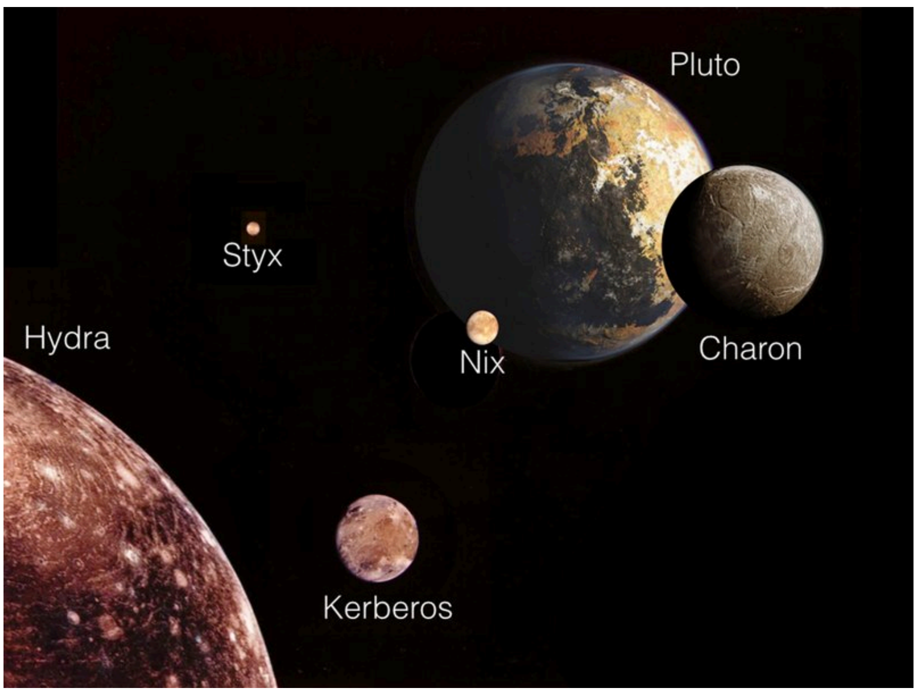
Unraveling the Complex Restricted Three-Body Dynamics of Pluto and its Moons



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Introduction

This project will use Python to perform a numerical analysis of an unmanned spacecraft under the influence of the gravitational forces created by Pluto and its orbiting moons using the restricted three body equation. Restricted three body equations are fundamental in solving orbital positions and behaviors of different planets in orbit with respect to each other, or how a spacecraft orbits two different planets after leaving the orbit of one of those planets. As a result, these equations provide a respected approximation for elliptical orbits within our Solar System. The restricted three body equations analyze only the masses, and thus the resulting gravitational potential, of the dominating bodies. This allows for the impact and comparison of gravitational potential energy and impacts of spheres of influences to be compared from planet to planet and utilized in order to perform maneuvers such as Hohmann transfers and gravity assist flybys.

Using the restricted three body equations in a cartesian coordinate system for 2 planets and 1 satellite. The behavior of a satellite around Pluto and one of its 5 moons can be approximated and altered based on a variety of parameters. Due to the nature of the restricted three body equations, the mass of the satellite or spacecraft can be ignored, leaving the only relevant points of information to be the masses of the planets, distances, and orbital elements of the burn that will cause the satellite or spacecraft to leave the lower Pluto orbit it currently resides in. Figure 1 illustrates a restricted three body problem in a cartesian coordinate system. For the Pluto-Moon-Spacecraft application, M1 is Pluto, M2 is the Moon, and M is the spacecraft or satellite. The origin of the plot, O, is the center of mass of the system.

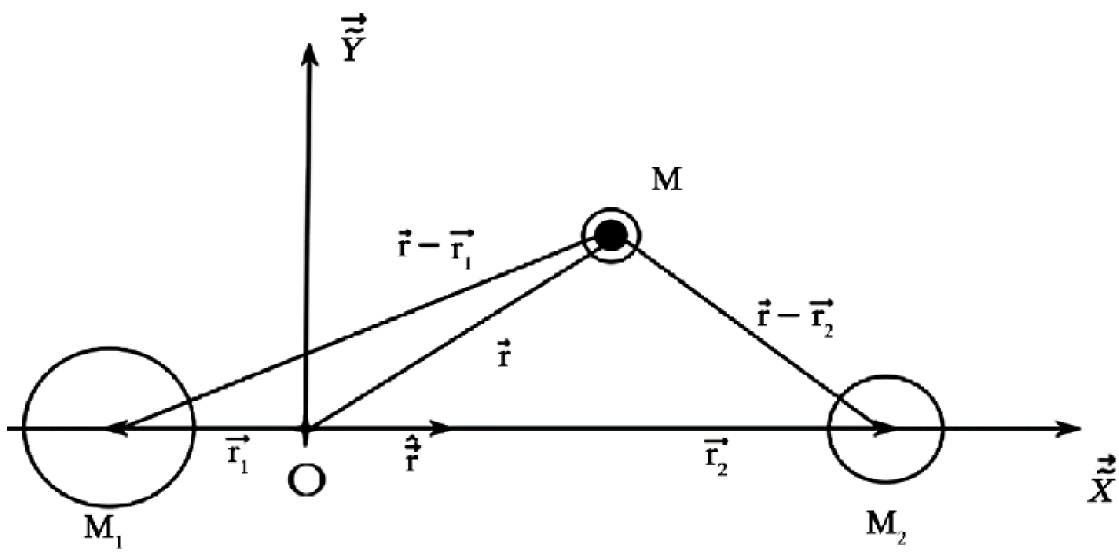


Figure 1: Restricted three body problem in a cartesian coordinate system for 2 planets and 1 satellite

From this, restricted three body scalar motions are created. These equations are highlighted in the equations below. Equation 1 relates the motion to x-direction acceleration. Equation 2 relates y-directions acceleration and Equation 3 relates z-direction acceleration. Equation 4 and Equation 5 solve for the specific mass relationship values of $\pi 1$ and $\pi 2$, respectively.

$$x'' - 2\Omega y' - \Omega^2 x = -\frac{\mu_1}{r_1^{\ 3}}(x + \pi_2 r_{12}) - \frac{\mu_2}{r_2^{\ 3}}(x + \pi_1 r_{12}) \quad \textbf{(1)}$$

$$y'' - 2\Omega x' - \Omega^2 y = -\frac{\mu_1}{r_1^{\ 3}} y - \frac{\mu_2}{r_2^{\ 3}} y \quad \textbf{(2)}$$

$$z'' = -\frac{\mu_1}{r_1^{\ 3}} z - \frac{\mu_2}{r_2^{\ 3}} z \quad \textbf{(3)}$$

$$\pi_1 = \frac{m_1}{m_1 + m_2} \quad \textbf{(4)}$$

$$\pi_2 = \frac{m_2}{m_1 + m_2} \quad \textbf{(5)}$$

Where:

x,y,z: position, and their corresponding derivatives for the prime and double prime values
Ω = Angular Velocity of the desired Moon [rad/s]
μ_1 = Mu of Pluto [km^3/sec^2]
μ_2 = Mu of the desired Moon [km^3/sec^2]
m_1 = Mass of the Pluto
m_2 = Mass of the desired Moon
r_{12} = Distance Pluto to desired Moon

The required mass values of Pluto and its 5 moons, along with their radius, angular velocity, and distance from Pluto, are housed in Table 2. Angular velocity was solved for using Equation 6. From the mass values in Table 2 and the Gravitational Constant of 6.67E-20 [km^3/(kg*s)], the Mu constants were calculated using Equation 7 and stored in Table 3. From these values, the trajectory of a mission from lower Pluto orbit around one of these moons can be accurately mapped. It is essential to recognize that Pluto is treated as a stationary planet with no angular velocity as its orbit around the Sun is not taken into account. Our launch will also take place over the south pole.

$$\omega = 2\pi/T \quad \textbf{(6)}$$

Where:
ω = Angular Velocity [rad/s]
T = Period [s]

Table 1: Physical Values of Pluto and its 5 moons.

