

# 1 Linear logic

## 1.1 Rule of linear sequent calculus

### 1.1.1 Additive

$$\begin{array}{c}
 \frac{\vdash \Gamma, A \quad \vdash \Gamma, B}{\vdash \Gamma, A \& B} \& \\
 \frac{\vdash \Gamma, A}{\Gamma, \forall x : A} \forall, x \text{ free in } \Gamma \\
 \frac{}{\vdash \Gamma, \top} \top \\
 \text{There is no for } 0.
 \end{array}
 \qquad
 \begin{array}{c}
 \frac{\vdash \Gamma, A_i}{\vdash \Gamma, A_1 \oplus A_2} \oplus_i \\
 \frac{\vdash \Gamma, A}{\Gamma, \exists x : A} \exists
 \end{array}$$

### 1.1.2 Multiplicative

$$\begin{array}{c}
 \frac{\vdash \Gamma, A \quad \vdash \Delta, B}{\vdash \Gamma, \Delta, A \otimes B} \otimes \\
 \frac{}{\vdash 1} 1 \\
 \text{There is usually no multiplicative quantifiers in linear logic.}
 \end{array}
 \qquad
 \begin{array}{c}
 \frac{\vdash \Gamma, A_1, A_2}{\vdash \Gamma, A_1 \wp A_2} \wp \\
 \frac{\vdash \Gamma}{\vdash \Gamma, \perp} \perp
 \end{array}$$

### 1.1.3 Exponentials

$$\begin{array}{c}
 \frac{\vdash \Gamma, ?A, ?A}{\vdash \Gamma, ?A} \text{contraction} \\
 \frac{\vdash \Gamma, A}{\vdash \Gamma, ?A} \text{Derilection}
 \end{array}
 \qquad
 \begin{array}{c}
 \frac{\vdash \Gamma}{\vdash \Gamma, ?A} \text{Weakening} \\
 \frac{\vdash ?\Gamma, A}{\vdash ?\Gamma, !A} \text{Promotion}
 \end{array}$$

### 1.1.4 Identity

$$\begin{array}{c}
 \frac{}{\vdash A, A^\perp} \text{axiom} \\
 \frac{A^\perp \vdash \Gamma}{\vdash A, \Gamma} R^\perp
 \end{array}
 \qquad
 \begin{array}{c}
 \frac{\vdash \Gamma, A \quad \vdash A^\perp, \Delta}{\vdash \Gamma, \Delta} \text{cut} \\
 \frac{\vdash A^\perp, \Gamma}{A \vdash \Gamma} L^\perp
 \end{array}$$