Final Fisica 1 SIBIS Fecha 2 Invierno 2015

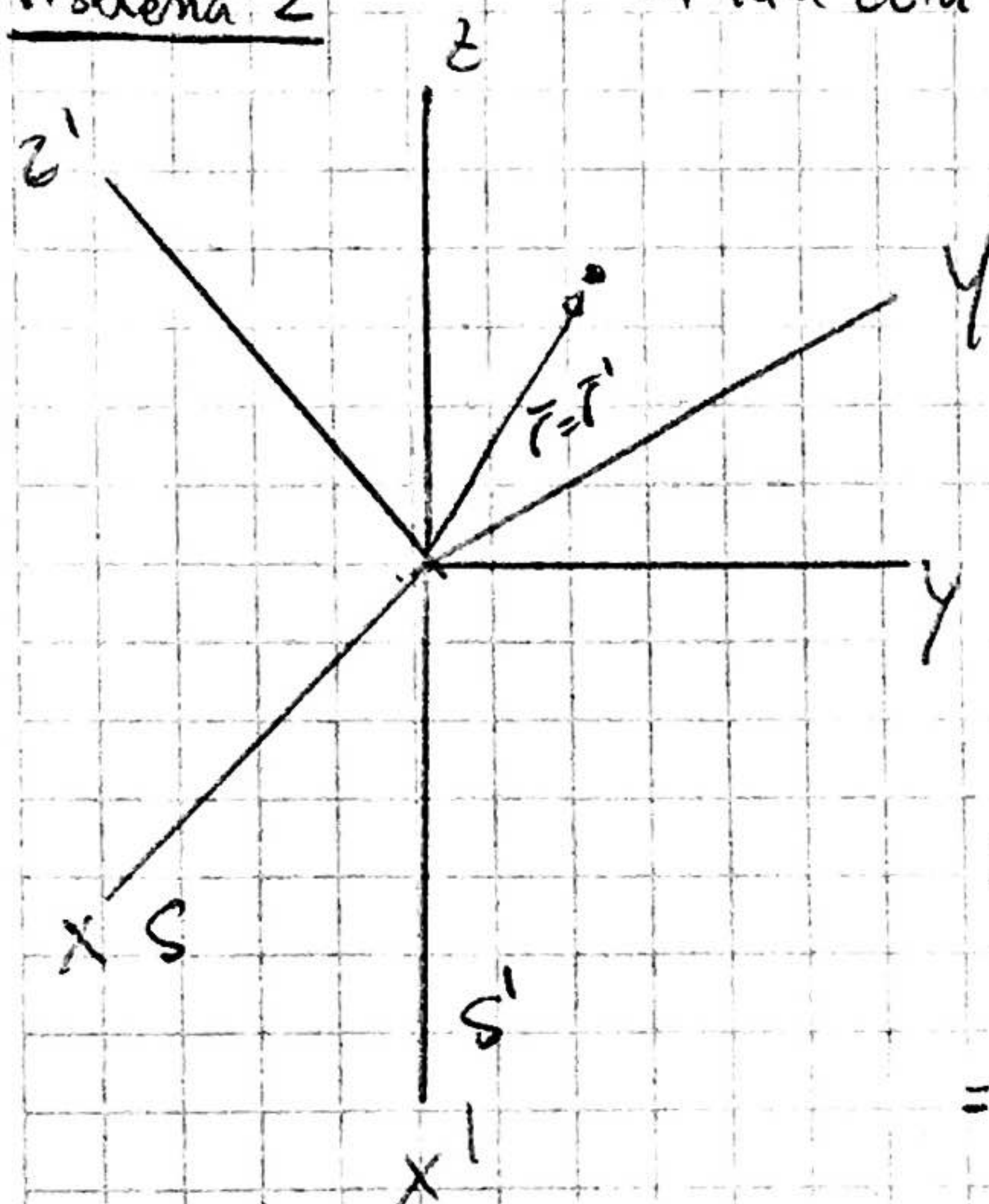
1) Oscilador amortiguado
2) Deducir las fuerzas de un SNI rotante
3) Deducir las ecuaciones que rigen el movimiento de un cuerpo rigido
4) Contraccion de lorentz

Una vez hecho, EXITOS!

HOJA N°

FECHA

Problema 2   S' rota con $\overline{\omega}$



$\bar r = \bar r'$

$\bar r = x\hat x + \hat y y + z\hat z$

$\bar r' = x'\hat x' + y'\hat y' + z'\hat z'$

$$\left.\frac{d\bar r}{dt}\right|_S = \left.\frac{d}{dt}\left(x'\hat x' + y'\hat y' + z'\hat z'\right)\right|_S$$

$$= \cancel{\dot{\bar r}' + \frac{d}{dt}}\ '$$

$$= \dot x'\hat x' + \dot y'\hat y' + \dot z'\hat z' + x\dot{\hat x}' + y\dot{\hat y}' + z\dot{\hat z}'$$

$$= \bar v' + \bar\omega \times \bar r'$$

$\dot{\hat x}'$ el versor mantiene su norma, es velocidad pura de rotación

$\dot{\hat x}' = \bar\omega \times \hat x'$

$$\Rightarrow \left.\frac{d}{dt}\right|_S = \left.\frac{d}{dt}\right|_{S'} + \bar\omega \times (\ )$$

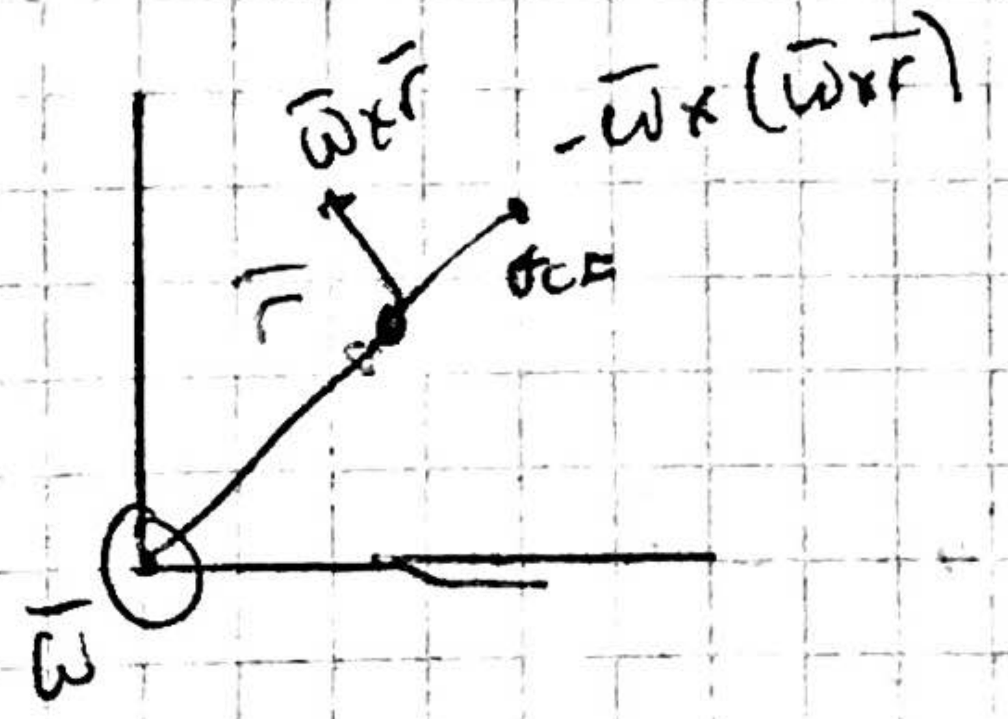
$$\left.\frac{d\bar v}{dt}\right|_S = \left.\frac{d}{dt}\left(\bar v' + \bar\omega \times \bar r'\right)\right|_{S'} + \bar\omega \times \left(\bar v' + \bar\omega \times \bar r'\right)$$