Body	Mass [kg]	Radius [km]	Pluto to Body [km]	Angular V. [rad/s]
Pluto	13.03E21	1187	X	X
Charon	1.5466E21	603.6	17536	1.139E-5
Styx	7.50E15	7.5	42000	3.6E-6
Hydra	9.8912E17	36	64749	1.904E-6
Kerberos	1.65E16	9	59000	2.2655E-6
Nix	4.50E16	25	48708	2.925E-6

$$\mu = GM \quad \textbf{(7)}$$

Where:
μ = Planetary Mu [km^3/s^2]
G = Gravitational Constant
M = Mass of desired Body

Table 2: Mu values for Pluto and its 5 moons

Body	μ
Pluto	8.69E2
Charon	1.0316E2
Styx	5.0025E-4
Hydra	6.5974E-2
Kerberos	1.101E-3
Nix	3.002E-3

Objectives

The objective of this project is to develop a trivial understanding of how launch velocity and flight path angle may need to be altered in order for a probe in low Pluto orbit to successfully traverse around each of Pluto's moons per mission. Because changing flight path angle and velocity impacts the orbit, an illustration of initial velocity and flight path angle can be seen in Figure 2. Another interest is the overall shape of the trajectory path for the probe between the two bodies and how angular velocity can impact this trajectory. Also, how the moon's mass and distance from Pluto can cause the trajectory to alter right after launch and well after launch.

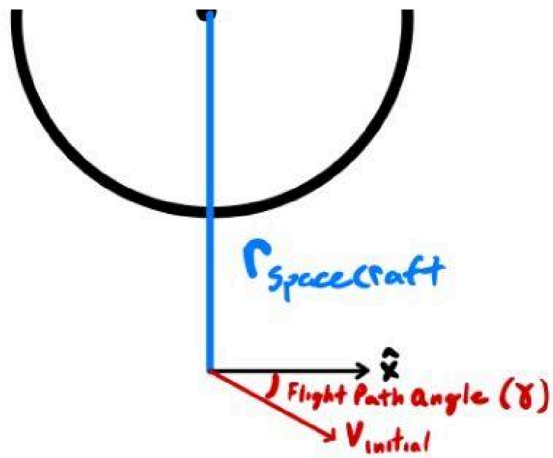


Figure 2: Relationship between cartesian coordinates, flight path angle, and initial velocity.

Performing orbital missions around Pluto and its moons will allow scientists to better understand how to travel between Pluto and its satellite planets. This breakthrough would mean that scientists could send probes into Pluto's orbit from Earth and then move them around each of Pluto's moons to collect data about the formation of the universe. The sheer distance of Pluto from the center of the solar system would also allow scientists to better understand interplanetary travel in deep space.

Approximation Acknowledgement

Although this is a scientific analysis, it must be recognized that many of these numbers are approximations due to the sheer distance Pluto's moons are from Earth. This distance means that distances across can be incorrect and the mass due to physical makeup can be incorrect. Another important note is that many of Pluto's moons are not circular, and are instead shaped more like an asteroid with non uniform dimensions. When this was the case, the largest dimension (X,Y,Z) was taken and used as the diameter.

Introduction to Numerical Methods

I will be examining the initial velocity and flight path angle case for the restricted three body scalar motion equations by altering these parameters. This results in the velocity vector being a non-set value that changes throughout the analysis. In order to solve this problem, conditions are induced onto the system to turn it into an initial value problem. An initial value problem is a type of differential equation that has a set starting condition. These create the starting point that the differential equation uses to fill in the values as time progresses. For the orbital analysis case, the initial conditions will be the initial velocity of the probe, its location with respect to the center of mass, and its flight path angle.

To perform the analysis of this initial value problem, the initial velocity and position of the probe will be stored within a list. These conditions must contain values for each cartesian coordinate X, Y, and Z. Next, applying Equations 1,2,3,4 and 5, the derivative vector can be created that holds position prime and velocity prime. The work used to create the position and velocity list can be seen in Figure 3, and the steps to solve the complete initial value problem are illustrated in Figure 4.

m_1 = Pluto m_2 = moon

Pluto – Charon

$M_2 r_{12} = -\left(\frac{m_2}{m_1+m_2}\right)|r|\hat{i} = -1860.60$ km X distance to CG

$M_1 r_{12} =$ distance $- |M_2 r_{12}| = 17536 - 1860.6 = 15675.4$ km X distance to CG

Pluto – Styx

$M_2 r_{12} = -0.0242$ km

$M_1 r_{12} = 41999.976$ km

Pluto – Hydra

$M_2 r_{12} = -4.915$ km

$M_1 r_{12} = 64744.085$ km

Pluto – Kerberos

$M_2 r_{12} = -0.0747$ km

$M_1 r_{12} = 58999.925$ km

Pluto – Nix

$M_2 r_{12} = -0.168$ km

$M_1 r_{12} = 48707.832$ km

Figure 3: Adjustment values to find the location of Pluto and each moon individually with respect to the center of gravity of that system.

2nd Order Differential Equations

$$\ddot{x} - 2\Omega\dot{y} - \Omega^2 x = -\mu_1/r_1^3 \left(x + r_2 \pi_{12}\right) - \frac{\mu_2}{r_2^3}\left(x - r_1 \pi_{12}\right)$$

$$\ddot{y} + 2\Omega\dot{x} - \Omega^2 y = -\mu_1/r_1^3 \left(y\right) - \frac{\mu_2}{r_2^3}\left(y\right)$$

$$\ddot{z} = \frac{-\mu_1}{r_1^3} z - \frac{\mu_2}{r_2^3} z$$

Position and velocity list

$$\left[r_1, r_2, r_3, V_1, V_2, V_3\right]$$

Initial position and velocity of spacecraft

$$\vec{r} = \left(M_2 r_{12},\ r_p + AH,\ 0\right) \text{ km}$$

$$\vec{V} = \left(V_i \cos\gamma,\ -V_i \sin\gamma,\ 0\right) \text{ km/s}$$

array of derivatives and subsequent values.

$$\left[\dot{r}_1, \dot{r}_2, \dot{r}_3, \dot{V}_1, \dot{V}_2\ \dot{V}_3\right]$$

$$\dot{r}_1 = V_1 \qquad \dot{r}_2 = V_2 \qquad \dot{r}_3 = V_3$$

$$\dot{V}_1 = 2\Omega\dot{y} + \Omega^2 x - \mu_1/r_1^3 \left(x + r_2 \pi_{12}\right) - \frac{\mu_2}{r_2^3}\left(x - r_1 \pi_{12}\right)$$

$$\dot{V}_2 = -2\Omega\dot{x} + \Omega^2 y - \mu_1/r_1^3 \left(y\right) - \frac{\mu_2}{r_2^3}\left(y\right)$$

$$\dot{V}_3 = -\frac{\mu_1}{r_1^3} z - \frac{\mu_2}{r_2^3} z$$

Figure 4: Solving IVP conditions for initial position and velocity and their corresponding derivatives.

