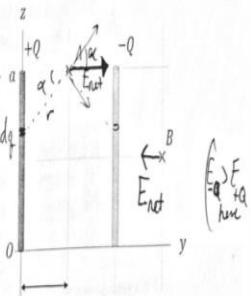


A positive charge  $+Q$  is distributed uniformly along the  $z$ -axis between  $z = 0$  and  $z = a$ . An equal magnitude but opposite charge  $-Q$  is distributed uniformly a distance  $y = 2d$  away, also between  $z = 0$  and  $z = a$ .

- a. Draw an arrow on the figure at both locations  $A$  and  $B$  showing the direction of the net electric field at those locations. Write ' $E = 0$ ' if the field is zero there.

- b. Calculate the net electric field produced by the charge distribution at location  $A$ . Set up the integral and simplify as much as possible up to the point where the next step would be to carry out the integration, but it is not necessary to solve it. Give your answer in terms of  $k$ ,  $Q$ , and any other constants. Show all work.



$$\begin{aligned}\vec{E} &= \int d\vec{E} = \int dE_y \cos \alpha \\ &= \int \frac{k dq}{r^2} \cdot \cos \alpha = \int \frac{k dq}{r^2} \cdot \frac{d}{r} \\ &= \int \frac{k \cdot \left(\frac{Q}{a}\right) dz \cdot d}{r^3} = \frac{k Q d}{a} \int_0^a \frac{dz}{(d^2 + (a-z)^2)^{3/2}}\end{aligned}$$

$$\boxed{\frac{2 k Q d}{a} \int_0^a \frac{dz}{(d^2 + (a-z)^2)^{3/2}}}$$

- c. True or False (circle one): In the limit  $a \gg d$ , the electric field strength produced by the wire, measured at location A will look like that of the sum of two infinite lines of charge:

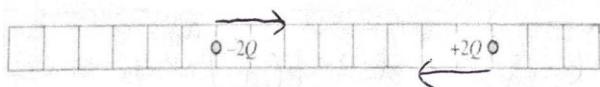
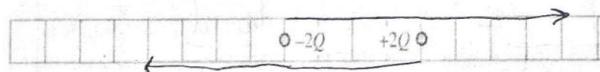
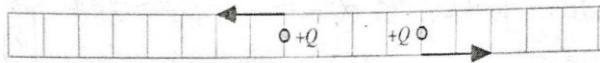
$$|\vec{E}| = \frac{\lambda}{\pi \epsilon_0 r}$$

Explain:

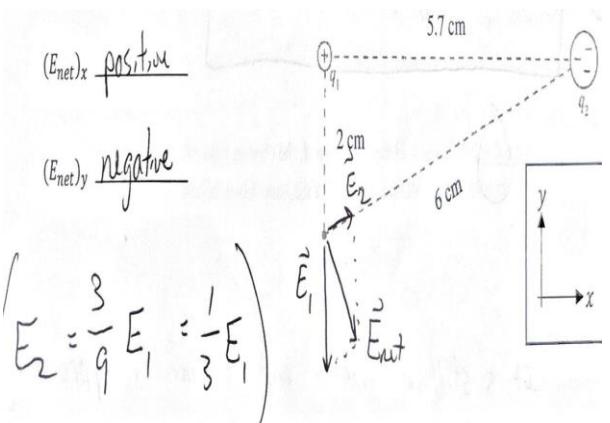
If it is still at wire's end, so infinite approximation fails.

(4 pts) You have a magic wand (plastic pipe) that has a net charge of +3.0 nC. You bring the magic wand next to a sphere made of an unknown material, and the sphere is pulled toward the wand. Of the following options, which could be true? (circle all that apply)

- a. The object could be a conductor.
- b. The object could be an insulator
- c. The object could be positively charged
- d. The object could be negatively charged
- e. The object could be neutral
- f. None of the above are possible



$$F \propto \frac{Q_1 Q_2}{r^2}$$

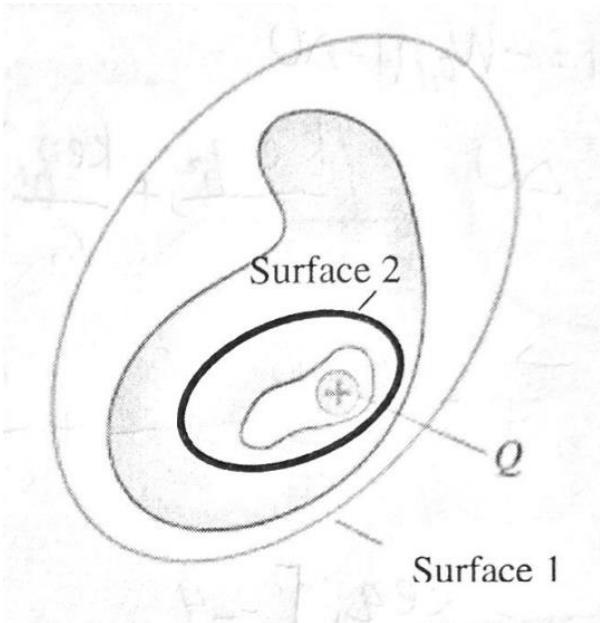


- a. What is the total charge on the inner surface of the cavity

$$[-Q]$$

$E = 0$  inside conductor,  
at surface 2. So  
Enclosed by Surface 2  
must be zero.

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

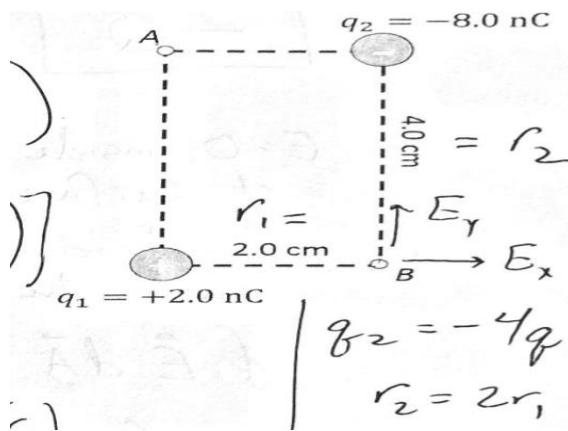


- b. The total flux through Surface 1 is 1,000 N·m<sup>2</sup>/C. What is the numerical value of the point charge, Q? Explain.

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$

$$Q_{\text{enc}} = Q = \epsilon_0 \Phi_E = (8.85 \cdot 10^{-12} \frac{\text{C}^2}{\text{Nm}^2})(1,000 \frac{\text{Nm}^2}{\text{C}})$$

$$\boxed{8.85 \text{ nC}}$$



$$\Delta U = \left( \frac{k_e q_2}{r_2} + \frac{k_e q_1}{r_1} \right) - \left( \frac{k_e q_2}{r_1} + \frac{k_e q_1}{r_2} \right)$$

$$= k_e \left[ \left( \frac{-4q_1}{2r_1} + \frac{q_1}{r_1} \right) - \left( \frac{-4q_1}{r_1} + \frac{q_1}{2r_1} \right) \right]$$

$$= \frac{k_e q_1}{r_1} \left[ -\frac{4}{2} + 1 + 4 - \frac{1}{2} \right]$$

$$S_{\text{keq}} = \frac{5(9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2})(1.6 \cdot 10^{-19} \text{C})(2 \cdot 10^8 \text{C})}{0.02 \text{m}} = 3.6 \cdot 10^{-6} \text{ J}$$

$$E_{\text{net},x} = \frac{kq_1}{r_1^2} \hat{i} \quad E_{\text{net},y} = \frac{kq_2}{r_2^2} \hat{j} = (E_{\text{net},x}) \hat{j}$$

$$= \frac{(9 \cdot 10^9 \frac{\text{Nm}^2}{\text{C}^2})(2 \cdot 10^8 \text{C})}{(0.02 \text{m})^2} = 45,000 \text{ N/C}$$

$$\boxed{E = 45,000 \frac{\text{N}}{\text{C}} (\hat{i} + \hat{j})}$$

(10 pts) A non-conducting, solid sphere of radius 5.00 cm carries a uniform volume charge density of  $18.0 \mu\text{C/m}^3$ . Use Gauss's Law to calculate the electric field at a distance of 2.00 cm from the center of the sphere. Show all work.

$2 \text{ cm} < 5 \text{ cm}$ , so we need expression for field inside sphere,  $r < 5.00 \text{ cm}$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$$



$$E(4\pi r^2) = \rho \left( \frac{4}{3}\pi r^3 \right)$$

$$E = \frac{\rho r}{3\epsilon_0} = \frac{(18 \cdot 10^{-6} \text{ C/m}^3)(0.02 \text{ m})}{3(8.85 \cdot 10^{-12} \frac{\text{C}^2}{\text{Nm}^2})} = \boxed{13,560 \frac{\text{N}}{\text{C}}}$$

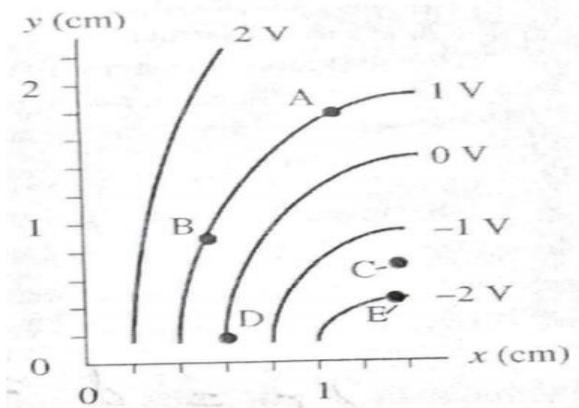
7. (10 pts) An alpha particle ( $q = +2e, m \approx 4m_{proton}$ ) is shot toward a uranium nucleus ( $q = +92e$ ) from far away. The alpha particle reaches a closest distance of 1.0 femtometers ( $10^{-15} \text{ m}$ ) before changing direction. What was the initial speed of the alpha particle? (Assume the uranium nucleus does not move, since it is so much more massive than the alpha particle).

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2} m_\alpha v^2 = \frac{k(+2e)(+92e)}{(1 \cdot 10^{-15} \text{ m})}$$

$$\frac{1}{2} \cdot 4 \cdot (1.67 \cdot 10^{-27} \text{ kg}) v^2 = \frac{(9 \cdot 10^9 \text{ Nm}^2/\text{C}^2) |8e|_e^2}{1 \cdot 10^{-15} \text{ m}}$$

$$v = 1.13 \cdot 10^8 \text{ m/s}$$



Is the electric field strength at location A greater than, less than, or equal to the field strength at location B? Circle one answer, and then explain your reasoning:

Equipotentials Farther apart,  
 $\vec{E} = -\vec{\nabla}V$

Is the electric field strength at location C greater than, less than, or equal to the field strength at location D? Circle one answer, and then explain your reasoning.

As accurately as is possible from the information in the figure, determine the electric field (magnitude and direction) at location D. Show all your work.

$$\vec{E} = -\vec{\nabla}V = -\frac{\Delta V}{\Delta X} = \left( \frac{-2V}{0.4\text{cm}} \right)$$

$$= +5.0 \frac{V}{\text{cm}}$$

Magnitude:	$5.0 \frac{V}{\text{cm}}$
Direction:	$\rightarrow$

As accurately as possible from the information in the figure, determine how much work is done by the electric field as a  $+600 \mu\text{C}$  point-charge moves from location D to location E. (At location D, its velocity is  $5.0 \text{ km/s}$ ). Show all your work.

$$W = -\Delta U = -q \Delta V$$

$$= -q(-2V) = (600 \cdot 10^{-6} \text{C})(2V)$$

$$= 1200 \cdot 10^{-6} \text{ J}$$

## ELECTRIC CHARGE AND FORCE

## Coulomb's Law

• Force between two point charges (in vacuum  $\approx$  air)

$$|\vec{F}_{12}| = k \frac{q_1 q_2}{r^2}$$



$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \cdot 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2$$

$$* \epsilon_0 = 8.854 \cdot 10^{-12} \text{ C}^2 / \text{Nm}^2$$

magnitude

$$|\vec{F}_{12}| = \frac{1}{4\pi\epsilon_0} \frac{|q_1 q_2|}{r^2}$$

direction

- Along the line connecting the two charges
- Attractive or repulsive

→ ← ← →

• Concept Test

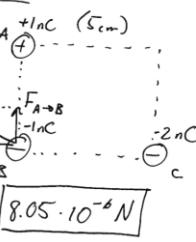
\* Mutual force

- Newton's 3rd Law Pair
- Similar to : Earth pulls on me with same grav. force that I pull on Earth.

fundamental charge  
(electron, proton)  $e = 1.602 \cdot 10^{-19} \text{ C}$

Example: Superposition of Forces 2

$$\begin{aligned} \vec{F}_{\text{net}} &= \vec{F}_{c \rightarrow b} + \vec{F}_{a \rightarrow b} \\ &= \frac{1}{4\pi\epsilon_0} \frac{q_B q_c}{(0.05\text{m})^2} (-\hat{i}) + \frac{1}{4\pi\epsilon_0} \frac{q_B q_A}{(0.05\text{m})^2} (+\hat{j}) \end{aligned}$$



$$|\vec{F}_{\text{net}}| = \sqrt{|\vec{F}_{c \rightarrow b}|^2 + |\vec{F}_{a \rightarrow b}|^2} \quad \text{(exercise for you)} \quad [8.05 \cdot 10^{-6} \text{ N}]$$

What if I replaced  $q_B$  with  $q_D = 3 \text{nC}$ ?

Most of the calculation would be the same.

$$\frac{\vec{F}_{\text{net}}}{q_B} = \frac{1}{4\pi\epsilon_0} \frac{q_c}{(0.05\text{m})^2} (-\hat{i}) + \frac{1}{4\pi\epsilon_0} \frac{q_A}{(0.05\text{m})^2} (+\hat{j}) \equiv \vec{E}_{\text{net}}$$

↑ created by  $q_A$  and  $q_c$ .

In other words,  $q_A$  and  $q_c$  create an electric field whether or not  $q_B$  or  $q_D$  ("test charge") are there to feel the force.

- A. Only conductors can be charged  
 B. Only insulators can be charged  
 C. Both conductors and insulators can be charged.

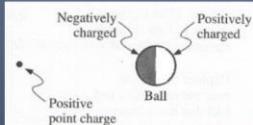
Both conductors and insulators can be charged/discharged. They differ in the ability of charge to **move freely**.

## Conductors: charges can move freely

A small ball with zero net charge is positively charged on one side, and equally negatively charged on the other side. The ball is placed near a positive point charge as shown.

Would the ball be...

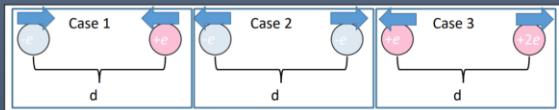
- A. Attracted toward  
 B. Repelled from  
 C. Unaffected by  
 ...the positive point charge?



The positive charges on the ball are farther away from the point charge than the negative charges on the ball.

The *net effect* is an attractive force  
 We'll see this in Coulomb's Law later today.

Each case below (1-3) shows two point charges separated by a distance  $d$ .



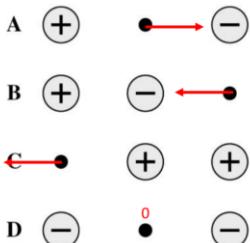
- a. For each charge in each case, find the *direction* of the electric force.  
 b. In which case (1-3) is the electric force on the *left* charge larger than the force on the *right* charge?

- A) Case 1  
 B) Case 2  
 C) Case 3  
 D) Two of the cases  
 E) None of the cases

**Two ways to see it:**  
 1. Newton's 3<sup>rd</sup> Law  
 2. Look at Coulomb's Law

- A)  $F_A > F_B > F_C > F_D$   
 B)  $F_A = F_B > F_C = F_D$   
 C)  $F_A > F_C > F_B > F_D$   
 D)  $F_A = F_D > F_B = F_C$   
 E)  $F_C = F_D > F_A = F_B$

selection and distance  
 both matter.



## ELECTRIC FIELD

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q_s}{r^2} \hat{r}$$

Handwritten notes on the equation:  
 - "Source charge" is written above  $q_s$ .  
 - "unit vector points from source charge to field location" is written next to  $\hat{r}$ .  
 - "distance from source charge to field location measured" is written next to  $r$ .

Example 21.33 Two options: ①  $\hat{r} = (\cos \theta)\hat{i} + (\sin \theta)\hat{j}$  ← this problem  
 (really the same) ②  $\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{|x|\hat{i} + |y|\hat{j}}{|\vec{r}|}$  ← See Example 21.6

A point charge is at the origin. With this point charge as the source point, what is the unit vector  $\hat{r}$  in the direction of the field...

- at  $x = 0, y = 2.1 \text{ m}$
- at  $x = -8.0 \text{ cm}, y = -8.0 \text{ cm}$
- at  $x = 0.9 \text{ m}, y = -2.7 \text{ m}$

Reflect on whether your answer to (c) is *plausible*, even if you aren't sure the numbers themselves are correct.

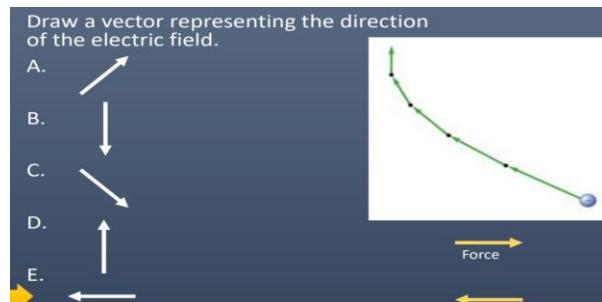
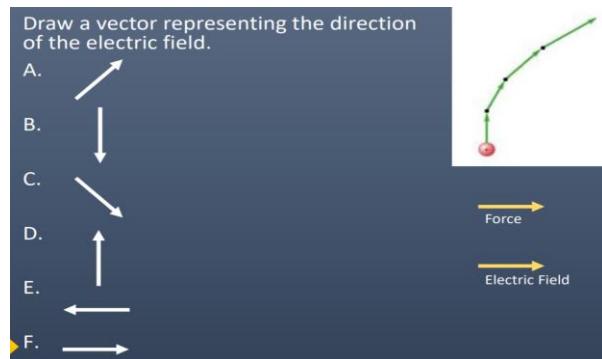
$$\tan \theta = \frac{y}{x} \rightarrow \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$a) \theta = \tan^{-1}\left(\frac{2.1 \text{ m}}{0 \text{ m}}\right) = \frac{\pi}{2} \text{ rad} \Rightarrow \boxed{\hat{r} = \hat{j}}$$

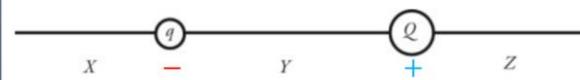
$$b) \theta = \tan^{-1}\left(\frac{-8.0 \text{ cm}}{-8.0 \text{ cm}}\right) = \frac{\pi}{4} \text{ or } \frac{5\pi}{4} \Rightarrow \boxed{\hat{r} = -\frac{\sqrt{2}}{2}\hat{i} - \frac{\sqrt{2}}{2}\hat{j}}$$

$$c) \theta = \tan^{-1}\left(\frac{-2.7 \text{ m}}{0.9 \text{ m}}\right) = -1.25 \text{ rad} \Rightarrow \boxed{\hat{r} = 0.32\hat{i} - 0.95\hat{j}}$$

## CONTINUOUS CHARGE DISTRIBUTION



than charge  $q$ . In which regions will there be a location at which the net electric field due to these two charges is zero?