$$\bar a = \bar a' + \dot{\bar\omega} \times \bar r' + \bar\omega \times \bar v' + \bar\omega \times \bar v' + \bar\omega \times (\bar\omega \times \bar r')$$

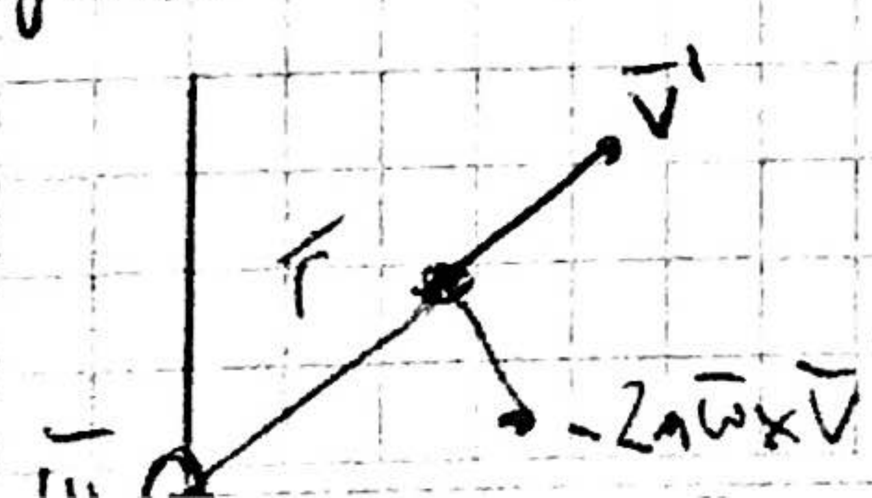
Luego

$$m\bar a' = \underbrace{m\bar a}_{\bar F} - m\bar\omega \times (\bar\omega \times \bar r') - 2m\bar\omega \times \bar v' - m\dot{\bar\omega} \times \bar r'$$

$f_{cf} = -m\bar\omega \times (\bar\omega \times \bar r)$ aparece siempre en un SNI rotante y apunta hacia afuera del eje de rotación



$f_{coriolis} = -2m\bar\omega \times \bar v'$ aparece si la masa tiene velocidad respecto al sistema rotante,



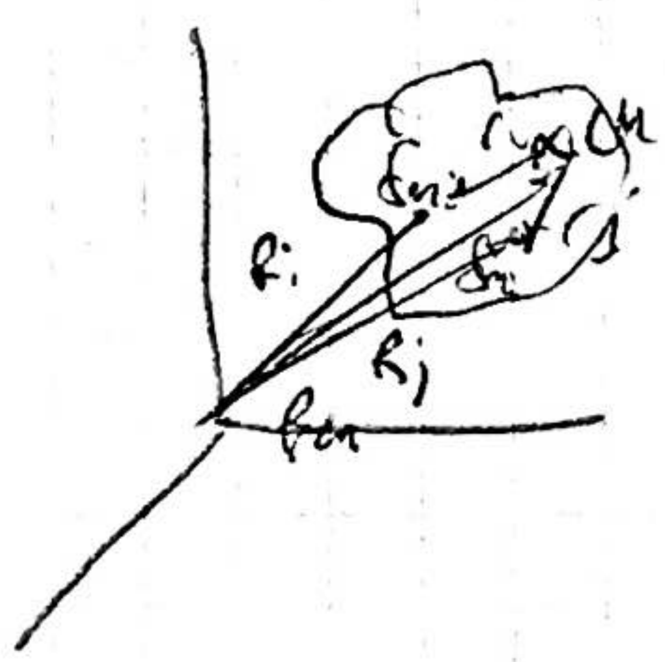
$f_{translación} = -m\dot{\hat{\omega}} \times \bar{r}$ aparece si la rotación ~~[illegible]~~ cambia (como vector)

<u>Problema 3</u>

Son tres ecuaciones $\bar{P}$, $\bar{I}$, $\bar{L}$

Empecemos

Condición de rigidez



$|R_j - R_i| = cte$ para ser rígido, o sino

$(\bar{R_j} - \bar{R_i})^2 = cte$

$\Rightarrow \frac{d}{dt}(\bar{R_j}-\bar{R_i})^2 = 0 = 2(\bar{R_j}-\bar{R_i})(\bar{V_j}-\bar{V_i}) = 0$

cte $(\bar{R_j}-\bar{R_i}) \perp (\bar{V_j}-\bar{V_i})$

$\bar{P_T} = \sum_{i=1}^{\hat{n}} \delta m_i \cdot \dot{\bar{R_i}} \Rightarrow \frac{dP_T}{dt} = \frac{d}{dt}\left(\sum_{i=1}^{\hat{n}} \delta m_i \cdot \dot{\bar{R_i}}\right)$

$= \sum_{i=1}^{\hat{n}} \delta m_i \frac{d\dot{\bar{R_i}}}{dt} = \sum_{i=1}^{\hat{n}} \delta m_i \bar{A_i} = \sum_{i=1}^{\hat{n}} \bar{F_i}^{ext} + \sum_{i \neq j}^{\hat{n}} \bar{F_{ij}}$

$= \sum_{i=1}^{\hat{n}} \bar{F_i}^{ext} + \underbrace{(\cancel{\bar{F}_{12}} + \bar{F}_{13} + \dots + \cancel{\bar{F}_{21}} + \bar{F}_{23} + \dots)}_{=\emptyset}$

$\Rightarrow \frac{dP_T}{dt} = \bar{F_i}^{ext}_T$

Si ahora lo veo desde el CM

$P_T = \sum_{i=1}^{\hat{n}} \delta m_i (\bar{V_{cm}} + \bar{\omega} \times \bar{r_i}) = \sum_{i=1} \delta m_i \bar{V_{cm}} + \sum_{i=1}^{\hat{n}} \delta m_i \bar{\omega} \times \bar{r_i}$

