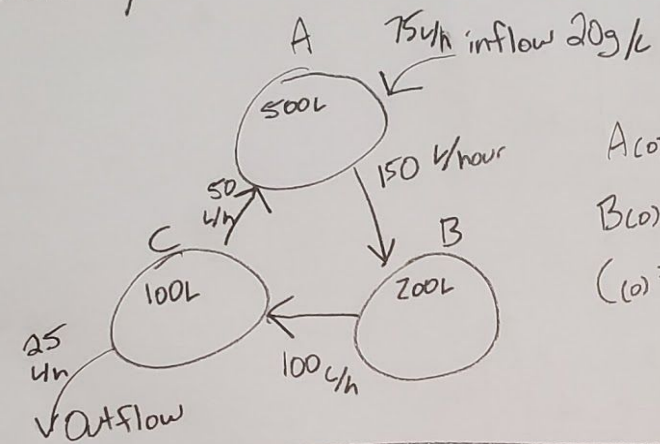
Project 3 - Just Chase (not Dominick)

Salty Tanks

Chase & Dominick



$A(0) = 30g$

$B(0) = 35g$

$C(0) = 40g$

$$\dot{A} = \left(75\,L/h \cdot 20\,g/L + 50\,L/h \cdot \frac{C}{100L}\right) - \left(150\,L/h \cdot \frac{A}{500L}\right)$$

$$\dot{B} = \left(150\,L/h \cdot \frac{A}{500L}\right) - \left(100\,L/h \cdot \frac{B}{200L}\right)$$

$$\dot{C} = \left(100\,L/h \cdot \frac{B}{200L}\right) - \left(25\,L/h \cdot \frac{C}{100L} + 50\,L/h \cdot \frac{C}{100L}\right)$$

$$A' = -.3A + 0B + .5C = 1500$$

$$B' = .3A - .5B + 0C = 0$$

$$C' = 0A + .5B - .75C = 0$$

$\dot{x} = Ax + b$

$$\dot{x} = \begin{bmatrix} -.3 & 0 & .5 \\ .3 & -.5 & 0 \\ 0 & .5 & -.75 \end{bmatrix} x + \begin{bmatrix} 1500 \\ 0 \\ 0 \end{bmatrix}$$

Eisen Using Wolfram

vector

$$\lambda_1 = -.75 + .33i \quad \begin{bmatrix} -.73 - .53i \\ .01 + .66i \\ 1 \end{bmatrix}$$

$$\lambda_2 = -.75 - .33i \quad \begin{bmatrix} -.73 + .53i \\ .01 - .66i \\ 1 \end{bmatrix}$$

$$\lambda_3 = -.06 \quad \begin{bmatrix} 2.05 \\ 1.38 \\ 1 \end{bmatrix}$$

$$\lambda_1 = e^{(-.75 + .33i)t} \begin{bmatrix} -.73 - .53i \\ .01 + .66i \\ 1 \end{bmatrix}$$

$$\lambda_1 = e^{-.75t}\left(\cos(.33t) + i\sin(.33t)\right)\begin{bmatrix} -.73 - .53i \\ .01 + .66i \\ 1 \end{bmatrix}$$

$$\lambda_1 = C e^{-.75t}\begin{bmatrix} -.73\cos(.33t) - .53\sin(.33t) \\ .01\cos(.33t) + .66\sin(.33t) \\ \cos(.33t) \end{bmatrix} \leftarrow \text{Real}$$

$$+$$

$$+ C_1 e^{-.75t}\begin{bmatrix} -.53\cos(.33t) - .73\sin(.33t) \\ .66\cos(.33t) + .01\sin(.33t) \\ \sin(.33t) \end{bmatrix} \leftarrow \text{Imaginary}$$

Solve for Initial Values

Row Reduce

$$\left[\begin{array}{ccc|c} -.3 & 0 & .5 & 1500 \\ .3 & -.5 & 0 & 0 \\ 0 & .5 & -.75 & 0 \end{array}\right] \Rightarrow \begin{array}{r} A(0) = \\ B(0) = \\ C(0) = \end{array}\begin{bmatrix} 15000 \\ 9000 \\ 6000 \end{bmatrix}$$

$$\begin{array}{ccccc} & \text{Real} & & \text{Img} & \\ \lambda_1 = C_1 & \begin{bmatrix} -.346 \\ .0065 \\ 1 \end{bmatrix} & + C_1 & \begin{bmatrix} -.252 \\ .312 \\ .0058 \end{bmatrix} & = C_1 \begin{bmatrix} -.598 \\ .319 \\ 1.0058 \end{bmatrix} \end{array}$$

$$\lambda_2 = e^{-.75}((\cos(-.33t) + i\sin(-.33t)) \begin{bmatrix} -.73 + .53i \\ .01 - .66i \\ 1 \end{bmatrix}$$

$$\lambda_2 = C_2 \begin{bmatrix} -.346 \\ .0065 \\ .472 \end{bmatrix} + C_2 \begin{bmatrix} .252 \\ -.307 \\ -.0027 \end{bmatrix} = C_2 \begin{bmatrix} -.094 \\ -.301 \\ .469 \end{bmatrix}$$

$$\lambda_3 = C_3 \begin{bmatrix} -.123 \\ -.083 \\ -.06 \end{bmatrix}$$

General Solution

$$\vec{X} = C_1 \begin{bmatrix} -.598 \\ .319 \\ 1.0058 \end{bmatrix} + C_2 \begin{bmatrix} -.094 \\ -.301 \\ .469 \end{bmatrix} + C_3 \begin{bmatrix} -.123 \\ -.083 \\ -.06 \end{bmatrix} + \begin{bmatrix} 15000 \\ 9000 \\ 6000 \end{bmatrix}$$

