Diffy Q

Final Project

Dampened Harmonic Oscillators

By

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Professor Brian Drake section 6

Due: 4/23/2020

Part 1:

1.

Suppose that m = 4, k = 25, and c = 12. The object is pulled 2 meters and then released. Solve the initial value problem, outlining the major steps needed to solve it by hand.

Mass = 4

Spring Constant $k = 25$

Damping Constant $c = 12$

Pulled 2m

$\rightarrow y(0) = 2$

$y'(0) = 0$

General form of spring differential equation

$$y'' + \frac{c}{m} y' + \frac{k}{m} y = 0$$

Plugging into equation

$$y'' + \frac{12}{4} y' + \frac{25}{4} y = 0$$

Solution in form $e^{\lambda T}$ for $ay'' + by' + cy = 0$

$$(e^{\lambda T})'' + 3(e^{\lambda T})' + 6.25 e^{\lambda T} = 0$$

$$\rightarrow e^{\lambda T}(\lambda^2 + 3\lambda + 6.25) = 0$$

$$\rightarrow \lambda_1 = -\frac{3}{2} + 2i$$

$$\rightarrow \lambda_2 = -\frac{3}{2} - 2i$$

for two complex roots where $\lambda_1 \neq \lambda_2$

$\lambda_1 = a + i\beta \qquad \lambda_2 = a - i\beta$

General Solution form

$$y = e^{aT}\left(C_1 \cdot \cos(\beta T) + C_2 \cdot \sin(\beta T)\right)$$

Putting into general form

$$y = e^{-\frac{3}{2}T} \cdot \left((C_1 \cdot \cos(2T) + C_2 \cdot \sin(2T)\right)$$

find Constants

$$y(0) = 2 \Rightarrow 2 = e^{-\frac{3}{2}\cdot 0}\left(C_1 \cdot \cos(2\cdot 0) + C_2 \cdot \sin(2\cdot 0)\right)$$

Solving $C_1 = 2$

Derivate $y = e^{-\frac{3}{2}T} \cdot \left(C_1 \cdot \cos(2T) + C_2 \cdot \sin(2T)\right)$

$$y' = C_1\left(-2e^{-\frac{3}{2}T} \cdot \sin(2T) - \frac{3}{2}e^{-\frac{3}{2}T} \cdot \cos(2T)\right)$$
$$+ C_2\left(2e^{-\frac{3}{2}T} \cdot \cos(2T) - \frac{3}{2}e^{-\frac{3}{2}T} \cdot \sin(2T)\right)$$