$= M\bar{V_{cm}} + \bar{\omega} \times \underbrace{\sum_{i=1}^{\hat{n}} \delta m_i \bar{r_i}}_{=0 \text{ respecto CM}}$

$\Rightarrow M\bar{V_{cm}} = P_T = \sum \delta m_i \bar{V_i}$

$y \ \frac{dP_T}{dt} = \bar{F_i}^{ext}_{TOT} = M\bar{A_{cm}}$

Finalmente $\boxed{P_T = M\bar{V_{cm}} = \sum_{i=1}^{\hat{n}} \delta m_i \bar{V_i} \qquad \frac{dP_T}{dt} = M\bar{A_{cm}} = \sum_{i=1}^{\hat{n}} F_i^{ext}}$

$$L_o = \sum_{i=1}^{n} \overline{R_i} \times \delta m_i \overline{V_i} \qquad \frac{d\overline{L_o}}{dt} = \frac{d}{dt}\left(\sum_{i=1}^{n} \overline{R_i} \times \delta m_i \overline{V_i}\right) = \sum_{i=1}^{n} \underbrace{\frac{d\overline{R_i}}{dt} \times \delta m_i \overline{V_i}}_{=0} + \sum_{i=1}^{n} \overline{R_i} \times \delta m_i \ldots$$

$$= \sum_{i=1}^{n} \overline{R_i} \times \left(\overline{F_i}^{ext} + \underbrace{\sum_{\substack{j=1 \\ j\neq i}}^{n} \overline{F_{ij}}}_{=0}\right)$$

$$\sum_{i=1}^{n} R_i \times \sum_{\substack{j=1 \\ j\neq i}}^{n} \overline{F_{ij}} = (\overline{R_1} - \overline{R_2}) \times \overline{F_{12}} = 0$$

por ser igual dirección

$$\Rightarrow \frac{d\overline{L_o}}{dt} = \sum_{i=1}^{n} \overline{R_i} \times F_i^{ext}$$

Viendolo desde cm $\overline{R_i} = \overline{R_{cm}} + \overline{r_i}$ $\quad \overline{V_i} = \overline{V_{cm}} + \overline{\omega} \times \overline{r_i}$

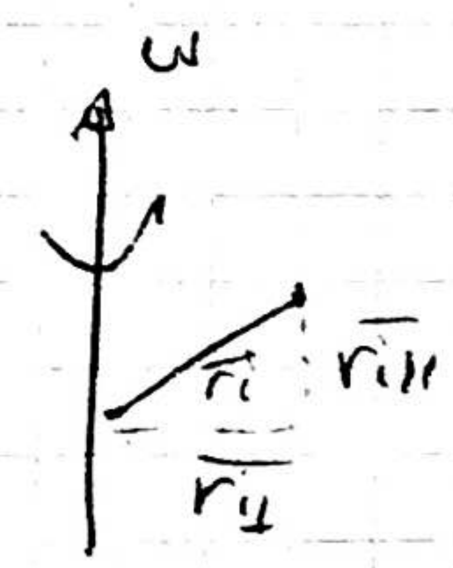
$$\Rightarrow \overline{L_o} = \sum_{i=1}^{n} (\overline{R_{cm}} + \overline{r_i}) \times \delta m_i (\overline{V_{cm}} + \overline{\omega} \times \overline{r_i})$$

$$= \underbrace{\sum_{i=1}^{n} \overline{R_{cm}} \times \delta m_i \overline{V_{cm}}}_{L_{cm}} + \sum_{i=1}^{n} \overline{R_{cm}} \times \overline{\omega} \times \overline{r_i}\, \delta m_i + \sum_{i=1}^{n} \overline{r_i} \times \delta m_i \overline{V_{cm}} + \underbrace{\sum_{i=1}^{n} \overline{r_i} \times \delta m_i (\overline{\omega} \times \overline{r_i})}_{L_{int}}$$

$$\overline{R_{cm}} \times \overline{\omega} \times \underbrace{\left(\sum_{i=1}^{n} \delta m_i \overline{r_i}\right)}_{=0} \qquad = \underbrace{\left(\sum_{i=1}^{n} \delta m_i \overline{r_i}\right)}_{=0} \times \overline{V_{cm}}$$

$$\Rightarrow \overline{L_o} = \overline{R_{cm}} \times M \overline{V_{cm}} + \sum_{i=1}^{n} \overline{r_i} \times \delta m_i (\overline{\omega} \times \overline{r_i})$$

$$\overline{r_i} \times \overline{\omega} \times \overline{r_i}$$



$$= \overline{r_i} \times \overline{\omega} \times (\overline{r_{i\perp}})$$

$$\sum_{i=1}^{n} \overline{r_i} \times \delta m_i (\overline{\omega} \times \overline{r_i})$$

$$\Rightarrow \overline{\omega}(\overline{r_i}\, \overline{r_{i\perp}}) - \overline{r_{i\perp}}(\overline{r_i} \overline{\omega}) = \overline{\omega}(r_{i\perp}^2) - \overline{r_{i\perp}} \underbrace{(\overline{r_\parallel} \omega)}_{=0}$$

vamos a trabajar con cuerpos de rotación axial $\Rightarrow$

$$\Rightarrow \overline{L_o} = \overline{R_{cm}} \times M \overline{V_{cm}} + \overline{\omega} \underbrace{\sum_{i=1}^{n} \delta m_i r_{i\perp}^2}_{I_{cm}}$$

$$\Rightarrow \overline{L_o} = \overline{R_{cm}} \times M \overline{V_{cm}} + \overline{\omega} I_{cm} \qquad \text{adm.} \quad \sum_{i=1}^{n} \overline{R_i} \times \overline{F_i}^{ext}$$

$$\frac{d\overline{L_o}}{dt} = \overline{R_{cm}} \times M \overline{A_{cm}} + \frac{d\overline{\omega}}{dt} I_{cm} \qquad = \overline{R_{cm}} \times \overline{F_{ext}} + \sum_{i=1}^{n} \overline{r_i} \times \overline{F_i}^{ext}$$

$$\boxed{\begin{gathered} \Rightarrow \frac{d\overline{L_o}}{dt} = \overline{R_{cm}} \times M \overline{A_{cm}} = \sum \overline{R_i} \times \overline{F_i}^{ext} = \overline{R_{cm}} \times \overline{F_{i\,ext}} + \sum_{i=1}^{n} \overline{r_i} \times \overline{F_{i\,ext}} \\ \Rightarrow \frac{d\overline{\omega}}{dt} I_{cm} = \sum_{i=1}^{n} \overline{r_i} \times \overline{F_{i\,ext}} \end{gathered}}$$

