Through the Wormhole: Using Ray Tracing to Visualize the Wormholes of Interstellar

by **Quinn White, Chase Hanson, and Lily Whitler**

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Problem

Using the equations from Einstein's theory of general relativity, we will be attempting to recreate the wormhole visuals found in the film *Interstellar*. To do this, we will be following the work done in a paper by physicist Kipp Thorne and his collaborators at the VFX studio *Double Negative*, where they describe how they built the visuals for the wormhole in the film. The necessary math can be summarized as needing to know the wormhole metric and the solutions to the geodesic equation. First, the metric for the wormhole we will use is written as[1]

$$ds^2 = -(1 + 2\Phi)dt^2 + d\ell^2 + r^2(d\theta^2 + \sin^2\phi) \tag{1}$$

The metric is crucial to general relativity, as it is what describes the geometry of our space. Following the metric, we turn to the geodesic equation.

$$\frac{d^2x^\alpha}{d\zeta^2} + \Gamma^\alpha_{\mu\nu}\frac{dx^\mu}{d\zeta}\frac{dx^\nu}{d\zeta} = 0 \tag{2}$$

For purposes of numerical calculations, we will use a rewritten version of this equation known as a superhamiltonian

$$H = \frac{1}{2}\left[-p_t^2 + p_\ell^2\frac{p\theta^2}{r(\ell)^2} + \frac{p_\phi^2}{r(\ell)^2\sin^2\theta}\right] \tag{3}$$

The superhamiltonian can be split into 5 separate differential equations that we will need to solve in order to describe the quantities necessary to recreate our desired image.

Approach

In order to reproduce visuals similar to those in the film, we will be required to create a ray tracing program in python. This will essentially allow us to follow the path of a light beam as it moves through curved space to our placed "camera". It is here that the solutions to the geodesic equation come in. Using an integrator (likely RK4), we will solve these 5 equations for each ray, which will allow us to describe their movement through time and recreate our desired image. By using parameters provided in the *Interstellar* paper, as well as images and material from the *Double Negative website*, we can recreate the images from the film using our engine.

Objectives

1.Successfully build a ray tracing engine that that returns expected visuals
2.Implement all necessary math to begin our first imaging
3.Recreate the Einstein rings described in the *Interstellar* paper
4.Do animated passes around and through the wormhole

[1] All variable definitions and other equations of importance are presented in the *Interstellar* paper.