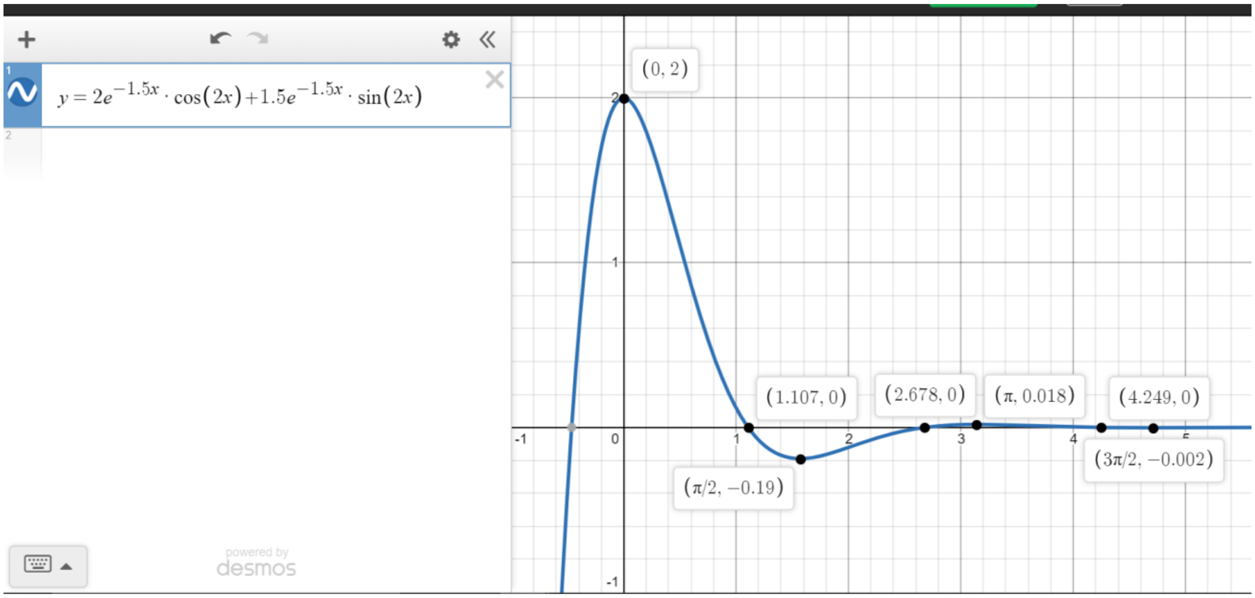
find C_2

$y'(0) = 0 \Rightarrow$ plug $T = 0$ into y'

$$0 = -\frac{3}{2}C_1 + 2C_2$$

$C_1 = 2$ so $C_2 = 1.5$ solving equation above

2 (a) By zooming in appropriately on a graph of the solution to part a), estimate the times and heights of two consecutive local maxima. (Include a snapshot of the relevant portion of your graph in your writeup.)



2 (b) Suppose the first is y(ta) = ya and the second is y(tb) = yb. Check that tb − ta = 2π/Ω, where Ω appears in your solution inside the trigonometric functions as cos(Ωt).

$$y(0) = 2$$

$$y(\pi) = 0.018$$

$$\pi - 0 = \frac{2\pi}{2}$$

$$\pi = \pi$$

2 (c)

Calculate

$$\ln\left(\frac{y_a}{y_b}\right).$$

This is called the *logarithmic decrement*. Check that the logarithmic decrement is equal to

$$\frac{\pi c}{m\Omega}.$$

$$\frac{12\pi}{4(2)} = \ln\left(\frac{2}{.018}\right)$$

$$4.71238898 \approx 4.71238888$$

2 (d) Choose two different consecutive local maxima and check that we get the same value of the logarithmic decrement using those maxima. (You may need to make the scale on the y-axis very small to find other local maxima.

$$Min\ y(3\pi/2) = -.018$$

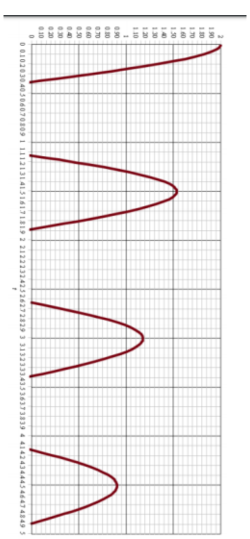
$$Min\ y(\pi/2) = -.19$$

$$\frac{3\pi}{2} - \frac{\pi}{2} = \frac{2\pi}{2}$$

3 Write a short paragraph explaining how the value of the logarithmic decrement tells you if oscillations will die out quickly or slowly. (Hint: The logarithmic decrement depends on c. You could think about what changing c would do to your system. Or you could try changing c and re-solving the problem. It's also possible to do this by thinking about ya/yb.)

The logarithmic decrement is represented by the equation $\frac{\pi c}{m\Omega}$ thus raising the damping constant c will increase the logarithmic decrement. The logarithmic decrement represents how quickly the oscillating spring will come to a stop. A higher logarithmic decrement means the spring's oscillations will die out quicker. Higher damping constant means higher logarithmic decrement which models the springs oscillations dying out quicker.

4 In applications, often the value of the damping constant c is unknown and must be determined experimentally. Suppose that an object with m = 17 is released. The graph of the resulting motion is on the last page. Estimate the value of c and write a short paragraph explaining how one can calculate c in such a problem.



