growth path is unsustainable, then emissions will grow ad infinitum even as the economy approaches its balanced growth path. But even in the unsustainable case the growth rate of emissions falls along the transition path until it approaches its balanced growth path rate from above. If growth is sustainable then T lies to the left of B and the time profile for emission levels depends on the location of  $k(0)$  relative to point T. If an economy starts with a small initial capital stock then emissions at first rise and then fall as development proceeds: i.e. we obtain an EKC profile for emissions. If initial capital is larger it is possible that the level of emissions falls monotonically as the economy moves towards its sustainable growth path. It is important to note while the level of emissions may rise and then fall over time, the growth rate of emissions is monotonically declining. This is apparent because emissions growth is rapid for countries a long way from point B, and slower for those near B regardless of the location of T. Finally when emissions peak depends on the relationship between points T and B. For example, if  $|G_E|$  is small, then T and B differ very little and emissions will only peak as the economy approaches its balanced growth path which may of course take a very long time. Since these are key results, we record them as a proposition.

**Proposition 1** *If growth is sustainable and  $k^T > k(0)$ , then the growth rate of emissions is at first positive but turns negative in finite time. If growth is sustainable and  $k(0) > k^T$ , then the growth rate of emissions is negative for all t. If growth is unsustainable, then emissions growth declines with time but remains positive for all t.*

Proof: See Appendix

Proposition 1 tells us about the shape of the emissions and income profile but says very little about the level of emissions and income per capita at the turning point. Although the model is simple, it can be deceptive in this regard. For example, it is a short step from knowing that  $k^T$  is unique to an assumption that income per capita at the turning point is unique. Similarly, it is easy to assume that the path for income growth and emissions is the same for countries sharing savings rates, population growth rates, etc. Both of these conjectures are wrong: although  $k^T$  is unique, the associated income per capita and emissions level at  $k^T$  are not.

**Proposition 2** *Economies with identical parameter values but different initial conditions produce different income per capita and emission profiles over time. The peak level of emissions and the level of income per capita associated with peak emissions are not unique.*

Proof: in text.

The intuition for this result is straightforward. The peak level of emissions is reached when the rate of emissions growth created by output growth equals the rate of technological progress in abatement. This occurs at a unique  $k^T$ . Take two economies with the same physical capital and assume both economies are at  $k^T$ . While these economies must have the same effective labor force at this point, one economy could have a highly efficient but small working population while the other had a less efficient but more numerous labor force. Clearly income per capita differ in these two economies even though each is at  $k^T$ .

The nonuniqueness of emissions follows for related reasons. An economy that is larger has greater emissions everywhere even though it may have the same capital per effective worker along the transition path as some hypothetical smaller economy (see 9). Less transparently an economy with a inferior abatement technology (a higher  $\Omega(0)$ ) will have a higher emissions per unit of output leading to a difference in peak emissions at the turning point and elsewhere.

These examples highlight an important point brought out by the Green Solow model. The current literature has tended to focus our attention on level variables - specifically the level of pollution against the level of income per capita. Even the "control variables" added to EKC regressions are often level variables such as population density, openness to trade, measures of democracy or deposits of coal. The Green Solow model refocuses our attention on growth rates since it is the equality of two growth rates that determines the turning

point in emissions. By doing so it shows how looking at the levels of variables can be misleading.

The non uniqueness of peak income and emission levels, offers a potential explanation for the contradictory and sometimes erratic empirical results found in the EKC literature. It is now well known that the shape of the estimated EKC can differ quite widely when researchers vary the time period of analysis, the sample of countries, the pollutant, or even the data source. For example, Harbaugh et al. (2002) reconsider Grossman and Krueger's specification and find little support for an EKC using newer updated data. Stern and Common (2001) employ a larger and different sulfur dioxide dataset and find no EKC. And the literature reviews by both Barbier (1997) and Stern (2003) note that published work differs greatly in the estimated turning points for the EKC, the standard errors on turning points are often very large, and empirical results differ widely across pollutants and countries. At the same time, plots of raw pollution data for the US and other countries often present a dramatic confirmation of the EKC.<sup>10</sup>

Proposition 2 offers a simple explanation for the seeming inconsistency between country level data and cross-country empirical results. If EKC profiles for even very similar countries are not unique because of differences in initial conditions, then unobserved heterogeneity is surely a problem. Unobserved heterogeneity could then account for the large standard errors on turning points and the sensitivity of results to the sample. While in theory conditioning on country characteristics could eliminate the problem of unobserved heterogeneity, existing work has focussed on additional controls that are level variables and not the rate of change variables suggested by our theory.

To be more precise concerning peak emission and income levels write income per capita at any time  $t$  as:

$$y^c(t) = k(t)^\alpha B(0)[1 + \theta] \exp[gt] \quad (12)$$

which is a function of  $k(t)$ , time, abatement and the initial condition  $B(0)$ . At the turning point, emissions growth is zero and solving for the  $k^T$  identified in Figure 6 yields:

$$k^T = \frac{s(1 + \theta)}{n + g + \delta + G_E/\alpha}^{1/(1 - \alpha)} \quad (13)$$

---

<sup>10</sup>For international evidence see Table 3 in Section 4.

To solve for the time - call it  $t = T$  - at which the economy's capital per effective worker reaches  $k^T$  solve the differential equation for  $k(t)$  to find:

$$k(t) = \frac{h}{k^a(1-i-\alpha)(1-i-\exp[i-\lambda t]) + k(0)^{(1-i-\alpha)}\exp[i-\lambda t]}^{1/(1-i-\alpha)} \quad (14)$$

$$k^a = \frac{s(1-i-\theta)}{n+g+\delta}^{1/(1-i-\alpha)} \quad (15)$$

As expected  $k(t)$  is an exponentially weighted average of initial capital per worker  $k(0)$  and its balanced growth path level  $k^a$  where the weight given to initial versus final positions is determined by the speed of adjustment in the Solow model  $\lambda = [1-i-\alpha][n+g+\delta]$ . We can now set  $k(t)$  equal to  $k^T$  yielding an implicit equation for the time it takes to reach the peak level of emissions.  $T$  is defined by:

$$T : k^T = \frac{h}{k^a(1-i-\alpha)(1-i-\exp[i-\lambda T]) + k(0)^{(1-i-\alpha)}\exp[i-\lambda T]}^{1/(1-i-\alpha)} \quad (16)$$

Note that  $k(0) = K(0)/B(0)L(0)$ . Income per capita at the peak is found by evaluating 12 using  $T$  from 16 and subbing in for  $k^T$  using 13. Peak emission levels follow similarly.

To verify that income per capita is not unique at the peak level of emissions note that 16 shows us that  $T$  is independent of variations in initial conditions that leave  $k(0)$  unchanged. At the same time, from 12 it is apparent that any variation in  $B(0)$  alters income per capita directly even if  $k(0)$  and  $T$  are left unchanged. To see that emissions are not unique substitute 13 into 9 and again consider variations in initial condition  $B(0)$  leaving  $k(0)$  unchanged. These variations have no effect on the time to peak emissions, but will affect emissions directly by altering effective labor. Note that  $\Omega(0)$  plays no role in determining  $k^T$  or  $T$ ; hence variations in it alter the peak level of emissions directly via 9. Even if we corrected for country size by measuring emissions per person, countries with a higher emissions per unit of output at time zero will have greater emissions as well.

Despite these indeterminacies it remains true that every economy will follow an EKC pattern as described in Proposition 1. Since empirical work regresses emissions on income per capita and not time as we have here, it is useful to make the connection between our theory and the existing empirical work precise. To

do so use 14 in 9 to find:

$$\begin{aligned} E(t) &= c_0 \exp[G_E t] \cdot k^{\alpha(1-\alpha)} (1 - \exp[-\lambda t]) + k(0)^{\alpha(1-\alpha)} \exp[-\lambda t]^{\frac{1-\alpha}{1-\alpha}} \\ c_0 &= B(0)L(0)\Omega(0)a(\theta) \end{aligned} \quad (17)$$

But from 14 and 12 it is apparent that  $y^c(t)$  is a strictly increasing function of time. We can therefore invert it finding  $t = \phi(y^c)$  and substitute for time in 17. This gives us a parametric relationship between aggregate emissions and income per capita. Establishing the properties of this relationship requires further work that we leave to the appendix, but we note here:

**Proposition 3** *There exists a parametric relationship between emissions  $E$  and income per capita  $y^c$  that we refer to as an EKC. If  $k^a > k^T > k(0)$ , then emissions first rise and then fall with income per capita. If  $k^a > k(0) > k^T$ , then emissions fall monotonically with income per capita.*

Proof: See Appendix

Proposition 3 is important in establishing that the Green Solow model reproduces an EKC relating emissions to per capita income. This EKC may take on a typical hump shape or it may be monotonically declining as some authors have found for some pollutants. It is important however to recognize that both income per capita and emissions are both functions of more primitive determinants such as initial conditions, savings rates, etc. Even though an EKC relationship exists in the Green Solow model, strictly speaking there is no causal relationship between income per capita and emission levels. Therefore, the typical processes held responsible for an EKC can be very weak or even non-existent and yet have researchers observe an EKC pattern in the data.

## 2.4 Comparative Steady State Analysis

Most of the empirical exercises investigating the EKC employ cross country data that includes both developed and developing countries and often both democracies and communist states. Clearly these economies differ in much more than just initial conditions, and this heterogeneity may further confound estimation. To investigate how differences in deep parameters affect our results we now consider the impact of changes in savings, abatement and rates of technological progress.

Consider the role of savings. An increase in the savings rate shifts the savings locus rightward raising both T and B in Figure 6. Greater savings raises capital per effective worker in steady state. The turning point for emissions rises because higher savings implies more rapid capital accumulation at each  $k$ . This in turn means faster output growth and faster emissions growth at any given  $k$ . The turning point can only be reached when diminishing returns lowers output growth to meet  $|g_A|$ ; greater savings makes this task harder and hence  $k^T$  rises.

To determine whether economies that save more will reach peak emissions at a higher or lower income per capita write income per capita at the peak as:

$$y^c(T) = [k^T]^\alpha B(0)[1 + \theta] \exp[gT]$$

Income per capita at the peak is rising in capital per effective worker at the peak and rising in the calendar time needed to reach the peak. The former determines the capital intensity of the economy, the latter determines how productive labor is when the transition point is reached. We have already shown  $k^T$  rises with  $s$ . To solve for the calendar time to transition, rearrange 16 to find:

$$T = \frac{1}{\lambda} \log \frac{k^{s(1-i-\alpha)} - k(0)^{1-i-\alpha}}{k^{s(1-i-\alpha)} + k^T(1-i-\alpha)} \quad (18)$$

The calendar time needed to reach peak emissions is declining in the convergence speed of the Solow model,  $\lambda$ , increasing in the gap between initial and final capital per effective worker, and is larger the closer is point T to B. If we substitute for  $k^s$  and  $k^T$  in 18 it is possible to show that an increase in the savings rate raises  $T$ . Since savings also raises capital per effective worker at the turning point we are done: an economy with a higher savings rate reaches its peak emissions level at a higher income per capita than otherwise.