Solve for Constants

Set $\vec{X} = \begin{bmatrix} 30 \\ 35 \\ 40 \end{bmatrix}$

$$\begin{array}{ccc}
C_1 & C_2 & C_3 \\
\end{array}$$

$$\left[\begin{array}{ccc|c}
-.598 & -.094 & -.123 & 14970 \\
.319 & -.301 & -.083 & 8965 \\
1.0058 & .469 & -.06 & 5960
\end{array}\right] \Rightarrow \begin{array}{l} C_1 = -1329 \\ C_2 = 742 \\ C_3 = -115,812 \end{array}$$

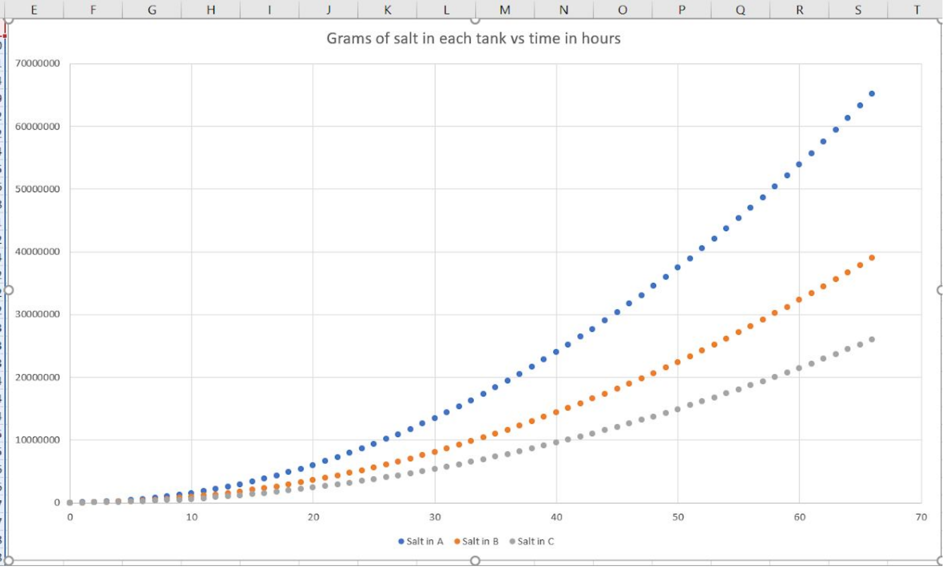
Salt in each tank over time

$$A(t) = (-1329 \cdot -.598t + 742 \cdot -.094t + -115812 \cdot -.123) \cdot t + 15000$$

$$B(t) = (-1329 \cdot .319t + 742 \cdot -.301t - 115812 \cdot -.083t) \cdot t + 9000$$

$$C(t) = (-1329 \cdot 1.0058t + 742 \cdot .469t - 115812 \cdot -.06t) \cdot t + 6000$$

Time in hours	Salt in A	Salt in B	Salt in C
0	15000	9000	6000
1	29969.87	17965.1	11960.01
2	74879.48	44860.41	29840.04
3	149728.8	89685.93	59640.09
4	254517.9	152441.6	101360.2
5	389246.8	233127.6	155000.2
6	553915.3	331743.7	220560.4
7	748523.6	448290	298040.5
8	973071.7	582766.6	387440.6
9	1227559	735173.3	488760.8
10	1511987	905510.3	602001
11	1826354	1093777	727161.2
12	2170661	1299975	864241.4
13	2544908	1524102	1013242
14	2949095	1766160	1174162
15	3383221	2026148	1347002
16	3847287	2304066	1531763
17	4341292	2599915	1728443
18	4865238	2913693	1937043
19	5419123	3245402	2157564
20	6002948	3595041	2390004
21	6616713	3962610	2634364
22	7260417	4348110	2890645
23	7934061	4751539	3158845
24	8637645	5172899	3438966
25	9371169	5612189	3731006
26	10134632	6069410	4034967
27	10928035	6544560	4350847
28	11751378	7037641	4678648
29	12604661	7548652	5018368



Conclusion

Having now finished this project I have a better understanding of how the inflow and outflows work to a salty tank system. At the beginning, I chose numbers that were all different and named the variables the names of their respective tanks for easily reading where the inflows and outflows were going. Solving for llamada and it's vectors was easy using wolfram and row reducing was easy using my calculator. The hardest part of this project was solving for the real and imaginary parts of the equations for the salt in each tank vs time. Use Euler's form of a complex number to solve!

Euler's form of a complex Number:

$$e^{ix} = \cos x + i \sin x$$

$$e^{ix} = cos\ x\ +\ i\ sin\ x \qquad (1)$$

Solving for constants from the initial values was easy after that. The plot made in excel shows how the salt in each tank keeps increasing over time and this is because we have a 75L/h inflow rate bringing in 20g/L of salt per hour. The inflow of 75L/h is far greater than the 25L/h leaving the system, so it makes sense that the concentration of salt in each tank is going to keep increasing over time. If this inflow rate was lesser than we would start to see more of a logistical growth to the salt in each tank over time. The logistic growth model looks like this: