Chase Jacobs

Math 411-01

9 hrs

1/5

C2-P5, P6, P7
-P8, P9, P10

C2-P5
2.2.4

Solve the system by finding the LU factorization and then carrying out the two-step back substitution

a) $\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$ $\begin{matrix} -R1+R3 \\ -2R1+R2 \end{matrix} \rightarrow \underset{U}{\begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}}\underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}}$

$LUx = b$

$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$ $\begin{matrix} c_1 = 0 \\ c_2 = 1 \\ c_3 = 3 \end{matrix}$ $\begin{bmatrix} 3 & 1 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}$ $\begin{matrix} x_1 = -1 \\ x_2 = 1 \\ x_3 = 1 \end{matrix}$

b) $\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ $\begin{matrix} -R1+R2 \\ -\frac{1}{2}R1+R3 \\ -\frac{1}{2}R2+R3 \end{matrix} \rightarrow \underset{U}{\begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}}\underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix}}$

$LUx = b$

$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ .5 & .5 & 1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ $\begin{matrix} c_1 = 2 \\ c_2 = 2 \\ c_3 = 4 \end{matrix}$ $\begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$ $\begin{matrix} x_1 = 1 \\ x_2 = -1 \\ x_3 = 2 \end{matrix}$