Differences in abatement intensity have a similar but opposite effect. An increase in abatement lowers  $(1 + \theta)$  and shifts the savings locus leftward. This reduces both T and B. Since more resources are devoted to abatement and less to savings, larger investments in abatement slow transitory growth and for any given  $k$ , they imply slower growth in emissions as well. Using 18 we can show that  $T$  falls as well. Therefore, peak emissions are reached at a lower level of income per capita when abatement is more aggressive. It is very important to note that emissions start to fall at a lower income per capita not because abatement lowers emissions per se although it does this in the level sense, but

because abatement uses up scarce resources that would otherwise have gone to investment. This reduces the rate of growth of output during the transition period. It is the impact of abatement on growth rates during the transition that alters  $k^T$ . Changes in the abatement intensity have no effect whatsoever on the economy's long run growth or on the long run growth rate of emissions.<sup>11</sup>

Finally consider the impact of changes in technological progress. Start with changes in the rate of progress in abatement,  $g_A$ . An increase in  $g_A$ , pushes emissions down faster and shifts the uppermost line in Figure 6 upwards lowering  $k^T$ . This lowers the growth rate of emissions for any  $k$ , and will likewise lower the growth rate of emissions in steady state. This change has no effect on the growth rate of output or on  $k^*$ . Not surprisingly using 18 we find that  $T$  is reduced. Putting these results together we find peak emissions are reached at a lower income per capita than otherwise.

Faster technological progress in goods production has a less clear cut effect. An increase in  $g$  shifts the uppermost line in Figure 6 downward raising  $T$  and the lowermost line upwards lowering  $B$ . The time to peak emissions could rise or fall, and hence income per capita at the peak may be higher or lower. All else equal income is higher since capital intensity at the peak has risen, but income may be lower if the calendar time needed to reach the peak is lower than before. A somewhat similar result arises from changes in population growth. Population growth lowers steady state capital per worker and this lowers transitional growth at any given  $k$ . But population growth raises emissions directly via a scale effect and this raises both emissions growth and the point at which emissions start to fall. Whether this new higher transition point is reached sooner or later in calendar time is indeterminate and hence so too is the associated income per capita.

These results demonstrate that there are three qualitatively different sets of parameters in the model. The first set are parameters (such as initial conditions) that affect emissions and income levels at their peak but have no effect on long run growth rates of emissions or ouput nor any affect on the steady state. A second set of parameters (such as savings rates and abatement intensities) affect both peak emissions and income levels, alter steady state levels and have an impact on transitional growth, but have no impact on long run growth rates. The final set of parameters (such as rates of technological progress or population growth) alter peak emissions and income, transitional growth and growth

---

<sup>11</sup>This doesn't mean more abatement has no costs: greater abatement lowers the level of income per capita along the balanced growth path.

along a balanced growth path. In short these results demonstrate that the relationship between income and pollution is exceedingly complex. And therefore it should come as no surprise that there are large standard errors on turning points and fragile coefficient estimates.

To our knowledge no empirical work examining the growth and environment relationship has used as controls savings rates, population growth rates, etc. that would be suggested by our analysis. But even with information on deep parameters we have already shown that since the EKC profile is reliant on initial conditions, estimation problems remain. One alternative that presents itself is to focus on model predictions that are tightly linked to parameters: that is focus on the relationship between the growth rate of emissions along the transition path rather than the parametric relationship between the level of emissions and income.

### 3 An Empirical Implementation

We have demonstrated why current empirical methods may have difficulty in estimating an EKC relationship. The income-emissions profile will differ across countries if they differ in initial conditions or in basic parameters such as savings or population growth rates. Criticism is of course much easier than creation, and while many authors have been critical of the EKC methodology, very little has been offered as a productive alternative.

In this section we present an alternative method to investigate the growth and environment relationship that draws on existing work in macroeconomics on absolute and conditional convergence. Our goal is to develop an explicit link between theory and empirical estimation since this link is largely absent in this literature. A secondary goal is to demonstrate Green Solow's ability to explain cross-country patterns of emissions growth with relatively few variables.

The Green Solow model contains two empirical predictions regarding convergence in emissions. The first is that a group of countries sharing the same parameter values - savings rates, abatement intensities, rates of technological progress etc. - but differing in initial conditions will exhibit convergence in a measure of their emissions. This is true even though each of these countries would typically exhibit a unique income and pollution profile over time. We will derive an estimating equation below to show that under the assumption of identical parameters values across countries, we obtain a prediction of *Absolute*

*Convergence in Emissions* or ACE. Under the ACE hypothesis differences in the pattern of cross-country growth of emissions per person, is fully explained by differences in initial emissions per person. This prediction follows from the familiar forces of diminishing returns plus an assumption that countries share the same steady state.

The second prediction is that a disparate group of countries will exhibit both very different pollution and income profiles and will not exhibit ACE. Disparate countries will grow outside of steady state at rates that are functions of both differences in initial conditions and differences in country characteristics. In theory if we condition on the right country characteristics, we could estimate a relationship predicting convergence in emissions per person. Since this convergence prediction is akin to the concept of conditional convergence in the macro literature we refer to it as *Conditional Convergence in Emissions*. Here we focus on the model's predictions for ACE within a sample of OECD countries, but also investigate how our results change when we allow countries to differ in savings rates etc. We do so in order to generate and implement a testable equation that may be of use to other researchers.

We conduct our empirical work with data on carbon dioxide emissions. We focus on carbon dioxide for several reasons. Carbon dioxide data exists for a large group of countries over a significant period of time. The large cross country coverage is important since it allows us to show that, as predicted, ACE does not hold over the entire universe of countries in our sample. This sample includes 139 developed and developing economies and this heterogeneity should lead to the failure of ACE.

As well, researchers have had great difficulty in making sense of the carbon data. Estimates of the turning point for carbon are often very high and variable, and hence carbon is one pollutant that may not follow an EKC. Given these difficulties, carbon offers a good testing ground for our approach.

Finally very little direct abatement of carbon emissions has occurred. Some reductions in carbon emissions have come about as a result of other pollution regulations, but much of the trend in carbon emissions per unit output is related to changes in the energy intensity of economies. But changes in energy use per unit output and emissions per unit energy are thought to be responsible for a majority of the reductions we have seen in the set of regulated pollutants.<sup>12</sup> Therefore while carbon is unlike other pollutants because it is unregulated, it is

---

<sup>12</sup>See for example Selden et al. (1999).

much like other air pollutants in that it is tightly tied to energy use.

### 3.1 Estimating Equation

We start with equation 9 for emissions but rewrite it in terms of emissions per person  $e^c(t) = E(t)/L(t)$ , and income per capita,  $y^c(t) = F(t)[1 + \theta]/L(t)$ . Using standard notation, this gives us:

$$e^c(t) = \Omega(t)\tilde{a}(\theta)y^c(t) \quad (19)$$

where  $\tilde{a}(\theta) = a(\theta)/[1 + \theta]$ . Differentiating with respect to time yields

$$\frac{\dot{e}^c}{e^c} = \dot{g}_A + \frac{\dot{y}^c}{y^c} \quad (20)$$

where we have made use of our assumption that the fraction of overall resources dedicated to abatement is constant over time. As shown, growth in emissions per person is the sum of technological progress in abatement plus growth in income per capita. Along the balanced growth path this is equal to  $\dot{g}_A + g$  which may be positive, negative, or zero; outside of the balanced growth path we will approximate the growth rate.

We make equation 20 operational in three steps. First approximate the growth rate of income per capita and emissions per person over a discrete time period of size N by their average log changes and rewrite the equation as:

$$[1/N] \log[e_t^c/e_{t+N}^c] = \dot{g}_A + [1/N] \log[y_t^c/y_{t+N}^c] \quad (21)$$

Second use the now standard procedures employed by Mankiw, Romer and Weil (1992) and Barro (1991) to approximate the discrete N period growth rate of income per capita near the model's steady state via a log linearization to obtain:<sup>13</sup>

$$[1/N] \log[y_t^c/y_{t+N}^c] = b + \frac{[1 - \exp[i - \lambda N]]}{N} \log[y_{t+N}^c] \quad (22)$$

where  $b$  is a constant (discussed in more detail below) and  $\lambda = [1 - \alpha][n + g + \delta]$  is the Solow model's speed of convergence towards  $k^*$ .

Finally substitute for income growth in 21 using 22 and substitute for initial

---

<sup>13</sup>See the appendix for a full derivation.

period income per capita using  $y_{t_i N}^c = e_{t_i N}/\Omega_{t_i N} a(\theta)$  from equation 19.

By making these substitutions we obtain a simple linear equation suitable for cross-country empirical work. It relates log changes in emissions per person across  $i$  countries (over a discrete period of length  $N$ ) to a constant and initial period emissions per person. We write this as a simple linear regression with error term  $\mu_{it}$ :

$$[1/N] \log[e_{it}^c / e_{it_i N}^c] = \beta_0 + \beta_1 \log[y_{t_i N}^c] + \mu_{it} \quad (23)$$

$$\begin{aligned} \beta_0 &= g_A + \frac{[1 - \exp[-\lambda N]]}{N} \log[y^a a(\theta)^{-\bar{\Omega}_{t_i N}^{-1} \bar{B}_{t_i N}^{-1}}] \\ \beta_1 &= -\frac{[1 - \exp[-\lambda N]]}{N} < 0 \\ \mu_{it} &= \frac{[1 - \exp[-\lambda N]]}{N} \log \Omega_{i,t_i N}^h B_{i,t_i N} - \bar{\Omega}_{t_i N}^{-1} \bar{B}_{t_i N}^{-1} \end{aligned}$$

We refer to the specification in 23 as the short specification. Somewhat heroic assumptions are needed to estimate 23 consistently with OLS. For example, if we assume countries share the same steady state  $y^a$ , then countries can only differ in their initial technology levels  $\Omega_{t_i N}$  and  $B_{t_i N}$ . While Mankiw, Romer and Weil (1992) assume the initial goods technology  $B_{t_i N}$  differs across countries by at most a idiosyncratic error term, this assumption has come under severe criticism on both econometric and theoretical grounds (see especially Durlauf and Quah (1999)). The primary econometric concern is that unobserved variation in initial technology in  $B_{i,t_i N}$  across  $i$  may be correlated with other right hand side variables determining  $y^a$ .

While unobserved heterogeneity is certainly a possibility here as well, it may pose less of a problem in our context. The reason is simply that a productive goods technology implies a large initial  $B_{t_i N}$ , while a productive emissions technology implies a small  $\Omega_{t_i N}$ . Therefore, a technologically sophisticated country at  $T_i N$  may have the same  $\Omega_{i,t_i N} B_{i,t_i N}$  as a technologically backward country at  $T_i N$  making unobserved heterogeneity in initial technology levels less of a problem.<sup>14</sup> We invoke this argument to justify our decomposi-

---

<sup>14</sup>In some circumstances this heuristic argument is exact. For example assume the initial technology levels were proportional to each country's initial "technological sophistication" at  $T_i N$ . Denote technological sophistication by  $S_{i,t_i N}$  and assume that initial productivity

tion of the unobservable country specific products  $\Omega_{i,t_1 N} B_{i,t_1 N}$  into an overall cross country mean we denote by  $\bar{\Omega}_{t_1 N} \bar{B}_{t_1 N}$  plus a country specific deviation. These country specific deviations plus standard approximation error in generating our linear form are contained in our error term  $\mu_{it}$  as shown above. OLS is consistent if the covariance of  $\mu_{it}$  and our right hand side variables is zero.