- Only X and Z
- Only X
- Only Y
- Only Z
- All three regions

### Quantitative Example 1

④ Make use of symmetry to make life easier

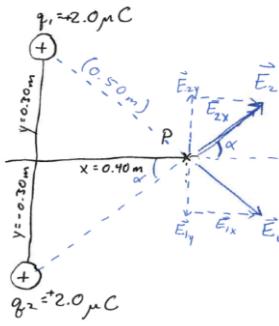
At location P,

$$|\vec{E}_1| = |\vec{E}_2| = \frac{1}{4\pi\epsilon_0} \frac{|q_1|}{r^2}$$

$$= (9 \cdot 10^9 \frac{\text{N}\cdot\text{C}^2}{\text{C}^2}) \cdot \frac{(2.0 \cdot 10^{-6} \text{C})}{(0.50 \text{m})^2}$$

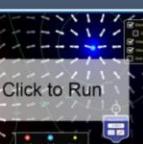
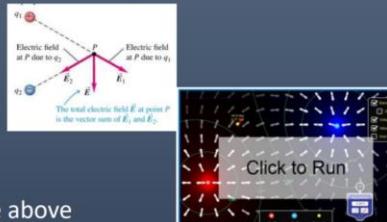
$$= 7.2 \cdot 10^4 \frac{\text{N}}{\text{C}}$$

(what each q would contribute if alone)



In the previous example, suppose the two point charges still had the same magnitude, but the top charge was positive, and the bottom charge was negative. Now the direction of the electric field at point P would be:

- A. Positive x
- B. Negative x
- C. Positive y
- D. Negative y**
- E. Zero
- F. None of the above



### Example: Uniformly charged rod

Find the electric field at location P (on the x-axis at  $x = d$ ) created by a 60-cm long rod of total charge  $Q$  and linear charge density  $\lambda$ . The rod lies on the y-axis between  $a = +30 \text{ cm}$  and  $-a = -30 \text{ cm}$ .

Prequel:

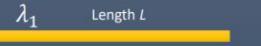
- What are the dimensions of  $\lambda$ ?

Core Learning Target #1 (C.1)

1. Cut up the charge distribution into pieces and draw  $\Delta\vec{E}$ .
2. Write an expression for the electric field due to one piece.
3. Add up the contributions of all the pieces.
4. Check the result.

Suppose we cut our line charge into two pieces. One piece is 1/3 the original length while the other is 2/3 the original length. Rank the three pieces (original and two smaller pieces) according to linear charge density (largest to smallest):

A.  $\lambda_1 > \lambda_2 > \lambda_3$



B.  $\lambda_1 > \lambda_3 > \lambda_2$



C.  $\lambda_2 > \lambda_3 > \lambda_1$



D.  $\lambda_3 > \lambda_2 > \lambda_1$

E.  $\lambda_1 = \lambda_2 = \lambda_3$

F. None of the above

**Consequence:** For uniform distribution, we can use same  $\lambda$  for  $dq$  as for entire line charge.

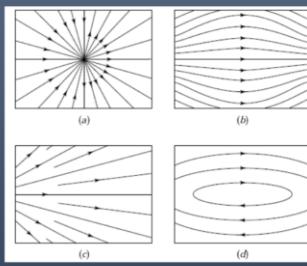
$$\lambda = \frac{Q}{L}$$

$$\lambda = \frac{dq}{dx}$$

### Distribution (detailed) -- Core LT: C.1

1. Cut up the charge distribution into pieces and draw  $\Delta\vec{E}$ .
  - Divide distribution up into pieces whose field is known (e.g. small pieces can be approximated by point particles)
  - Pick a representative piece, and at the location of measurement, draw vector  $\Delta\vec{E}$  showing contribution to E-field from that piece.
2. Write an expression for the electric field due to one piece.
  - Establish coordinate system and origin, and show on your diagram.
  - Draw vector  $\vec{r}$  from source piece to measurement location. Write algebraic expressions for  $\vec{r}$  and  $\hat{r}$ .
  - Write algebraic expression for magnitude  $|\Delta\vec{E}|$  contributed by representative piece. Multiply by  $\hat{r}$  to get  $\Delta\vec{E}$ , from which you can get components  $\Delta E_x$ ,  $\Delta E_y$ ,  $\Delta E_z$ . Your expressions should contain one or more integration variables related to coordinates of piece of charge.
  - Write the amount of charge  $\Delta q$  in terms of your variables.
  - For integration, algebraic expressions should include small increments of the integration variable. (e.g. if integration variable is  $y$ , expressions must be proportional to  $\Delta y$ ).
3. Add up the contributions of all the pieces.
  - The net electric field is the sum of contributions of all the pieces.
  - To write sum as a definite integral, you must include limits given by range of the integration variable.
  - If the integral can be done symbolically, do it (Today's example).
  - If not, choose a finite number of pieces and do the sum with a calculator or a computer (This will be Lab 3).
4. Check the result.
  - Check that the direction of the net field is qualitatively correct.
  - Check that the units of your result are correct (N/C).
  - Look at special/limiting cases. For example:
    - If net charge is nonzero, result should look like point charge very far away
    - For numerical integration on computer, check that computation gives correct result for special cases that can be calculated by hand.

no charges in the regions shown, which of the patterns represent(s) a possible electrostatic field:

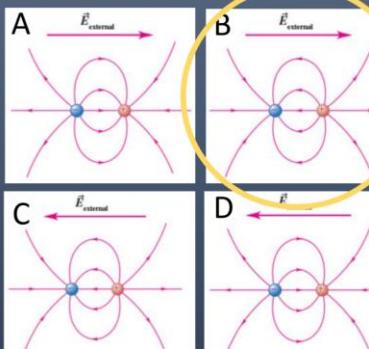


- A. (a)  
 B. (b)  
 C. (b) and (d)  
 D. (a) and (c)  
 E. (b) and (c)  
 F. None of the above.

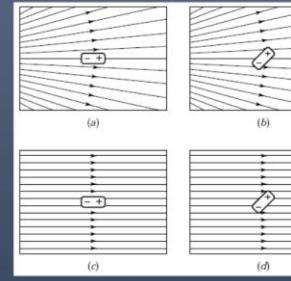
Consider an electric dipole. What can you say about the electric field at point P? [halfway between charges]

- A.  $E=0$  because the net charge is zero.
- B. E points down because the vector sum points down.
- C.  $E=0$  because point P is the same distance from each charge. So the vector sum of the field from the positive charge, plus the field from the negative charge will be zero.
- D. None of the above are true.

Choose a figure that correctly shows the dipole's own electric field and the external electric field for the stable equilibrium position.

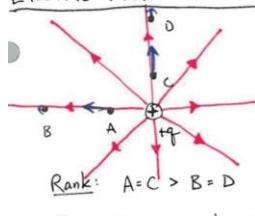


An electrically neutral dipole is placed in an external field. In which situation(s) is the net force on the dipole zero?



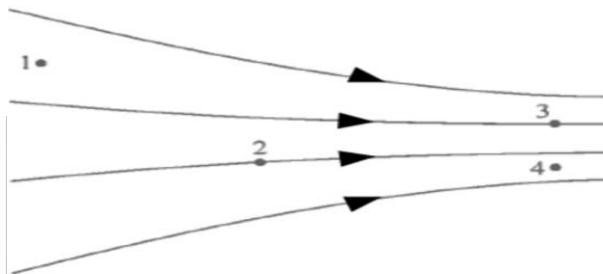
- A. (a)
- B. (c)
- C. (b) and (d)
- D. (a) and (c)
- E. (c) and (d)
- F. none of the above

### Electric Field Lines



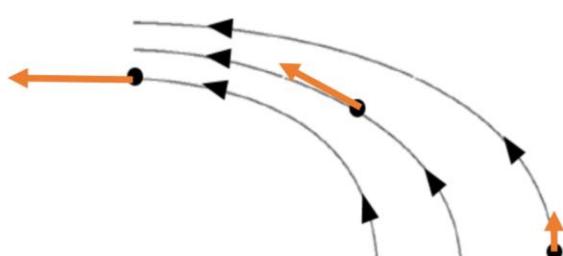
### - Another pictorial representation

- electric field lines are:
- continuous curves tangent to the electric field vectors
- Closely spaced lines: greater field strength
- Field lines start on positive charges [or infinity] and end on negative charges [or infinity]
- Field lines never cross  
(-because what is not field?)



$$4 = 3 > 2 > 1$$

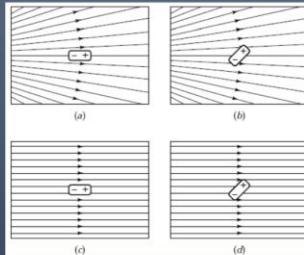
**field lines.** The relative length and direction should be accurate.



Consider an electric dipole. What can you say about the electric field at point P? [halfway between charges]

- A.  $E=0$  because the net charge is zero.
- B. E points down because the vector sum points down.
- C.  $E=0$  because point P is the same distance from each charge. So the vector sum of the field from the positive charge, plus the field from the negative charge will be zero.
- D. None of the above are true.

field. In which situation(s) is the net force on the dipole zero?



- A. (a)
- B. (c)
- C. (b) and (d)
- D. (a) and (c)
- E. (c) and (d)
- F. none of the above

Check Q: Dipole in external field

Non-uniform field:  $\vec{F}_{\text{net}} \neq 0$  on a dipole  
Uniform field:  $\vec{F}_{\text{net}} = 0$  on a dipole

but  $T_{\text{net}} \neq 0$

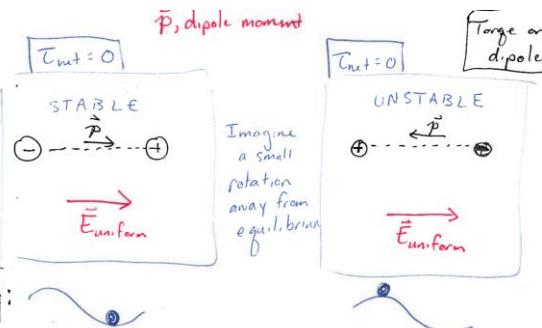
$$\vec{T} = \vec{r} \times \vec{F}$$

$$|T| = r F \sin \phi$$

$$|T_{\text{net}}| = \frac{d}{2} (q_1 E) \sin \phi + \frac{d}{2} (q_2 E) \sin \phi$$

$$|q_1| = |q_2|$$

$$= q d E \sin \phi = p E \sin \phi = \boxed{\vec{p} \times \vec{E}}$$



$$|E_+| = |E_-| = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} = \frac{1}{4\pi\epsilon_0} \left( \frac{1}{(d/2)^2 + x^2} \right)$$

$$\vec{E}_{\text{net}} = 2E_y = \frac{2q}{4\pi\epsilon_0} \left( \frac{1}{(d/2)^2 + x^2} \right) \cdot \frac{d/2}{\sqrt{(d/2)^2 + x^2}}$$

$$\vec{E}_{\text{net}} = \frac{-q d}{4\pi\epsilon_0 [(d/2)^2 + x^2]^{3/2}} \hat{y}$$

$$E_{\text{dipole}} \propto \frac{1}{r^3} \quad (\text{any direction})$$

For  $x \gg d$  (far away)

$$x^2 \gg (d/2)^2 \Rightarrow \boxed{E_{\text{dipole}} = \frac{qd}{4\pi\epsilon_0 X^3} = \frac{p}{4\pi\epsilon_0 X^3}}$$

example 21.14  $\Rightarrow$  along y-axis,  $E_{\text{dipole}} = \frac{p}{2\pi\epsilon_0 Y^3}$

$E_{\text{point charge}} \propto \frac{1}{r^2}$

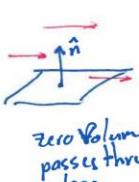
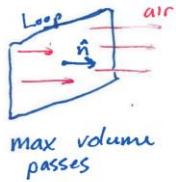
$E_{\text{Quadrupole}} \propto \frac{1}{r^4} \quad \left( \begin{array}{c} \oplus r \\ \ominus -r \\ \oplus -r \end{array} \right)$

$E_{\text{Octupole}} \propto \frac{1}{r^5}$

Solids: Atoms  
Field very small unless practically in "Contact"  
so normal force, other contact forces are electrostatic in nature.

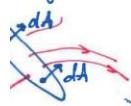
## SYMMETRY OF ELECTRIC FIELD

- fan blowing air through loop.



$$\text{volume flow rate: } V_1 A = V A \cos \theta \\ (\text{units m}^3/\text{s}) \quad = \vec{V} \cdot \vec{A}$$

- generally.. Flux in non-uniform field

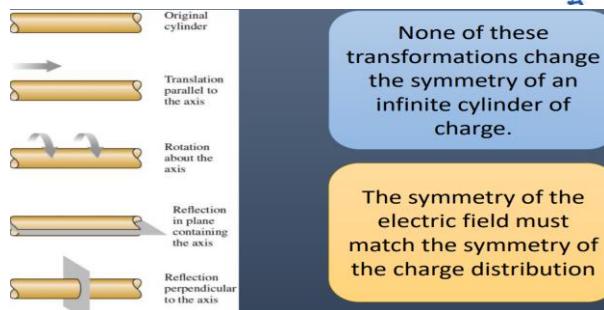


Add up all  $\vec{E} \cdot d\vec{A}$

$$\Rightarrow \Phi_E = \int_{\text{surface}} \vec{E} \cdot d\vec{A}$$

But choose wisely. Choose surface where  $E$  is same at every point.

$$\int_{\text{surface}} \vec{E} \cdot d\vec{A} = \int_{\text{surface}} E \cos \theta dA = E \cos \theta \int dA$$



## GAUSS LAW

### FLUX

Electric Flux  
E-field doesn't literally flow, but we can apply same idea.

$$\Phi_E = E_A = \vec{E} \cdot \vec{A}$$

Area vector is  $\perp$  to surface  $A$

implies

surface: disk radius  $r = 5\text{ mm}$   
 $\theta = 45^\circ$

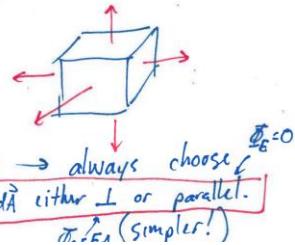
$$\Phi_E = EA \cos 45^\circ$$

$$= (5.65 \cdot 10^4 \frac{\text{N}}{\text{C}})(\pi(0.005\text{m})^2) \cos 45^\circ$$

$$\Phi_E = 3.14 \text{ Nm}^2/\text{C}$$

### Gauss's Law

Book derives Gauss's law based on this idea that flux / charge inside are related.



We will exploit symmetry  $\rightarrow$  always choose surfaces for which  $\vec{E} \cdot d\vec{A}$  either  $\perp$  or parallel to  $\vec{E}_A$  (simpler!)

A Gaussian surface is most useful when it...

- ...matches the shape of the field (so that E is either perpendicular or parallel to the surface),
- ...is chosen such that field is the same magnitude everywhere on the surface.

So that:  $\Phi_{elec} = \oint \vec{E} \cdot d\vec{A} = EA \cos \theta = EA$

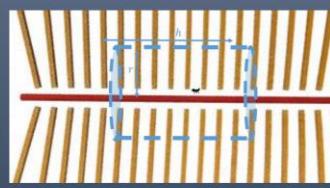
$$\text{Gauss's Law: } \Phi_{elec} = \oint \vec{E} \cdot d\vec{A} = \frac{\sum q_{\text{enclosed}}}{\epsilon_0}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{\sum q_{\text{enclosed}}}{\epsilon_0}$$

$$EA = \frac{\sum q_{\text{enclosed}}}{\epsilon_0}$$

$$EA_{\text{cylinder}} = \frac{\lambda h}{\epsilon_0}$$

$$E(2\pi rh) = \frac{\lambda h}{\epsilon_0}$$

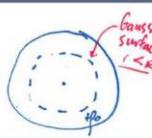


$$\Rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2k\lambda}{r}$$

Got this with much more effort integrating Coulomb's Law last week

### Uniformly charged sphere

solution: split to two parts.  
 i)  $r < R$  (because  $Q_{\text{enclosed}}$  depends on size of Gaussian)  
 ii)  $r > R$  ( $Q_{\text{enclosed}}$  same for any Gaussian)



Inside the sphere,  $r < R$

$$i) r < R: \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0} \quad \text{won't be total charge here}$$

$$E(4\pi r^2) = \frac{1}{\epsilon_0} \int p \cdot dV$$

$$E(4\pi r^2) = \frac{p_0 (\frac{4}{3}\pi r^3)}{\epsilon_0}$$

$$E = \frac{p_0 r}{3\epsilon_0} = \frac{(Q_{\text{enclosed}})}{3\epsilon_0} \cdot r$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{Qr}{R^2} \hat{r} \quad (\text{linear})$$



### Outside the sphere, $r > R$

$$ii) r > R: \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0} \quad \text{now, total } Q \text{ is } Q_{\text{enclosed}}$$

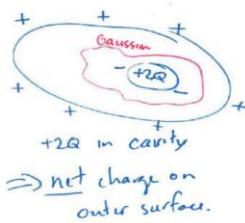
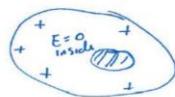
$$EA = \frac{Q}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r} \quad \text{from symmetry}$$

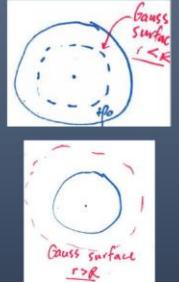
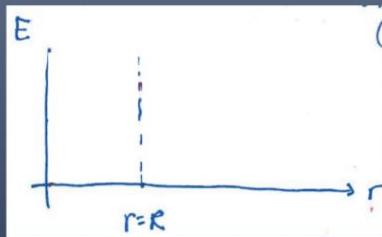
Same  $\vec{E}$  as for point charge.  
 (and same symmetry.)

use cavity  
 so all charge is on external surface



- Need to keep  $E=0$  inside conductor
- Gauss's Law gives  $Q_{\text{enclosed}}$  must net to zero.
- So inner surface of cavity balances charge

Exercise: Plot the electric field as a function of  $r$

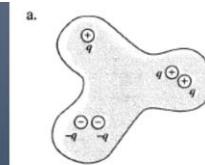


What is the flux through the surface?

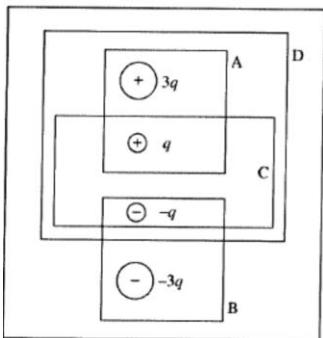
$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{\sum q_{\text{enclosed}}}{\epsilon_0}$$

Integral would be pretty complicated.  
 Look at enclosed charge instead.