After a list is created for the prime values of position and velocity, the ode.int() function can be used to solve for positions and velocity values. This function must take in that prime list, an array of the initial conditions, and an array that holds the time values that position and velocity are to be found at.

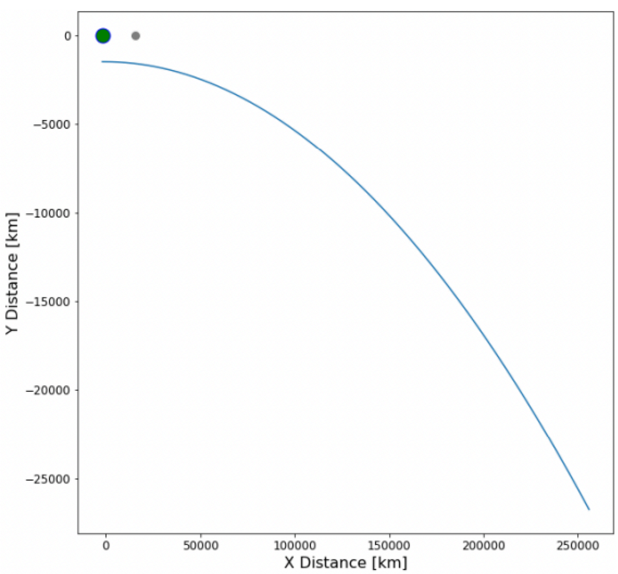
Results

I plan on running 2 different tests, plotting the trajectory at different times for 30 [km/s] 0.0 [deg] and 50 [km/s] 0.05 [deg]. From this comparison two different angle and velocity combinations can be analyzed. Also, the trajectory due to mass, distance, and angular velocity will be examined.

Charon

Of the five moons, Charon is the largest, and thus has the greatest impact on the center of gravity. The center of gravity however is still heavily influenced by Pluto, evident in the upcoming figures.

When leaving a 300 [km] low Pluto orbit, the first attempt to reach a distance close to Charon was to have a 0 degree flight path angle and a tangential velocity change of 30 [km/s]. The resulting orbit 0.1 Earth Days after the velocity impulse can be seen in Figure 5. Here, the closest the probe got to Charon's surface from Pluto was 994 [km]. Figure 6 uses the same parameters but expands the time bounds to 5 Earth days.



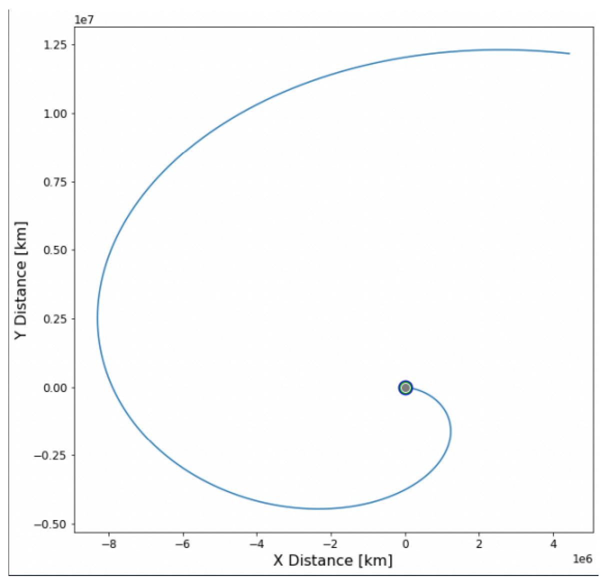
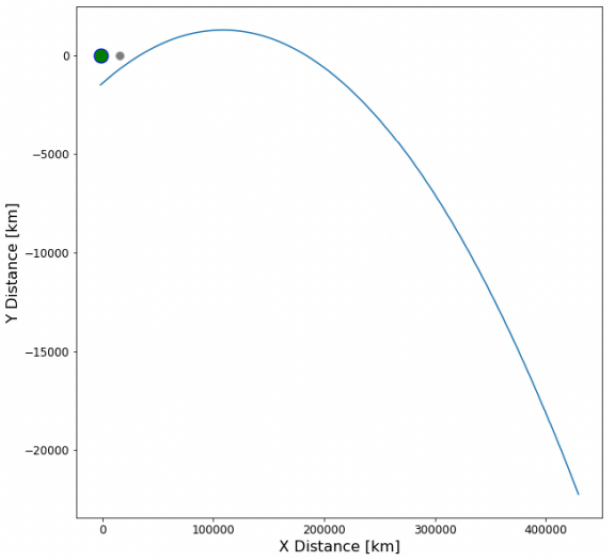


Figure 5 (Left): Pluto Charon interaction on Probe 0.1 days after impulse (30 [km/s] 0.00 [deg])
Figure 6 (Right): Pluto Charon interaction on Probe 5 days after impulse (30 [km/s] 0.00 [deg])

From the distance data collected in Figure 5, it can be assumed that such flight parameters would not be appropriate to get a probe near Charon to collect data. This is further supported by Figure 6 as it shows the continuing orbit spirling farther away from Pluto and not getting any closer to the moon. In order to get The probe near Charon, an expensive secondary impulse would need to be used to change the probe's trajectory again.

The second attempt to reach a distance close to Charon used a 0.05 degree flight path angle and a tangential velocity change of 50 [km/s]. Unlike the flight angle pictured in Figure 2, this 0.05 degrees was above the X axis.

The resulting orbit 0.1 Earth Days after the velocity impulse with new values can be seen in Figure 7. Here, the closest the probe got to Charon from Pluto was 86.65 [km]. Figure 7 uses the same parameters but expands the time bounds to 5 Earth days.



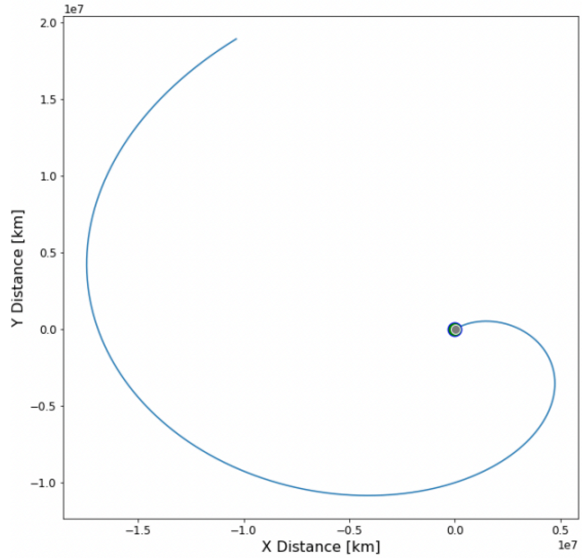


Figure 7 (Left): Pluto Charon interaction on Probe 0.1 days after impulse (50 [km/s] 0.05 [deg])
Figure 8 (Right): Pluto Charon interaction on Probe 5 days after impulse (50 [km/s] 0.05 [deg])

From the distance data collected in Figure 7, the flight parameters supplied, 50 km/s at 0.05 degrees above the x axis when launched from lower Pluto Orbit under the South Pole would allow the probe to get within 100 km of Charon. This impulse would be expensive due to the scale of the delta V and inclination change needed. Figure 8 continues the trend set by Figure 6 as the probe would spiral out further away from both Pluto and Charon. What this means is that as the delta V is supplied, the probe would be teetering on the exit velocity required to leave the influence of Pluto as its kinetic energy is out-weighing the gravitational energy. This idea is reinforced in Figure 9, which is an impulse of 50 [km/s] at 0.05 [deg] plotted over 50 days.