Since individual elements making up the constant term in 23 are not identified we have no prediction concerning the sign. The growth rate of emissions per person should however fall with higher initial period emissions per person and this is reflected in the prediction of  $\beta_1 < 0$ . The intuition for this is simple. Holding the technology levels and abatement intensity fixed, a lower emissions per person  $e_{t_1 N}^c$  corresponds to a lower initial capital per effective worker  $k_{t_1 N}$ . This then implies a rapid rate of growth in aggregate emissions  $E$  and hence a rapid rate of growth of emissions per person. It is also useful to note that since  $N$  is given, any estimate of  $\beta_1$  carries with it an implicit estimate of the rate of convergence of the Solow model,  $\lambda$ . Since  $N$  is fixed, we can back out these estimates of  $\lambda$  and check them against those provided by the cross-country growth literature.

While the short specification is simple it is also unsatisfactory. It is unsatisfactory because the Green Solow's key predictions on emissions follow from the new element  $g_A$  and the reliance of  $y^a$  and  $a(\theta)$  on abatement. Thus far we have assumed  $g_A$  and  $\theta$  are both constant over time and exhibit no cross country variation. But data on these variables are available. Cross country data on the share of pollution abatement costs in GDP shows  $\theta$  varies little over time, but exhibits substantial cross country variation. Moreover if we take our model literally then  $g_A$  equals the rate at which carbon emissions to output falls over time. This ratio is both observable and does vary across countries.

To carry forward these new elements into empirical work we now construct the long specification of our estimating equation. To do so it proves useful to assume  $a(\theta) = (1 + \theta)^\epsilon$  where  $\epsilon > 1$ . This formulation follows from a constant returns abatement function, and like all isoelastic functions it is quite useful in empirical work. To generate our long specification we return to our short specification in 23. Let savings, abatement,  $g_A$ , and the effective depreciation rate  $(n + g + \delta)$  vary across countries by taking on an  $i$  subscript, and then

---

is proportional to technological sophistication; that is,  $\Omega_{i,t_1 N} = a/S_{i,t_1 N}$  and  $B_{i,t_1 N} = bS_{i,t_1 N}$  for some  $a$  and  $b$  positive. Note the product  $\Omega_{i,t_1 N} B_{i,t_1 N} = ab$  which is now independent of  $i$ .

substitute for the determinants of  $y^a$  to write the long specification as:

$$[1/N] \log[e_{it}^c/e_{it_i-N}^c] = \beta_0 + \beta_1 \log[e_{it_i-N}^c] + \beta_2 [g_{Ai}] \quad (24)$$

$$+ \beta_3 \log[s_i] + \beta_4 \log[1 - \theta_i]$$

$$+ \beta_5 \log[(n + g + \delta)]_i + \mu_{it}$$

$$\beta_0 = g + \frac{[1 - \exp[i - \lambda N]]}{N} [\log \frac{\Omega_{t_i-N}}{B_{t_i-N}}]$$

$$\beta_1 = i \frac{[1 - \exp[i - \lambda N]]}{N} < 0,$$

$$\beta_2 = i - 1 < 0$$

$$\beta_3 = [\alpha/(1 - \alpha)] \frac{[1 - \exp[i - \lambda N]]}{N} > 0$$

$$\beta_4 = [\alpha/(1 - \alpha) + \epsilon_i - 1] \frac{[1 - \exp[i - \lambda N]]}{N} > 0$$

$$\beta_5 = i - \beta_3 < 0$$

where  $g_{Ai}$  is a country specific estimate of technological progress in abatement (see below), while  $s_i$ ,  $[1 - \theta_i]$  and  $[(n + g + \delta)]_i$  are the time-averaged country specific savings rate, abatement, and effective depreciation rate respectively. Savings and depreciation are familiar from growth regressions as are several of the restrictions theory imposes on coefficient magnitudes. The long specification does however add new parameters to estimate and provides two new testable restrictions.

Given our earlier discussions it should be apparent that an increase in savings raises the growth rate of emissions per person by raising the steady state capital stock. This positive transitional effect of savings on emissions growth is captured by  $\beta_3 > 0$ . Since a reduction in abatement raises the economy's steady state capital intensity and raises emissions via reduced abatement, we

also have  $\beta_4 > 0$ . The coefficient sign of  $\beta_5$  follows directly from the role "effective depreciation" plays in determining a country's steady state capital intensity.

Apart from the usual parameter restrictions contained in the Solow model ( $\beta_5 + \beta_3 = 0$ , and  $\alpha$  should be close to capital's share) the Green Solow model contains the additional restriction of a unitary coefficient on  $g_A$  and  $\beta_4 \neq \beta_3 > 0$ . Moreover, by judicious use of estimates for  $\beta_1, \beta_3$  and  $\beta_4$  an estimate for  $\epsilon$  can be constructed. Again, since  $N$  is known, we can recover an estimate of  $\lambda$  as well.

There are however new econometric and data complications introduced by the long specification. One concern is a common one. We do not have data for either  $g$  or  $\delta$ , and hence we have to construct our regressor  $(n + g + \delta)$  using alternative means. Here we follow the literature in assuming  $g + \delta = .05$ , and then use observed population growth rates for  $n$  to construct the regressor. A second complication arises because strictly speaking once we admit population differences across countries, we should also admit differences across countries in the parameter  $\lambda$  as well. Since doing so would exhaust our degrees of freedom we again follow the literature in treating  $\lambda$  as constant across countries. Since this is somewhat unsatisfactory we will investigate how the inclusion of effective depreciation affects our results.

Finally since technological progress in abatement is a key part of our theory we construct a measure of  $g_{i,A}$ . This is somewhat problematic since our theory takes  $g_{i,A}$  to be exogenous and hence is uninformative on its determinants.<sup>15</sup> But since we are taking the intensity of abatement as fixed over time, we could in theory obtain estimates for  $g_{i,A}$  by regressing  $\ln(E_{it}/Y_{it})$  on a constant and a time trend. This specification follows directly from our theory's prediction for E/Y as given for example in 9. It seems likely however that although technological progress may be the largest force affecting the time profile for

---

<sup>15</sup>One generalization that may be worth pursuing is to assume that final output is produced by a continuum of inputs each with a different pollution intensity. In this case we can then write an economy's overall emission intensity as:  $E/Y = \int \Omega(z, t)a(z, \theta) \frac{p(z)y(z)}{Y} dz$  where the integral is defined over the set of available inputs;  $\Omega(z, t)a(z, \theta)$  is emission per dollar of final output in sector  $z$ , and  $p(z)y(z)$  is total dollar value added in this sector. Let industry shares be given by  $b(z, t) = p(z)y(z)/Y$ , with  $\int b(z, t)dz = 1$ . Then straightforward differentiation will show that even if the rate of technological progress in abatement is identical across sectors (i.e. assume  $\Omega(z, t)$  is independent of  $z$ ), then the rate of change in  $E/Y$  will differ from  $g_A$  if there are compositional changes in the economy. This suggests that our crude method for estimating  $g_A$  could be improved by employing information on sectoral shares and carbon intensity. We intend to investigate this possibility in the future.

$E_{it}/Y_{it}$ , it is not the only force. Compositional changes, regulations, oil price shocks etc. may all play a role as well. Of these energy prices is probably the most important for our carbon data, and since we do not wish to attribute to technological change what is a compositional change caused by rising energy prices, we allow for energy prices to affect the carbon intensity of production. For each country,  $i$ , we obtain our estimates of  $g_{i,A}$  by estimating the following (country by country) using OLS:

$$\log(E/Y)_{it} = \gamma_{i0} + \gamma_{i1} \log p_{oil,t}^w + \gamma_{i2} \hat{time} + \epsilon_{it} \quad (25)$$

where  $\gamma_{i0}$  is equal to  $\log[\Omega_{iT_1} \tilde{Na}(\theta_{iT_1} N)]$ ,  $g_{i,A}$  is given by  $\gamma_{i2}$ , and  $t$  runs from 1960-1998.  $p_{oil,t}^w$  is the real US dollar world price for oil, and  $(E/Y)_{it}$  is emissions per dollar of real gdp also measured in US dollars. While more sophisticated models for  $g_{i,A}$  may improve our estimates, our goal here is to provide preliminary evidence while staying as close as possible to the direct implications of our theory.

### 3.2 Data

Obtaining good cross country data on pollution emissions is difficult. We used the World Bank's Development Indicator's 2002 for data on carbon emissions per capita, carbon per dollar of GDP, population size, and investment rates. The data starts in 1960 and we take 1998 as our terminal year. Our focus is the OECD sample comprised of 22 countries for which we have a relatively complete set of data from 1960 to 1998. The countries are: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, and the United States.<sup>16</sup> We follow the common practice of employing time average values for savings, population growth rates and abatement intensities. Savings is proxied by the average investment to GDP ratio over the 1960-1998 period. Population size is measured using actual population figures (using the working age population has little effect on our results). Data on the share of abatement in GDP is especially spotty. The OECD publishes data on the share of pollution abatement costs in GDP for many OECD countries, but the country coverage is not complete. In theory these data include both public and private sector expenditures, and span the

---

<sup>16</sup>Germany was excluded because of extensive border changes.

years 1985 to 1998, but few countries have complete coverage.<sup>17</sup> Accordingly we employ averages over the longest period possible and employ the widest measure for pollution abatement cost available (public plus private expenditures when available). For some countries the time averaged estimates reflects relatively few observations. In order not to lose degrees of freedom we calculated the share for Luxembourg directly from OECD sources, and assumed New Zealand had the same abatement intensity as its neighbor Australia. Given the quality of this data the reader is cautioned from drawing strong conclusions from our results.

### 3.3 Results

We start by examining the possibility of absolute convergence in emissions. It would of course be surprising to find ACE supported across anything but the most homogenous of country groupings. We expect ACE to fail miserably because any broad set of countries will differ greatly in their rates of savings, population growth and technological progress.

As a starting point for our analysis we present in Figure 7 the yearly average log changes in emissions per person against the log of initial levels for 139 countries available in the World Bank development indicators.<sup>18</sup> As the plot shows there is little apparent relationship between the two series.

---

<sup>17</sup>This data is available from the OECD publication "Pollution abatement and control expenditures in OECD Countries", Paris: OECD Secretariat. We present our pollution abatement cost data in the appendix.

<sup>18</sup>This plot includes all countries in the database for which there is data in 1960. Emissions are measured in lbs of emissions per capita for ease of reading.