$\hat{\ } \; m_i \bar{V}_i \; \frac{dL}{dt} \ldots \sum$

$$L_0 = \sum_{i=1}^{n} \bar{R}_i \times \delta m_i \bar{V}_i \qquad \frac{d\bar{L}_0}{dt} = \frac{d}{dt}\left(\sum_{i=1}^{n} \bar{R}_i \times \delta m_i \bar{V}_i\right) = \sum_{i=1}^{n} \underbrace{\frac{d\bar{R}_i}{dt} \times \delta m_i \bar{V}_i}_{=0} + \sum_{i=1}^{n} \bar{R}_i \times \delta m_i \bar{A}_i$$

$$= \sum_{i=1}^{n} \bar{R}_i \times \Big(\bar{F}_i^{ext} + \underbrace{\sum_{\substack{j=1 \\ j\neq i}}^{n} \bar{F}_{ij}}_{=0}\Big)$$

$$\sum_{i=1}^{n} \bar{R}_i \times \sum_{\substack{j=1 \\ j\neq i}}^{n} \bar{F}_{ij} = (\bar{R}_1 - \bar{R}_2) \times \bar{F}_{12} = 0$$

por ser igual dirección

$$\Rightarrow \frac{d\bar{L}_0}{dt} = \sum_{i=1}^{n} \bar{R}_i \times F_i^{ext}$$

Viéndolo desde cm $\quad \bar{R}_i = \bar{R}_{cm} + \bar{r}_i \quad \bar{V}_i = \bar{V}_{cm} + \bar{\omega} \times \bar{r}_i$

$$\Rightarrow \bar{L}_0 = \sum_{i=1}^{n} (\bar{R}_{cm} + \bar{r}_i) \times \delta m_i (\bar{V}_{cm} + \bar{\omega} \times \bar{r}_i)$$

$$= \underbrace{\sum_{i=1}^{n} \bar{R}_{cm} \times \delta m_i \bar{V}_{cm}}_{L_{cm}} + \sum_{i=1}^{n} \bar{R}_{cm} \times \bar{\omega} \times \bar{r}_i \delta m_i + \sum_{i=1}^{n} \bar{r}_i \times \delta m_i \bar{V}_{cm} + \underbrace{\sum_{i=1}^{n} \bar{r}_i \times \delta m_i (\bar{\omega} \times \bar{r}_i)}_{L_{int}}$$

$$\bar{R}_{cm} \times \bar{\omega} \times \Big(\underbrace{\sum_{i=1}^{n} \delta m_i \bar{r}_i}_{=0}\Big) \qquad = \Big(\underbrace{\sum_{i=1}^{n} \delta m_i \bar{r}_i}_{=0}\Big) \times \bar{V}_{cm}$$

$$\Rightarrow \bar{L}_0 = \bar{R}_{cm} \times M\bar{V}_{cm} + \sum_{i=1}^{n} \bar{r}_i \times \delta m_i (\bar{\omega} \times \bar{r}_i)$$

$$\bar{r}_i \times \bar{\omega} \times \bar{r}_i$$

$$= \bar{r}_i \times \bar{\omega} \times (\bar{r}_{i\perp})$$

$$\sum_{i=1}^{n} \bar{r}_i \times \delta m_i (\bar{\omega} \times \bar{r}_i)$$

$$\Rightarrow \bar{\omega}(\bar{r}_i \bar{r}_{i\perp}) - \bar{r}_{i\perp}(\bar{r}_i \bar{\omega}) = \bar{\omega}(r_{i\perp}^2) - \underbrace{\bar{r}_{i\perp}(\bar{r}_{\parallel} \omega)}_{\to 0}$$

vamos a trabajar con cuerpos de rotación axial $\Rightarrow$

$$\Rightarrow \bar{L}_0 = \bar{R}_{cm} \times M\bar{V}_{cm} + \bar{\omega} \underbrace{\sum_{i=1}^{n} \delta m_i r_{i\perp}^2}_{I_{cm}} - 0$$

$$\Rightarrow \bar{L}_0 = \bar{R}_{cm} \times M\bar{V}_{cm} + \bar{\omega} I_{cm} \qquad \text{adem.} \; \sum_{i=1}^{n} \bar{R}_i \times \bar{F}_i^{ext}$$

$$\frac{d\bar{L}_0}{dt} = \bar{R}_{cm} \times M\bar{A}_{cm} + \frac{d\bar{\omega}}{dt} I_{cm} \qquad = \sum_{i=1}^{n} \bar{R}_{cm} \times \bar{F}_{ext} + \sum_{i=1}^{n} \bar{r}_i \times \bar{F}_i^{ext}$$

$$\boxed{\Rightarrow \frac{d\bar{L}_0}{dt} - \bar{R}_{cm} \times M\bar{A}_{cm} = \sum \bar{R}_i \times \bar{F}_i^{ext} = \bar{R}_{cm} \times \bar{F}_{ext} + \sum_{i=1}^{n} \bar{r}_i \times \bar{F}_i^{ext} \qquad \Rightarrow \frac{d\bar{\omega}}{dt} I_{cm} = \sum_{i=1}^{n} \bar{r}_i \times \bar{F}_i^{ext}}$$

CIUIS

Final Invierno Fecha 1

1) choque en un sistema aislado
a) se conserva la energia
b) el choque es plastico
Datos m1 m2 v1i v2i
Encontrar las velocidades finales

2) A partir del trabajo mecánico derive la conservación de la energía mecánica (y discuta bajo qué condiciones se conserva). Analice un ejemplo donde la energía mecánica no se conserva.

3) Describa el argumento de Newton que conduce a la expresión de la fuerza gravitatoria Fg = -Gm1m2 /r^2
Mencione el punto de partida y supuestos adicionales.

4) Derive la expresión de la energia cinetica de un cuerpo rigido suponiendo que rota alrededor de su eje de simetria.
a) Escriba la expresion considerando rotacion en torno a un eje que pasa por el cm
b) Escriba la expresion considerando que gira en torno a un eje instantaneo o permanente de rotacion.

(Planteas que la energia cinetica del cuerpo rigido es igual a la sumatoria de cada energia cinetica)

Problema 1

Datos: $m_1, m_2, \overline{V_{10}}, \overline{V_{20}}$

Choque aislado

1) Se conserva la energía $\Rightarrow$

sea t_0 antes de chocar y t_F luego

$\Rightarrow$ $\overline{P_0} = m_1\overline{V_{10}} + m_2\overline{V_{20}}$ por sistema aislado $\overline{P_F} = m_1\overline{V_{1F}} + m_2\overline{V_{2F}}$

y $T_0 = \frac{1}{2}m_1\overline{V_{10}}^2 + \frac{1}{2}m_2\overline{V_{20}}^2$ como se conserva la energía $T_F = T_0 = \frac{1}{2}m_1\overline{V_{1F}}^2 + \frac{1}{2}m_2\overline{V_{2F}}^2$

$\Rightarrow$ (1) $\frac{m_1\overline{V_{10}}^2}{\cancel{2}} + \frac{m_2\overline{V_{20}}^2}{\cancel{2}} = \frac{m_1\overline{V_{1F}}^2}{\cancel{2}} + \frac{m_2\overline{V_{2F}}^2}{\cancel{2}}$