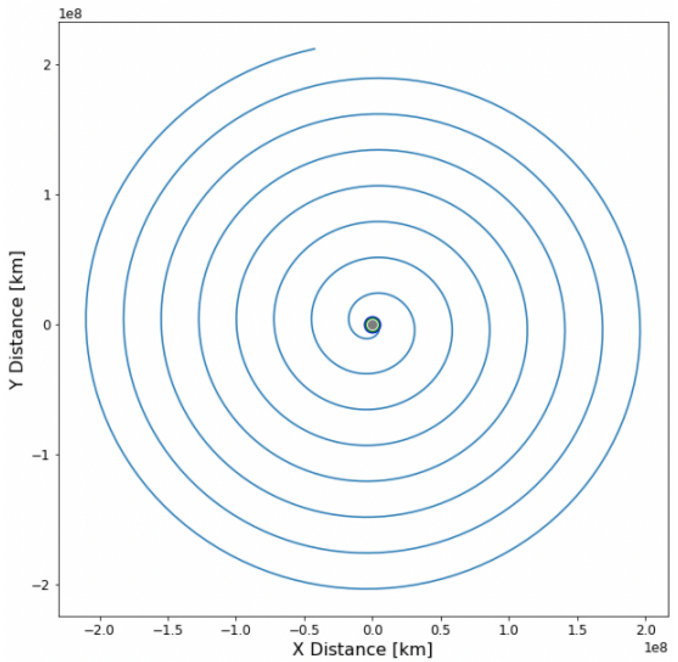
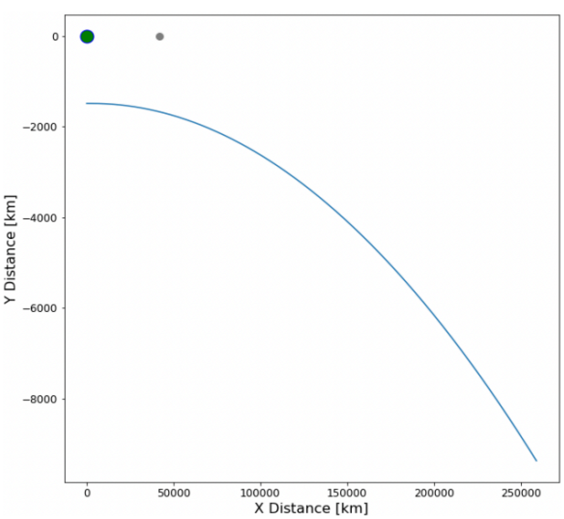


Figure 9: Pluto Charon interaction on Probe 50 days after impulse (50 [km/s] 0.05 [deg])

Styx

The second moon of Pluto that will be analyzed, and is the smallest moon in orbit around Pluto in terms of radius and mass. For a test launch from 300 km below Pluto's South Pole, 30 [km/s] and 0 [deg] will again be used in order to show the impact of the center of gravity and distance between the two bodies. Figure 10 shows the resulting orbit 0.1 days after impulse and Figure 11 shows the same orbit 5 days into its motion. During this, the closest the probe would get is 1665.2 [km], a relatively large value that would make data collection hard.



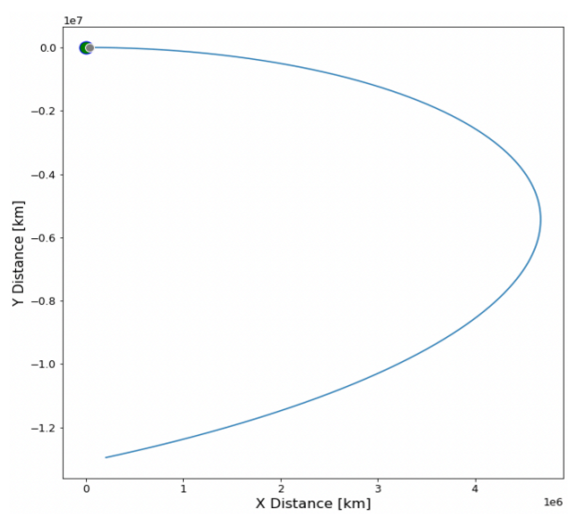
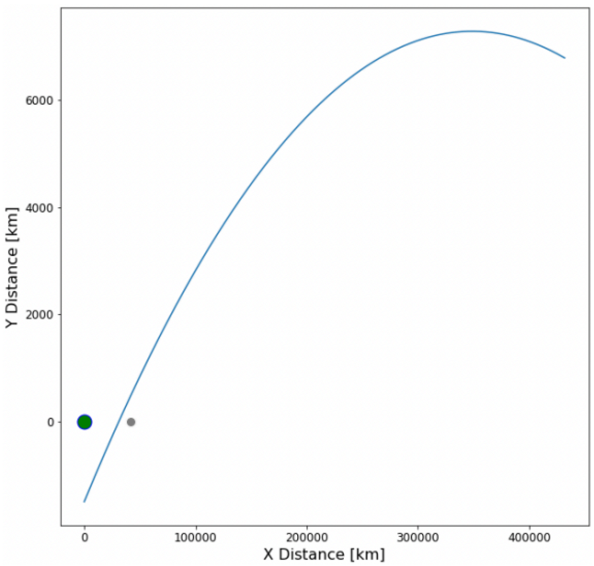


Figure 10 (Left): Pluto Styx interaction on Probe 0.1 days after impulse (30 [km/s] 0.00 [deg])
Figure 11 (Right): Pluto Styx interaction on Probe 5 days after impulse (30 [km/s] 0.00 [deg])

The second attempt to reach a distance close to Styx once again used a 0.05 degree flight path angle and a tangential velocity change of 50 [km/s]. The resulting orbit 0.1 Earth Days after the velocity impulse with new values can be seen in Figure 12. Here, the closest the probe got to Styx from Pluto was 496.89 [km]. Figure 13 uses the same parameters but expands the time bounds to 5 Earth days. Figure 14 shows the trajectory 50 days after impulse.