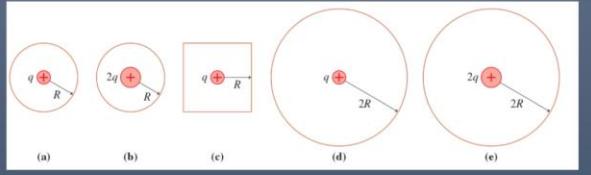
$$\frac{\sum q_{\text{enclosed}}}{\epsilon_0} = \frac{3q - 2q}{\epsilon_0} = \frac{q}{\epsilon_0}$$



<https://www.glowscript.org/#/user/matterandinteractions/folder/matterandinteractions/program/13-fields>



- $\Phi_A = 4q/\epsilon_0$
- $\Phi_B = -4q/\epsilon_0$
- $\Phi_C = 0$
- $\Phi_D = 3q/\epsilon_0$
- $\Phi_E = 0$



These are two-dimensional cross sections through 3D closed spheres and a cube. Rank in order, from largest to smallest, the electric fluxes through each surface.

- A. (b) > (a) > (c) > (e) > (d)  
B. (b) = (e) > (a) = (c) = (d)  
C. Neither of the above

The total flux does not depend on the radius of the surface.

It only depends on the enclosed charge.

In a thunderstorm, charge builds up on the water droplets or ice crystals in a cloud. Thus, the charge can be considered to be distributed uniformly throughout the cloud. For the purposes of this problem, take the cloud to be a sphere of diameter 1.00 kilometer. The point of this problem is to estimate the maximum amount of charge that this cloud can contain, assuming that the charge builds up until the electric field at the surface of the cloud reaches the value at which the surrounding air breaks down. This breakdown means that the air becomes highly ionized, enabling it to conduct the charge from the cloud to the ground or another nearby cloud. The ionized air will then emit light due to the recombination of the electrons and atoms to form excited molecules that radiate light. In addition, the large current will heat up the air, resulting in its rapid expansion. These two phenomena account for the appearance of lightning and the sound of thunder. Take the breakdown electric field of air to be

$$E_b = 3.00 \times 10^6 \text{ N/C}$$

Part A

Estimate the total charge  $q$  on the cloud when the breakdown of the surrounding air is reached.

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N} \cdot \text{m}^2)$$

Express your answer numerically, to three significant figures, using

**22.53 ++ CALC** A nonuniform, but spherically symmetric, distribution of charge has a charge density  $\rho(r)$  given as follows:

$$\rho(r) = \rho_0 \left(1 - \frac{r}{R}\right) \quad \text{for } r \leq R$$

$$\rho(r) = 0 \quad \text{for } r \geq R$$

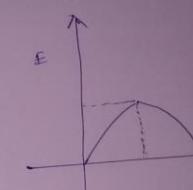
where  $\rho_0 = 3Q/\pi R^3$  is a positive constant. (a) Show that the total charge contained in the charge distribution is  $Q$ . (b) Show that the electric field in the region  $r \geq R$  is identical to that produced by a point charge  $Q$  at  $r = 0$ . (c) Obtain an expression for the electric field in the region  $r \leq R$ . (d) Graph the electric-field magnitude  $E$  as a function of  $r$ . (e) Find the value of  $r$  at which the electric field is maximum, and find the value of that maximum field.

$$\frac{1}{3} - \frac{2r}{3R} = 0$$

$$\frac{2r}{3R} = \frac{1}{2}$$

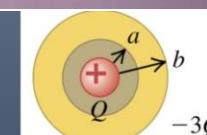
$$r = \frac{R}{2}$$

At  $r = R/2$  field is uniform  
out side the sphere  $E = 0$   
because Gaussian surface encloses no charge



A conducting spherical shell with inner radius  $a$  and outer radius  $b$  has a positive point charge  $Q$  located at its center. The total charge on the shell is  $-3Q$ , and it is insulated from its surroundings.

a) Find the electric field in the three regions:  $r < a$ ,  $a < r < b$ , and  $r > b$ .



$$-\frac{Q}{4\pi a^2}$$

b) What is the surface charge density on the inner surface of the conducting shell?

$$-\frac{2Q}{4\pi b^2}$$

c) What is the surface charge density on the outer surface of the conducting shell?

$$\oint \vec{E} \cdot d\vec{s} = q/\epsilon_0$$

at an inside point

$$q = 4\pi \epsilon_0 \left[ \frac{r^3}{3} - \frac{r^3}{3R} \right]$$

total charge in the sphere

$$q = \int_0^R 4\pi r^2 dr \left[ 1 - \frac{4r}{3R} \right] 4\pi R^2 dr$$

$$q = 8\pi \epsilon_0 \left[ \frac{R^3}{3} - \frac{4R^3}{3R} \right]$$

$$q = 4\pi \epsilon_0 \left[ \frac{R^3}{3} - \frac{R^3}{3} \right]$$

at an outside point

$$q = 4\pi \epsilon_0 \left[ \frac{r^3}{3} - \frac{R^3}{3R} \right]$$

field from Gauss's law

$$\oint \vec{E} \cdot d\vec{s} = q/\epsilon_0$$

$$E(r) = \frac{q}{\epsilon_0} \left[ \frac{r^3}{3} - \frac{R^3}{3R} \right]$$

if field is uniform

$$E = \text{const}$$

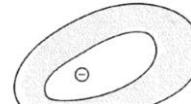
$$\frac{dE}{dr} = 0$$

$$\frac{d}{dr} \left[ \frac{r^3}{3} - \frac{R^3}{3R} \right] = 0$$

Net charge =  $-5 \text{ nC}$

A  $-8 \text{ nC}$  point charge is inside a hole in a conductor. The conductor has no net charge.

- a. What is the total charge on the inside surface of the conductor? Explain.  $+8 \text{ nC}$   
b. What is the total charge on the outside surface of the conductor? Explain.



$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

$$EA = \frac{+Q}{\epsilon_0}$$

$$E = \frac{+Q}{\epsilon_0 A} = \frac{+Q}{\epsilon_0 \cdot 4\pi r^2} \frac{kQ}{r}$$

point charge!  
radially outward

Gaussian surfaces

$a < r < b$  This is a conductor,  
so  $E = 0$  inside.

What if it was an insulator?

$$r > b \quad \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

$$EA = \frac{+Q - 3Q}{\epsilon_0} = \frac{-2Q}{\epsilon_0 A}$$

$$E = \frac{-2Q}{\epsilon_0 A} = \frac{-2Q}{4\pi\epsilon_0 r^2} \hat{r} \quad \text{radially inward}$$

From part (b) that inner surface has  $-Q$ .

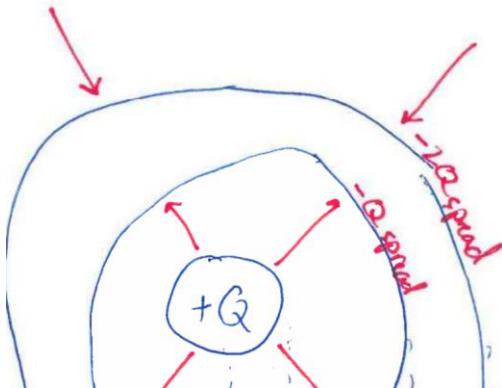
$$-Q_{\text{inner}} + \frac{-Q_{\text{outer}}}{\text{outer}} = -\frac{3Q}{\text{total}}$$

for  $a < r < b$ , must enclose zero net charge since  $E=0$  inside conductor.  
 $\Rightarrow$  total charge on inner surface is  $-Q$  (opposite of  $+Q$ ).  
 and surface charge density  $\sigma_{\text{inner}} = \frac{-Q}{4\pi a^2}$

) Net charge on shell is  $-\frac{1}{4\pi b^2} \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$

$$\sigma_{\text{outer}} = \frac{Q_{\text{outer}}}{4\pi b^2} = \left| \frac{-2Q}{4\pi b^2} \right|$$

$$\sigma_{\text{outer}} = \frac{Q_{\text{outer}}}{4\pi b^2} = \left| \frac{-2Q}{4\pi b^2} \right|$$



### ELECTRIC POTENTIAL ENERGY

$$E_{\text{mech}} = K + U$$

$\uparrow$  total mechanical energy

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = 0$$

Energy Conservation

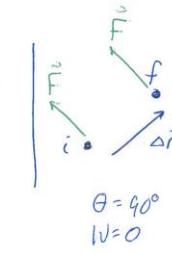
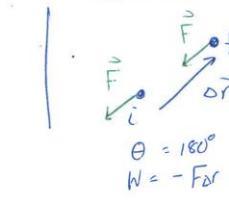
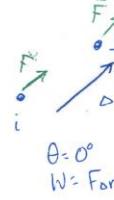
$$\left\{ \begin{array}{l} \text{kinetic energy} \quad K = \frac{1}{2}mv^2 \\ \text{potential energy} \end{array} \right.$$

$$\Delta U = -W_{\text{interaction}} \quad (\text{grav, electric})$$

$$\text{If force is not constant, then more generally } W = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x} = \int_{x_i}^{x_f} F(x) \cos\theta dx = \int_{r_i}^{r_f} F(r) \cos\theta dr.$$

If force is constant, then

$$W = \vec{F} \cdot \vec{\Delta r} = F \cos\theta$$



interactions	energy
Between two or more charges:	force, $\vec{F}$
Property of source charges : only	electric field, $\vec{E} = \frac{\vec{F}}{q_{\text{test}}}$

$$\text{potential energy, } U$$

$$\text{electric potential, } V = \frac{U}{q_{\text{test}}}$$

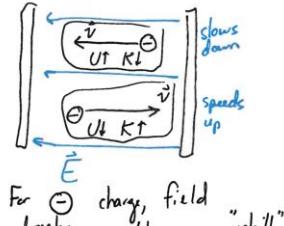
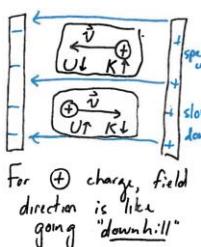
A Grav. field does work  $W_g = -\Delta U_g$   $U_f, K \uparrow$

B  $U_{\text{grav}} \xrightarrow{\text{transformed}} \text{Kinetic}$

$W_g = \int \vec{F}_g \cdot d\vec{r} = \vec{F}_g \cdot \Delta \vec{r} \cos\theta = mg|\Delta r|$

$$= mg(y_f - y_i) = mg y_f - mg y_i$$

$$U_f - U_i = mg y_f - mg y_i = -W_g$$

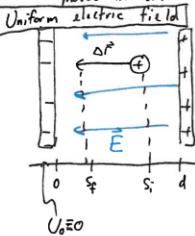


Important geometries: ① Uniform field (parallel-plate capacitor)

② Point charge } (spherical symmetry)

③ sphere (Uniform field: Special case)

A charged particle exchanges  $K, U$  as it moves in an electric field.



$$\text{Particle "falling" in direction of } \vec{E}.$$

$$W_{\text{elec}} = \int \vec{F}_e \cdot d\vec{r} = F_e \Delta \vec{r} \cos 0^\circ = qE|s_f - s_i|$$

$$W_{\text{elec}} = -\Delta U_{\text{elec}}$$

$$\Rightarrow \Delta U_{\text{elec}} = qE(s_f - s_i)$$

$$U_{\text{elec}} = qES + U_0 \quad \text{(after } U(0) = 0 \text{)}$$

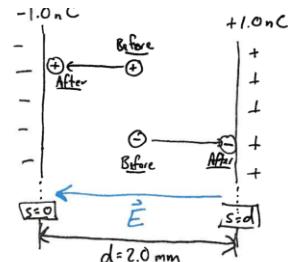
### Example 1

• Mech. Energy is conserved, and field is uniform for parallel-plate capacitor.

a) electron

$$\begin{aligned} E_{\text{mech}} &= K_i + U_i \\ &= 0 - eE\left(\frac{1}{2}d\right) \\ &= 0 - e\left(\frac{Q}{\epsilon_0 A}\right)\left(\frac{1}{2}d\right) \end{aligned}$$

field inside parallel plate capacitor (Ch. 22)



$E_{\text{kin}} = -4.5 \cdot 10^{-17} \text{ J}$  electron       $E_{\text{kin}} = +4.5 \cdot 10^{-17} \text{ J}$  proton

same magnitude

$$= -(1.602 \cdot 10^{19} \text{ C}) \left( \frac{1.0 \cdot 10^{-9} \text{ C}}{(8.99 \cdot 10^9 \frac{\text{N} \cdot \text{C}^2}{\text{Coul}^2}) \cdot (0.002 \text{ m})} \right) \left( \frac{1}{2} (0.002 \text{ m}) \right)$$

$$\Rightarrow \begin{cases} K_f + U_f = K_i + U_i = E_{\text{kin}} \\ \frac{1}{2} m v_f^2 + 0 = E_{\text{kin}} \quad (\text{hits plate}) \Rightarrow (v_f)_p = \sqrt{\frac{2 E_{\text{kin}}}{m_p}} = 2.3 \cdot 10^5 \text{ m/s} \\ \frac{1}{2} m v_f^2 + q Ed = E_{\text{kin}} \\ \frac{1}{2} m v_f^2 - e Ed = E_{\text{kin}} \Rightarrow (v_f)_e = \sqrt{\frac{-2 E_{\text{kin}}}{m_e}} = 1.0 \cdot 10^5 \text{ m/s} \\ \frac{1}{2} m v_f^2 + 2 E_{\text{kin}} = E_{\text{kin}} \end{cases}$$

smaller speed  
larger mass

larger speed  
smaller mass

point charges:  $W = \int \vec{F} \cdot d\vec{r}$ ;  $\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r}$

spherical symmetry: By definition,  $U = 0$  for  $r \rightarrow \infty$

$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$

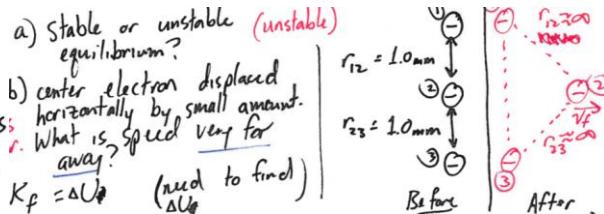
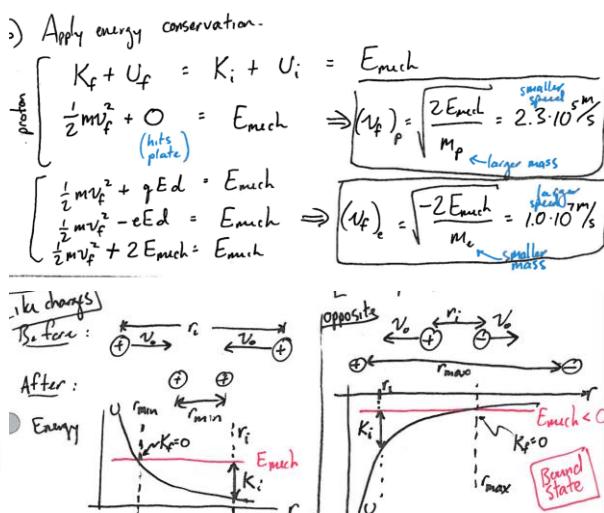
it up: Energy Conservation

$K_i + U_i = K_f + U_f$

$\frac{1}{2} m v_i^2 + 0 = 0 + \frac{1}{4\pi\epsilon_0} \frac{q_p q_s}{r_f}$

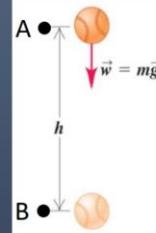
$\frac{1}{2} (1.67 \cdot 10^{-27} \text{ kg}) V_i^2 = \frac{k (1.6 \cdot 10^{-19} \text{ C}) (100 \cdot 10^{-9} \text{ C})}{0.0005 \text{ m}} \Rightarrow V_i = 1.86 \cdot 10^5 \text{ V}$

P.E. between two point charges  $q_1, q_2$  separated by distance  $r$ .



In moving from point A to point B, does the potential energy of this baseball:

- A. Increase
- B. Decrease
- C. Remain the same
- D. Not enough information
- E. I have no idea where to start



$$\Delta U_f = \frac{k q_1 q_2}{r_{12}} + \frac{k q_2 q_3}{r_{23}} = \frac{k e^2}{r_{12}} + \frac{k e^2}{r_{23}} = k e^2 \left( \frac{1}{r_{12}} + \frac{1}{r_{23}} \right)$$

c) Work by field:  $W = -\Delta U$  save yourself time.

Show that  $|v_f| = 1000 \text{ m/s}$

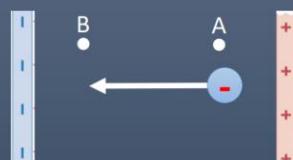
In moving from point A to point B, does the potential energy of this particle:

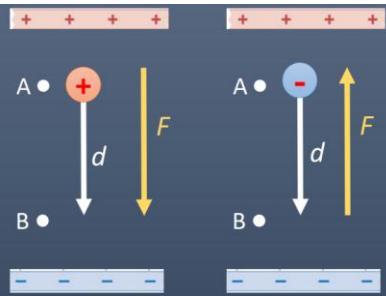
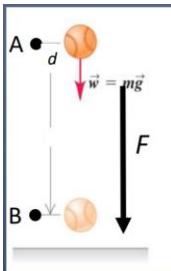
- A. Increase
- B. Decrease
- C. Remain the same
- D. Not enough information
- E. I have no idea where to start

What about

In moving from point A to point B, does the potential energy of this particle:

- A. Increase
- B. Decrease
- C. Remain the same
- D. Not enough information
- E. I have no idea where to start





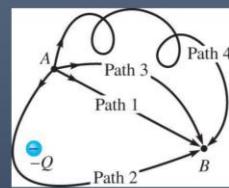
$W_{grav} = Fd$  is **positive**  
( $F, d$  same direction)  
 $\Delta U < 0$

$W_{Elec} = Fd$  is **positive**  
( $F, d$  same direction)  
 $\Delta U < 0$

$W_{Elec} = Fd$  is **negative**  
( $F, d$  opposite direction)  
 $\Delta U > 0$

A large negative charge  $-Q$  is located in the vicinity of points A and B. Suppose a positive charge  $+q$  is moved at a constant speed from A to B by an external agent. Along which of the paths shown in the figure will the work done by the electric field be greatest?

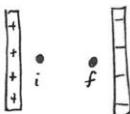
- A. Path 1
- B. Path 2
- C. Path 3
- D. Path 4
- E. Work is the same along all four paths.



The electric force is a conservative force.  
Work done by field is path-independent

In the figure, a particle is between two parallel, uniformly charged plates. It begins at position  $i$  and ends at position  $f$ . Suppose the electric field magnitude is 190 N/C. The distance between  $i$  and  $f$  is 1.5 cm.

- What is the change in electric potential energy of the system during this process?
- What is the change in kinetic energy of the proton during this process? Explain.
- If the proton were initially at rest at location  $i$ , what is its speed when it reaches location  $f$ ?
- Suppose instead that the particle were an electron instead of a proton. Would the change in potential energy  $\Delta U$  be positive, negative, or zero? Explain.



