Algorithm 1 Cycle-Aware Battery Discharge Simulation

Input: Initial SOC_0, initial capacity $Q_{\max}^{(0)}$, initial resistance $R_0^{(0)}$, number of cycles N_{cyc}

Output: Time-to-empty $\mathrm{TTE}^{(k)}$, critical SOC $\mathrm{SOC}_{\mathrm{crit}}^{(k)}$, and shutdown mode for each cycle

for $k = 1, \ldots, N_{\mathrm{cyc}}$ **do**
    Update aging parameters:
    $Q_{\max}^{(k)} \leftarrow \mathcal{A}_Q\big(Q_{\max}^{(k-1)}\big),\ R_0^{(k)} \leftarrow \mathcal{A}_R\big(R_0^{(k-1)}\big)$
    Initialize fast-time states $(\mathrm{SOC}, t, V_{c1}, V_{c2})$
    while power feasibility condition $\Delta > 0$ is satisfied **do**
        Solve algebraic power constraint for current I
        Integrate the coupled DAE system over one time step
    end while
    Record $\mathrm{TTE}^{(k)}$, $\mathrm{SOC}_{\mathrm{crit}}^{(k)}$, and shutdown mode
end for