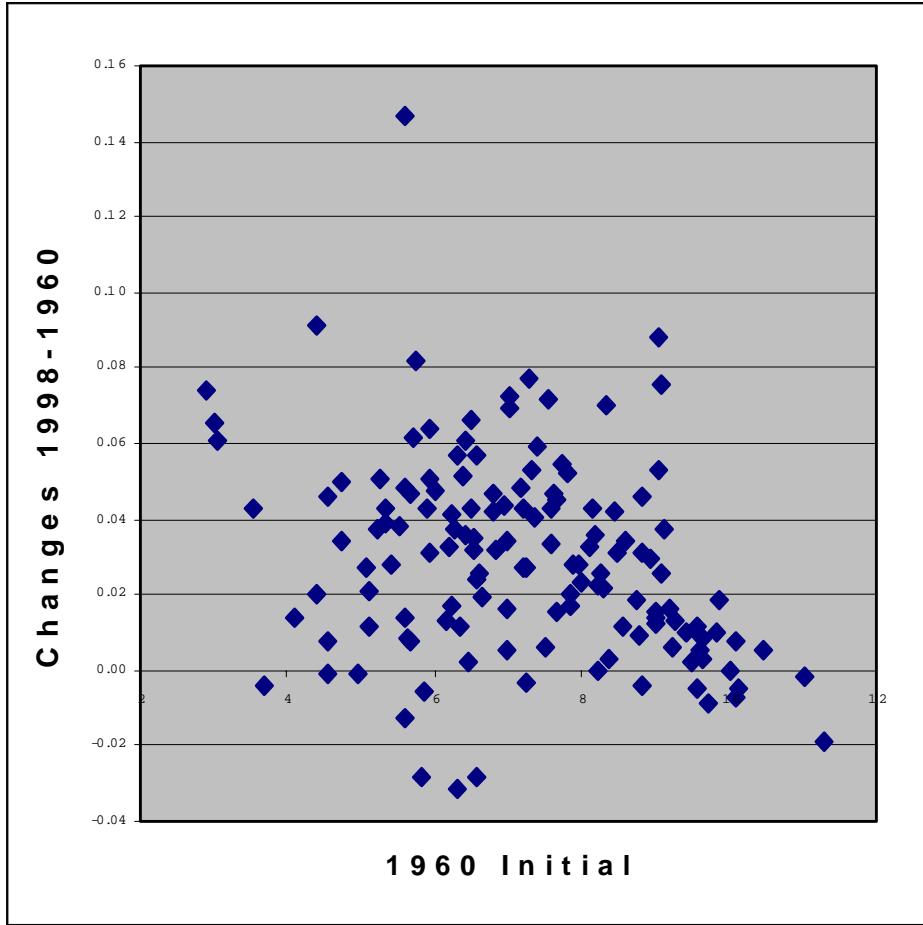


Figure 7: World CO2 Convergence

In Figure 8 we construct the same plot for the sample of OECD countries that we expect are similar in parameter values. The difference is striking. There is obviously a strong negative and very tight relationship between the growth of emissions per person over the 1960-1998 period and 1960 emissions per capita. This is true even though the figure does not correct for any of the differences across countries allowed for in our long specification. The figure shows countries with small emissions per person experienced rapid emissions growth while those with large emissions per person grew far more slowly.

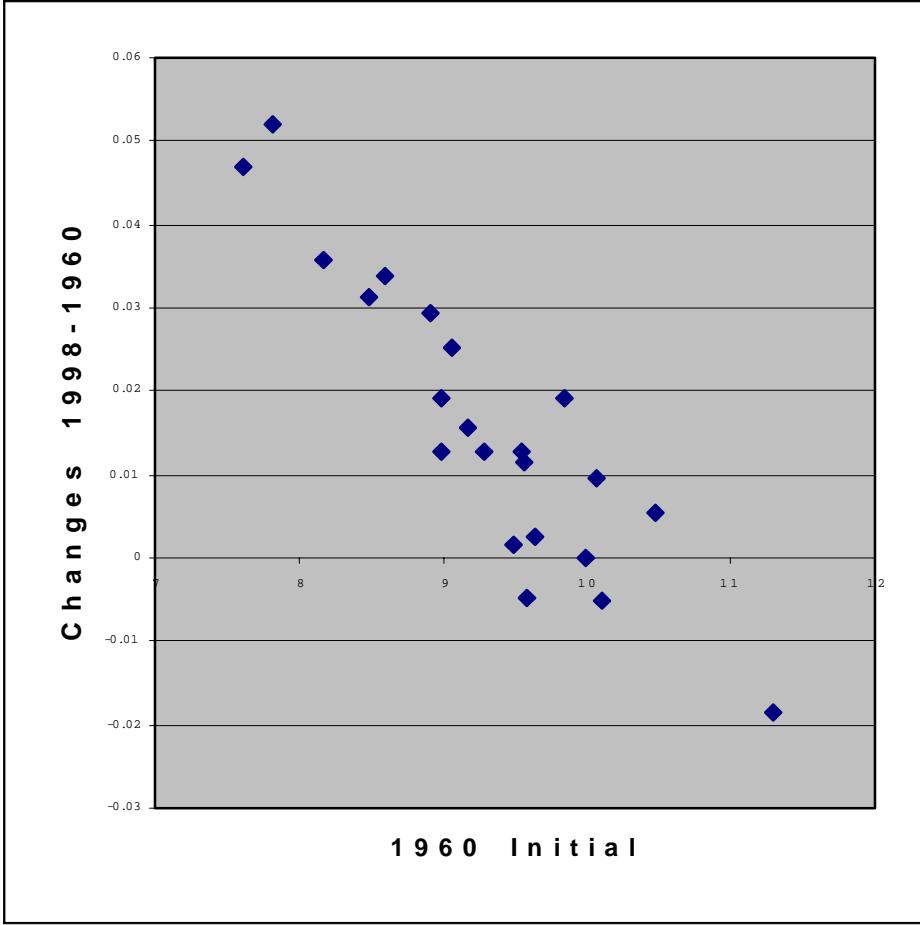


Figure 8: OECD Convergence

To go further we present estimates of our short and long specifications in Table 1.<sup>19</sup> In the first column we report estimates for the short specification. Not surprisingly, initial emissions per person has a negative effect on growth as shown in Figure 8. What is surprising is the goodness of fit, with over 80% of the variation in emissions per person being explained by initial emissions per person alone. This is far above the typical explanatory power of unconditional Solow type regressions.

---

<sup>19</sup>Following Barro (1991), Durlauf and Johnson (1995) and others we employ heteroscedasticity corrected standard error estimates.

Table 1: Convergence Across the OECD

Variables	(1)	(2)	(3)	(4)	(5)
cons	.045	.041	.005	-.025	.014
	(18.5)	(14.6)	(.11)	(-.58)	(.25)
$\log e_{it_1 N}^c$	-.018	-.011	-.010	-.010	-.009
	(-11.9)	(-4.1)	(-3.5)	(-3.4)	(-2.5)
$\mathbf{j} g_A$	..	-.49	-.49	-.49	-.54
		(-2.7)	(-2.8)	(-2.7)	(-2.8)
$\log s$			.010	.011	.015
			(.75)	(.79)	(.93)
$\log[1 \mathbf{j} \theta]$				.30	.33
				(.98)	(1.1)
$\log(n + g + \delta)$					-.012
					(-.47)
No. obs	22	22	22	22	22
Adj. R <sup>2</sup>	.82	.88	.88	.88	.87

Notes: t-statistics are in parentheses. Each column estimates a version of our long specification under various restrictions.

$$\begin{aligned}[1/N] \log[e_{it}^c / e_{it_1 N}^c] = & \beta_0 + \beta_1 \log[e_{it_1 N}^c] + \beta_2 [g_A] + \\ & \beta_3 \log[s_i] + \beta_4 \log[1 \mathbf{j} \theta]_i + \beta_5 \log[(n + g + \delta)]_i + \mu_{it}. \end{aligned}$$

The dependent variable is the average growth rate of emissions per capita over the 1998-1960 period.  $e_{it_1 N}^c$  is emissions per capita in 1960,  $\mathbf{j} g_A$  is the country specific estimate for the rate of technological progress in abatement,  $s$  is the average investment to GDP ratio over the 1960-1998 period,  $[1 \mathbf{j} \theta]$  is one minus the ratio of pollution abatement costs to GDP also averaged over the period, and  $(n + g + \delta)$  is average population growth over the period plus .05.

In column (2) we add our measure of technological progress in abatement. This variable enters significantly and with the expected negative sign, although its coefficient estimate is far from the  $-1$  value predicted by theory. Given the method by which we constructed this regressor it is tempting to attribute this to attenuation bias. In column (3) we add the savings rate which enters positively as predicted by theory, but is not significant at conventional levels. In column (4) we add pollution abatement costs. The measure of pollution abatement costs enters positively as predicted by theory, and its magnitude is above that of savings also as predicted by theory. Our measure of abatement costs is however not statistically indistinguishable from zero. Finally for completeness in column (5) we add the final Solow regressor - effective depreciation. This regressor enters negatively as predicted, but likewise enters insignificantly. We can reject a joint F-test that our four added variables are jointly zero ( $F(4,16) = 2.82$ ) at the 6% level, or reject that our two new Green Solow variables ( $g_A$ , or  $\theta$ ) are jointly zero ( $F(2,16)=4.7$ ) at the 5% level.

Overall the results from Table 1 are encouraging. Even the simple specification explains over 80% of the variation in carbon per capita growth rates over 1960-1998. While the addition of technological progress in abatement adds something to the model's explanatory power, the remaining regressors lower the adjusted  $R^2$ . While the significance level of several key regressors is low, this may be due to the small sample size and poor data quality. Despite the lack of significance for several regressors, point estimates are in most cases reasonable and in line with those reported in related empirical work. For example, the implied rate of convergence  $\lambda$  varies from a high of 1.4% in column (1) to a low of .8% per year in column (5). And using our final regression in column (5) we find the implied share of capital in GDP is approximately .6. These two results, a slow rate of convergence and a too large capital share are of course the same as those reported by Barro (1991) from an estimation of the standard Solow model. Moreover, if we use the estimates from column (5) on savings, abatement and initial emissions per person, we can develop an estimate for  $\epsilon$  in the abatement technology. In theory  $\epsilon$  must exceed one. Its point estimate is approximately 35.

Underlying the estimates in Table 1 are of course our estimates of  $g_{i,A}$ , which we report in Table 2 together with their 95% confidence intervals.

Table 2: Technical Progress Estimates

Country	$\hat{g}_{i,A}$	95% C.I.
Australia	-.41	(-0.52, -0.31)
Austria	-1.66	(-1.89, -1.43)
Belgium	-2.95	(-3.21, -2.70)
Canada	-1.39	(-1.59, -1.18)
Denmark	-1.19	(-1.58, -.81)
Finland	-.23	(-.88, .41)
France	-3.01	(-3.34, -2.68)
Greece	2.23	(2.02, 2.47)
Iceland	-2.19	(-2.45, -1.93)
Ireland	-1.31	(-1.60, -1.03)
Italy	-.48	(-.90, -.07)
Japan	-1.35	(-1.67, -1.03)
Luxembourg	-4.72	(-5.10, -4.33)
Netherlands	-1.31	(-1.54, -1.08)
New Zealand	.57	(.36, .79)
Norway	-1.33	(-2.05, -.61)
Portugal	.88	(.64, 1.12)
Spain	-.04	(-.41, .32)
Sweden	-3.03	(-3.57, -2.50)
Switzerland	-.65	(-.97, -.33)
United Kingdom	-2.59	(-2.68, -2.49)
United States	-1.71	(-1.90, -1.52)

Note: t-statistics are in parentheses. For each country we estimate  $\log(E/Y)_{it} = \gamma_{i0} + \gamma_{i1} \log p_{oil,t}^w + \gamma_{i2} \hat{time} + \epsilon_{it}$  over the 1960-1998 period. The dependent variable is the log of emissions to GDP measured in US dollars. The estimates for  $\hat{g}_{i,A}$  above, are the coefficients  $\gamma_{i2}$ .