The change in kinetic energy = Work done

$$\text{So } \Delta K = F \Delta x = q E \Delta x$$

$$\text{for proton } q = 1.6 \times 10^{-19} \text{ C} \quad E = 190 \text{ N/C} \quad \Delta x = 1.5 \times 10^{-2} \text{ m}$$

$$\Delta K = (1.6 \times 10^{-19})(190)(1.5 \times 10^{-2}) = [4.56 \times 10^{-19} \text{ J}]$$

Initially  $V_i = 0$   $V_f = ?$

$$\Rightarrow \frac{1}{2} m v_f^2 - 0 = 4.56 \times 10^{-19} \text{ J}$$

Change in electric potential energy

$$\Delta V = V_f - V_i = \frac{\text{Work done}}{\text{charge}}$$

If  $q$  is the charge the work done

$$W = F \cdot \Delta x$$

$$\text{here } \Delta x = 1.5 \text{ cm} \quad \& \quad F = q E = q (190) \text{ N}$$

$$\text{so } \Delta V = \frac{q (190) \Delta x}{V} = (190)(1.5 \times 10^{-2})$$

$$\boxed{\Delta V = 2.85 \text{ V}}$$

$$V_f = \sqrt{\frac{2 \times 4.56 \times 10^{-19}}{9.1 \times 10^{-31}}} = [1.001 \times 10^6 \text{ m/s}]$$

If particle were an electron then  $q$  is negative  
so  $V$  is also negative positive since it is independent of charge

Hence, electric field set up  $E = \frac{F}{q} = \frac{q}{2C}$

Now, change in potential energy of each charge  $= q \frac{Ed}{2}$ , since they have travelled under electric force  $q, E$  (where  $q$  is the charge of each particle) a distance of  $\frac{d}{2}$ .

$$\therefore \text{Change in potential energy} = \frac{2qEd}{2} = \frac{q_1 q_2}{2C d} = \frac{q_1 q}{2C} = \frac{q_1 q}{260A}$$

$$\text{Now } d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}, E = 8.85 \times 10^{-12} \text{ N}^{-1} \text{ m}^{-2}, q_1 = 1.6 \times 10^{-19} \text{ C}$$

$$q = 1 \text{ nC} = 1 \times 10^{-9} \text{ C}, A = (2 \times 2) \text{ cm}^2 = 4 \text{ cm}^2 = 4 \times 10^{-4} \text{ m}^2$$

$$\therefore \text{Potential energy change} = \frac{8 \times 10^{-9} \times 1.6 \times 10^{-19} \times 2 \times 10^{-3}}{2 \times 8.85 \times 10^{-12} \times 4 \times 10^{-4}} = 0.045 \times 10^{-15} \text{ J}$$

A 2.0 cm x 2.0 cm parallel plate capacitor with a 2.0 mm spacing is charged to  $\pm 1.0 \text{ nC}$ . First a proton and then an electron are released from rest at the midpoint of the capacitor.

- What is each particle's energy?

Electron:  $E_{mech} = -4.5 \times 10^{-17} \text{ J}$   
Proton:  $E_{mech} = +4.5 \times 10^{-17} \text{ J}$

- What is each particle's speed as it reaches the plate?

Are your answers to part (b) plausible?

Electron:  $v_f = 2.3 \times 10^5 \text{ m/s}$   
Proton:  $v_f = 1.0 \times 10^7 \text{ m/s}$

Let  $v$  be the speed of electron and  $v_i$  be that of proton on reaching the respective plate,

$$\text{For electron, } \frac{1}{2} m_e v^2 = 0.045 \times 10^{-15} \Rightarrow v^2 = \frac{2 \times 0.045 \times 10^{-15}}{9.1 \times 10^{-31}} = 0.009830 \times 10^{16}$$

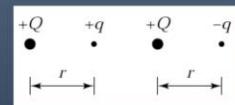
$$\therefore \text{Speed of electron, } v = 0.099 \times 10^8 = 9.9 \times 10^6 \text{ m/s}$$

$$\text{For proton, } \frac{1}{2} m_p v_i^2 = 0.045 \times 10^{-15} \Rightarrow v_i^2 = \frac{2 \times 0.045 \times 10^{-15}}{1.67 \times 10^{-27}} = 0.05389 \times 10^{12}$$

$$\therefore \text{Speed of proton, } v_i = 0.232 \times 10^6 \text{ m/s} = 2.32 \times 10^5 \text{ m/s}$$

Two test charges are brought separately into the vicinity of a charge  $+Q$ . First, test charge  $+q$  is brought to a point a distance  $r$  from  $+Q$ . Then this charge is removed and test charge  $-q$  is brought to the same point. The electrostatic potential energy of which test charge is greater:

- A.  $+q$
- B.  $-q$
- C. the same for both



Potential energy between 2 point charges depends on sign of both point charges.

$$U_{12} = \frac{k q_1 q_2}{r_{12}}$$

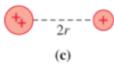
Rank in order, from largest to smallest, the potential energies  $U_a$  to  $U_d$  of these four pairs of charges. Each + symbol represents the same amount of charge.



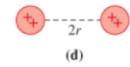
(a)



(b)



(c)



(d)

A proton is fired from very far away at a glass sphere ( $d = 1.00 \text{ mm}$ ) charged to  $+100 \text{ nC}$ . What initial speed is needed so that the proton just reaches the surface of the glass?

$$b = d > a = c$$

$$U_{12} = \frac{kq_1q_2}{r_{12}}$$

Model: Point Charges

$$v_i = 1.86 \times 10^7 \text{ m/s}$$

$$\frac{1}{2}m_p v^2 = \frac{K q_p q_s}{r}$$

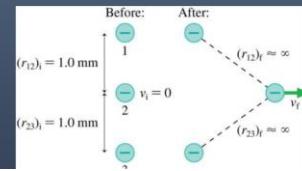
$$\frac{1}{2} \times 1.67 \times 10^{-27} \times v^2 = \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 100 \times 10^{-9}}{0.5 \times 10^{-3}}$$

$$v = 1.85 \times 10^7 \text{ m/s} \quad \underline{\text{Ans}}$$

Three electrons are spaced 1.0 mm apart along a vertical line. The outer two electrons are fixed in position. unstable

- Is the center electron at a point of stable or unstable equilibrium?
- If the center electron is displaced horizontally by a small distance, what will its speed be when it is very far away?

$$v_f = 1,000 \text{ m/s}$$



Electric potential at point 2 due to outermost electrons is  $V_2 = kq_1/r_1 + kq_3/r_3 = 2 \times 8.99 \times 10^9 \times 1.6 \times 10^{-19} / 0.001 = 2.8768 \times 10^{-6} \text{ V}$

=> Electric potential energy of electron  $E = qV = 1.6 \times 10^{-19} \times 2.8768 \times 10^{-6} = 4.60288 \times 10^{-25} \text{ J}$

=> from conservation of energy Electric PE at point 2 = KE at infinity

=>  $v = \sqrt{2 \times \text{KE}/m} = \sqrt{2 \times 4.60288 \times 10^{-25} / (9.1 \times 10^{-31})} = 1005.79 \text{ m/s}$

## ELECTRIC POTENTIAL

Electric Potential

Property of source charges

$$V = \frac{U}{q_0} \quad (\text{definition})$$

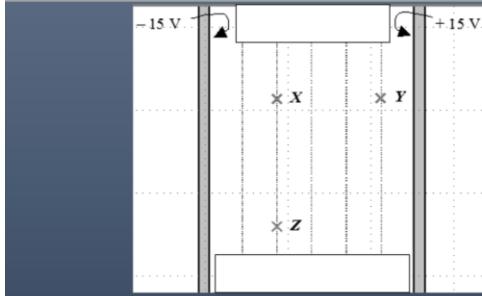
$1 V = 1 \text{ J/C}$

If  $W_{a \rightarrow b} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b q_0 \vec{E} \cdot d\vec{l}$

(2) *disturb energy in space somehow*

Then  $\frac{W_{a \rightarrow b}}{q_0} = -\Delta U_a = -\Delta V_{ba} = \int_a^b \vec{E} \cdot d\vec{l}$

so  $\Delta V_{ab} = V_a - V_b = + \int_a^b \vec{E} \cdot d\vec{l}$



rank  $\Delta U$  for each pair of points (for proton, then electron).

Proton:  $\Delta U_{YX} = \Delta U_{YZ} > \Delta U_{XZ}$ ; Electron:  $\Delta U_{YX} = \Delta U_{YZ} < \Delta U_{XZ}$

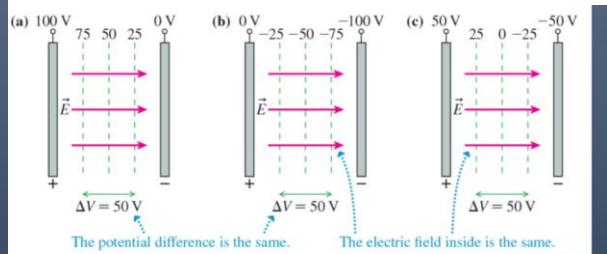
rank  $\Delta V$  for each pair of points (for proton, then electron).

Proton:  $\Delta V_{YX} = \Delta V_{YZ} > \Delta V_{XZ}$ ; Electron: Same.

- A positive charge slows down as it moves toward higher electric potential.
- The potential difference is positive.
- Lower potential  $\xrightarrow{\hspace{10em}}$  Higher potential
- Direction of increasing  $V$
- A positive charge speeds up as it moves toward lower electric potential.
- The potential difference is negative.

### Charged particles moving in an electric potential

Electric potential		
	Increasing ( $\Delta V > 0$ )	Decreasing ( $\Delta V < 0$ )
+ charge	Slows down	Speeds up
- charge	Speeds up	Slows down



For each diagram, which direction will (proton/electron) released from rest accelerate?

A proton is released from rest at the left plate. For which diagram (a, b, or c) will the proton be moving fastest at the right plate?

**Change** in potential is what matters

# Electric Potential of Point Charges

$$V_{point} = \frac{kq}{r}$$

$q$ : source charge;  
 $r$ : distance to measurement

Ex. Point Charge

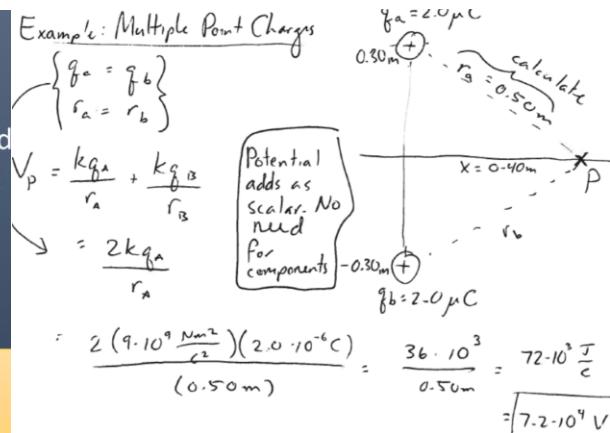
$$\begin{aligned} V(r) &= V(r) - 0 = V(r) - V(\infty) \\ &= \int_r^\infty \vec{E} \cdot d\vec{l} = \int_r^\infty \frac{kq}{r^2} \hat{r} \cdot \hat{r} dr \\ &= \int_r^\infty \frac{kq}{r^2} dr = -\left. \frac{kq}{r} \right|_r^\infty = 0 - \left( -\frac{kq}{r} \right) = \boxed{\frac{kq}{r}} \text{ V for point charge} \end{aligned}$$

Two charges, each  $+2.0 \mu\text{C}$ , are on the  $y$ -axis at  $y = +30 \text{ cm}$  and  $y = -30 \text{ cm}$ , respectively.

A couple of weeks ago, we found: electric field (magnitude and direction) at location  $P$ , lying on the  $x$ -axis, at  $x = 40 \text{ cm}$ . Pointed to the right

- a. Find the electric potential at the same location.

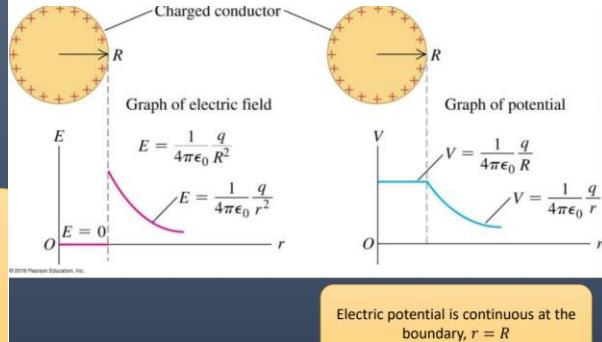
$$\text{Multiple Point Charges: } V = \sum_i \frac{kq_i}{r_i}$$



## Electric Potential of Continuous Charge Distributions

$$\text{Multiple Point Charges: } V = \sum_i \frac{kq_i}{r_i}$$

$$\text{Continuous distribution: } V = \int dV = \int \frac{k dq}{r}$$



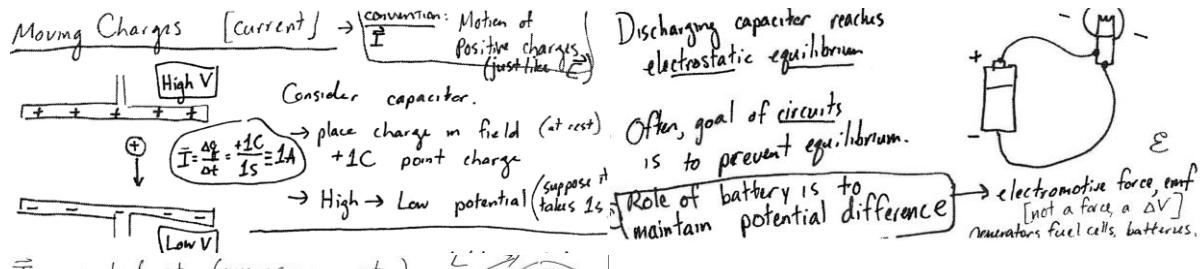
## Electric Field and Potential

$$\vec{E} = -\vec{\nabla}V$$

$$= \frac{\partial V}{\partial x} \hat{i} + \frac{\partial V}{\partial y} \hat{j} + \frac{\partial V}{\partial z} \hat{j}$$

$$\Delta V_{ab} = \int_a^b \vec{E} \cdot d\vec{l}$$

## RESISTANCE AND CAPACITANCE

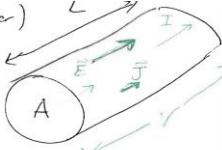


$\vec{J}$ : current density (microscopic, vector)

$I$ : current (macroscopic)

$\vec{J} = nqVd$  drift speed

What should resistance be proportional to?



Thinking microscopically...

Segment of a wire.

$$\vec{E} = \rho \vec{J}$$
 (Ohm's Law)
 

$\downarrow$

resistivity (material property)  
microscopic

$$\frac{\Delta V}{L} = \rho \frac{I}{A} \quad \rightarrow \quad \Delta V = \frac{\rho L}{A} I$$

\* Book says  $V$  instead of  $\Delta V$  for shorthand

macro what should  $I$  be proportional to?

$$I = \frac{\Delta V}{R} \quad \rightarrow \quad \Delta V = R I$$

Compare  $R = \frac{\rho L}{A}$  compare to predictions for  $R$  above.

Lab question.

Ex When connected to 120 V line, 60 W bulb carries ideal wire  
0.50-A current.  
 $\Rightarrow R = \frac{\Delta V_{\text{line}}}{I} = \frac{120 \text{ V}}{0.50 \text{ A}} = 240 \Omega$  resistance of the bulb's filament.

(\*) Note: tungsten resistor wires

Define Resistance  $I \propto \Delta V; R = \frac{\Delta V}{I}$  - measure of how hard to push charges through wire.

- $I = \frac{\Delta V_{\text{wire}}}{R}$  establishing potential difference creates electric field  $\rightarrow$  current
- \* smaller  $R \Rightarrow$  greater  $I$ .
- SI unit for  $R$  is ohm  $1 \text{ ohm} = 1 \Omega = 1 \text{ V/A}$

resistivity: material property (resistivity  $\rightarrow$  resistance as Young's mod.  $\rightarrow$  spring consta.)  
long/short; fat/thin  $\Rightarrow R = \frac{\rho L}{A}$  (Young's mod.  $\rightarrow$  spring consta.)

(\*) Note: tungsten resistor wires

More charge  $\Rightarrow$  more  $\Delta V$ .

$$\Delta V \propto Q ; \text{ proportion is } \frac{1}{C}$$

$$\Delta V = \frac{1}{C} \cdot Q ; C \text{ is capacitance}$$

Normally written:  $Q = C \Delta V$ . All capacitors

Capacitors store electrical potential energy

Demo: Put Ball inside charged capacitor.