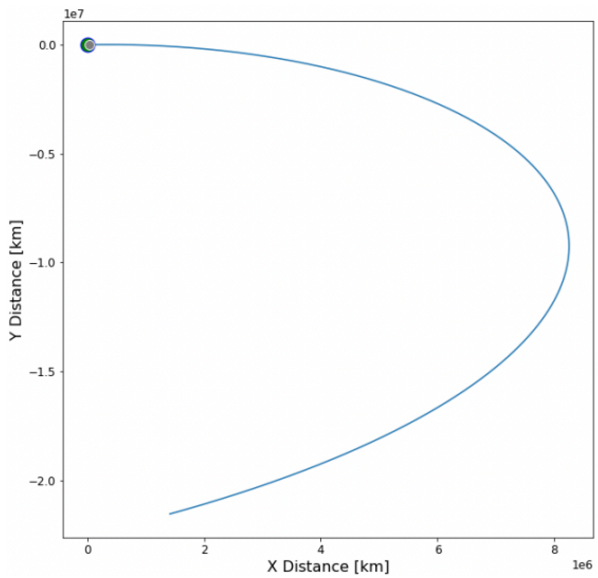


Figure 12 (Left): Pluto Styx interaction on Probe 0.1 days after impulse (50 [km/s] 0.05 [deg])
Figure 13 (Right): Pluto Styx interaction on Probe 5 days after impulse (50 [km/s] 0.05 [deg])

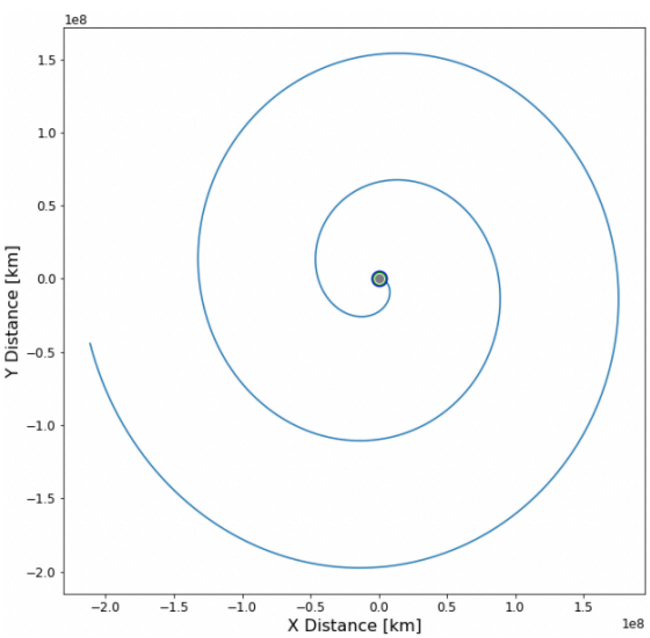
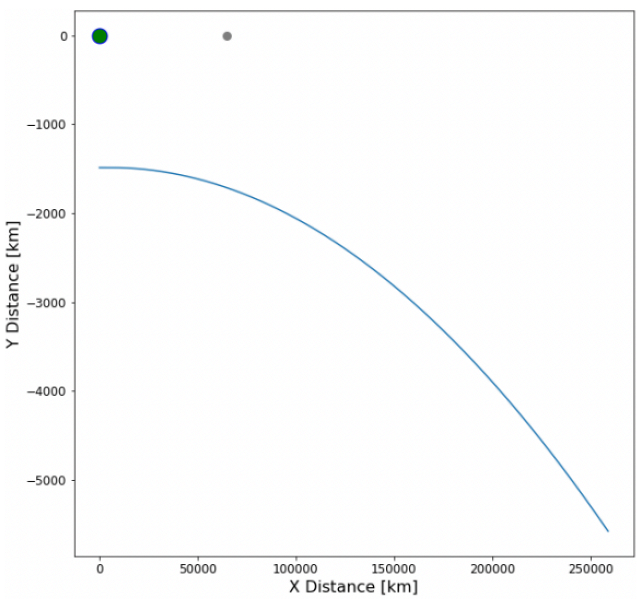


Figure 14: Pluto Styx interaction on Probe 50 days after impulse (50 [km/s] 0.05 [deg])

From the distance data collected in Figure 12, the flight parameters supplied, 50 km/s at 0.05 degrees above the x axis when launched from lower Pluto Orbit under the South Pole would allow the probe to get within 500 km of Styx. This impulse would be again expensive due to the scale of the delta V and inclination change needed and doesn't get the same closeness as Charon. Already after 5 days the difference between the two systems' trajectory is clear in Figure 8 versus Figure 13 as Figure 13 contains more of a sweep parabola compared to Figure 8's onset of a swirl. In a span of 50 days for Figure 14 versus Figure 9, Charon puts the probe in a tighter swirl pattern containing more revolutions whereas the Styx system contains far less revolutions and each revolution is less tight.

Hydra

The third moon of Pluto, Hydra, is the 3rd largest in both mass and size. Again, the probe was launched from 300 km below Pluto's South Pole at 30 [km/s] and 0 [deg]. Figure 15 shows the resulting orbit 0.1 days after impulse and Figure 16 shows the same orbit 5 days into its motion. During this, the closest the probe would get is 1678.45 [km].



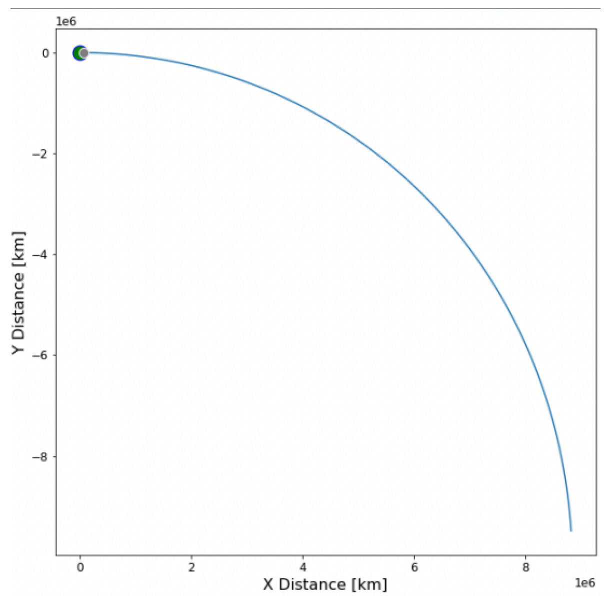
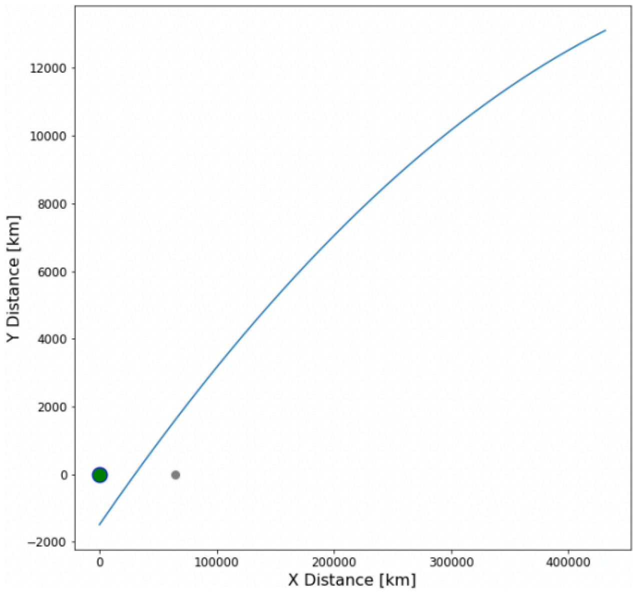


Figure 15 (Left): Pluto Hydra interaction on Probe 0.1 days after impulse (30 [km/s] 0.00 [deg])
Figure 16 (Right): Pluto Hydra interaction on Probe 5 days after impulse (30 [km/s] 0.00 [deg])

Again, a second attempt to reach a distance close to Hydra was again used with a 0.05 degree flight path angle and a tangential velocity change of 50 [km/s]. The resulting orbit 0.1 Earth Days after the velocity impulse with new values can be seen in Figure 17. Here, the closest the probe got to Hydra from Pluto was 1574.03 [km]. Figure 18 uses the same parameters but expands the time bounds to 5 Earth days. Figure 19 shows the trajectory 50 days after impulse. These values are also not too acceptable compared to the previous distances to the moon's surface, however, even when the values were altered more, the distance changed by a miniscule amount, meaning that a secondary burn is necessary to get a probe in a respectable distance of Hydra.



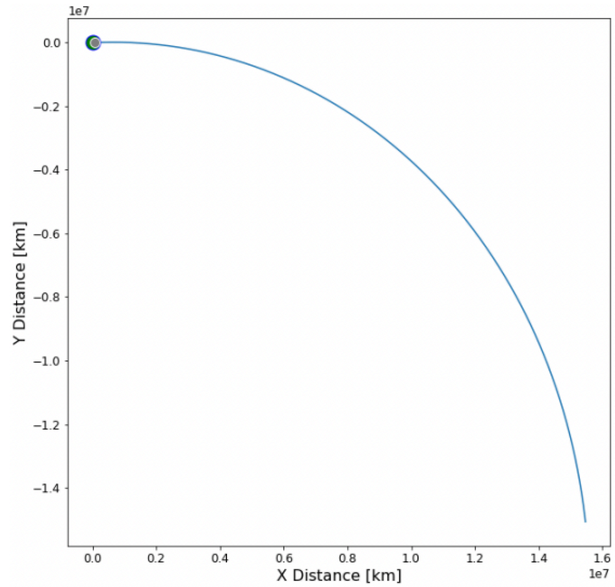


Figure 17 (Left): Pluto Hydra interaction on Probe 0.1 days after impulse (50 [km/s] 0.05 [deg])
Figure 18 (Right): Pluto Hydra interaction on Probe 5 days after impulse (50 [km/s] 0.05 [deg])

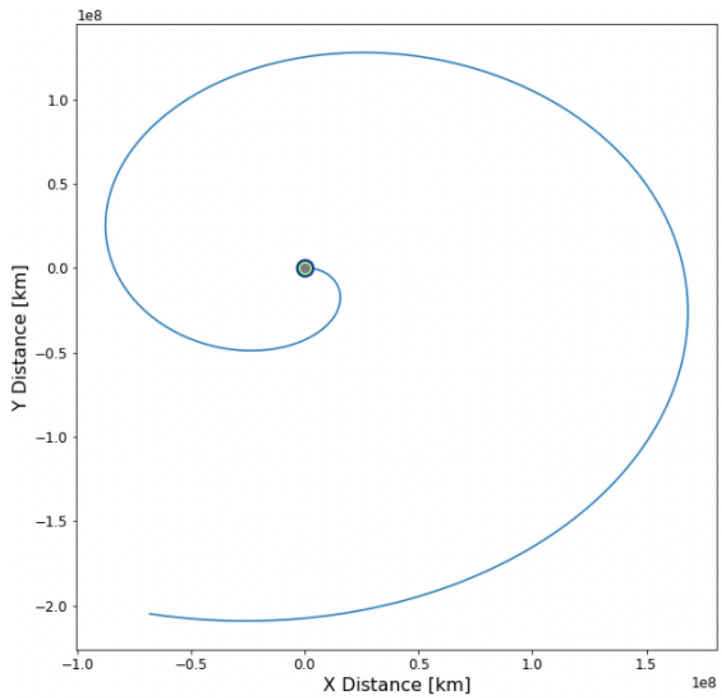
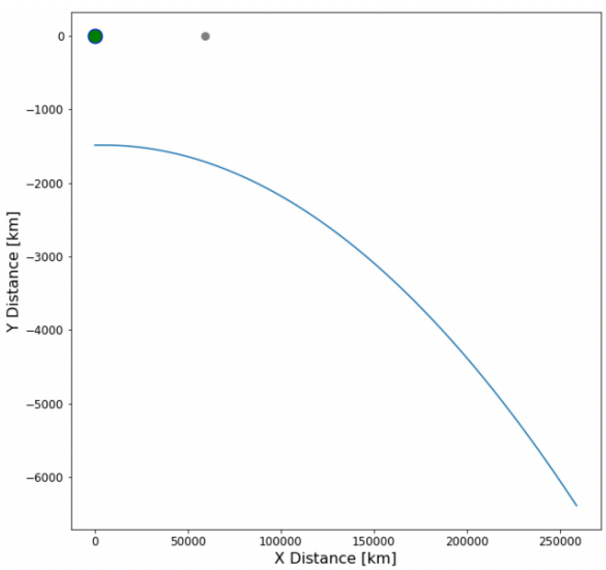


Figure 19: Pluto Hydra interaction on Probe 50 days after impulse (50 [km/s] 0.05 [deg])

Although 50 km/s at 0.05 degrees above the x axis when launched from lower Pluto Orbit under the South Pole would allow the probe to get closer than if launched at 30 km/s with 0 degrees of elevation, the differences in mass, distance, and angular velocity have caused Pluto to massively out influence Hydra. This is evident in Figure 19 compared to Figure 9 and Figure 14 as Hydra, with slightly more mass, but also more distance and less angular velocity has caused there to be less revolutions around the system and loser revolutions. Because of the inability to get within a respectable distance of Hydra, a secondary burn would need to be applied to attempt to enter Hydras sphere of influence.

Kerberos

The fourth moon of Pluto, Kerberos, will have the probe sent near it after being launched from 300 km below Pluto's South Pole at 30 [km/s] and 0 [deg]. Figure 20 shows the resulting orbit 0.1 days after impulse and Figure 21 shows the same orbit 5 days into its motion. During this, the closest the probe would get is 1707.42 [km].



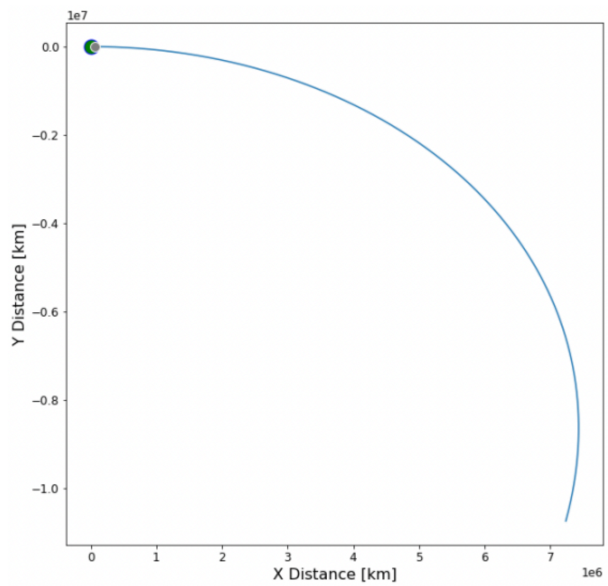
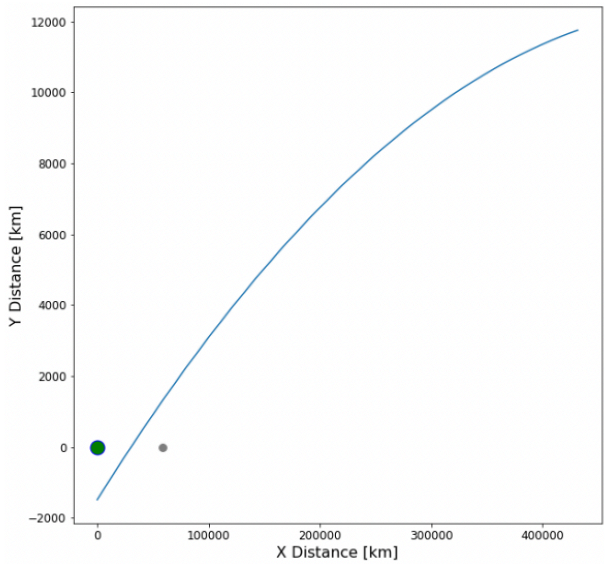


Figure 20 (Left): Pluto Kerberos system on Probe 0.1 days after impulse (30 [km/s] 0.00 [deg])
Figure 21 (Right): Pluto Kerberos system on Probe 5 days after impulse (30 [km/s] 0.00 [deg])

A 0.05 degree flight path angle and a tangential velocity change of 50 [km/s] was also used for Kerberos. The resulting orbit 0.1 Earth Days after the velocity impulse with new values can be seen in Figure 22. Here, the closest the probe got to Kerberos from Pluto was 1327.42 [km]. Figure 23 uses the same parameters but expands the time bounds to 5 Earth days. Figure 24 shows the trajectory 50 days after impulse. This distance is not too promising considering the delta V used; however, a secondary burn, which would add more delta V, can be used to get closer to Kerberos. Figure 19 shows the trajectory 50 days after impulse.



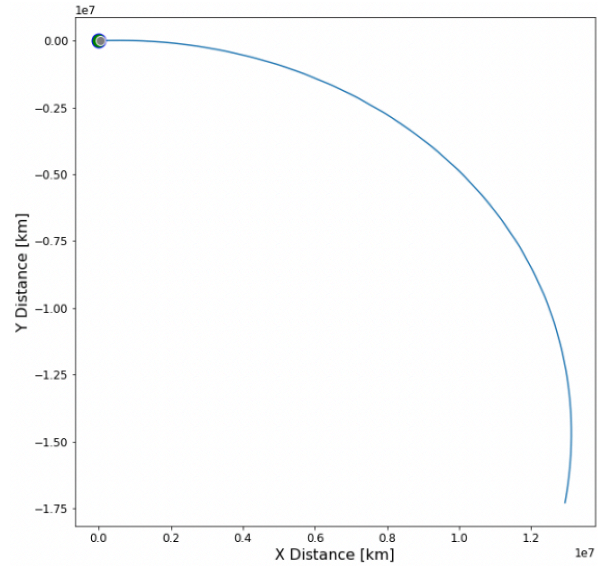


Figure 22 (Left): Pluto Kerberos system on Probe 0.1 days after impulse (50 [km/s] 0.05 [deg])
Figure 23 (Right): Pluto Kerberos system on Probe 5 days after impulse (50 [km/s] 0.05 [deg])

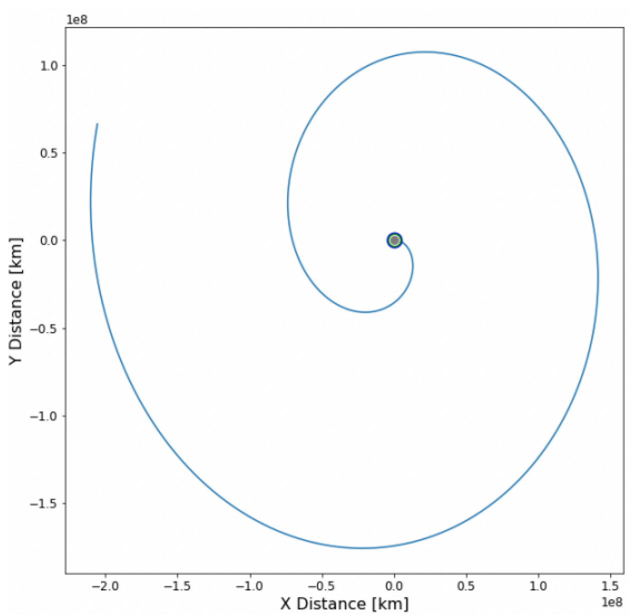
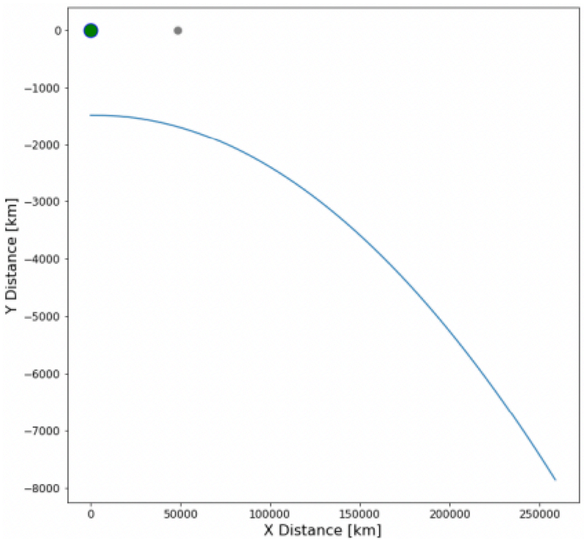


Figure 23: Pluto Kerberos system on Probe 50 days after impulse (50 [km/s] 0.05 [deg])

Again, the smaller mass moon proves to be difficult to reach with an appropriate altitude with the single burn in low Pluto orbit. The increase in angular velocity shows more revolutions completed compared to the previous moon rotations; however, because there is a similar ratio between mass and distance from Pluto within a certain threshold, the spacing between each spiral is similar.

Nix

The last moon of Pluto, Nix, will have the last probe sent near it after being launched from 300 km below Pluto's South Pole at 30 [km/s] and 0 [deg]. Figure 24 shows the resulting orbit 0.1 days after impulse and Figure 25 shows the same orbit 5 days into its motion. During this, the closest the probe would get is 1663.66 [km].



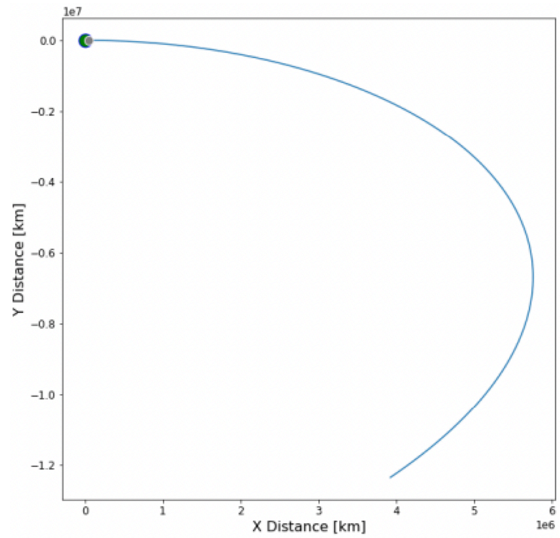
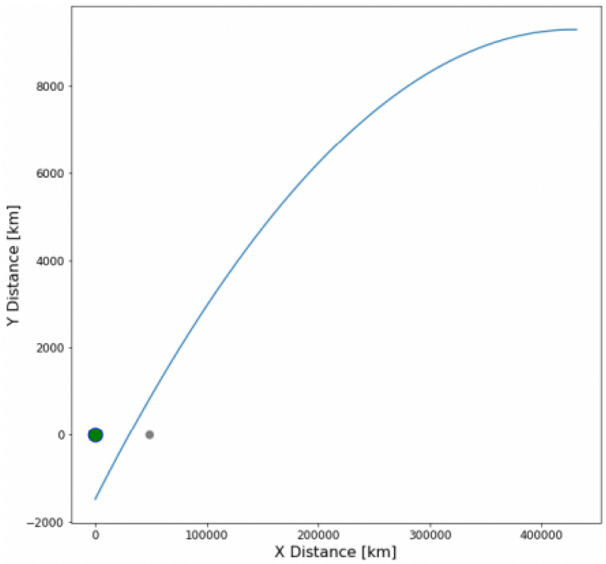


Figure 24 (Left): Pluto Nix system on Probe 0.1 days after impulse (30 [km/s] 0.00 [deg])
Figure 25 (Right): Pluto Nix system on Probe 5 days after impulse (30 [km/s] 0.00 [deg])

Nix will also be subjected to the probe leaving at a 0.05 degree flight path angle and a tangential velocity change of 50 [km/s]. The resulting orbit 0.1 Earth Days after the velocity impulse with new values can be seen in Figure 26. Here, the closest the probe got to Kerberos from Pluto was 806.98 [km]. Figure 27 uses the same parameters but expands the time bounds to 5 Earth days. Figure 28 shows the trajectory 50 days after impulse.



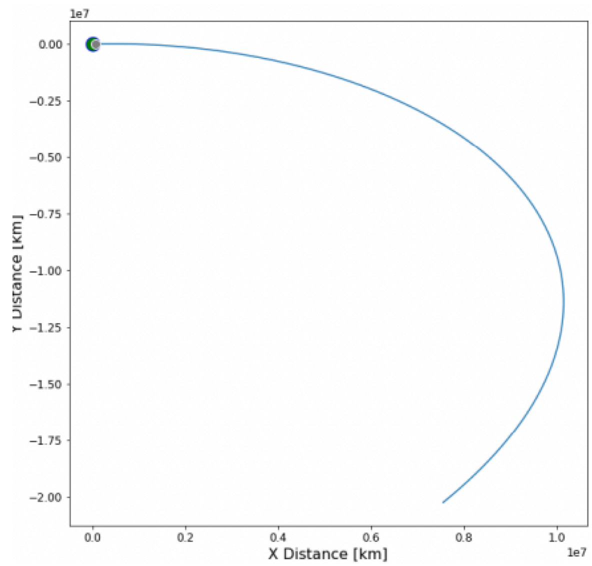


Figure 26 (Right): Pluto Nix system on Probe 5 days after impulse (50 [km/s] 0.05 [deg])
Figure 27 (Right): Pluto Nix system on Probe 5 days after impulse (50 [km/s] 0.05 [deg])

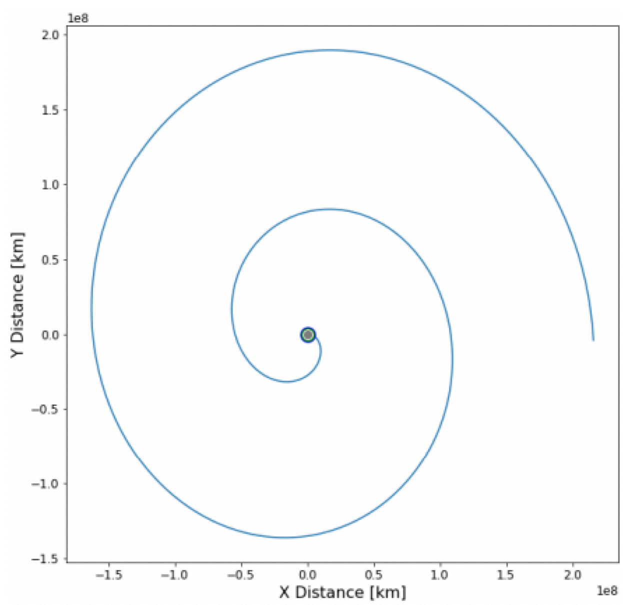


Figure 28: Pluto Nix system on Probe 50 days after impulse (50 [km/s] 0.05 [deg])

With a heavier moon, the probe was able to reach an altitude closer than the previous few moons; however, it is still a respectable distance away, most likely requiring a second burn to get near the surface. The revolutions of the spiral are somewhat consistent with previous trajectories, leading to some conclusions regarding the relationship between these spirals and angular velocity, and the distance between spirals is also very similar to other trajectories.

Conclusion

As a result of analyzing the trajectory of a probe sent close to each of Pluto's moons using the restricted three body equations, the impacts of mass, distance from pluto, and angular velocity of the moon with respect to pluto can be realized. The importance of flight path angle and impulse velocity can also be analyzed.

For flight path angle and initial velocity, it is evident that these parameters are only so effective and are heavily influenced by the mass of the orbiting moon and the distance from Pluto. This can be seen in Charon as the large mass and relatively close distance meant that with the 0.05 degrees of inclination with a 50 [km/s] speed, the mass of Charon pulled the probe towards itself and down over its north pole. This trend continues with Styx as its mass is smaller than Charon and farther away, meaning that the probe did not get as close, but it still got closer to the surface compared to the remaining moons which had much smaller masses and larger distances. With the smaller masses, the probe experiences less gravitational pull due to the moon and is pulled more up towards Pluto itself. This pull towards Pluto combined with the larger distances meant that as the Probe passed over the moon, it was much farther away from its surface. This is further supported by the 5 day plots as the larger moons pulled the probe back creating a more full parabolic shape, and the smaller mass moons mulled the probe less farther back in the same time window.

When 30 [km/s] and 0.0 [deg] were used, it was clear from those corresponding plots that the mass of Pluto dominated the system no matter the scale of the opposing moon and resulted in the probe being pulled down and back towards Pluto. From the flight path and initial angle tests, it can be concluded that a positive inclination with a large velocity impulse would be required to reach the moons like Charon and Styx, and that reaching Hydra, Nix, and Kerberos would require a secondary impulse.

When analyzing the 50 day plots for each moon, it can be concluded that the angular velocity of the orbiting moon about Pluto correlates to the number of spirals the probe complets around the 2 bodies. In order from highest angular velocity to lowest angular velocity is Charon, Styx, Nix, Kerberos, and Hydra. From the 50 day plot, this is also the order of the most completed spiral revolutions. Styx, Nix, Kereberos, and Hydra are very close in revolution count due to the same magnitude of power their angular velocity has; however, the much larger angular velocity of Charon causes the number of revolutions to be much larger. Also noticeable from this series of plots is that as the moon's mass increases and noticeable impacts the center of mass, the spirals of the probe are much tighter with less distance in between each revolution. This is noticeable with Charon and Styx, and not as noticeable with the other 3 moons due to the scale of their mass versus Pluto.

Thus, a mission around Pluto and its moons would require scientists to identify the importance of the moon's mass, distance from Pluto, and angular velocity. From this, it can be determined if a secondary burn would be required to get the probe near the moon's surface.

References

Christie, J. (n.d.). *By the Numbers | Charon – NASA Solar System Exploration*. NASA Solar System Exploration. Retrieved May 7, 2023, from https://solarsystem.nasa.gov/moons/pluto-moons/charon/by-the-numbers/

Pluto - Pluto's moons | Britannica. (n.d.). Encyclopedia Britannica. Retrieved May 7, 2023, from https://www.britannica.com/place/Pluto-dwarf-planet/Plutos-moons

Weber, B. (n.d.). *Circular Restricted Three-Body Problem — Orbital Mechanics & Astrodynamics*. Orbital Mechanics & Astrodynamics. Retrieved May 7, 2023, from https://orbital-mechanics.space/the-n-body-problem/circular-restricted-three-body-problem.html