4 solving for damping constant

$$\frac{\pi c}{17(2)} = \ln\left(\frac{2}{1.55}\right) = .255$$

$$c = \frac{.255 * (17 * 2)}{\pi} = 2.75974671$$

Part 2

Spring System Diffenential Eq Laplace Trans form

Mass = 1

Spring Constant K = 7

Damping Constant c = 8

Pulled 2m

$\Rightarrow y(0) = 2$

$\dot{y}(0) = 0$

After 5sec it's hit providing 6 unit impulse

General form

$$y'' + 8\dot{y} + 7y = 6\delta(T-5)$$

Laplace Trans form

$$L[y''] = s^2(L[y]) - s(y(0)) - \dot{y}(0)$$

$$\Rightarrow (s^2 L[y] - s(2) - 0) + 8(sL[y] - 2) + 7L[y] = 6e^{-5s}$$

$$\Rightarrow L[y] = \frac{2s+16}{s^2+8s+7} + 6e^{-5s} \cdot \frac{1}{s^2+8s+7}$$

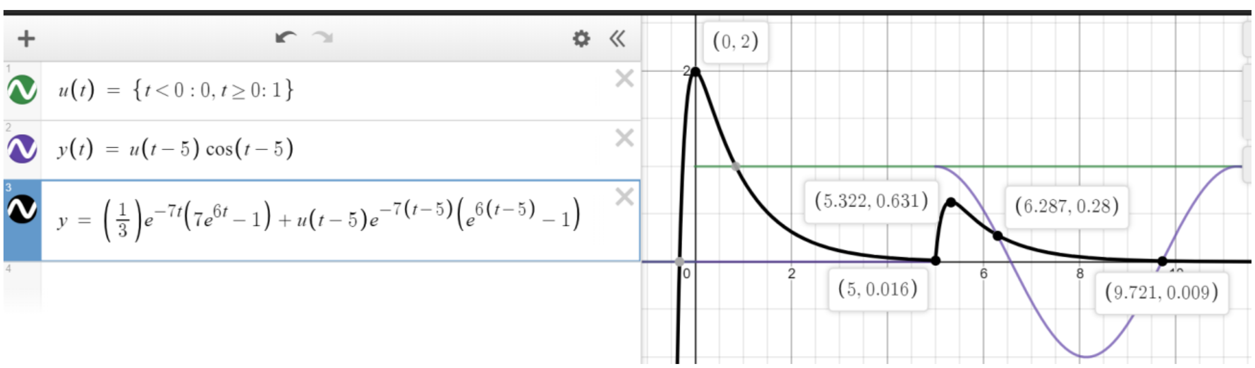
$$\Rightarrow \frac{2s+16}{s^2+8s+7} = \frac{1}{3}e^{-7T}\cdot\left(7e^{6T}-1\right)$$

$$\Rightarrow 6e^{-5s}\cdot\frac{1}{s^2+8s+7} = \delta(T-5)\,e^{-7(T-5)}\cdot\left(e^{6(T-5)}-1\right)$$

Final Solution Laplace Transform Method

$$y = \frac{1}{3}e^{-7T}\cdot\left(7e^{6T}-1\right)+\delta(T-5)\,e^{-7(T-5)}\cdot\left(e^{6(T-5)}-1\right)$$

6 Graph your solution to the previous problem and write some interpretation. You might compare the strength of the damping in this problem to the problems from Part 1, or you might write 2 about the effect of the hammer strike



6 The graph above models the Spring Mass System. Notice the spring starts displaced 2 units at the beginning and then gets hit with the hammer at t = 5 seconds. The damping constant was much higher in this system, so we do not see as much spring oscillation.

Part 3

Two mathematical ideas I want to take with me is solving for eigen vectors and eigen values. When you explained in class how google used eigen stuff to make their search engine better it got me interested in how as a Computer Engineer I could definitely be using eigen stuff again someday if I worked somewhere like google. Also, solving systems of equations using matrices is very useful and I have already utilized those skills in solving many circuits this semester.

Change nothing. You have been the best! 😊