Fix to parallel plates

$$E = \frac{Q}{\epsilon_0 A} , \Delta V_c = Ed$$

$$\Rightarrow Q = \frac{\epsilon_0 A}{d} \Delta V_c \Rightarrow C = \frac{\epsilon_0 A}{d}$$

$$C = \frac{Q}{\Delta V} ; 1 \text{ F} = 1 \text{ farad} = 1 \frac{C}{V} = 1 \frac{C^2}{J}$$

( $\mu\text{F}, \text{pF}$  more common)

e.g. Fix  $\Delta V$ . Greater  $C \Rightarrow$  more  $Q$  (for given  $\Delta V$ )  $\Rightarrow$  more energy.

parallel plates,

$$C_{\text{plate}} = \frac{K\epsilon_0 A}{d}$$

(depends only on geometry)  
 $\Rightarrow \epsilon_0 = 8.85 \cdot 10^{-12} \frac{\text{F}}{\text{m}}$

e.g. Spacing of keys on computer keyboard  
 is capacitor. Push  $\Rightarrow d \downarrow \Rightarrow C \uparrow$

→ a) from definition of  $C_{\text{plate}}$   $\rightarrow A = \frac{d \cdot C}{K\epsilon_0} = \frac{(5 \cdot 10^{-5} \text{m})(1.0 \cdot 10^{-6} \text{F})}{1(8.85 \cdot 10^{-12} \frac{\text{F}}{\text{m}})}$   
 Vacuum  $\Rightarrow K=1$   
 $A = 5.65 \text{ m}^2$  Enormous! How would a dielectric help?

$$\begin{aligned} 1) Q &= C \Delta V = (1.0 \cdot 10^{-6} \text{F})(1.5 \text{V}) \\ &= 1.5 \cdot 10^{-6} \frac{C}{V} \cdot V = [1.5 \cdot 10^{-6} \text{C}] \\ &= [1.5 \mu\text{C}] \end{aligned}$$

Energy Each time I add a dq, voltage goes up.  
 → spring ... stretch  $dx \Rightarrow$  force goes up.

$$\Delta U = dW_{\text{ext}} = F_{\text{ext}} dx = kx dx$$

$$\Delta U = W = k \int_{x_0}^{x'} x' dx' = \frac{1}{2} k(x')^2$$

$$\Delta U_{\text{capacitor}} = dW_{\text{ext}} = \cancel{dq} \frac{q}{C} dq = \frac{q}{C} dq$$

$$= \frac{1}{C} \int_0^Q q dq = \frac{1}{2} \frac{Q^2}{C}$$

$$1) U_{\text{capacitor}} = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} C \Delta V^2 = \frac{1}{2} Q \Delta V$$

across plates

"spring-loaded" vs "capacitor-loaded" (flash / defibrillator)

) energy density in electric field

$$\Delta V_c = Ed \quad (\text{uniform field})$$

$$C = \frac{K\epsilon_0 A}{d}$$

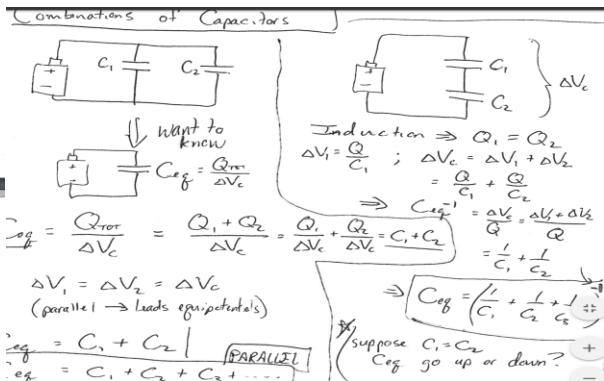
$$\Rightarrow U_c = \frac{1}{2} C(\Delta V_c)^2 = \frac{1}{2} \frac{K\epsilon_0 A}{d} (Ed)^2 = \frac{K\epsilon_0}{2} (Ad) E^2$$

but energy density:

$$u = \frac{U_c}{\text{Volume}} = \frac{K\epsilon_0}{2} (Ad) E^2$$

$$u_E = \frac{1}{2} K\epsilon_0 E^2$$

$$U_E (\text{in vacuum}) = \cancel{\frac{1}{2} K\epsilon_0 E^2}$$



Example 2

$$a) U_c = \frac{1}{2} C(\Delta V)^2 = \frac{1}{2} (220 \cdot 10^{-6} \text{F})(330 \text{V})^2 = [12 \text{kJ}]$$

$$(1 \text{F} = 1 \frac{\text{C}}{\text{V}})$$

b) discuss.  $\rightarrow \approx 1 \text{ms}$

$$c) P = \frac{\Delta U}{\Delta t} = \frac{12 \text{kJ}}{0.001 \text{s}} = [12,000 \text{kW}]$$

to 12 kJ is like raising  $\frac{1 \text{kg}}{1,000 \text{kg}}$  to  $h = 1.2 \text{m}$ . What if you had to do it in 1 ms? vs. 1 s

fibrillator: Online Example (see slide)

$$C = 100 \mu\text{F} \quad \Delta V = 2000 \text{V}$$

$$d = 0.01 \text{mm} \quad K = 300$$

$$a) C = \frac{K\epsilon_0 A}{d} \Rightarrow A = \frac{C d}{K\epsilon_0} = \frac{(100 \cdot 10^{-6} \text{F})(0.01 \cdot 10^{-3} \text{m})}{(300)(8.85 \cdot 10^{-12} \frac{\text{N} \cdot \text{C}^2}{\text{C}^2})}$$

$$A = 0.377 \text{ m}^2 = 3,770 \text{ cm}^2$$

$$b) u = \frac{U}{\text{Volume}} = \frac{\frac{1}{2} C \Delta V^2}{A \cdot d} = \frac{\frac{1}{2} (100 \cdot 10^{-6} \text{F})(2000 \text{V})^2}{(0.377 \text{ m}^2)(0.01 \cdot 10^{-3} \text{m})}$$

$$u = 5.3 \cdot 10^7 \frac{\text{J}}{\text{m}^3}$$

Notice, can write  $u$  in terms of  $E$ -field.

$$u = \frac{1}{2} \frac{C \Delta V^2}{A \cdot d} = \frac{1}{2} \frac{K \epsilon_0 A}{d} \cdot \frac{\Delta V^2}{A \cdot d} = \frac{1}{2} K \epsilon_0 \left( \frac{\Delta V}{d} \right)^2$$

$$u = \frac{1}{2} K \epsilon_0 E^2$$

The energy is stored in the electric field.

happens to the capacitance of the plates beneath it?  
(which is then detected by the rest of the circuit)

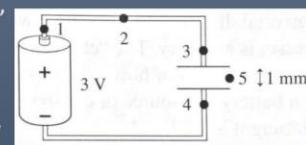
- A. Capacitance decreases
- B. Capacitance increases
- C. Capacitance remains the same



In the capacitance equation, there are three variables.

First, identify what remains constant ( $Q$ ,  $C$ , or  $V$ )

Then use equations for how capacitance changes to see how others change.



What happens if you pull the plates slightly apart while the battery is attached? Do the following increase, decrease, or stay the same?

$$\begin{matrix} \Delta V_{cap} \\ Q \\ C \end{matrix}$$

Which of the following is true about a battery connected to a circuit?

- A. The battery keeps electric potential (voltage) constant.
- B. The battery is a source of constant current; it keeps constant the charge flowing through the circuit.

The flash unit in a camera uses a 3.0 V battery to charge a capacitor. The capacitor is then discharged through a flash lamp. The discharge takes 10  $\mu$ s, and the average power dissipated in the flash lamp is 10 W.

- (a) [17 pts.] What is the energy stored in the charged capacitor?  
(b) [17 pts.] What is the capacitance of the capacitor?

(a)  $P_{avg} = \frac{\Delta E}{\Delta t}$  Energy stored in the capacitor

so  $\Delta E = P_{avg} \Delta t = (10 \text{ W})(10 \mu\text{s}) = 10^{-4} \text{ J}$

b)  $\Delta E = \frac{1}{2} Q V = \frac{1}{2} C V^2$

$C = \frac{Q}{V}$  so we can solve for  $C$  noting that  $V = 3 \text{ V}$

$$C = \frac{2 \Delta E}{V^2} = 2.2 \times 10^{-5} \text{ F}$$

Consider a capacitor made of two parallel metallic plates separated by a distance  $d$ . The top plate has a surface charge density  $+\sigma$ , the bottom plate  $-\sigma$ . A slab of metal of thickness  $l < d$  is inserted between the plates, not connected to either one. Upon insertion of the metal slab, the potential difference between the plates

- 1. increases.
- 2. decreases.
- 3. remains the same.

Consider a simple parallel-plate capacitor whose plates are given equal and opposite charges and are separated by a distance  $d$ . Suppose the plates are pulled apart until they are separated by a distance  $D > d$ . The electrostatic energy stored in the capacitor is

- 1. greater than
- 2. the same as
- 3. smaller than

before the plates were pulled apart.

A defibrillator unit contains a  $150 \mu\text{F}$  capacitor that is charged to  $2000 \text{ V}$ . The capacitor plates are separated by a  $0.010\text{-mm-thick}$  dielectric with  $\kappa = 300$ .

- a) What is the total area of the capacitor plates?
- b) What is the energy density stored in the electric field with the capacitor is charged?

A parallel-plate capacitor is attached to a battery that maintains a constant potential difference  $V$  between the plates. While the battery is still connected, a glass slab is inserted so as to just fill the space between the plates. The stored energy

- 1. increases.
- 2. decreases.
- 3. remains the same.

$$C = k * A * \epsilon / d$$

$$k = \text{permittivity of dielectric} = 300$$

$$\epsilon = 8.85 * 10^{-12} \text{ F/m}$$

$$d = 0.010 \text{ mm} = 0.010 * 10^{-3} \text{ m}$$

(I think there may have been mistake in units of t  
any variations)

$$\text{therefore } A = C * d / (k * \epsilon)$$

$$(b) A = 5.65 \times 10^3 \text{ m}^2 \text{ (answer)}$$

$$(a) E = V/d = Q/(A\epsilon)$$

$$Q = V \cdot A \cdot \epsilon / d$$

$$Q = 2000 \times 5.65 \times 10^3 \times 8.85 \times 10^{-12} / 0.010 \times 10^{-3}$$

$$Q = 10 \text{ C (answer)}$$

$$(c) \text{energy density} = (1/2)k\epsilon E^2$$

$$E = V/d = 2 \times 10^8$$

$$\text{energy density} = 5.31 \times 10^7 \text{ J/m}^3$$

Which of the following is true about a battery connected to a circuit?

- A. The battery keeps electric potential (voltage) constant.
- B. The battery is a source of constant current; it keeps constant the charge flowing through the circuit.

## MASTERING PHYSICS CORE TARGETS

Three point charges are arranged on a line. Charge  $q_1 = +5.00 \text{ nC}$  is at the origin. Charge  $q_2 = -2.00 \text{ nC}$  and is at  $x = 3.50 \text{ cm}$ . Charge  $q_3$  is at  $x = 1.00 \text{ cm}$ .

You may want to review (Pages 690 - 695).

For related problem-solving tips and strategies, you may want to view a Video Tutor Solution of Vector addition of electric forces on a line.

### Part A

What is  $q_1$  (magnitude and sign) if the net force on  $q_3$  is zero?

$$q_1 = 0.163 \text{ nC}$$

[Previous Answers](#)

Correct

IDENTIFY: Apply Coulomb's law. The two forces on  $q_3$  must have equal magnitudes and opposite directions.

SET UP: Like charges repel and unlike charges attract.

$$F_2 = k \frac{|q_1 q_3|}{r_2^2} \text{ and is in the } +x\text{-direction.}$$

$$F_1 \text{ must be in the } -x\text{-direction, so } q_1 \text{ must be positive. } F_1 = F_2 \text{ gives } k \frac{m_1 |q_1|}{r_1^2} = k \frac{|q_2 q_3|}{r_2^2}.$$

$$|q_1| = |q_2| \left( \frac{r_2}{r_1} \right)^2 = (2.00 \text{ nC}) \left( \frac{1.00 \text{ cm}}{3.50 \text{ cm}} \right)^2 = 0.163 \text{ nC.}$$

EVALUATE: The result for the magnitude of  $q_1$  doesn't depend on the magnitude of  $q_3$ .

Use the equilibrium force condition in the  $y$  direction.

Substitute  $T_y - F_g$  for  $\sum_y F$  in the equation  $\sum_y F = 0$  to solve for the tension force.

$$T_y - F_g = 0$$

$$T_y = F_g$$

Substitute  $T \cos \theta$  for  $T_y$  and  $mg$  for  $F_g$  in the equation  $T_y = F_g$ .

$$T \cos \theta = mg$$

$$T = \frac{mg}{\cos \theta}$$

Use the equilibrium force condition in the  $x$  direction.

Substitute  $F_E - T_x$  for  $\sum_x F$  in the equation  $\sum_x F = 0$  to solve for the electric force.

$$F_E - T_x = 0$$

$$F_E = T_x$$

A small 12.8 g plastic ball is tied to a very light 25.2 cm string that is attached to the vertical wall of a room. (See the figure.) A uniform horizontal electric field exists in this room. When the ball has been given an excess charge of  $-1.20 \mu\text{C}$ , you observe that it remains suspended, with the string making an angle of 17.4 degrees with the wall.



a) Find the magnitude of the electric field in the room.

b) Find the direction of the electric field in the room.

Substitute  $qE$  for  $F_E$  and  $T \sin \theta$  for  $T_x$  in the equation  $F_E = T_x$ .

$$qE = T \sin \theta$$

Substitute  $\frac{mg}{\cos \theta}$  for  $T$  in the equation  $qE = T \sin \theta$  and solve for electric field  $E$ .

$$qE = \left( \frac{mg}{\cos \theta} \right) \sin \theta$$

$$E = \frac{mg}{q} \tan \theta$$

Substitute  $-1.20\mu\text{C}$  for  $q$ ,  $12.8\text{g}$  for  $m$ ,  $9.8\text{m/s}^2$  for  $g$ , and  $17.4^\circ$  for  $\theta$  in the equation  $E = \frac{mg}{q} \tan \theta$ .

$$E = \frac{(12.8\text{g})(9.8\text{m/s}^2)}{(-1.20\mu\text{C})} \tan 17.4^\circ$$

$$= \frac{\left(12.8\left(\frac{10^{-3}\text{kg}}{1\text{kg}}\right)\right)(9.8\text{m/s}^2)}{\left(-1.20\mu\text{C}\left(\frac{10^{-6}\text{C}}{1\mu\text{C}}\right)\right)} \tan 17.4^\circ$$

$$= -3.28 \times 10^4 \text{N/C}$$

The magnitude of the electric field is  $3.28 \times 10^4 \text{N/C}$ .

The Tension force is along the string. Refer the diagram, the tension is at  $17.4^\circ$  from the vertical. The  $y$  component of the Tension force is upward so positive and force of gravity  $F_g$  is downward so negative. The  $x$  component of the Tension force is leftward so negative and force due to electric field is rightward so positive. Rearrange the equation to solve for the electric field. The magnitude of the electric field is absolute value of the electric field.

The relation used for unit conversion is as follows:

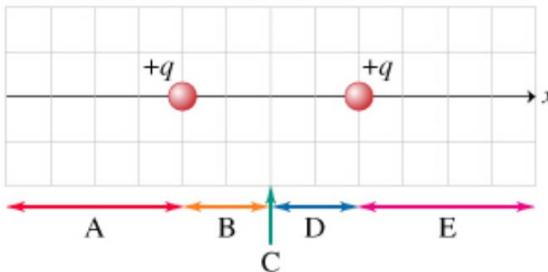
$$1\text{g} = 10^{-3}\text{kg}$$

$$1\mu\text{C} = 10^{-6}\text{C}$$

#### Step 2 of 2 ^

(b)  
The sign of charge is negative and direction of electric force is towards right. The electric field expression is negative. So, the direction of electric field is opposite to the direction of the electric force that is electric field points left.

Part b  
The direction of electric field is towards **Left**.



For the charge distribution provided, indicate the region (A to E) along the horizontal axis where a point exists at which the net electric field is zero. ([Figure 1](#))

If no such region exists on the horizontal axis choose the last option (nowhere).

► [View Available Hint\(s\)](#)

- A
- B
- C

For the charge distribution provided, indicate the region (A to E) along the horizontal axis where a point exists at which the net electric field is zero. ([Figure 2](#))

If no such region exists on the horizontal axis choose the last option (nowhere).

► [View Available Hint\(s\)](#)

- A
- B

For the charge distribution provided, indicate the region (A to E) along the horizontal axis where a point exists at which the net electric field is zero. (Figure 3)

If no such region exists on the horizontal axis choose the last option (nowhere).

► View Available Hint(s)

- A
- B
- C
- D
- E
- nowhere

For the charge distribution provided, indicate the region (A to E) along the horizontal axis where a point exists at which the net electric field is zero. (Figure 4)

► View Available Hint(s)

- A

A proton is placed in a uniform electric field of 3000 N/C.

You may want to review (Pages 695 - 699).

For related problem-solving tips and strategies, you may want to view a Video Tutor Solution of Electron in a uniform field.

▼ Part A

Calculate the magnitude of the electric force felt by the proton.

$$F = 4.80 \times 10^{-16} \text{ N}$$

Submit

Previous Answers

$$\text{a)} \quad F = qE \\ = 1.6 \times 10^{-19} \times 3000 = 4.8 \times 10^{-16} \text{ N}$$

$$\text{b)} \quad a = \frac{qE}{m} = \frac{4.8 \times 10^{-16}}{1.67 \times 10^{-27}} = 2.87 \times 10^{11} \text{ m/s}^2$$

$$\text{c)} \quad u = 0 \quad a = 2.87 \times 10^{11} \text{ m/s}^2 \quad t = 1.7 \times 10^{-6} \text{ s}$$

$$v = u + at$$

$$= 2.87 \times 10^{11} \times 1.7 \times 10^{-6}$$

$$v = 4.879 \times 10^5 \text{ m/s}$$

Flux is the amount of a vector field that "flows" through a surface. We now discuss the electric flux through a surface (a quantity needed in Gauss's law).  $\Phi_E = \int \vec{E} \cdot d\vec{A}$ , where  $\Phi_E$  is the flux through a surface with differential area element  $d\vec{A}$ , and  $\vec{E}$  is the electric field in which the surface lies. There are several important points to consider in this expression:

1. It is an integral over a surface, involving the electric field at the surface.
  2.  $d\vec{A}$  is a vector with magnitude equal to the area of an infinitesimal surface element and pointing in a direction normal (and usually outward) to the infinitesimal surface element.
  3. The scalar (dot) product  $\vec{E} \cdot d\vec{A}$  implies that only the component of  $\vec{E}$  *normal* to the surface contributes to the integral. That is,  $\vec{E} \cdot d\vec{A} = |\vec{E}| |d\vec{A}| \cos(\theta)$ , where  $\theta$  is the angle between  $\vec{E}$  and  $d\vec{A}$ .
- When you compute flux, try to pick a surface that is either parallel or perpendicular to  $\vec{E}$ , so that the dot product is easy to compute.

(Figure 1)  
Two hemispherical surfaces, 1 and 2, of respective radii  $r_1$  and  $r_2$ , are centered at a point charge and are facing each other so that their edges define an annular ring (surface 3), as shown. The field at position  $r$  due to the point charge is:

$$\vec{E}(\vec{r}) = \frac{C}{r^2} \hat{r}$$

where  $C$  is a constant proportional to the charge,  $r = |\vec{r}|$ , and  $\hat{r} = \vec{r}/r$  is the unit vector in the radial direction.

What is the electric flux  $\Phi_3$  through the annular ring, surface 3?

Express your answer in terms of  $C$ ,  $r_1$ ,  $r_2$ , and any constants.

[View Available Hint\(s\)](#)

$\Phi_3 = 0$

[Submit](#) [Previous Answers](#)

**Correct**

▼ Part B

What is the electric flux  $\Phi_1$  through surface 1?

Express  $\Phi_1$  in terms of  $C$ ,  $r_1$ ,  $r_2$ , and any needed constants.

[View Available Hint\(s\)](#)

$\Phi_1 = 2\pi C$

[Submit](#) [Previous Answers](#)

**Correct**

▼ Part C

What is the electric flux  $\Phi_2$  passing outward through surface 2?

Express  $\Phi_2$  in terms of  $r_1$ ,  $r_2$ ,  $C$ , and any constants or other!

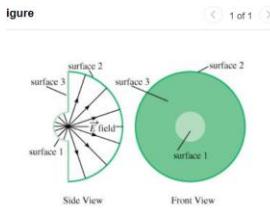
[View Available Hint\(s\)](#)

$\Phi_2 = 2\pi C$

[Submit](#) [Previous Answers](#)

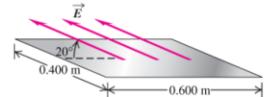
**Correct**

Observe that the electric flux through surface 1 is the same think in terms of field lines, this means that there is the same square,  $\frac{1}{r^2}$ , behavior of the electric field surrounding a point number of field lines that pass through the surface.



A flat sheet is in the shape of a rectangle with sides of length 0.400 m and 0.600 m. The sheet is immersed in a uniform electric field of magnitude 72.0 N/C that is directed at 20° from the plane of the sheet.

[Intro 1](#)



Find the magnitude of the electric flux through the sheet.

Calculate the magnitude of the electric flux through the plate.

The expression for the magnitude of the electric flux through the plate in the uniform electric field is,

$$\Phi_E = EA \cos \theta$$

Here,  $E$  is the magnitude of the uniform electric field,  $A$  is the area of the rectangular plate, and  $\theta$  is the angle between the electric field vector and area vector.

[Explanation](#) | [Common mistakes](#) | [Hint for next step](#)

The magnitude of the electric flux through the plate is equal to the scalar product between the electric field vector and area vector.

$$\Phi_E = \vec{E} \cdot \vec{A}$$

According to the definition of the dot product between the two vectors, the dot product between the electric field vector and area vector is as follows:

$$\vec{E} \cdot \vec{A} = EA \cos \theta$$

Substitute  $EA \cos \theta$  for  $\vec{E} \cdot \vec{A}$  in the equation  $\Phi_E = \vec{E} \cdot \vec{A}$  and solve for  $\Phi_E$ .

$$\Phi_E = EA \cos \theta$$

Step 2 of 2 ^

The area of the rectangular plate is,

$$A = lb$$

Substitute 0.600 m for  $l$  and 0.400 m for  $b$ .

$$A = (0.600 \text{ m}) (0.400 \text{ m})$$

$$= 0.240 \text{ m}^2$$

The angle between the electric field vector and area vector is equal to 70°.

$$\theta = 70^\circ$$

Substitute 72.0 N/C for  $E$ , 0.240 m<sup>2</sup> for  $A$ , and 70° for  $\theta$  in the equation  $\Phi_E = EA \cos \theta$  and solve for  $\Phi_E$ .

$$\Phi_E = (72.0 \text{ N/C}) (0.240 \text{ m}^2) \cos 70^\circ$$

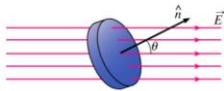
$$= 5.91 \text{ N} \cdot \text{m}^2/\text{C}$$

The magnitude of the electric flux through the sheet is 5.91 N · m<sup>2</sup>/C.

Suppose a disk with area  $A$  is placed in a uniform electric field of magnitude  $E$ . The disk is oriented so that the vector normal to its surface,  $\hat{n}$ , makes an angle  $\theta$  with the electric field, as shown in the figure. (Figure 1)

igure

1 of 1



Part A

What is the electric flux  $\Phi_E$  through the surface of the disk that is facing right (the normal vector to this surface is shown in the figure)? Assume that the presence of the disk does not interfere with the electric field.

Express your answer in terms of  $E$ ,  $A$ , and  $\theta$

[View Available Hint\(s\)](#)

$$\Phi_E = EA \cos \theta$$

[Submit](#)

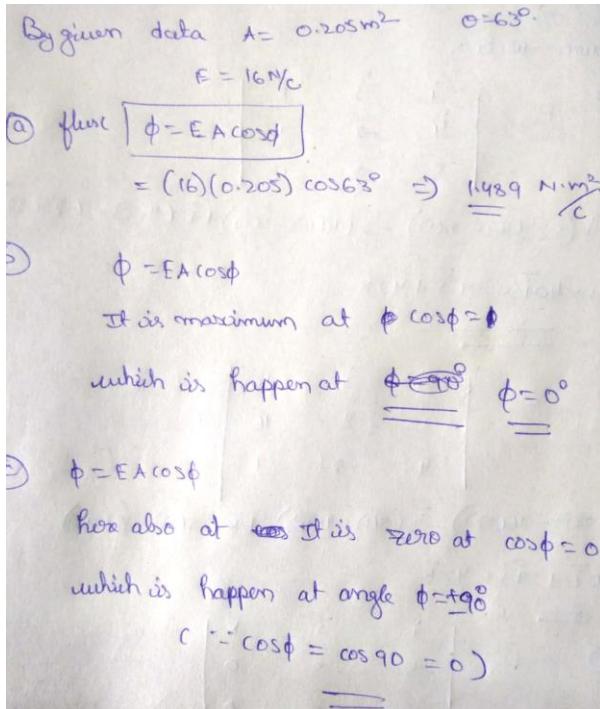
[Previous Answers](#)

Correct

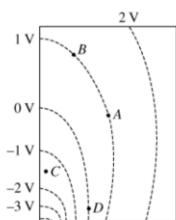
[Return to Assignment](#)

[Provide Feedback](#)

A flat sheet of paper of area  $0.295 \text{ m}^2$  is oriented so that the normal to the sheet is at an angle of  $59^\circ$  to a uniform electric field of magnitude  $11 \text{ N/C}$ .



The dashed lines in the diagram represent cross sections of equipotential surfaces drawn in  $1\text{-V}$  increments



What is the work  $W_{AB}$  done by the electric force to move a  $1\text{-C}$  charge from A to B?

What is the work  $W_{AD}$  done by the electric force to move a  $1\text{-C}$  charge from A to D?

The magnitude of the electric field at point C is

a) greater than the magnitude of the electric field at point B.

b) less than the magnitude of the electric field at point B.

c) equal to the magnitude of the electric field at point B.

d) unknown because the value of the electric potential at point C is unknown

The potential difference can be expressed as follows:

$$\Delta V = \frac{\Delta U}{q}$$

Here, q is the charge.

Rearrange the above equation for  $\Delta U$ .

$$\Delta U = q(\Delta V)$$

The change in the electric potential between the points A and B is,

$$\Delta V = V_B - V_A$$

Here,  $V_B$  is the potential at the point B and  $V_A$  is the potential at the point A.

Substitute  $V_B - V_A$  for  $\Delta V$  in the equation  $\Delta U = q(\Delta V)$  and solve for  $\Delta U$ .

$$\Delta U = q(V_B - V_A)$$

Replace  $\Delta U$  with  $q(V_B - V_A)$  in  $W = -\Delta U$ .

$$W = -q(V_B - V_A)$$

Since the points A and B are on the same potential line, they both will have equal amount of electric potential.

Substitute 1 C for q, 1 V for  $V_B$ , and 1 V for  $V_A$ .

$$W_{AB} = -(1C)(1V - 1V)$$

$$= 0$$

Thus, the work done by electric force to move a 1 C of charge from point A to point B is 0 J.

The electric field need not be constant over the equipotential surface. The electric field strength is inversely proportional to the spacing between the equipotential surfaces.

The equipotential lines are closer at point C than the equipotential lines at point B.

Hence, the magnitude of electric field at point C is not less than or equal to the magnitude of electric field at point B.

The field strength is stronger where the equipotential lines are closer together and weaker where they are farther apart.

The equipotential lines are closer at point C than that of at point B.

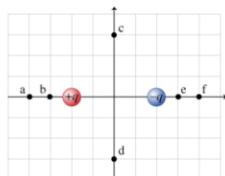
The value of electric potential at point C can be calculated and hence it is a known value.

The equipotential lines are closer at point C than the equipotential lines at point B.

Hence, the magnitude of electric field at point C is greater than the magnitude of electric field at point B.

Thus, the statement magnitude of electric field at point C is greater than the magnitude of electric field at point B is correct.

In the diagram below, there are two charges of  $+q$  and  $-q$  and six points (a through f) at various distances from the twocharges. (Intro 1 figure) You will beasked to rank changes in the electric potential along paths betweenpairs of points.



Q: Using the diagram to the left, rank each of the given pathson the basis of the change in electric potential. Rank the largest-magnitude positive change (increase in electric potential)as largest and the largest-magnitude negative change (decrease in electric potential) as smallest.

Rank from largest to smallest. To rank items as equivalent, overlap them.

- a. from c to b
- b. from c to d
- c. from c to e
- d. from d to a
- e. from b to a
- f. from f to e

Since, the given charge distribution represents a dipole and point c and d is at the equatorial position of dipole. The electric potential at the equatorial position of dipole is zero. Therefore, potential at point c and d is zero.

$$V_c = V_d = 0$$

The electric potential at point a due to two charges is given as follows:

$$V_a = \frac{kq}{2r} - \frac{kq}{6r}$$

$$= 0.33 \frac{kq}{r}$$

The electric potential at point b due to two charges is given as follows:

$$V_b = \frac{kq}{r} - \frac{kq}{5r}$$

$$= 0.8 \frac{kq}{r}$$

The electric potential at point e due to two charges is given as follows:

$$V_e = \frac{kq}{5r} - \frac{kq}{r}$$

$$= -0.8 \frac{kq}{r}$$

The electric potential at point f due to two charges is given as follows:

$$V_f = \frac{kq}{6r} - \frac{kq}{2r}$$

$$= -0.33 \frac{kq}{r}$$

The electric potential from c to b is given as follows:

$$V_{cb} = V_b - V_c$$

Substitute  $0.8 \frac{kq}{r}$  for  $V_b$  and 0 for  $V_c$  in the expression  $V_{cb} = V_b - V_c$ .

$$V_{cb} = 0.8 \frac{kq}{r} - 0$$

$$= 0.8 \frac{kq}{r}$$

The electric potential from c to d is given as follows:

$$V_{cd} = V_d - V_c$$

Substitute 0 for  $V_d$  and 0 for  $V_c$  in the expression  $V_{cd} = V_d - V_c$ .

$$V_{cd} = 0 - 0$$

$$= 0$$

The electric potential from c to e is given as follows:

$$V_{ce} = V_e - V_c$$

Substitute  $-0.8 \frac{kq}{r}$  for  $V_e$  and 0 for  $V_c$  in the expression  $V_{ce} = V_e - V_c$ .

$$V_{ce} = -0.8 \frac{kq}{r} - 0$$

$$= -0.8 \frac{kq}{r}$$

Therefore, the electric potential is given as follows:

$$V_{cb} = 0.8 \frac{kq}{r}$$

$$V_{cd} = 0$$

$$V_{ce} = -0.8 \frac{kq}{r}$$

$$V_{da} = 0.333 \frac{kq}{r}$$

$$V_{ba} = -0.467 \frac{kq}{r}$$

$$V_{fe} = -0.467 \frac{kq}{r}$$

On comparing, the rank of electric potential is given as follows:

$$V_{cb} > V_{da} > V_{cd} > (V_{ba} = V_{fe}) > V_{ce}$$

The rank of electric potential from largest to smallest is  $V_{cb} > V_{da} > V_{cd} > (V_{ba} = V_{fe}) > V_{ce}$ .

The electric field at the surface of a charged, solid, copper sphere with radius 0.250 m is 3900 N/C directed toward the center of the sphere. What is the potential at the center of the sphere, if we take the potential to be zero infinitely far from the sphere?

View comments (2) >

## Expert Answer



Luke Adams answered this

2,530 answers

Was this answer helpful? 9

Radius  $r = 0.21$  m

Electric field  $E = 3380$  N / C

The electric field is expressed as follows:

$$E = k \frac{q}{r^2}$$

Or

$$Er = \frac{kq}{r}$$

We know potential at the surface of the sphere is,

$$V = k \frac{q}{r}$$

Therefore, potential at the surface of the sphere is,

$$V = -Er$$

We know potential at any point inside the sphere is equal to the potential at the surface of the sphere.

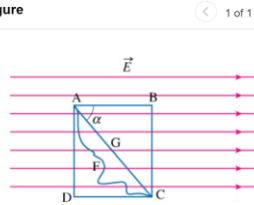
Therefore, potential at the center of the sphere is  $-975V$ .

To review the concept of conservative forces and to understand that electrostatic forces are, in fact, conservative.

As you may recall from mechanics, some forces have a very special property, namely, that the work done on an object does not depend on the object's trajectory; rather, it depends only on the initial and the final positions of the object.

Such forces are called *conservative forces*. If only conservative forces act within a closed system, the total amount of mechanical energy is conserved within the system (hence the term "conservative"). Such

#### Figure



#### Part C

Calculate the total amount of work  $W_{ABC}$  done by the electrostatic force on the charged particle as it moves from A to B to C.

Express your answer in terms of some or all the variables  $E$ ,  $q$ ,  $L$ , and  $\alpha$ .

$$W_{ABC} = qEL$$

[Submit](#) [Previous Answers](#)

Correct

#### Part D

Now assume that the particle "chooses" a different way of traveling. Calculate the total amount of work  $W_{ADC}$  done by the electrostatic force on the charged particle as it moves from A to D to C.

Express your answer in terms of some or all the variables  $E$ ,  $q$ ,  $L$  and  $\alpha$ .

$$W_{ADC} = qEL$$

[Submit](#) [Previous Answers](#)

Correct

Since  $W_{AB} = W_{BC}$  and  $W_{BC} = W_{CD}$ , it is clear that  $W_{ABC} = W_{ADC}$ . It appears that the work done by the electrostatic force on the particle is the same for both paths that begin at point A and end at point C. We now have a reasonable suspicion that this force may, in fact, be conservative. Let us check some more.

$$W_{AGC} = qEL$$

[Submit](#) [Previous Answers](#)

Correct

Though we have not proved it, it can be shown that the Coulomb force is indeed conservative. This implies that the amount of work  $W_{AFC}$  done by the electrostatic force on the charged particle as it moves in a curved path from A to F to C is also equal to  $qEL$ .

With the knowledge that the Coulomb force is conservative, and again referring to the diagram, answer the following questions. These questions are meant to highlight some important properties of conservative forces.

#### Part F

Find the amount of work  $W_{BA}$  done by the electrostatic force on the charged particle as it moves along the straight path from B to A.

Express your answer in terms of some or all the variables  $E$ ,  $q$ ,  $L$ , and  $\alpha$ .

[View Available Hint\(s\)](#)

$$W_{BA} = -qEL$$

[Submit](#) [Previous Answers](#)

Correct

The angle  $\theta$  between the force and the displacement is  $180^\circ$  here, so  $\cos \theta = -1$ , and the general formula for work becomes  $W = -Fd$ . Note that  $W_{BA} = -W_{AB}$ .

The amount of work  $W_{ABA}$  done by the electrostatic force on the charged particle as it moves from A to B to A is equal to

$$\begin{aligned} W_{ABA} &= W_{AB} + W_{BA} \\ &= W_{AB} + (-W_{AB}) \\ &= 0. \end{aligned}$$

#### Part G

Find the amount of work  $W_{ABCDA}$  done by the electrostatic force on the charged particle as it moves from A to B to C to D to A.

Express your answer in terms of some or all the variables  $E$ ,  $q$ ,  $L$  and  $\alpha$ .

$$W_{ABCDA} = 0$$

[Submit](#) [Previous Answers](#)

Correct

Another important property of conservative forces, which can be very helpful in problem solving, is that the total work done by a conservative force over a closed path is zero.

Three equal point charges, each with charge  $1.60 \mu\text{C}$ , are placed at the vertices of an equilateral triangle whose sides are of length  $0.700 \text{ m}$ . What is the electric potential energy  $U$  of the system? (Take as zero the potential energy of the three charges when they are infinitely far apart.)

Use  $\epsilon_0 = 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}$  for the permittivity of free space.

[View Available Hint\(s\)](#)

$$U = 9.86 \times 10^{-2} \text{ J}$$

[Submit](#) [Previous Answers](#)

Correct

The potential energy is usually written

$$U = \sum_{i < j} \frac{q_i q_j}{4\pi\epsilon_0 r_{ij}}$$

This means that all pairs of charges (1-2, 1-3, and 2-3) will interact, but no charge can interact with itself ( $i = j$ ), nor can any pair be counted twice as a result of the condition  $i < j$  for all possible pairs. For example,  $i = 1, j = 2$  will be counted, while  $i = 2, j = 1$  will not.

5. A particle with charge  $+7.60 \text{ nC}$  is in a uniform electric field directed to the left. Another force, in addition to the electric force, acts on the particle such that when it is released from rest, it moves to the right. After it has moved  $8.0 \text{ cm}$ , the additional force has done  $6.50 \times 10^{-5} \text{ J}$  of work and the particle has  $4.35 \times 10^{-5} \text{ J}$  of kinetic energy. (a) What work was done by the electric force? (b) What is the potential of the starting point with respect to the end point? (c) What is the magnitude of the electric field?

Charge of the particle is  $q = +7.60 \text{ nC}$

$$= 7.60 \times 10^{-9} \text{ C}$$

Direction of the electric field is towards left.

Now let us consider a co-ordinates system so that  $+x$  axis is towards right and negative  $x$  axis is towards left.

Here the applied force is acting on the particle towards right.

Workdone by the electric field on the charged particle is

$$W_{\text{electric}} = -qEd < 0$$

Workdone by the applied force on the charged particle is

$$W_{\text{applied}} = Fd = 6.50 \times 10^{-5} \text{ J}$$

Kinetic energy of the charged particle is  $K = 4.35 \times 10^{-5} \text{ J}$

According to the law of conservation of energy,

$$W_{\text{electric}} + W_{\text{applied}} = K$$

$$W_{\text{electric}} = K - W_{\text{applied}}$$

$$= 4.35 \times 10^{-5} \text{ J} - 6.50 \times 10^{-5} \text{ J}$$

$$= -2.15 \times 10^{-5} \text{ J}$$

The difference in **electric** potential is the work done by the **particle** on the **field** normalized by its **charge**  $q$  is

$$\Delta V = -\frac{W_{\text{electric}}}{q}$$

$$= -\frac{-2.15 \times 10^{-5} \text{ J}}{7.60 \times 10^{-9} \text{ C}}$$

$$= 2.83 \times 10^3 \text{ V}$$

$$= 2.83 \text{ kV}$$

(c)

Magnitude of the workdone by the electric field is

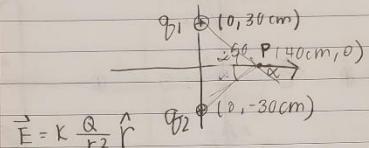
$$W_{\text{electric}} = qEd$$

$$E = \frac{W_{\text{electric}}}{qd} = \frac{2.15 \times 10^{-5} \text{ J}}{(7.60 \times 10^{-9} \text{ C})(0.08 \text{ m})}$$

$$= 35.4 \times 10^3 \text{ V/m}$$

$$= 35.4 \text{ kV/m}$$

### Quantitative Problem



$$k = 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}$$

$$\vec{E}_1 = \frac{2 \times 10^{-9}}{(5)^2} \left( 8.99 \times 10^9 \right) = 71.92 \quad E_{1y} = -E_{2y}$$

$$2 |E_1| \cos \alpha$$

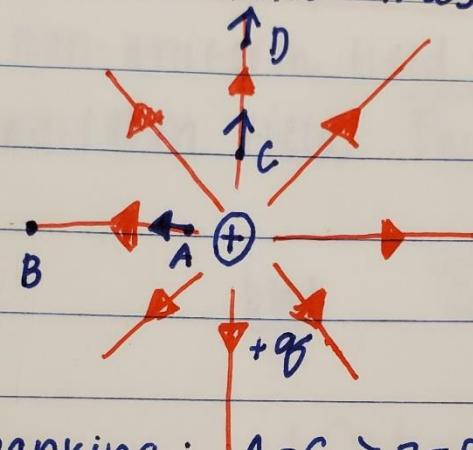
$$2 |E_1| \frac{4}{5}$$

$$2 \left( \frac{k q_1}{r^2} \right) \left( \frac{40 \text{ cm}}{50 \text{ cm}} \right) \left( \frac{4}{5} \right) \left[ 11.52 \times 10^4 \frac{\text{N}}{\text{C}^2} \right]$$

$$E_{1x} = E_{2x}$$

$$E_{\text{net}} = 2 |E_{1x}|$$

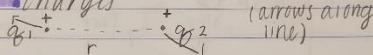
### Electric Field lines -



Ranking:  $A=C > B=D$

### Polarized

CAVIOMO'S LAW - ELECTRIC FORCE BETWEEN TWO POINT CHARGES



$$|\vec{F}_{1 \rightarrow 2}| = \frac{k q_1 q_2}{r^2} = \frac{k |q_1| |q_2|}{r^2} \quad \text{magnitude}$$