There are three features of the estimates worth noting. First, most but not all of the estimates are negative. The estimates for Greece, Portugal and New Zealand are positive and significantly so. And we cannot rule out a zero rate of technological progress in either Finland or Spain. Taking our model literally a positive estimate implies technological regress. A more reasonable interpretation is that factors other than oil prices and technological progress are driving the emissions to output ratio in some of these economies. Second, the estimates are quite precise which makes concerns over measurement error in our proxy for technological progress less of an issue. Third, if we ignore outliers, the estimates indicate a rate for  $g_A$  of perhaps 1.5 to 2% per year. If we couple this average estimate with average population growth of say 1% per year and per capita income growth of 2%, it is apparent that carbon emissions should not exhibit an EKC pattern. In terms of our theory, point T is to the right of point B.

With these empirical results in hand return to Figure 8. One explanation of the tight relationship shown in Figure 8 is that convergence in the Solow model is generating the result, although this would only literally be true if emissions per unit of output were constant over time. Our results in Table 2 suggest otherwise, but clearly the convergence properties of the Solow model are helping. A further contributing factor may be that unobserved cross country heterogeneity is playing less of a role here than it does in the typical Solow framework. Since a country with a unusually productive goods technology may also be one with an unusually productive abatement technology, unmeasured technological differences may to some extent be netting out in the wash. Finally, there is some evidence in Table 2 of a weak relationship between a country's development level and its estimated  $g_A$ . The three countries with positively estimated coefficients had low incomes in 1960. The positive estimates for these countries could reflect a strong compositional shift towards energy and hence carbon intensive manufactures at the earliest stage of development. If this is true, then countries with low incomes and little carbon per person will see emissions rise from both rapid growth via Solow, and a compositional shift towards heavy industry. Countries with high incomes and relatively high emissions per person will see lower growth and a shift away from heavy industry. As a consequence, the convergence predictions of the model may be reinforced by compositional shifts along the development path. Alternatively, relatively poor countries in 1960 may also be relatively slow at assimilating and implementing new abatement technologies. Under this scenario, low income countries countries would

see a large increase in carbon per capita because of relatively fast growth but relatively slow progress in adopting new abatement technologies. Together these four reasons - output convergence, compositional shifts, reduced heterogeneity and technology catch up- may explain the strength of our goodness of fit statistics and the tightness of the relationship depicted in Figure 8

Although the evidence for convergence in Figure 8 seems undeniable, it is well known that cross-sectional tests such as ours may indicate convergence while time series tests find no such evidence (See for eg. Bernard and Durlauf 1995). Fortunately, in a series of prescient papers John List and a series of coauthors<sup>20</sup> have explored the time series properties of several pollutants to examine convergence in pollution levels across both states and countries. In Stracizich and List (2003) the authors examine the convergence properties of CO2 over a panel of 21 industrial countries from 1960-1997. When the authors estimate a relationship equivalent to our short regression they find evidence very similar to our OECD regression in column (1). The authors then add a series of conditioning variables (temperature, energy prices, and the level and square of per capita income and population density ) to allow for conditional convergence but have little success. This is perhaps not surprising in light of our theory since these are not variables determining steady states in the Green Solow model. Stacizich and List supplement their cross-country regressions with a time series test of convergence using a panel unit root test. This time-series test also strongly supports convergence. The authors conclude there is significant evidence that CO2 emissions per capita have converged. Further work by Lee and List (2002) and Bulte et al. (2003) employ newer time series tests or examine new data sources. Overall, their results demonstrate that there is considerable evidence of convergence in pollution levels across both countries (for CO2) and across states (for both SO2 and NOx) although convergence may be stronger over the last 30 years.

---

<sup>20</sup>See List (1999), Lee and List (2002), Strazicich and List (2003), and Bulte, Strazicich and List (2003). This work is largely empirical arguing for a convergence specification by analogy with the Solow model. Bulte et al. (2003) contains theory that extends the Andreoni and Levinson (2001) model to a dynamic environment to derive a testable equation. The resulting derivation is however problematic. Equation (5) of Bulte et al (2003), which gives the balanced growth path level of pollution, produces negative pollution for finite  $t$  (when there are increasing returns to abatement which is their standard case and necessary to produce the EKC in the model). Pollution goes to negative infinity as time progresses. When there are constant returns to abatement the EKC is no longer a prediction and their equation (5) yields negative pollution levels for all  $t$  when  $\alpha > \beta$ . These problems seem to have arisen from mapping the strong increasing returns to abatement in the Andreoni and Levinson (2001) model into a model where investment in abatement rises lockstep with aggregate output.

Convergence is also apparent in the earlier work of Holtz-Eakin and Selden (1995). These authors examined whether carbon per capita followed an EKC. They found that the carbon EKC turned quite late, if at all, at income per capita levels ranging from 35,000 (1986 US Dollars) to above \$8 million per capita depending on the specification. A key finding was that the marginal propensity to emit (the change in emissions per person for a given change in income per capita) fell with income levels but that overall emissions were forecast to grow. These findings are consistent with the Green Solow model when  $g_A < n + g$ . Under these circumstances, convergence in emissions per person still obtains, but emissions still grow along the balanced growth path.

In total there is considerable evidence of convergence in measures of pollution emissions. What the Green Solow model offers to this body of work is a theoretical structure that links the strength of convergence to observable variables, makes explicit and testable connections between theory and empirical work, and offers a new method for learning about the growth and environment relationship.

## 4 Discussion and Extensions

We have presented a very simple theory linking growth rates, income levels and environmental quality. In doing so we have made a host of simplifying assumptions some of which may appear quite limiting. In this section we discuss these assumptions, provide further empirical evidence supporting our approach and develop methods for extending our results.

### 4.1 Sample Selection, Galton's Fallacy and $\sigma$ Convergence

Our empirical methods are closely related to those employed in the cross-country growth literature where variation in cross-country growth rates over some period of time are explained by initial income plus other controls. The cross-country growth literature is voluminous and controversial. It started with the work of Baumol (1986), was formalized and extended by the important contributions of Barro (1991) and Barro and Sala-i-Martin (1992), and it played an lead role in Mankiw, Romer and Weil (1992). Durlauf and Quah (1999) provide an excellent critical review of the empirical literature.

Since our methods are similar, some - but not all - of the criticisms of cross-country growth regressions are relevant to the estimates we provided here. The

earliest critique came from DeLong (1988) who argued that Baumol's (1986) original finding of convergence in productivity levels across 16 currently rich countries was the result of sample selection and measurement error. Measurement error was a potentially important econometric problem since the data spanned the 1870 -1979 period, with the quality of the 1870 data quite uncertain. As DeLong noted measurement error in the estimates of 1870 values worked towards the convergence finding. Since our data only spans the 1960-1998 period we think the measurement error issues discussed by DeLong are largely irrelevant here.

Sample selection may however be an issue, and DeLong argued that the 16 countries chosen by Baumol were ex post winners who undoubtedly differed in their productivity levels in earlier years; as a consequence the convergence finding was all but guaranteed. Sample selection could be an issue with our dataset. For example, the convergence we find and attribute to the interplay of diminishing returns and technological progress, could arise from convergence in environmental policies driven by income convergence across the rich OECD countries in our sample. While this is a possibility two pieces of evidence work in our favor. The first is that carbon emissions are largely unregulated and have been largely unregulated for many years. Therefore convergence in carbon emissions is unlikely to arise from forces causing convergence in environmental policies amongst the OECD.<sup>21</sup>

The second piece of evidence is shown in Figure 9 below. Here we follow DeLong's advice and extend our sample of rich OECD countries to include all other countries that as of 1960 were at least as well off as the poorest OECD member included (Portugal). Extending the sample in this way gives us a sample of 32 countries; ten of which do not appear in the high income OECD in 1998.<sup>22</sup> As shown by Figure 9, the strong convergence properties remain with the larger sample.

---

<sup>21</sup>While some carbon abatement occurs as a joint product of other abatement efforts, the time profiles for carbon and the set of highly regulated pollutants are very different.

<sup>22</sup>The additional countries are Venezuela, Uruguay, South Africa, Saudi Arabia, Puerto Rico, Israel, Hong Kong, Barbados, Bahamas and Argentina.

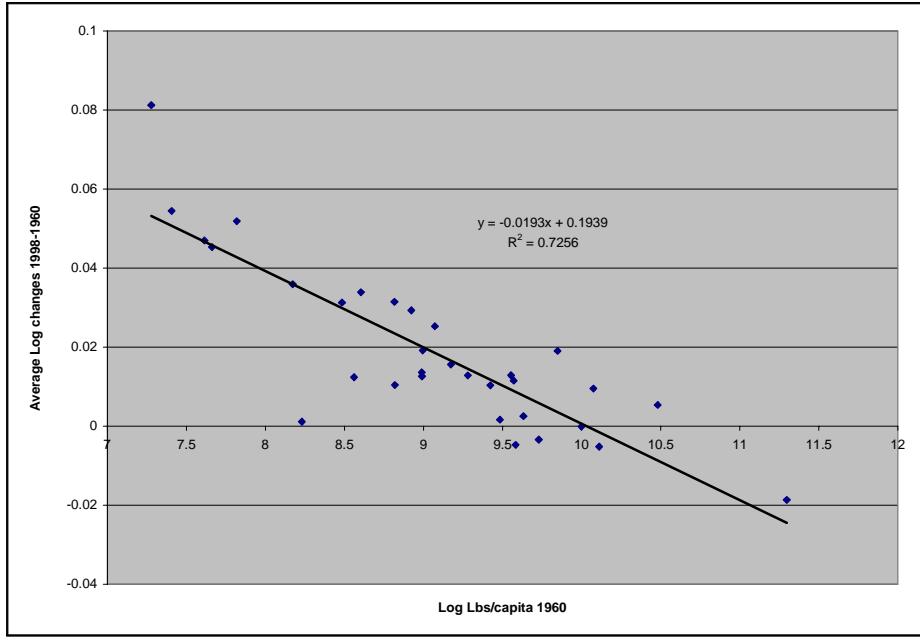


Figure 9: DeLong's Critique

But even if we take figure 9 at face value, it may in fact reveal nothing causal about convergence but instead be a manifestation of regression towards the mean. This critique, was initially put forward by Friedman (1992), and was developed more fully in a series of papers by Danny Quah (see especially Quah (1993)) and Durlauf and Quah (1999). The basic criticism (in terms of our variable emissions per person) is that if our cross-country observations on emissions per person were independent draws from a common and time invariant distribution, then countries having a high draw in 1960 are likely to have a lower draw in 1998. Countries with a low draw in 1960 are likely to have a higher draw in 1998. As a consequence a scatter plot of country emission growth rates against initial 1960 values will show a negative relationship but this " $\beta$  convergence result" is consistent with many stable and non-degenerate long run distributions for emissions per unit output. Friedman notes that if regression towards the mean was the only force operating then a scatter plot of growth rates against terminal 1998 values should show a strong positive relationship. To investigate we followed Friedman (1992) and plotted emission per capita growth rates against 1998 levels. The relationship is still strongly negative. To go further we followed both Quah (1993) and Friedman (1992) and investigated other moments of the distribution. We examined the time profiles for log emissions per

person and calculated the point in time variances across the sample. These two exercises showed a great deal of convergence in the distribution.

