# 1 Linear logic

## 1.1 Rule of linear sequent calculus

### 1.1.1 Additive

$$\frac{\vdash \Gamma, A \qquad \vdash \Gamma, B}{\vdash \Gamma, A \,\&\, B}\,\& \qquad \frac{\vdash \Gamma, A_i}{\vdash \Gamma, A_1 \oplus A_2}\,\oplus_i$$

$$\frac{\vdash \Gamma, A}{\Gamma, \forall x : A}\,\forall,\ x \text{ free in } \Gamma \qquad \frac{\vdash \Gamma, A}{\Gamma, \exists x : A}\,\exists$$

$$\frac{}{\vdash \Gamma, \top}\,\top$$

There is no for 0.

### 1.1.2 Multiplicative

$$\frac{\vdash \Gamma, A \qquad \vdash \Delta, B}{\vdash \Gamma, \Delta, A \otimes B}\,\otimes \qquad \frac{\vdash \Gamma, A_1, A_2}{\vdash \Gamma, A_1 \parr A_2}\,\parr$$

$$\frac{}{\vdash 1}\,1 \qquad \frac{\vdash \Gamma}{\vdash \Gamma, \bot}\,\bot$$

There is usually no multiplicative quantifiers in linear logic.

### 1.1.3 Exponentials

$$\frac{\vdash \Gamma, ?A, ?A}{\vdash \Gamma, ?A}\,\text{contraction} \qquad \frac{\vdash \Gamma}{\vdash \Gamma, ?A}\,\text{Weakening}$$

$$\frac{\vdash \Gamma, A}{\vdash \Gamma, ?A}\,\text{Derilection} \qquad \frac{\vdash ?\Gamma, A}{\vdash ?\Gamma, !A}\,\text{Promotion}$$

### 1.1.4 Identity

$$\frac{}{\vdash A, A^\perp}\,\text{axiom} \qquad \frac{\vdash \Gamma, A \qquad \vdash A^\perp, \Delta}{\vdash \Gamma, \Delta}\,\text{cut}$$

$$\frac{A^\perp \vdash \Gamma}{\vdash A, \Gamma}\,R^\perp \qquad \frac{\vdash A^\perp, \Gamma}{A \vdash \Gamma}\,L^\perp$$