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \quad \text{SF UNIT}$$

$$8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2}$$

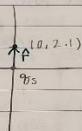
electrons have the fundamental charge, which is

$$e = 1.6 \times 10^{-19} \text{ C}$$

$q_0$  = test charge  
 $q_s$  = source charge

What is a field?  $\vec{F} = \frac{\vec{r}}{r^2}$

### Quantitative Example (21.33)



$$a) |\vec{r}| = \sqrt{(0)^2 + (2.1)^2} = 2.1 \quad \vec{r} = \frac{0\hat{i} + 2.1\hat{j}}{2.1} = \hat{j}$$

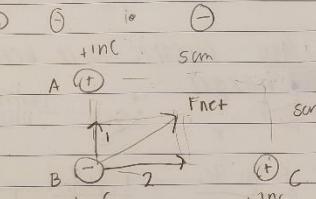
$$b) \vec{r} = \frac{-0.8\hat{i} - 0.8\hat{j}}{\sqrt{0.0128}} \quad |\vec{r}| = \sqrt{(0.8)^2 + (0.8)^2} = \sqrt{0.0128}$$

$$\vec{r} = -0.707\hat{i} - 0.707\hat{j}$$

$$c) |\vec{r}| = \sqrt{9^2 + (2.1)^2} = \sqrt{8.1}$$

$$\vec{r} = \frac{0.9\hat{i} - 2.1\hat{j}}{\sqrt{8.1}}$$

$$\vec{r} = 0.316\hat{i} - 0.487\hat{j}$$



$$\vec{F}_{\text{net}} = \vec{F}_{C \rightarrow B} + \vec{F}_{A \rightarrow B}$$

$$\vec{F}_{\text{net}} = \frac{k q_1 q_2 (-\hat{i})}{(0.05)^2} + \frac{k q_1 q_2 (\hat{j})}{(0.05)^2}$$

$$|\vec{F}| = \sqrt{|\vec{F}_{C \rightarrow B}|^2 + |\vec{F}_{A \rightarrow B}|^2} = 8.05 \times 10^{-6} \text{ N}$$

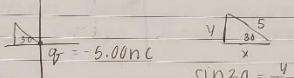
### Guided Practice Exercise

$$1) \vec{E} = k \frac{Q}{r^2} \hat{r}$$

$$2) \vec{E} = \frac{\vec{F}}{q_0 \text{test}} = \frac{0\hat{i} - 9.6 \times 10^{-17} \text{ N} \hat{j}}{2.0(1.6 \times 10^{-19} \text{ C})} = \frac{-9.6 \times 10^{-17} \text{ N} \hat{j}}{3.2 \times 10^{-19} \text{ C}} = -300 \text{ N/C}$$

3) i) electric field lines point different directions.  $E$  is smaller near + charge, larger near - charge.

4a)



$$y = 2.5$$

$$x = \frac{5\sqrt{3}}{2}$$

$$\sin 30 = \frac{y}{5}$$

$$5 \sin 30 = 2.5$$

$$\cos 30 = \frac{x}{5}$$

$$5 \cos 30 = x$$

$$\vec{r} = 2.5\hat{i} - \frac{5\sqrt{3}}{2}\hat{j}$$

$$\vec{r} = \frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}$$

Proportional reasoning with E-field

- draw field vectors  $\vec{E}_1$  and  $\vec{E}_2$  due to  $q_1$  and  $q_2$ , and the net  $E_{\text{net}}$ .

$$\vec{E}_1 = \frac{k(1/2)}{5} = \frac{-2k}{5} \hat{i}$$

$$|\vec{r}| = \sqrt{(2)^2 + 1^2} = \sqrt{5}$$

$$\vec{E}_{\text{net}} = m\vec{a} = q_0 \vec{E}$$

$$\vec{a} = \frac{q_0 \vec{E}}{m}$$

- b) suppose a point charge,  $q_{20}$ , is placed at location P. Find its acceleration if:
- $q_{20} = +4.0 \mu\text{C}$
  - $q_{20} = -4.0 \mu\text{C}$

$$a) |E_1| = |E_2| = \frac{1}{4\pi\epsilon_0} \frac{q_1}{r_1^2} = \frac{1}{4\pi\epsilon_0} \frac{q_2}{r_2^2}$$

$$E_{\text{net}} = 2E_y = \frac{2q_2}{4\pi\epsilon_0} \left( \frac{1}{d/2} + \frac{1}{x} \right) \frac{d/2}{\sqrt{(d/2)^2 + x^2}} \hat{j}$$

$$\vec{E}_{\text{net}} = \frac{-qd}{4\pi\epsilon_0 [(d/2)^2 + x^2]^{3/2}} \hat{j}$$

(B) For  $x \gg d$  (far away)

$$x^2 \gg (d/2)^2$$

$$\Rightarrow E_{\text{faraway}} = \frac{qd}{4\pi\epsilon_0 x^2} = \frac{P}{4\pi\epsilon_0 x^2}$$

$x \gg d$

example 21.14  $\Rightarrow$  along y-axis  $E_{\text{faraway}} = \frac{P}{2\pi\epsilon_0 y^3}$

we use linear charge density such as

$$\lambda = \frac{Q}{L} \quad \sigma = \frac{Q}{A} \quad \rho = \frac{Q}{V}$$

$$dE = \frac{k dq}{r^2} \quad \text{* all x comp. cancel}$$

$$(dE_x) = \frac{k dq}{r^2} \sin\theta$$

$$dE_x = \frac{k dq}{r^2} \sin\theta$$

$$E_y = + \int \frac{k dq}{r^2} \sin\theta$$

$$dq = \lambda dx \quad \lambda = \frac{Q}{L} = \frac{dQ}{dx}$$

$$E_y = \int \frac{k \lambda dx}{r^2} \cdot \frac{R}{\sqrt{R^2 + x^2}}$$

$$r = \sqrt{R^2 + x^2} \quad \sin\theta = \frac{R}{\sqrt{R^2 + x^2}}$$

$$E_y = k \lambda R \left( \frac{x}{R^2 + x^2} \right)^{1/2}$$

\* plug in

$$= k \lambda R \left( \frac{L}{R^2 + (R^2 + \frac{L^2}{4})} \right)^{1/2}$$

$$= k \lambda \left( \frac{L}{R \sqrt{R^2 + \frac{L^2}{4}}} \right)$$

$$= \frac{k Q}{L} \left( \frac{R}{R \sqrt{R^2 + \frac{L^2}{4}}} \right)$$

$$E_y = \frac{k Q}{R \sqrt{R^2 + \frac{L^2}{4}}} \quad \leftarrow \text{electric field at point}$$

(like video notes)  
Uniformly charged rod

FIND THE electric field at location P (on the x-axis at  $x=d$ ) created by a 60 cm long rod of total charge  $Q$ .

$$r = \sqrt{x^2 + y^2}$$

$$\vec{E} = \frac{k Q}{r^2} \hat{r}$$

$$\lambda = \frac{Q}{L}$$

$$dQ = \lambda dx \quad \lambda = \frac{Q}{L}$$

$$r = \frac{dx}{L} = \frac{dQ}{dQ} \cdot \frac{dy}{dQ}$$

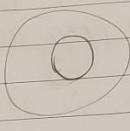
GROUP QUESTION

a)  $\Phi_3 = 0$

b)  $\Phi_2 = 2\pi C$

c)  $\Phi_1 = 2\pi C$

$$\Phi_3 = EA \cos \phi$$



$$\Phi_3 = \frac{c}{r^2} r^2 \cdot \pi \cos \phi$$

$$\Phi_1 = \frac{c}{r^2} r^2 \cdot \pi / 2 \cos \phi = \pi r^2 C$$

$$\Phi_2 = \frac{c}{r^2} r^2 \cdot 2\pi r$$

a)  $\phi = 90^\circ, \cos 90^\circ = 0$

b)  $\phi = 0^\circ, \cos 0^\circ = 1$

c)  $\phi = 0^\circ, \cos 0^\circ = 1$

$$\Delta E_x = \frac{k \Delta Q}{x^2 + y^2} \frac{x}{\sqrt{x^2 + y^2}} \left( \frac{k A Q d}{(d^2 + y^2)^{3/2}} \right) = \frac{k x A y d}{(d^2 + y^2)^{3/2}}$$

$$\Delta E_y = - \frac{k \Delta Q}{x^2 + y^2} \frac{y}{\sqrt{x^2 + y^2}} \left( \frac{-k A Q y}{(d^2 + y^2)^{3/2}} \right) = \frac{k x A y y}{(d^2 + y^2)^{3/2}}$$

out

$$3) \sum_{i=1}^{10} \Delta E_{x,i} = \sum_{i=1}^{10} \frac{k x d \Delta y}{(d^2 + y^2)^{3/2}}$$

$$k x d \int_{-a}^a \frac{dy}{(d^2 + y^2)^{3/2}} = \frac{k x d (2a)}{d \sqrt{d^2 + a^2}} \stackrel{1}{=} \vec{E}$$

$$b) \cos 0^\circ \int F dA = E \cos 0^\circ 2\pi r^2$$

$$\frac{c}{r^2} \cdot x \cdot 2\pi r^2 = \boxed{2\pi C}$$

Group Problem - 22.4 9.10.19

$$\lambda = 4.60 \quad E = \frac{\lambda}{2\pi\epsilon_0 r} \quad r = 0.135 \text{ m} \quad l = .5 \text{ m}$$

a)

$$\Phi_E = \int \vec{E} \cdot d\vec{A}$$

$$= \frac{4.60}{2\pi\epsilon_0 r^2} \cdot \int d\vec{A}$$

$$4.60 \mu\text{C/m} = \frac{4.60(l+r)}{\epsilon_0} = 4.60(5)$$

$$\epsilon_0$$

$$\Phi_E = 2.6 \times 10^{-5} \frac{\text{N} \cdot \text{m}^2}{\text{C}}$$

b) same answer ) doesn't depend on radius

c)  $\frac{2l}{\epsilon_0} = \frac{4.6 \times 10^{-5}}{8.85 \times 10^{-12}} \text{ m}$

GAUSS'S LAW  $\oint \vec{E} \cdot d\vec{A} = \frac{\Sigma \text{enclosed}}{\epsilon_0}$

Determine 3 symmetries for which Gauss's law may be useful

a plane      sphere  
a cylinder      point charge

Example 2 - A charged, insulating sphere 9.12.19

- radius of sphere is  $R$

- positively charged, total charge  $Q$

- uniform charge density,  $\rho = \rho_0$

Find the field both inside and outside the sphere, as a function of distance,  $r$ .

Solution: split to two parts.

- i)  $r < R$  ( $Q_{\text{enc}} \text{ depends on size of Gaussian}$ )
- i)  $r > R$  ( $Q_{\text{enc}} \text{ same for any Gaussian}$ )

i)  $r < R$ :  $\oint \vec{E} \cdot d\vec{A} = Q_{\text{enc}} \frac{r^2 \pi}{\epsilon_0}$  (inside sphere)

$$\vec{E} \frac{4\pi r^2}{\epsilon_0} = \frac{1}{\epsilon_0} \cdot \frac{Q}{r^3} \cdot \vec{r} d\Omega, \text{ but in this example, } \rho = \rho_0 \text{ is constant}$$

$$E(\text{inside}) = \frac{\rho_0}{\epsilon_0} \left( \frac{4}{3} \pi r^3 \right) \cdot \frac{1}{r}$$

$$E = \frac{\rho_0 r}{3\epsilon_0} = \left( \frac{1}{3} \pi r^3 \right) \cdot \frac{3\epsilon_0}{r}$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{Qr}{R^3} \cdot \frac{1}{r} \quad (\text{linear})$$

ii)  $r > R$   $\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$  now, total  $Q = Q_{\text{enc}}$

$$EA = \frac{Q}{\epsilon_0}$$

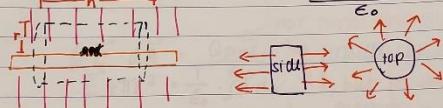
$$E(4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{4\pi r^2 \epsilon_0} \quad (\text{from symmetry})$$

Gauss surface  $r > R$

2. is chosen such that the field is the same magnitude everywhere on the surface:

such that:  $\Phi_{\text{ext}} = \oint \vec{E} \cdot d\vec{A} = EA \cos 0^\circ = EA = \frac{\Sigma \text{enclosed}}{\epsilon_0}$



$$\oint \vec{E} \cdot d\vec{A} = \frac{\Sigma \text{enclosed}}{\epsilon_0}$$

$$\vec{E} \oint d\vec{A} = \frac{\Sigma \text{enclosed}}{\epsilon_0}$$

$$\vec{E} \text{ cylinder} = \frac{2\pi r}{\epsilon_0}$$

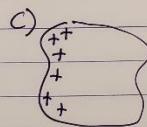
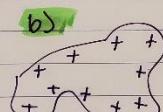
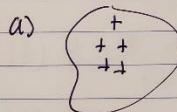
$$E(2\pi r) = "$$

$$E 2\pi r = \frac{\lambda}{\epsilon_0}$$

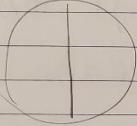
$$E = \frac{\lambda}{2\pi\epsilon_0 r} \cdot \boxed{\frac{2\pi r}{r}}$$

### conductors in electrostatic equilibrium

which one is in equilibrium? (no external field)



## charge on Thundercloud

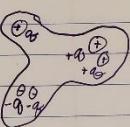


$$d = 1 \text{ km} \quad r = 5 \text{ km} \\ = 500 \text{ m}$$

Example - GP exercise  
what is the flux through surface?

integral would be complicated. a.  
look at enclosed charge instead.

$$\Phi = \frac{\Sigma q_{\text{enclosed}}}{\epsilon_0} = \frac{3q - 2a}{\epsilon_0} = \frac{q}{\epsilon_0}$$



Guided PROB - Questions

$$p(r) = p_0 \left(1 - \frac{r}{R}\right)$$

$$p(R) = 0$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{1}{\epsilon_0} \int p dV$$

$$E(4\pi r^2) = \frac{1}{\epsilon_0} \int p_0 \left(1 - \frac{r}{R}\right) dV$$

$$= \frac{p_0}{\epsilon_0} \int \left(\frac{1}{R}r - \frac{1}{R}\right) \int r dV$$

$$Q = \int p(r) dV$$

$$Q = p_0 \int \left(1 - \frac{r}{R}\right) r^2 \sin\theta dr d\theta d\phi$$

$$Q = p_0 \int \left(\frac{1}{R}r^2 - \frac{r^3}{R^2}\right) \sin\theta dr$$

(a) i  $r < a$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

$$EA = \frac{+Q}{\epsilon_0}$$

$$E = \frac{+Q}{\epsilon_0 A} = \frac{+Q}{\epsilon_0 4\pi r^2}$$

point charge  
radially outward  
Gaussian surface

ii  $a < r < b$

This is a conductor

so  $E = 0$  inside.

What if it has an insulator?

iii  $r > b$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

$$EA = \frac{+Q - 3Q}{\epsilon_0}$$

$$E = \frac{-2Q}{\epsilon_0 A} = \frac{-2Q}{4\pi\epsilon_0 r^2}$$

radially inward

b) for  $a < r < b$ , must enclose zero net charge since  $E = 0$  inside conductor  $\Rightarrow$  total charge on inner surface is  $-Q$  (opposite of  $Q$ ) and surface charge density  $\sigma_{\text{inner}} = \frac{-Q}{4\pi a^2}$

$$E_b = 3.00 \times 10^6 \text{ N/C}$$

$$3.00 \times 10^6 = \frac{kQ}{(500\text{m})^2} \quad Q = 83.4 \text{ C}$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

$$Q_{\text{enclosed}} = \epsilon_0 E_b (4\pi r^2)$$

only vertical components add

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

$$E(2\pi r^2) = \frac{\sigma A}{\epsilon_0}$$

$$E(2\pi r^2) = \frac{\sigma (4\pi r^2)}{2\epsilon_0}$$

add top plate to bottom plate, so  $\times 2$   $\leftarrow$  infinite sheet

$$\text{so, } E = \frac{\sigma}{\epsilon_0}$$

inside capacitor

c) Net charge on shell is  $-3Q$ . We know from part (b) that inner surface is  $-Q$ .

$$-Q \underset{\text{inner}}{+} \underset{\text{outer}}{-} = -3Q \underset{\text{total}}{+}$$

$$\sigma_{\text{outer}} = -2Q$$

$$\Rightarrow \sigma_{\text{outer}} = \frac{\sigma_{\text{outer}}}{4\pi b^2} = \frac{-2Q}{4\pi b^2}$$

b) Apply energy conservation

$$\begin{cases} K_f + U_f = K_i + V_i \\ \frac{1}{2} m v_f^2 + 0 = E_{\text{mech}} \\ (\text{with plate}) \end{cases} \Rightarrow \frac{(V_f)_p}{m_p} = \sqrt{\frac{2E_{\text{mech}}}{m_p}} = 2.3 \times 10^7 \text{ m/s}$$

↑ smaller speed  
↓ larger mass

$$\frac{1}{2} m v_f^2 + q Ed = E_{\text{mech}}$$

$$\frac{1}{2} m v_f^2 - eEd = E_{\text{mech}} \Rightarrow (V_f)_c = \sqrt{\frac{2E_{\text{mech}}}{m_e}} = 1.0 \times 10^7 \text{ m/s}$$

$$\frac{1}{2} m v_f^2 + 2E_{\text{mech}} = E_{\text{mech}}$$

↑ smaller mass

$$-(V_f/3 - (V_{12i} + V_{13i} + V_{23i}))$$

$$V_{12i} + V_{23i} = \frac{kq_1q_2}{r_{12}} + \frac{kq_2q_3}{r_{23}}$$

$$= \frac{kq_2}{r_{12}(q_1 + q_3)}$$

$$\frac{1}{2} m(v_f^2 - v_i^2) = q \cdot 10^9 (-e)(-e - e)$$

$$V_f = 1000 \text{ m/s}$$

9.17.19

How to break free from bound state?

9.17.19

$\Rightarrow$  Large  $K_f$ . Fire faster  $\rightarrow E_{\text{mech}} > 0$   
so that

[Ex] proton fired from far away towards charged +100nC ( $d=1\text{m}$ )  
what initial speed needed so proton just reaches  
surface of glass?