(2) $m_1\overline{V_{10}} + m_2\overline{V_{20}} = m_1\overline{V_{1F}} + m_2\overline{V_{2F}}$

(1) lo reescribo como $m_1(\overline{V_{10}} - \overline{V_{1F}})(\overline{V_{10}} + \overline{V_{1F}}) = m_2(\overline{V_{2F}} - \overline{V_{20}})(\overline{V_{2F}} + \overline{V_{20}})$

(2) $m_1(\overline{V_{10}} - \overline{V_{1F}}) = m_2(\overline{V_{2F}} - \overline{V_{10}})$

divido $\frac{(1)}{(2)}$ $\Rightarrow$ $\overline{V_{10}} + \overline{V_{1F}} = \overline{V_{2F}} + \overline{V_{20}}$

Luego (1) $\overline{V_{10}} + \overline{V_{1F}} = \overline{V_{2F}} + \overline{V_{20}}$ $\Rightarrow$ $\overline{V_{1F}} = \overline{V_{2F}} + \overline{V_{20}} - \overline{V_{10}}$

(2) $m_1(\overline{V_{10}} - \overline{V_{1F}}) = m_2(\overline{V_{2F}} - \overline{V_{20}})$

Luego $m_1\overline{V_{10}} - m_1(\overline{V_{2F}} + \overline{V_{20}} - \overline{V_{10}}) = m_2(\overline{V_{2F}} - \overline{V_{20}})$

$\Rightarrow$ $2m_1\overline{V_{10}} - m_1\overline{V_{20}} + m_2\overline{V_{20}} = (m_1 + m_2)\overline{V_{2F}}$

$$\boxed{\overline{V_{2F}} = \frac{m_1(2\overline{V_{10}} - \overline{V_{20}}) + m_2\overline{V_{20}}}{m_1 + m_2}}$$

$$\overline{V_{1F}} = \frac{m_1(2\overline{V_{10}} - \overline{V_{20}}) + m_2\overline{V_{20}} + (m_1 + m_2)(\overline{V_{20}} - \overline{V_{10}})}{m_1 + m_2}$$

$$\boxed{\overline{V_{1F}} = \frac{m_1(\overline{V_{10}}) + m_2(2\overline{V_{20}} - \overline{V_{10}})}{m_1 + m_2}}$$

b) El choque es plástico

$$\Rightarrow \cancel{\frac{1}{2}m_1 V_{10}^2 + \frac{1}{2} m_2 V_{20}^2 = \frac{1}{2}(m_1+m_2)V_F^2} \quad \text{no}$$

$$m_1\bar{V}_{10} + m_2\bar{V}_{20} = (m_1+m_2)\bar{V}_F$$ conservación de $\bar{p}$

$$\Rightarrow \boxed{\bar{V}_F = \frac{m_1\bar{V}_{10}+m_2\bar{V}_{20}}{m_1+m_2} = \bar{V}_{cm}}$$

Problema 2

$$\Delta W_i = \bar{F}\cdot\Delta\bar{r}$$

$$\Rightarrow \Delta W_{1,2} = \sum_{i=1}^{N} F_i \Delta\bar{l} \Rightarrow \lim_{\Delta\bar{l}\to 0} = W_{1,2} = \int_{1\to 2}\bar{F}\cdot d\bar{l}$$

$$W_{1,2} = \int_{1\to 2}\bar{F}\cdot d\bar{l} = \int_{1\to 2} m\bar{a}\cdot d\bar{l} = \int_{1\to 2} m\frac{d\bar{v}}{dt}\cdot\bar{v}\,dt = \int_{1\to 2} m(V_x, V_y, V_z)(dV_x, dV_y, dV_z)$$

$$= \frac{m}{2}(V_{2x}^2+V_{2y}^2+V_{2z}^2) - \frac{1}{2}m(V_{1x}^2+V_{1y}^2+V_{1z}^2) = \frac{m}{2}(V_2^2-V_1^2) = \Delta T$$

$$\Rightarrow W_{1,2} = \Delta T = \int_{1\to 2}\bar{F}_{explícita}\cdot d\bar{l} = \int_{1\to 2}\bar{F}_{cons}\cdot d\bar{l} + \int_{1\to 2}\bar{F}_{Nocons}\cdot d\bar{l}$$

Si $\bar{F}_{cons} \Rightarrow \bar{F}_{cons} = -\nabla U$

$$\Rightarrow \Delta T = \int_{1\to 2} -\left(\frac{\partial U}{\partial x}, \frac{\partial U}{\partial y}, \frac{\partial U}{\partial z}\right)\cdot(dx, dy, dz) + \int_{1\to 2}\bar{F}_{nocons}\cdot d\bar{l}$$

$$\Leftrightarrow \Delta T = \underbrace{U_1 - U_2}_{\Delta U} + \int_{1\to 2}\bar{F}_{nocons}\cdot d\bar{l}$$

Luego $\underbrace{\Delta T + \Delta U}_{\Delta E_{mec}} = \int_{1\to 2}\bar{F}_{nocons}\cdot d\bar{l}$

$$\boxed{\Delta E_{mec} = 0 \Leftrightarrow \int_{1\to 2}\bar{F}_{nocons}\cdot d\bar{l} = 0}$$

$$E_{mec} = \frac{1}{2}mV^2 + U$$

Luego la energía mecánica se conserva cuando las fuerzas no conservativas (por ej, el roz) no hacen trabajo

Por ejemplo, el caso anterior

$$\Delta E_{mec} = -\left(\frac{1}{2}m_1V_{10}^2 + \frac{1}{2}m_2V_{20}^2\right) + \frac{1}{2}(m_1+m_2)\left(\frac{m_1V_{10}+m_2V_{20}}{m_1+m_2}\right)^2$$

$$\Rightarrow \frac{1}{2(m_1+m_2)}\left(m_1^2 V_{10}^2 + 2m_1m_2V_{10}V_{20} + m_2 V_{20}^2\right) - \frac{1}{2}m_1V_{10}^2 - \frac{1}{2}m_2V_{20}^2$$

• si $m_1 = m_2 = m$

$$\Rightarrow \frac{1}{4\cancel{m}}\left(m^{\cancel{2}}(V_{10}^2 + V_{20}^2 + 2V_{10}V_{20})\right) - \frac{1}{2}mV_{10}^2 - \frac{1}{2}mV_{20}^2$$

$$\Rightarrow \frac{m}{4}V_{10}^2 + \frac{m}{4}V_{20}^2 + \frac{m}{2}V_{10}V_{20} - \left(\frac{m}{2}V_{10}^2 + \frac{m}{2}V_{20}^2\right) =$$

