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Chase Jacobs

C2-pg
2.4.4a

Solve the system by finding $PA = LU$ and then carrying out two-step back substitution

a) $\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

$\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} \xrightarrow[\text{R1 R2}]{\text{switch}} \begin{bmatrix} 4 & 4 & 2 \\ 4 & 2 & 0 \\ 2 & 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 4 & 2 \\ 1 & -2 & -2 \\ 2 & 2 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 4 & 4 & 2 \\ 1 & -2 & -2 \\ 1/2 & 0 & 2 \end{bmatrix}$

$\underset{P}{\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \underset{A}{\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix}} = \underset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1/2 & 0 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} 4 & 4 & 2 \\ 0 & -2 & -2 \\ 0 & 0 & 2 \end{bmatrix}}$

$Lc = Pb$

$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1/2 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}$ $c_1 = 4$, $c_2 = -2$, $c_3 = 4$

$Ux = c$

$\begin{bmatrix} 4 & 4 & 2 \\ 0 & -2 & -2 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 4 \end{bmatrix}$ $x_1 = 1$, $x_2 = -1$, $x_3 = 2$