In total, these additional checks make us reasonably confident that the convergence we find in the data is not due to sample selection, measurement error, or regression towards the mean. The concerns of Durlauf and Quah (1999) and Durlauf and Johnson (1995) regarding convergence across a very heterogenous worldwide sample are largely moot here given our select sample of countries. The additional issues raised by Durlauf and Quah (1999) regarding the interpretation of cross country growth results as tests of new versus old growth theory, the treatment of endogenous regressors, the fragility of estimates, and the addition of ad hoc regressors to proxy for the free parameters of the production function would of course be relevant to any extension of our work.

## 4.2 International Evidence

The starting point for our analysis was three observations drawn from U.S. data: emissions per unit of output have been falling for lengthy periods of time; these reductions predate reductions in the absolute level of emissions; and abatement costs are a relatively small share of overall economic activity. In Table 3 we present the available evidence from European countries on these same three statistics for four of the pollutants we considered in Figure 1.

We focus on European evidence because of data limitations. The table presents summary statistics for the average yearly percentage change in emissions per unit GDP over the 1980-2001 period. As well, where possible, the table indicates when aggregate emissions peaked but in many cases this is prior to the start of the sample as indicated by the entry "< 1980". In the last column we list the country averages for pollution abatement costs as a fraction of GDP over the 1990-2000 period for these same countries.

There are two remarkable features of the data. The first is the massive reduction in emissions per unit of output over the period. These reductions are on the order of 4-5% per year for nitrogen oxides, carbon monoxide, and volatile organic compounds but closer to 10% per year for sulfur. The second is, of course, the relatively small pollution control costs shown in the last column. On average these costs are only between 1 and 2% of GDP.

Table 3: International Evidence

Countries	NOx	Peak	SOx	Peak	CO	Peak	VOC	Peak	$\theta$ Share
Austria	-2.8	<1980	-13.4	<1980	-5.5	<1980	-4.2	1990	1.6
Finland	-3.8	1990	-11.6	<1980	-2.9	<1980	-3.8	1990	1.4
Czech Rep.	-7.6	<1980	-18.6	1985	-4.8	1990	-6.5	1990	2.0
France	-3.8	<1980	-10.0	<1980	-6.4	<1980	-4.2	1985	1.2
Germany	-5.4	<1980	-3.1	<1980	-7.0	<1980	-2.6	1985	1.6
Italy	-2.7	1990	-9.5	<1980	-3.7	1990	-3.8	1995	.9
Ireland	-2.7	2000	-7.8	<1980	-7.0	1990	-6.3	1990	.6
Poland	-7.5	1985	-9.9	1985	-10.1	1990	-6.6	<1980	1.6
Slovak Rep.	-4.7	1990	-10.0	<1980	-4.2	1990	-7.5	1985	1.5
Sweden	-4.2	1985	-12.1	<1980	-3.4	1990	-5.1	1985	1.0
Switzerland	-4.4	1985	-9.5	<1980	-6.9	<1980	-5.1	1985	2.1
Netherlands	-4.1	1985	-10.6	<1980	-6.5	<1980	-6.1	<1980	1.7
Hungary	-3.0	<1980	-7.7	<1980	-3.7	<1980	-2.3	1985	.6
Portugal	1.0	2000	-2.5	1999	-3.4	1995	1.1	1997	.6
U.K.	-4.5	<1980	-9.4	<1980	-5.9	<1980	-4.9	1990	1.5
Average	-4.0	n.a.	-9.7	n.a.	-5.4	n.a.	-4.5	n.a.	1.3

Notes: Data on particulates is unavailable. Table 3 is constructed using three data sources. Data on European pollution emissions comes from the monitoring agency for LRTRAP available at <http://www.emep.int/>. Real GDP data is taken from the World Bank's Development Indicators 2002 on CD Rom. Pollution abatement costs are taken from the OECD publication "Pollution abatement and control expenditures in OECD countries", Paris: OECD Secretariat, See the data appendix for details.

The table also gives, where possible, the peak year for emission levels. In many cases these peaks occur before 1980, and with the exception of Portugal and one pollutant for Ireland, the remaining peaks in emissions occurred in the 1980s or early 1990s. Since emissions are now declining for these pollutants and countries, this European data offers strong confirmation that each country pollutant pair exhibits a time profile for emissions roughly consistent with an EKC.<sup>23</sup> We have of course argued that the first two features of the data imply a large role for technological progress in abatement, but rapid technological progress in abatement, when coupled with the convergence properties of the Solow model, produce the third feature of the data.

### 4.3 What is $g_A$ ?

Aggregate models of economic activity make heroic assumptions to bring into sharp focus relationships that may otherwise be obscured. Our analysis is no different. Our use of an aggregate measure for technological progress in both abatement and goods production surely hides many processes at work in the economy. Changes in the composition of national output and private consumption, fuel mix changes, and changes in factor quality over time are all partly responsible for the time profile of emissions to GDP that we have observed over the last fifty years. And many sorts of changes, including regulatory ones, lie behind what we have called technological progress in abatement. But whether this is a good or misleading way to think about the growth and environment relationship does not rest on whether this characterization is literally true, but whether it helps us identify a key force at work.

Our review of current empirical evidence suggests a key role for technological progress in abatement. We note that most if not all EKC studies find a strong and persistent time effect driving emissions downward. These time effects are not small and reduce emissions by significant amounts each year.<sup>24</sup> More direct evidence is contained in studies that decompose the change in pollution

---

<sup>23</sup>Because, in the words of Andreoni and Levinson, what is now coming down must have first gone up.

<sup>24</sup>The interested reader should take his or her favourite EKC study and conduct the following experiment. Calculate the number of years it would take for an average developing country to move from low income to high income status if growth in per capita income were rapid - say 5% per year. Calculate what the implied income gain would mean in terms of reduced emissions/concentrations. Then calculate using the coefficient estimate on time in the same EKC regression the implied reduction in emissions/concentrations that would occur via time related effects over this same interval. Compare the magnitude of these two changes. In the cases we have investigated, time related effects are often ten times larger than the changes created by income growth.

emissions into scale, composition and technique effects. For example, Selden et al. (1999) provide a decomposition of the change in US air pollution emissions over the 1970 to 1990 period. Using data on 6 criteria pollutants they divide the change in emissions into scale, composition and three types of technique effects. For all six criteria pollutants (lead, sulfur dioxide, nitrogen oxide, volatile organic compounds, carbon monoxide and particulates) reductions in emissions per unit fuel combusted, and reductions in emissions per unit output were key in driving emissions downwards. While changes in the composition of output lowered emissions for some pollutants, it raised them for volatile organic compounds and carbon monoxide. And while changes in energy intensity and the mix of energy sources helped lower the emissions of some pollutants, for all pollutants studied emissions would have risen in the absence of the change in techniques discussed above.

Related work by Bruvoll and Medin (2003) find similar results using Norwegian data. They report that their analysis reveals that "air pollution has not followed the pace of economic growth. This is mainly due to new technologies"; and that "changes in production structure or composition of energy types have been of less importance to the development of energy related emissions", p. 42.

Overall these results suggest to us that a large component of the change in the emissions to output ratio must be technological progress. Composition changes have not been large enough, technology or technique effects have been found to be key, and abatement costs are just too small to be largely responsible for the large reductions in emissions per unit of output experienced.

#### 4.4 Optimization and Functional Forms

To what extent are our assumptions of fixed abatement or savings rates required for our results? It is well known that allowing for optimal consumption complicates but does not reverse the Solow model's convergence properties. With optimal consumption the savings rate now varies over the transition path and this may hasten or delay the speed of adjustment to the steady state. One concern may be that optimal abatement could shift over the transition path in such a way as to rule out the EKC profile we derived. To investigate note that if the optimal  $k$  is bounded, then emissions have to fall in the long run if  $G_E < 0$ . Therefore, falling emissions and rising environmental quality are guaranteed. In the short run emissions will at first rise as long as the abatement is not initially too aggressive relative to the pace of growth. Many different assumptions will

generate this result. In our fixed rule case we generated this result by assuming the initial capital stock  $k(0)$  was less than  $k^T$ . The small initial capital ensured growth was rapid and this overwhelmed technological progress in abatement. In an optimizing framework similar forces are at work but we need to add assumptions on abatement (to ensure its growth is not too rapid), consumption (to make sure output growth is rapid enough initially) and marginal damage (to control the planner's response to rising pollution).

One method is to assume marginal damage from emissions is not too high when  $k(0)$  is small so that abatement is not undertaken initially. For example we could assume the marginal product of abatement is bounded above and the damage from pollution convex in emissions as in Stokey (1998) or Brock and Taylor (2003a). In this situation the first unit of emissions has zero marginal damage while abatement has a finite cost (determined by the shadow value of capital). These authors show emissions at first rise only to be offset by abatement or a combination of abatement plus technological progress in the future.

Alternatively, we could let the abatement function satisfy Inada type conditions so that abatement is always undertaken, but then adopt assumptions to ensure that sacrifices in consumption are not too costly (so growth is initially rapid) and pollution not too damaging (to make policy responses weak). In short many sets of reasonable assumptions on abatement and utility will generate the result that pollution can at first rise with growth. Assuming sufficiently strong technological progress in abatement ensures that pollution will fall eventually.

A final concern of readers may be our use of a Cobb-Douglas aggregator for output. Although this functional form is commonly used in growth theory and elsewhere, it is important to understand its limitations. Its benefit to us outweighed its costs because it allowed us to derive simple closed form solutions for quantities of interest. In many cases our results carry through although they are more difficult to prove and less transparent to the reader. For example, our key result that an EKC relationship arises from the interplay of diminishing returns and technological progress in abatement remains true. To verify consider the dynamics of environmental quality  $X$  and income per capita with a general intensive production function  $f(k)$ . Assume  $f(k)$  satisfied the Inada conditions, and write the dynamic system for  $k$  and  $X$  as:

$$\overset{2}{k} = sf(k)[1 - \theta] + [\delta + n + g]k \quad (26)$$

$$\overset{2}{X} = c_0 \exp[G_E t] f(k) + \eta X \quad (27)$$

where  $c_0 > 0$  and  $G_E < 0$ . To show the environment must at first worsen evaluate 27 at  $t = 0$ . At  $t = 0$  the environment is initially pristine,  $X(0) = 0$ , and the initial capital stock is not zero,  $k(0) > 0$ . Equation 27 shows the environment must at first worsen.  $X$  has to be growing at least initially. To examine the rest of the transition path recall  $k(t)$  is increasing in time until it reaches  $k^*$ . We can use this fact to bound the path for  $X$  noting:

$$\overset{2}{X} = c_0 \exp[G_E t] f(k) + \eta X < \overset{2}{X} = c_0 \exp[G_E t] f(k^*) + \eta X \quad (28)$$

For any  $t \geq 0$ ,  $\overset{2}{X}(t)$  must be below the solution to the ordinary differential equation  $\overset{2}{X} = c_0 \exp[G_E t] f(k^*) + \eta X$ ,  $X(0) = 0$ . This ordinary differential equation has a closed form solution showing  $X(t)$  tends to zero as  $t$  goes to infinity. Using the inequality in 28 we conclude  $X$  must at first rise, but then fall as before. Proving a similar result for  $E(t)$  is left to the reader. The EKC prediction of the model is not limited to our Cobb-Douglas formulation.