Set it up: Energy conservation

$$K_i + V_i = K_f + U_f$$

$$\frac{1}{2} m v_i^2 + 0 = 0 + \frac{1}{4\pi\epsilon_0} \frac{q_p q_s}{r_i} \quad \begin{matrix} q_p = \text{charge} \\ r_i \approx \infty \quad \text{from around point} \\ \text{radius of glassphere} \end{matrix}$$

$$(\frac{1}{2})(1.67 \cdot 10^{-27} \text{ kg}) v_i^2 = \frac{k(1.6 \cdot 10^{-19} \text{ C})(1.0 \cdot 10^{-19} \text{ C})}{0.005 \text{ m}} \Rightarrow v_i = 1.86 \cdot 10^7 \text{ m/s}$$

\* smaller  $R \Rightarrow$  greater  $I$

SI unit for  $R$  is Ohm

$$1 \text{ ohm} = 1 \Omega = 1 \text{ V/A}$$

[Ex] When connected to 120V line, how bulb carries

50A current

$$\Rightarrow R = \frac{AV_{\text{wire}}}{I} = \frac{120 \text{ V}}{50 \text{ A}} = 240 \Omega \quad \text{resistance of the bulb's filament}$$

Resistivity: material property  $\Rightarrow$  resistivity  $\propto$  spring constant  
 $\frac{\text{long/short}}{\text{fat/thin}} = \frac{R}{\frac{PL}{A}} \propto \frac{1}{A} \quad \text{resistivity} \quad \text{size doesn't matter for } P$

GUIDED PREP

LT.1 E S P I

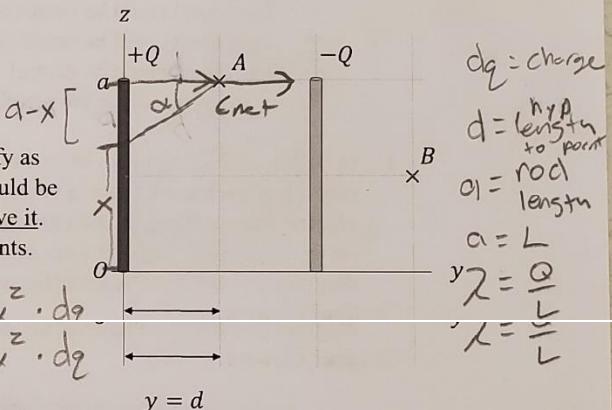
A positive charge  $+Q$  is distributed uniformly along the z-axis between  $z = 0$  and  $z = a$ . An equal magnitude but opposite charge  $-Q$  is distributed uniformly a distance  $y = 2d$  away, also between  $z = 0$  and  $z = a$ .

- Draw an arrow on the figure at both locations A and B showing the direction of the net electric field at those locations. Write ' $E = 0$ ' if the field is zero there.
- Calculate the net electric field produced by the charge distribution at location A. Set up the integral and simplify as much as possible up to the point where the next step would be to carry out the integration, but it is not necessary to solve it. Give your answer in terms of  $k$ ,  $Q$ , and any other constants. Show all work.

Show all work.

$$\int dE = \int \frac{k \cdot dq}{dz}$$

$$\cancel{\lambda dx = a^z + x^z \cdot dq} \quad \cancel{\lambda dx = a^z + x^z \cdot dq}$$



$$E = \int_0^a \frac{k \cdot da}{dz} \cdot \cos\alpha$$

$$E = \int_0^a \frac{k \cdot 2 \cdot dx}{(\sqrt{d^2 + (a-x)^2})^2} \cdot \frac{d}{\sqrt{d^2 + (a-x)^2}}$$

$$E = \int_0^a \frac{k \cdot Q/a \cdot d \cdot dx}{(d^2 + (a-x)^2)^{3/2}} = \boxed{\frac{k \cdot Q \cdot d}{a} \int_0^a \frac{dx}{(d^2 + (a-x)^2)^{3/2}} \text{ for one rod at } z}$$

$$\boxed{E = \frac{2k \cdot Q \cdot d}{a} \int_0^a \frac{dx}{(d^2 + (a-x)^2)^{3/2}} \text{ for both rods}}$$

As accurately as possible from the information in the figure, determine how much work is done by the electric field as a  $+600 \mu\text{C}$  point-charge moves from location D to location E. (At location D, its velocity is  $5.0 \text{ km/s}$ ). Show all your work.

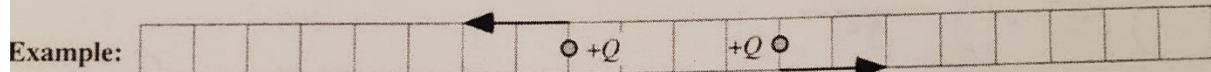
Final Answer:

$$1200 \mu\text{J}$$

$$\begin{aligned}
 W_{\text{elec}} &= -q \Delta V \\
 &\equiv -q(V_2 - V_1) \\
 &= -600 \times 10^{-6} \text{ C} (-2 - 0) \\
 &= -600 \times 10^{-6} \text{ C} (-2) \\
 &\boxed{1200 \times 10^{-6} \text{ J}}
 \end{aligned}$$

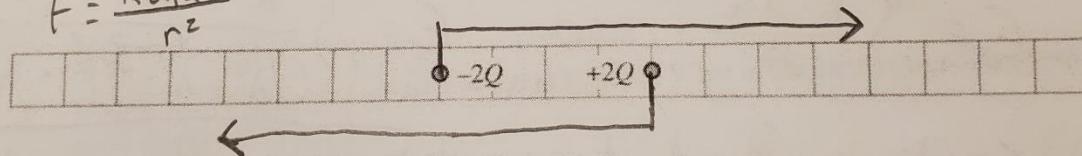
3

your answers.

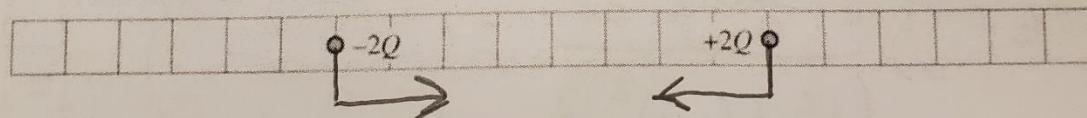


$$F = \frac{kQ_1 Q_2}{r^2}$$

a.)



b.)





Surface 1

- b. The total flux through Surface 1 is  $1,000 \text{ N}\cdot\text{m}^2/\text{C}$ . What is the numerical value of the point charge,  $Q$ ? Explain.

$$\text{flux} = \text{gauss law} = \frac{Q_{\text{enclosed}}}{\epsilon_0} = \frac{1_{\text{charge}}}{\epsilon_0} = 1000$$

$$\Rightarrow 1000 \cdot 8.85 \text{ C}^{-1} = 8.85 \text{ nC}$$

$$Q_{\text{enclosed}} = \epsilon_0 \cdot \text{flux}$$

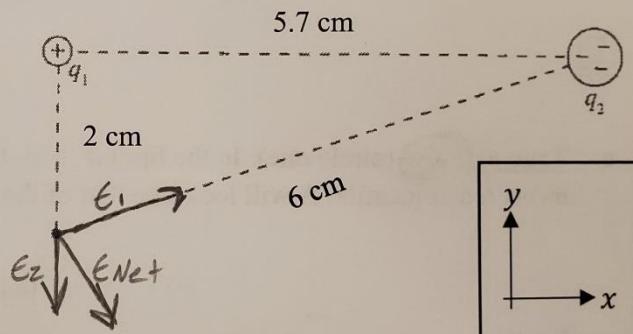
(4 pts) At the position of the dot, draw and label field vectors  $\vec{E}_1$  and  $\vec{E}_2$  due to  $q_1$  and  $q_2$ , and the net electric field  $\vec{E}_{\text{net}}$ . (The lengths of all three vectors must be proportional to their relative magnitudes.) Then, in the blanks, state whether the  $x$ - and  $y$ -components of  $\vec{E}_{\text{net}}$  are positive or negative.

$$(E_{\text{net}})_x \text{ Positive}$$

$$(E_{\text{net}})_y \text{ Negative}$$

$$E_1 = \frac{kq}{2^2} = \frac{1}{4}$$

$$E_2 = \frac{k \cdot 3q}{6^2} = \frac{1}{9}$$



5. (20 pts) Two point charges are fixed at the corners of a rectangle as shown in the figure.

- a. How much work is required by an external force to move an electron from location *A* to location *B*? Neglect gravity.

$$e_{\text{electron}} = 1.6e^{-19} \text{ C}$$

$$\text{Work}_k = -\Delta \text{ potential energy} \quad K = 9e^9$$

$$W = -(U_2 - U_1)$$

$$U_i = q_i \cdot V_i + q_z \cdot V_z = \frac{kq_1 \cdot e}{r_1} + \frac{kq_2 \cdot e}{r_2}$$

$$\Delta pe = \left( \frac{k \cdot e \cdot q_2}{r_2} + \frac{k \cdot e \cdot q_1}{r_1} \right) - \left( \frac{k \cdot e \cdot q_2}{r_1} + \frac{k \cdot e \cdot q_1}{r_2} \right)$$

$W = 3.6e^{-16} \text{ J}$

Diagram showing two charges  $q_1 = +2.0 \text{ nC} = 2e^{-9}$  and  $q_2 = -8.0 \text{ nC} = -8e^{-9}$  at the corners of a rectangle. The distance between them is  $r_1 = 2.0 \text{ cm}$ . The distance from  $q_1$  to point *A* is  $r_2 = 4.0 \text{ cm}$ . The distance from  $q_2$  to point *B* is also  $r_2 = 4.0 \text{ cm}$ .

- b. What is the electric field (magnitude and direction) at location *B*? You may give your answer either in component notation or as a magnitude and angle.

$$E_y = \frac{k \cdot |q_2|}{r_2^2} = \frac{9e^9 \cdot |8e^{-9}|}{0.04^2} = 45000 \frac{\text{N}}{\text{C}} \hat{j}$$

$$E_x = \frac{k \cdot |q_1|}{r_2^2} = \frac{9e^9 \cdot |2e^{-9}|}{0.04^2} = 45000 \frac{\text{N}}{\text{C}} \hat{i}$$

5. (10 pts) A non-conducting, solid sphere of radius 5.00 cm carries a uniform volume charge density of  $18.0 \mu\text{C/m}^3$ . Use Gauss's Law to calculate the electric field at a distance of 2.00 cm from the center of the sphere. Show all work.

$$\text{density } P = \frac{Q}{V} = 18\text{c}^{-6}$$

$$\oint E \cdot dI = \frac{Q_{\text{enclosed}}}{\epsilon_0} \Rightarrow E \cdot \text{area} = \frac{18\text{c}^{-6} \cdot \text{Volume}}{\epsilon_0}$$

$$E (\cancel{4\pi r^2}) = \frac{18\text{c}^{-6} \cdot \cancel{4\pi} r^2}{\epsilon_0} \Rightarrow E = \frac{18\text{c}^{-6} \cdot \frac{1}{3} \cdot r}{\epsilon_0}$$

$$E = \frac{18\text{c}^{-6} \cdot r}{3\epsilon_0} \Rightarrow \frac{18\text{c}^{-6} \cdot 0.02}{3 \cdot 8.85\text{c}^{-12}} = 13,560 \text{ N/C}$$

(10 pts) An alpha particle ( $q = +2e$ ,  $m \approx 4m_{\text{proton}}$ ) is shot toward a uranium nucleus ( $q = +92e$ ) from far away. The alpha particle reaches a closest distance of 1.0 femtometers ( $10^{-15} \text{ m}$ ) before changing direction. What was the initial speed of the alpha particle? (Assume the uranium nucleus does not move, since it is so much more massive than the alpha particle).  $k = 9\text{c}^9$

$$\text{mass of proton} = 1.67\text{c}^{-27} \Rightarrow \text{mass alpha particle} = 4 \cdot 1.67\text{c}^{-27}$$

$$q \cdot V = \frac{q_1 \cdot q_2 \cdot k}{r} = 6.68\text{c}^{-27}$$

$$r = 10\text{c}^{-15}$$

$$\frac{1}{2}mv^2 = q \cdot V$$

$$\frac{1}{2}(6.68\text{c}^{-27})v^2 = \frac{(2 \cdot 1.6\text{c}^{-19}) \cdot (92 \cdot 1.6\text{c}^{-19}) \cdot (9\text{c}^9)}{10\text{c}^{-15}}$$

$$e = 1.6\text{c}^{-19}$$

$$V = 1.13\text{c}^8 \text{ m/s}$$



Guided Preparation 4.2 - El Guided Preparation 4

[npaHn80LcAGWuub7h4r7jZRSvyj830HzDQ\\_E\\_ovg/view](#)

✓ Question 1 \*
1/1

a.) The change in electric potential is negative because the positive proton going away from the positive plate is similar to a ball rolling down a hill. The ball "wants" to roll down and the proton "wants" to go towards the negative. When it is going in the direction it "wants" to go, the PE is changing to KE, meaning the change in EP will be negative too divided by a positive test charge.

b.) The change in electric potential is also negative when it is a proton because the proton is not going the way it "wants" to, meaning that KE is being converted to PE resulting in a positive PE. The change in EP will be this positive change in PE divided by the negative test charge, resulting in a negative electric potential too.

**Feedback**

a) negative. Starts high near + plate; lower near - plate.  
 b) negative. Potential is independent of the test charge.

✓ Question 2 \*
1/1

iii.) It moves towards A with an increasing speed because A is at a lower EP than B, and the proton will move from high to low EP. It is at an increasing speed because the force on the proton is a function of distance, and as the proton gets closer, the force on it gets bigger.

**Feedback**

Moves toward A with increasing speed. The kinetic energy is increasing, so its speed is increasing. Also, there is a net force on the proton, so it is also accelerating.  $F=ma$

✓ Question 3 \*
1/1

v.) It moves towards B with an increasing speed because B is at a lower EP than C, and the electron will move from high to low EP. It is at an increasing speed because the force on the electron is a function of distance, and as the electron gets closer, the force on it gets bigger.

**Feedback**

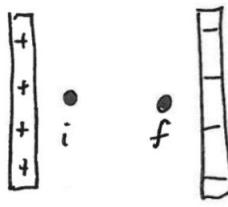
Moves toward C with increasing speed. Electrons feel force toward higher potential. Kinetic energy increases, so its speed is also increasing.

**Guided Preparation 4.2 - E**   **Guided Preparation 4.1 -**

mPMhSTnphaHn80LcAGWuub7h4r7jZRSvyj830HzDQ\_E\_0vg/viewscore

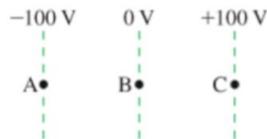
<b>Feedback</b> Moves toward C with increasing speed. Electrons feel force toward higher potential. Kinetic energy increases, so its speed is also increasing.	1/1
<b>✓ Question 4a *</b> $V_a = (k \cdot 1.6E-19C) / (0.01m)$ $V_b = (k \cdot 1.6E-19C) / (0.03m)$	1/1
<b>Feedback</b> If the orange charge is a proton... $V_a = ke/r = ke/(1 cm)$ $V_b = ke/r = ke/(3 cm)$ Note that $V_b < V_a/3$	
<b>✓ Question 4b *</b> $V_b - V_a = (1 \cdot 1.6E-19C) / (1.01m) - 1 / (0.03m)$	1/1
<b>Feedback</b> $\Delta V = V_a - V_b$	
<b>✓ Question 4c *</b> $P_E = (k^2 \cdot 1.6E-19C \cdot 1.6E-19C) / (0.01m)$	1/1
<b>Feedback</b> $U = qV = eV$	
<b>✓ Question 4d *</b> $(1/2)(1.57E-27Kg)(v_f^2 - 4E5m/s) = (k^2 \cdot (1.6E-19)^2) / (0.03m) (k^2 \cdot (1.6E-19)^2) / (0.01m)$ $v_f = 632m/s$	1/1
<b>✓ Question 4e *</b> This is the same magnitude, but in the opposite direction. $v_f = 632m/s$	1/1

1. In the figure, a particle begins at position *i* and ends at position *f*.
- Is the change in electric potential  $\Delta V$  positive, negative or zero if the particle is a **proton**? Explain.
  - Is the change in electric potential  $\Delta V$  positive, negative or zero if the particle is an **electron**? Explain.



2. **Multiple Choice +Explain.** Use the equipotential diagram to the right. A *proton* is released from rest at location *B*, where the electric potential is 0 V. Afterward, the proton:

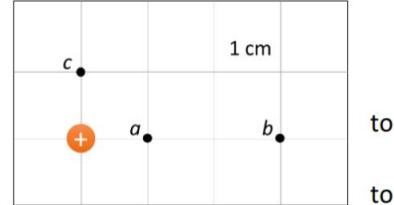
- Remains at rest at *B*.
- Moves toward *A* with a steady speed.
- Moves toward *A* with an increasing speed.
- Moves toward *C* with a steady speed.
- Moves toward *C* with an increasing speed.



3. **Multiple Choice +Explain.** Answer Problem 2 now for if an *electron* is released from rest at *B*.

4. Calculate the following quantities. Explain or show steps in the Google form response box.

- Electric potential at locations *a* and *b*.
- Potential difference between *a* and *b*.
- Potential energy of a proton at *a* and *b*.
- Speed at location *b* of a proton that was moving the right at location *a* with  $v = 4.0 \cdot 10^5$  m/s
- Speed at location *a* of a proton that was moving the left at location *b* with  $v = 4.0 \cdot 10^5$  m/s



to  
to

Guided Preparation 4.2 - E   X   Guided Preparation 4.1 -

8UY7DbgMgHqBso1-9aHS2RezoNDA6qdTb5e5DZnQ/views

✓ Question 1, part a \*

1/1

Change in PE =  $q|E| \Delta S$  =  $q(190 \text{ N/C})(0.015 \text{ m}) = 2.85 \text{ q J}$ . The problem does not say if the particle was a proton or electron yet so, in terms of q.

Feedback

$\Delta U = W$ . For a uniform field, as in this problem,  $\Delta U = -qEd = -eEd = -4.57 \times 10^{-19} \text{ J}$

✓ Question 1, part b \*

1/1

The change in kinetic energy is  $1/2m(v_f^2 - v_i^2)$ .

Feedback

$\Delta K = -\Delta U = +4.57 \times 10^{-19} \text{ J}$

✓ Question 1, part c \*

1/1

$(V_f)p = \sqrt{(-2E_{\text{mech}}/m)} = \sqrt{(-2(-eE(1/2)d)/m)} = 1.65 \text{ E6 m/s}$

Feedback

Call  $v_f$  the final velocity;  $v_i$  the initial velocity;  
 $\Delta K = 0.5m(v_f^2 - v_i^2) = 0.5m(v_f^2) = 0.5m(v_f^2)$   
solving for  $v_f$ :  
 $2.34 \times 10^6 \text{ m/s}$

✓ Question 1, part d \*

1/1

The mechanical energy is the same, so the change in potential energy would be the same but negative.

Feedback

Positive. The charge has the opposite sign, so  $\Delta U = -qEd = +eEd$

✓ Question 2b \*

1/1

i) from c to b and c to a the net work would be positive.  
ii) from b to c and a to c the net work would be negative.  
iii) from a to b and b to a, the net work would be zero.