Feo, mejor

$\rightarrow \overline{V_0} = v$

$$\Rightarrow m\ddot{x} = -f_{RD} = -\mu_D N = -\mu_d m g$$

$$\Rightarrow \dot{x} = -\mu_d g t + V_0$$

$t_0 = 0$

$t_1 = t_1 \qquad \Rightarrow \frac{1}{2}m(-\mu_d g t_1 + V_0)^2 - \frac{1}{2}mV_0^2$

$t_1 = t_f \qquad \frac{V_0}{\mu_d g} = t \qquad \Rightarrow \boxed{0 - \frac{1}{2}mV_0^2 < 0}$

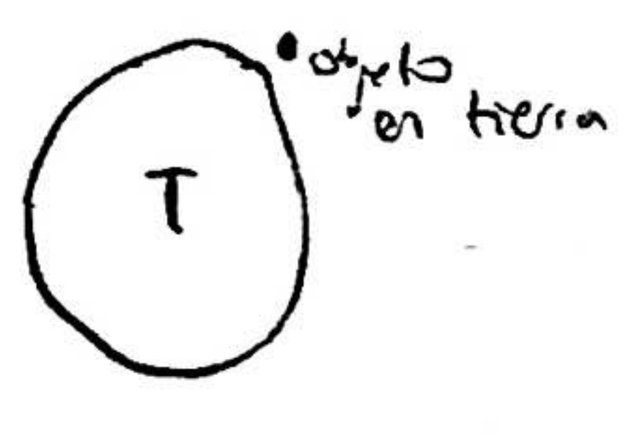
Problema 3: Newton lo ve como en un proceso de "3 pasos"

Primero, Galileo dedujo que despreciando los efectos del aire todo cuerpo cerca de la tierra cae con igual aceleración g

$$\Rightarrow m\overline{a} = \overline{F} = \overline{P} \simeq m\overline{g} \quad \Rightarrow \overline{a} = \overline{g}$$

Segundo

$$a_T = \frac{V_{T0}^2}{R_T} = \omega_0^2 R_T$$



$$a_L = \frac{V_{TL}^2}{R_L} \approx \frac{V_{TL}^2}{60 R_T} = \omega_L^2 R_T 60$$

$$\tau^2 = \frac{4\pi^2}{\omega^2} \Rightarrow \cancel{a = \omega^2} R \Rightarrow \tau^2 = \frac{4\pi^2 R_T}{a_T}$$

$$\Rightarrow a_T = \frac{4\pi^2 R_T}{\tau^2} \quad \text{y usando kepler } \tau^2 \sim k r^3$$

$$\Rightarrow a_T \approx \frac{4\pi^2 \, 1}{k R_T^2} \qquad a_L \approx \frac{4\pi^2}{k(R_T 60)^2}$$

$$\Rightarrow a_T = g \qquad \tau_L = 27{,}3 \text{ días}$$

$$a_L = \frac{g}{60^2}$$

Tercero

Tomo un caso por ejemplo sol tierra

$$a_T = \frac{4\pi^2}{k R_T^2} = \frac{Q_S}{R_T^2}$$

F_{ST} F_{TS}

$$|m_S a_S| = |m_T a_T|$$

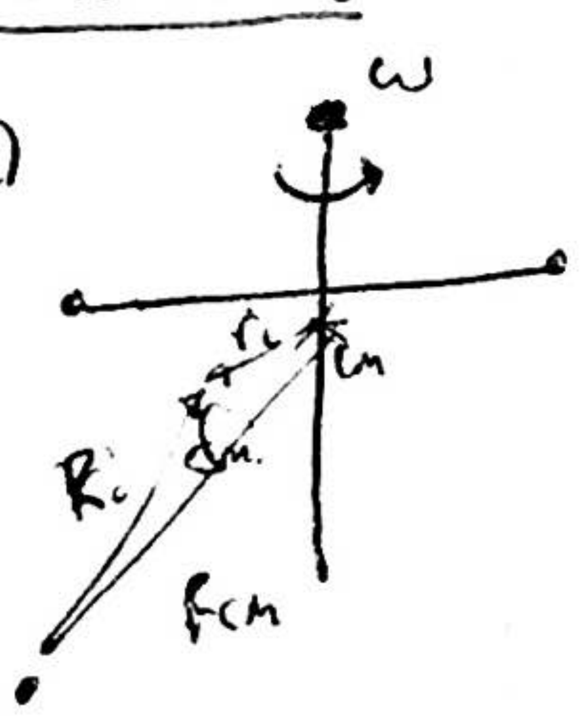
$$\Rightarrow m_S \frac{Q_T}{\cancel{R^2}} = m_T \frac{Q_S}{\cancel{R^2}}$$

$$\Rightarrow \frac{Q_T}{m_T} = \frac{Q_S}{m_S} = \frac{Q_P}{m_P} = G$$

$$\boxed{\Rightarrow |F_{ST}| = m_S a_S = m_S \frac{Q_T}{R_T^2} = \frac{m_S m_T G}{R^2}}$$

Problema 4

a)



$$\cancel{T = \sum_{i=1}^{N} \frac{\delta m_i}{2} \dot{R}_i^2 = \sum_{i=1}^{N} \frac{\delta m_i}{2} (\dot{R}_{cm} + \dot{r}_i)^2 = \sum_{i=1}^{N} \frac{\delta m_i}{2} \dot{R}_{cm}^2 + \sum_{i=1}^{N} \delta m_i \dot{R}_{cm} \dot{r}_i + \sum_{i=1}^{N} \frac{\delta m_i}{2} \dot{r}_i^2}$$

$$\overline{R_i} = \overline{R_{cm}} + \overline{r_i}$$

$$\overline{V_i} = \overline{V_{cm}} + \overline{\omega} \times \overline{r_i}$$

$$\Rightarrow T = \sum_{i=1}^{N} \frac{\delta m_i}{2} \overline{V_i}^2 = \frac{1}{2} \sum \delta m_i (\overline{V_{cm}} + \overline{\omega} \times \overline{r_i})^2 = \frac{1}{2}\Big(\sum \delta m_i \overline{V_{cm}}^2 + 2\sum \delta m_i \overline{V_{cm}}(\overline{\omega} \times \overline{r_i}) + \sum \delta m_i (\overline{\omega} \times \overline{r_i})^2 \Big)$$

$$= \frac{1}{2} M \overline{V_{cm}}^2 + \text{cosas} \qquad \overline{\omega} \times \overline{r_i} = \overline{\omega} \times \overline{r_{i\perp}}$$

$$2\sum \delta m_i \overline{V_{cm}} (\overline{\omega} \times \overline{r_i}) = 2\overline{V_{cm}}\big(\overline{\omega} \times \underbrace{\sum \delta m_i \overline{r_i}}_{=0}\big)$$

vista desde cm

$$\sum \delta m_i (\overline{\omega} \times \overline{r_i})^2 \qquad (\overline{\omega} \times \overline{r_i})^2 = (\overline{\omega} \times \overline{r_{i\perp}})^2 = |\overline{\omega}\, \overline{r_{i\perp}}|^2 - \underbrace{(\overline{\omega} \cdot \overline{r_{i\perp}})}_{=0}$$

$$\Rightarrow T = \frac{1}{2} M \overline{V_{cm}}^2 + \frac{1}{2} \sum \delta m_i \overline{\omega}^2 \overline{r_{i\perp}}^2 = \frac{1}{2} M \overline{V_{cm}}^2 + \frac{\overline{\omega}^2}{2} \sum \delta m_i r_{i\perp}^2 =$$

$$\boxed{\frac{1}{2} M \overline{V_{cm}}^2 + \frac{\overline{\omega}^2}{2} I_{cm}}$$

b) $\overline{V_{cm}} = \overbrace{\overline{V_0}}^{=0} + \overline{\omega} \times \overline{R_{cm}} = \overline{\omega} \times \overline{R_{cm}}$