## 5 Conclusion

This paper presented a simple growth and pollution model to investigate the relationship between economic growth and environmental outcomes. A recent and very influential line of research centered on the empirical finding of an EKC has, for the last ten years, dominated the way that economists and policymakers think about the growth and environment interaction. Numerous empirical researchers have sought to validate or contradict the original EKC findings by Grossman and Krueger (1994, 1995), while theorists have contributed to this explosion of research by presenting a myriad of possible explanations for the empirical result.

This paper makes three contributions to this line of enquiry. First and foremost it suggests that the most important empirical regularity found in the environment literature - the EKC - and the most influential model employed in the macro literature - the Solow model - are intimately related. While one hesitates to see "Solow everywhere", we have argued that the forces of dimin-

ishing returns and technological progress identified by Solow as fundamental to the growth process, may also be fundamental to the EKC finding.

In support of our argument we marshalled several pieces of evidence. We presented evidence that the U.S. emission to output ratio has fallen at a roughly constant rate for almost 50 years; that this reduction predates the peak of emissions; and that abatement expenditures while growing since the early 1970s have remained a fairly constant fraction of economic activity. These data are in many cases inconsistent with current explanations for the EKC.

Our second contribution was to develop a simple extension of the Solow model where the interplay of diminishing returns to capital formation and technological progress in abatement produced a time profile for emissions, abatement costs, and emissions to output ratios that are in accord with U.S. data. We also argued that this model could provide a natural explanation for the sometimes confusing and heterogenous results found in the empirical literature.

Finally we developed an empirical methodology that flowed very naturally from our model. By exploiting known results in the macro literature we developed a simple estimating equation predicting convergence in emissions per capita across countries. The model produced several testable restrictions and led to the estimation of key parameters. While we view our empirical work as preliminary, it lends further support to our view that the same ongoing dynamic processes responsible for income growth and convergence are also at play in determining the EKC finding and emission convergence. The evidence for convergence is quite strong, and well in accord with the theoretical predictions of the Green Solow Model.

The very simplicity of our model calls out for future work to qualify, elaborate or perhaps refute our thesis. The model is singularly successful in identifying technological progress in abatement as a potential key to much of the income and pollution data, but no theory of innovation nor optimal regulation was provided. Our formulation is consistent with a world where governments gradually tighten emission standards over time, but we can only speculate as to whether this gradual march forward in regulation is caused by income growth and whether gradually tightening standards are the impetus for ongoing technical improvements in abatement. Our empirical work, as we state, is preliminary and our method of estimating technological progress in abatement crude. Moving forward on both empirical and theoretical fronts is surely a worthy goal for future research.

## 6 Appendix

### 6.1 Constant costs and rising standards

It proves useful to adopt a specific abatement production function and consider the firm's problem. To that end, let the intensive abatement production function be given by  $a(\theta) = [1 + \theta]^\varepsilon$  where  $\varepsilon > 1$ . Assume the government imposes a technology restriction requiring emissions per unit of output not exceed  $\mu(t)$ ; that is,  $E(t)/Y(t) \leq \mu(t)$ . If firm's rent capital at rate  $r$ , hire labor at the wage  $w$ , and face the technology standard  $\mu(t)$ , then the firm's problem becomes one of maximizing profits by choice of labor, capital and abatement inputs

$$\underset{fk, l, \theta}{\text{Max}} \Pi = Y + wL + rK \quad (29)$$

$$s.t. Y = (1 + \theta)F(K, BL) \quad (30)$$

$$E = \Omega[1 + \theta]^\varepsilon F, \quad (31)$$

$$E/Y \leq \mu \quad (32)$$

Since abatement is costly in terms of foregone output firms will only just meet the technological standard. Using this information we can substitute the constraints into the objective to rewrite the firm's problem as simply

$$\underset{fk, l, g}{\text{Max}} \Pi = (\mu/\Omega)^{1/(\varepsilon - 1)} F(K, BL) + wL + rK$$

which is just a straightforward problem of input choice. The firm's allocation of labor and capital to abatement is determined by the technological standard. Algebra shows

$$(\mu/\Omega) = (1 + \theta)^{(\varepsilon - 1)} \quad (33)$$

which solves for the intensity of abatement implicitly. Note if  $\mu = \Omega$ , no abatement is necessary and  $\theta = 0$ , but as standards tighten  $\mu$  falls and  $\theta$  must rise. Suppose the technology standard is tightened slowly over time. Differentiating with respect to time we obtain:

$$\frac{\frac{d\mu}{dt}}{\mu} + g_A = (\varepsilon - 1) \frac{d}{dt} \ln(1 + \theta)$$

which indicates that the intensity of abatement rises or falls over time as technological progress outstrips or falls behind the steady march of rising technology standards. If the technology standard becomes tighter over time at rate  $g_A$  then cost minimizing firms meet the ever tightening standard by allocating the constant fraction  $\theta$  of their inputs to abatement. Therefore our fixed  $\theta$  corresponds to a world where imperfect governments have been raising emission standards slowly over time while firms have been minimizing costs in meeting them. Despite the rising standards, technological progress in abatement has kept pollution control costs roughly constant as a fraction of overall activity.

## 6.2 Proofs to Propositions

**Proposition 1.** If growth is sustainable and  $k^T > k(0)$ , then the growth rate of emissions is at first positive but turns negative in finite time. If growth is sustainable and  $k(0) > k^T$ , then the growth rate of emissions growth is negative for all  $t$ . If growth is unsustainable, then emissions growth declines with time but remains positive for all  $t$ .

**Proof:** From 10 and 5 the growth rate of emissions is declining in  $k$ . By definition emissions growth is zero at  $k^T$ . Therefore, if  $k^T > k(0)$  growth is positive but declines with  $k$ ; if  $k^T < k(0)$  growth is negative and declines with  $k$ . When growth is sustainable  $k^T < k^a$ . The solution for  $k(t)$  in 14 shows  $k^T$  is reached in finite time from  $k(0) < k^T$ . If growth is not sustainable,  $k^T > k^a$ . The solution for  $k(t)$  shows it converges to  $k^a$  as time goes to infinity. This implies  $k^T > k^a$  always, and by definition of  $k^T$  emission growth remains positive.

**Proposition 2.** Proof in the text

**Proposition 3.** There exists a parametric relationship between emissions  $E$  and income per capita  $y^c$  that we refer to as an EKC. If  $k^a > k^T > k(0)$ , then emissions first rise and then fall with income per capita. If  $k^a > k(0) > k^T$ , then emissions fall monotonically with income per capita.

**Proof:** We note from the text that

$$\begin{aligned} E(t) &= c_0 \exp[G_E t] - k^a (1 + \exp[i \lambda t]) + k(0)^{1-i} \exp[i \lambda t]^{i \alpha / (1-i \alpha)} \\ y^c(t) &= k(t)^\alpha B(0) [1 + \theta] \exp[gt] \end{aligned}$$

We have already shown that for any  $k(0) < k^a$ ,  $k(t)$  is increasing in time. Given the properties of  $\exp$  we can then conclude that  $y^c(t) = [1 + \theta] k(t)^\alpha B(0) \exp[gt]$

is strictly increasing in time when the conditions of the proposition are met. This allows us to invert and obtain  $t = \varphi(y^c)$  where  $\varphi^0 > 0$ . Substitute for time in  $E(t)$ . Now differentiate this parametric function  $E(\varphi(y^c))$  with respect to  $y^c$  to obtain  $E^0(\varphi(y^c))\varphi^0(y^c)$ . Note that if  $k^s > k^T > k(0)$  then  $E^0(\varphi(y^c))$  is positive for  $t < T$ , zero at  $t = T$ , and negative for  $t > T$ .  $\varphi^0(y^c)$  is always strictly positive and hence emissions at first rise and then fall with income per capita. If  $k^s > k(0) > k^T$ , then  $E^0(\varphi(y^c))$  is always negative. This implies  $E^0(\varphi(y^c))\varphi^0(y^c)$  is always negative as required.

### 6.3 Derivation of Estimating Equation

Start with the capital accumulation equation in 5 and use the following log linearization

$$\frac{\overset{z}{k}}{k} = sk^{\alpha i - 1}[1 + \theta] \downarrow [\delta + n + g] \quad (34)$$

$$\cong s[k^s]^{\alpha i - 1}[1 + \theta][1 + (\alpha - 1)\overset{\wedge}{k}] \downarrow [\delta + n + g] \quad (35)$$

$$\overset{\wedge}{k} = \log k(t) \downarrow \log k^s \quad (36)$$

Rearrange and use the definition of  $k^s$  and  $\lambda$  to obtain the simpler form

$$\frac{\overset{z}{k}}{k} = i(1 + \alpha)[n + g + \delta]\overset{\wedge}{k} = i\lambda\overset{\wedge}{k} \quad (37)$$

where we have used the fact that  $\overset{z}{k}$  is zero at  $k^s$ . Note that  $\overset{z}{k} = \log[k(t)/k^s]$  and the left hand side of 37 is just the time derivative of  $\log k(t)$ . To change this differential equation in  $\log k(t)$  to one over  $y(t)$  use  $y = (1 + \theta)k^\alpha$  and then rewrite it out more completely as

$$\frac{d}{dt}[\log y(t)] = i\lambda \log y(t) + \lambda \log y^s \quad (38)$$

This equation is easily solved to find

$$\log y(t) = \log y(0)e^{i\lambda t} + \log y^s[1 + e^{i\lambda t}] \quad (39)$$

where  $y(0)$  is income per effective worker at  $t = 0$ . Evaluate 39 at  $T$  and  $T \downarrow N$ . Note  $T \downarrow N$  is our initial period and corresponds to  $t = 0$ , since  $y(0)$

is period  $T \in N$  income per effective worker. Doing so we obtain

$$\log y(T) + \log y(T \in N) = \log[y(T \in N)/y^a][1 + e^{i \lambda N}] \quad (40)$$

Note that income per effective worker is related to income per worker by  $y(t) = y^c(t)/B(t)$  where  $B(t)$  is the index of labor augmenting technological progress. Making these substitutions leads to

$$\log y^c(T) + \log y^c(T \in N) = Ng + \log[y^c(T \in N)/B(T \in N)y^a][1 + e^{i \lambda N}] \quad (41)$$

Divide both sides by  $N$  to obtain the average log changes over the period and rearrange slightly to obtain

$$\frac{\log[y^c(T)/y^c(T \in N)]}{N} = g + \log B(T \in N) \frac{[1 + e^{i \lambda N}]}{N} + \log[y^a] \frac{[1 + e^{i \lambda N}]}{N} - \log[y^c(T \in N)] \frac{[1 + e^{i \lambda N}]}{N}$$

which is reported in 22 where the constant  $b$  represents the first three terms in brackets.