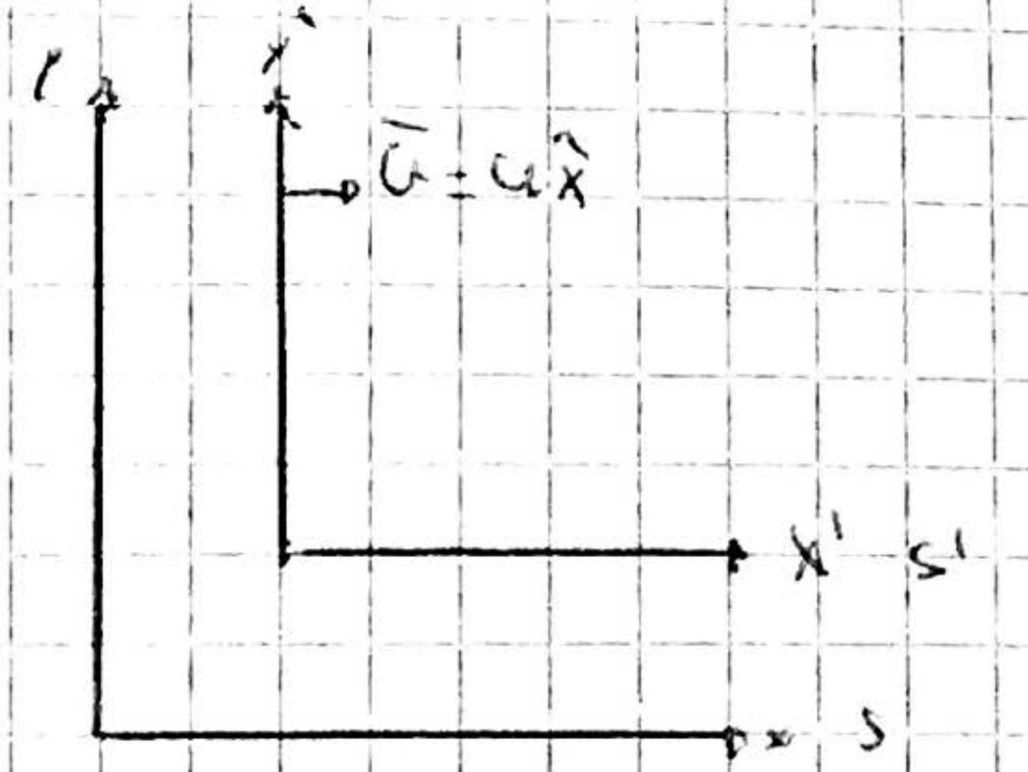
$$\Rightarrow T = \frac{1}{2} M \left(\overline{\omega} \times \overline{R_{cm}} \right)^2 + I_{cm} \frac{\omega^2}{2}$$

$$= \frac{1}{2} M \left(\left| \overline{\omega}^2 \overline{R_{cm}}^2 \right| - \underbrace{\left(\overline{\omega}\, \overline{R_{cm}} \right)^2}_{\Rightarrow 0 \quad \overline{\omega} \perp \overline{R_{cm}}} \right) + \frac{1}{2} \omega^2 I_{cm}$$

$$= \boxed{\frac{1}{2} \omega^2 \left(I_{cm} + M R^2 \right)}$$

Ambas ecuaciones suponen simetría axial

Problema 3) Usando la ley de adición de velocidades, demuestre que el cuadrivector $\left(\frac{E}{c}, \bar{p}\right)$ se transforma como $(ct, \bar{r})$



Utilizando la transformación de Lorentz

$$x = \gamma(x' + ut) \qquad y' = y \qquad t = \gamma\left(t' + \frac{\beta}{c}x'\right)$$
$$x' = \gamma(x - ut) \qquad z' = z \qquad t' = \gamma\left(t - \frac{\beta}{c}x\right)$$

Luego $V_x = \frac{dx}{dt}$ $\qquad dx = \gamma(dx' + u\,dt')$

$$dt = \gamma\left(dt' + \frac{\beta}{c}dx'\right)$$

Luego $V_x = \dfrac{\cancel{\gamma}(dx' + u\,dt')}{\cancel{\gamma}(dt' + \frac{\beta}{c}dx')} = \dfrac{\cancel{dt'}\left(\frac{dx'}{dt'} + u\right)}{\cancel{dt'}\left(1 + \frac{\beta}{c}\frac{dx'}{dt'}\right)} = \dfrac{V_x' + u}{1 + \frac{\beta}{c}V_x'}$

y analogamente $V_y = \dfrac{V_y'}{\gamma(1 + \frac{\beta}{c}V_x')}$

Finalmente

$$\boxed{\begin{array}{ll} V_x = \dfrac{V_x' + u}{1 + \frac{\beta}{c}V_x'} & V_x' = \dfrac{V_x - u}{1 - \frac{\beta}{c}V_x} \\ V_y = \dfrac{V_y'}{\gamma(1 + \frac{\beta}{c}V_x')} & V_y' = \dfrac{V_y}{\gamma(1 - \frac{\beta}{c}V_x)} \\ V_z = \dfrac{V_z'}{\gamma(1 + \frac{\beta}{c}V_x')} & V_z' = \dfrac{V_z}{\gamma(1 - \frac{\beta}{c}V_x)} \end{array}}$$

Analicemos el impulso. Si decimos $\bar{p} = m\bar{v}$ las cuentas no dan

$\Rightarrow \bar{p} = \dfrac{m_0 \bar{V}}{\left(1 - \frac{V^2}{c^2}\right)^{1/2}}$ $\qquad$ Veamos la energía

$$dW = \bar{F} \cdot d\bar{r} = \frac{d\bar{p}}{dt} \cdot \bar{v}\,dt$$

$$\frac{d}{dt}\left[\frac{m_0 \bar{V}}{(1 - \frac{V^2}{c^2})^{1/2}}\right] = m_0\left[\frac{\dot{\bar{V}}}{(1 - \frac{V^2}{c^2})^{1/2}} + \frac{\bar{V}\,V\dot{V}/c^2}{(1 - \frac{V^2}{c^2})^{3/2}}\right] = \frac{m_0}{(1 - V^2/c^2)^{3/2}}\left[\dot{\bar{V}}\left(1 - \frac{V^2}{c^2}\right) + \frac{\bar{V}V\dot{V}}{c^2}\right]$$