## 7 Data

We have obtained our data from several sources. Data on carbon emissions, carbon per capita, carbon per dollar GDP, population size, and investment as a share of GDP was obtained from the World Bank Development Indicators 2002 available on CD-ROM. The data in Figure 7 is drawn from this source. The countries that appear in Figure 7 are: Afghanistan, Albania, Algeria, Angola, Antigua and Barbuda, Argentina, Australia, Austria, Bahamas, Bahrain, Barbados, Belgium, Belize, Benin, Bolivia, Brazil, Brunei, Bulgaria, Burkina Faso, Cambodia, Cameroon, Canada, Cape Verde, Chad, Chile, China, Colombia, Congo Dem. Rep., Congo Rep., Costa Rica, Cote d'Ivoire, Cuba, Cyprus, Denmark, Djibouti, Dominica, Dominican Republic, Ecuador, Egypt, Arab Rep., El Salvador, Equatorial Guinea, Ethiopia, Fiji, Finland, France, French Polynesia, Gabon, Gambia, Ghana, Greece, Greenland, Grenada, Guam, Guatemala, Guinea, Guinea-Bissau, Guyana, Haiti, Honduras, Hong Kong, China, Hungary, Iceland, India, Indonesia, Iran, Islamic Rep., Iraq, Ireland, Israel, Italy, Jamaica, Japan, Jordan, Kenya, Korea, Dem. Rep., Korea, Rep., Kuwait, Lao PDR, Lebanon, Liberia, Libya, Luxembourg, Macao, China, Madagascar, Mali, Malta, Mauritania, Mauritius, Mexico, Mongolia, Morocco, Mozambique, Myanmar, Nepal, Netherlands, New Caledonia, New Zealand, Nicaragua, Niger, Nigeria, Norway, Papua New Guinea, Paraguay, Peru, Philippines, Poland, Portugal, Puerto Rico, Qatar, Romania, Samoa, Sao Tome and Principe, Saudi Arabia, Senegal, Sierra Leone, Singapore, Solomon Islands, South Africa, Spain, Sri Lanka, St. Lucia, St. Vincent and the Grenadines, Sudan, Suriname, Sweden, Switzerland, Syrian Arab Republic, Thailand, Togo, Tonga, Trinidad and Tobago, Tunisia, Turkey, Uganda, United Arab Emirates, United Kingdom, United States, Uruguay, Venezuela, RB, Virgin Islands (U.S.).

Data on US emissions of the criteria pollutants graphed in Figures 1 and 2 come from the US E.P.A. The long series of historical data presented in the figures is taken from the EPA's 1998 report National Pollution Emission Trends, available at <http://www.epa.gov/ttn/chief/trends/trends.98>

Data on European pollution emissions given in Table 3 comes from the monitoring agency for LRTRAP available at <http://www.emep.int/>.

Data on pollution abatement costs came from the 1996 and 2003 OECD publication Pollution Abatement and Control Expenditures in OECD Countries, Paris: OECD Secretariat. Since this data is difficult to get we have given the exact method of construction and the data used in the table below.

OECD Pollution Abatement Cost Estimates and Sources				
Country	$\theta$	Group	Year	OECD Source
Netherlands	1.7	P&P	1985, 1987, 1989-1992	1996
Japan	1.1	P&P	1985-1990	1996
Italy	.9	P&P	1989	1996
Ireland	.6	P&P	1998	2003
Greece	.5	Pub.	1985-1991	1996
France	1.2	P&P	1985-1992	1996
U.S.A.	1.7	P&P	1985-1992	1996
Luxembourg	.4	Pub.	1997	2003
U.K.	1.5	P&P	1990	1996
Switzerland	2.1	P&P	1992	1996
Sweden	1.2	P&P	1991	1996
Spain	.5	Pub.	1987-1991	1996
Portugal	.6	P&P	1988-1991	1996
Norway	1.2	P&P	1990	1996
New Zealand	.9	P&P	1990	1996
Finland	1.4	P&P	1992	1996
Denmark	.6	Pub.	1985-1991	1996
Canada	.9	P&P	1989	1996
Belgium	1.4	P&P	1996-2000	2003
Austria	1.6	P&P	1985,1987,1988, 1990-1991	1996
Australia	.9	P&P	1991	1996
Iceland	.3	Pub.	1985-1992	1996

Notes: P&P refers to both public and private expenditures. OECD source refers to whether the figures come from the 1996 or 2003 OECD study. In constructing these data two rules were followed. First we relied on the 1996 study as it had the longest time series and the time frame fit closer to the middle of our sample period. Second, in some cases data was not available in the 1996 study. In these cases, we then used the 2003 study. This was true for example for Belgium, Luxembourg, and Ireland. Third, we used the most inclusive measure reported. Public and Private is more inclusive than just private, although the definitions of public and private differ across countries. Finally, in some cases we calculated the figures ourselves. New Zealand was given the same ratio of expenditures as Australia, and Luxembourg's ratio was calculated by hand using numbers from the OECD (2003) publication plus data on real GDP in 1997.

## 8 References

- Aghion, P. and P. Howitt (1998), Endogenous Growth Theory, MIT Press, Cambridge, MA.
- Andreoni, J. and A. Levinson (2001), "The Simple Analytics of the Environmental Kuznets Curve", *Journal of Public Economics*, 80:269-86.
- Barbier, E.B. (1997), "Introduction to the environmental Kuznets curve special issue", *Environment and Development Economics* 2, 369-381.
- Barro, R.J. (1991), "Economic Growth in a Cross Section of Countries", *Quarterly Journal of Economics*, 106, 407-443.
- Barro, R.J. and X. Sala-i-Martin (1992), "Convergence across States and Regions", *Brookings Papers on Economic Activity* 1, 107-182.
- Baumol, W. (1986), "Productivity Growth, Convergence and Welfare", *American Economic Review*, Dec. 76, 1072-85.
- Bernard, A.B. and S.N. Durlauf (1995), "Convergence in International Output", *Journal of Applied Econometrics*, Vol. 10, 97-108.
- Bovenberg, A.L. and S. Smulders (1995), "Environmental Quality and pollution augmenting technological change in a two sector endogenous growth model", *Journal of Public Economics*, 57, 369-391.
- Brock, W.A. and M. Scott Taylor, (2003a), "The Kindergarten Rule of Sustainable Growth", NBER working paper no. 9597, Cambridge, MA. Revised and retitled as "Economic Growth and the Environment: Matching the Stylized Facts", University of Wisconsin, Social Science Research Institute working paper No. 2003-16 (August).
- Brock, W.A. and M. S. Taylor (2003b), "Economic Growth and the Environment: A Review of Theory and Empirics", prepared for the Handbook of Economic Growth, S. Durlauf and P. Aghion eds. North Holland.
- Bruvoll, A. H. Medin (2003), "Factors behind the environmental Kuznets curve: a decomposition of the changes in air pollution", *Environmental and Resource Economics*, 24, 27-48.
- Bulte, E., J.A. List and M.C. Strazicich (2003), Regulatory Federalism and the Distribution of Air Pollutant Emissions", Working paper University of Maryland.
- Copeland, B.R. and M.S. Taylor (1994), "North-South Trade and the Global Environment", *Quarterly Journal of Economics*, 109:755-87.
- Copeland, B.R. and M. S. Taylor (2004), "Trade, Growth and the Environment", *Journal of Economic Literature*, March, 7-71.

- Copeland, B.R. and M. S. Taylor (2003), *Trade and the Environment: Theory and Evidence*, Princeton University Press, Princeton, NJ.,
- DeLong, J.B., (1988), "Productivity Growth, Convergence, and Welfare: Comment", *American Economic Review*, Vol. 78, No. 5, 1138-1154.
- Dewey, S.H. (2000), *Don't Breathe the Air: Air Pollution and U.S. Environmental Politics: 1945-1970*, No. 16, *Environmental History Series*, D.L. Flores, editor, Texas A&M publisher, College Station, TX.
- Durlauf, S.N. and P.A. Johnson, (1995), "Multiple Regimes and Cross-Country Growth Behaviour", *Journal of Applied Econometrics*, Vol. 10, 365-384.
- Durlauf, S. and D. Quah, (1999), "The New Empirics of Economic Growth", *Handbook of Macroeconomics*, North Holland, J. Taylor and M. Woodford eds.
- Friedman, M. (1992), "Do Old Fallacies Ever Die?", *Journal of Economic Literature*, Vol. XXX, Dec. 2129-2132.
- Grossman, G.M. and A.B. Krueger (1994), "Environmental Impacts of a North American Free Trade Agreement" in P. Garber (Editor) *The US-Mexico Free Trade Agreement*, MIT Press, Cambridge MA.
- Grossman, G.M. and A.B. Krueger (1995), "Economic growth and the environment", *Quarterly Journal of Economics*, 110, 353-377.
- Harbaugh, W., A.M. Levinson, and D.M. Wilson (2002), "Reexamining the empirical evidence for an environmental Kuznets curve", *Review of Economics and Statistics*, 84, 541-551.
- Holtz-Eakin, D. and T.M. Selden (1995), "Stoking the fires? CO<sub>2</sub> emissions and economic growth", *Journal of Public Economics*, 57, 85-101.
- Lee, J. and J.A. List (2002), "Examining Trends of Criteria Air Pollutants: Are the effects of government intervention transitory", mimeo June.
- List, J.A. (1999), "Have Air Pollutant Emissions converged amongst US Regions? Evidence from Unit root Tests", *Southern Economic Journal* 66, 144-155.
- List, J.A. and C.A. Gallet, (1999), "The environmental Kuznets curve: does one size fit all?", *Ecological Economics*, 31, 409-423.
- Lopez, R. (1994). "The Environment as a Factor of Production: the effects of Economic growth and trade liberalization", *Journal of Environmental Economics and Management* 27:163-84.
- Mankiw, N.G., D. Romer and D.N. Weil (1992), "A Contribution to the Empirics of Economic Growth", *Quarterly Journal of Economics* 107, 407-438.

- Quah, D. (1993), "Galton's Fallacy and Tests of the Convergence Hypothesis", Scandinavian Journal of Economics, 95, 427-443.
- Selden, T.M., A.S. Forrest, J.E. Lockhart (1999), "Analyzing reductions in U.S. air pollution emissions: 1970 to 1990", Land Economics, 5, 1-21.
- Solow, R.M. (1956) "A Contribution to the Theory of Economic Growth", Quarterly Journal of Economics, LXX, 65-94.
- Stern, D.I., (2003), "The Rise and Fall of the Environmental Kuznets Curve", Rensselaer Polytechnic Institute, Working Paper No. 0302, October.
- Stern, D.I. and M.S. Common (2001), "Is there an environmental Kuznets curve for sulfur?" Journal of Environmental Economics and Management, 41, 162-178.
- Stokey, N., (1998), "Are there Limits to Growth", International Economic Review, vol. 39, No. 1, 1-31.
- Strazicich, M. C. and John List (2003), "Are CO<sub>2</sub> Emission levels converging among Industrial countries", Environmental and Development Economics, 24, 263-271, 2003.
- Xepapadeas, A. (2003), "Economic Growth and the Environment", in preparation for the Handbook of Environmental Economics, J. Vincent and Karl Goran Maler eds.
- Vogan, C.R. (1996) "Pollution Abatement and Control Expenditures: 1972-94", Survey of Current Business, Sept. 48-67.