Luego $\bar{V} \cdot \dfrac{d}{dt}(\dots) = \dfrac{m_0 \dot{V} V}{(1 - V^2/c^2)^{3/2}}\left[\left(1 - \frac{V^2}{c^2}\right) + \frac{V^2}{c^2}\right] = \dfrac{m_0 \dot{V} V}{(1 - V^2/c^2)^{3/2}}$

$= \dfrac{d}{dt}\left[\dfrac{m_0 c^2}{(1 - \frac{V^2}{c^2})^{1/2}}\right]$ $\qquad$ Luego $dW = \dfrac{d}{dt}\left[\dfrac{m_0 c^2}{(1 - \frac{V^2}{c^2})^{1/2}}\right] dt$

$$\Rightarrow W = \int^{V} d\left(\frac{m_0 c^2}{(1 - \frac{V^2}{c^2})^{1/2}}\right) = m_0 c^2 (\gamma_B - 1)$$

Luego = $\Delta T = T$

Definimos la energía relativista $E = \dfrac{m_0 c^2}{\left(1 - \dfrac{v^2}{c^2}\right)^{1/2}}$ si $v << c \to$ Taylor

$$E \simeq m_0 c^2 \left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$E \simeq m_0 c^2 + \underbrace{\frac{1}{2} m_0 v^2}_{E_{cin}}$$

Finalmente tenemos

$$\overline{P} = \frac{m_0 \overline{v}}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} \qquad \frac{E}{c} = \frac{m_0 c}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}}$$

Ahora queremos ver que este cuadrivector se comporta como el primero

Si tenemos

$$x' \to p_x' \qquad y' \to p_y' \qquad z' \to p_z' \qquad ct' \to \frac{E'}{c}$$

La norma de este cuadrivector es $\sqrt{\dfrac{m_0^2 c^2 - m_0^2 v^2}{\left(1 - \frac{v^2}{c^2}\right)}} = \sqrt{\dfrac{m_0^2 \cancel{(c^2 - v^2)}}{\frac{1}{c^2}\cancel{(c^2 - v^2)}}} = m_0 c$

$$p_x' = \frac{m_0 v_x'}{\left(1 - \frac{v'^2}{c^2}\right)^{1/2}} \qquad 1 - \frac{v'^2}{c^2} = 1 \frac{- v_x'^2 - v_y'^2 - v_z'^2}{c^2}$$

$$1 - \frac{v_x'^2}{c^2} = 1 - \frac{(v_x - u)^2}{c^2\left(1 - \frac{u}{c^2}v_x\right)^2} = \frac{\left(1 - \frac{u}{c^2}v_x\right)^2 - (v_x - u)^2/c^2}{\left(1 - \frac{u v_x}{c^2}\right)^2} = \frac{\left(1 - \cancel{\frac{2u}{c^2}v_x} + \frac{u^2 v_x^2}{c^4}\right) - \left(\frac{v_x^2}{c^2} - \cancel{\frac{2u v_x}{c^2}} + \frac{u^2}{c^2}\right)}{\left(1 - \frac{u v_x}{c^2}\right)^2}$$

$$1 - \frac{v_x'^2}{c^2} = \frac{\left(1 - \frac{v_x^2}{c^2}\right)\left(1 - \frac{u^2}{c^2}\right)}{\left(1 - \frac{u v_x}{c^2}\right)^2}$$

$$-\frac{v_y'^2}{c^2} = \frac{-\frac{v_y^2}{c^2}\left(1 - \frac{u^2}{c^2}\right)}{\left(1 - \frac{u v_x}{c^2}\right)^2} \qquad -\frac{v_z'^2}{c^2} = \frac{-\frac{v_z^2}{c^2}\left(1 - \frac{u^2}{c^2}\right)}{\left(1 - \frac{u v_x}{c^2}\right)^2}$$

Luego $1 - \dfrac{v'^2}{c^2} = \dfrac{\left(1 - \frac{u^2}{c^2}\right)}{\left(1 - \frac{u v_x}{c^2}\right)^2}\left(1 - \dfrac{v_x^2}{c^2} - \dfrac{v_z^2}{c^2} - \dfrac{v_y^2}{c^2}\right)$

$$\frac{\;}{c^2} = \frac{(\quad c^2)}{\left(1-\frac{uV_x}{c^2}\right)^2}\left(1-\frac{\;}{c^2}\right) \qquad = (V_x$$

Luego
$$p_x' = \frac{m_0 V_x'}{\left(1-\frac{V'^2}{c^2}\right)^{1/2}} = m_0 \frac{\cancel{\left(1-\frac{uV_x}{c^2}\right)}}{\left(1-\frac{u^2}{c^2}\right)^{1/2}\left(1-\frac{V^2}{c^2}\right)^{1/2}} \cdot \frac{(V_x-u)}{\cancel{\left(1-\frac{u V_x}{c^2}\right)}} =$$

$$= \frac{1}{\left(1-\frac{u^2}{c^2}\right)} \cdot \left(\frac{m_0 V_x - m_0 u}{\left(1-\frac{V^2}{c^2}\right)^{1/2}}\right) = \gamma\left(p_x - \beta \frac{E}{c}\right)$$

$$p_y' = \frac{m_0 V_y'}{\left(1-\frac{V'^2}{c^2}\right)^{1/2}} = m_0 \frac{V_y}{\cancel{\gamma\left(1-\frac{uV_x}{c^2}\right)}} \cdot \frac{\cancel{\left(1-\frac{uV_x}{c^2}\right)}}{\cancel{\left(1-\frac{u^2}{c^2}\right)^{-1}}\left(1-\frac{V^2}{c^2}\right)^{-1}} = p_y$$

Analogo $p_z' = p_z$

Como hasta ahora

$$\boxed{p_x' = \gamma\left(p_x - \beta\frac{E}{c}\right),\ p_y' = p_y,\ p_z' = p_z,\ \text{Basta ver que } E' = \gamma(E-\beta p_x)}$$

$$E' = \frac{m_0 c^2}{\left(1-\frac{V'^2}{c^2}\right)^{1/2}} = \frac{m_0 c^2}{\left(1-\frac{u^2}{c^2}\right)^{1/2}\left(1-\frac{V^2}{c^2}\right)^{1/2}} \cdot \left(1-\frac{uV_x}{c^2}\right) = \gamma\left(\frac{m_0 c^2 - m_0 u V_x}{\left(1-\frac{V^2}{c^2}\right)^{1/2}}\right) = \boxed{\gamma(E-\beta p_x)}$$