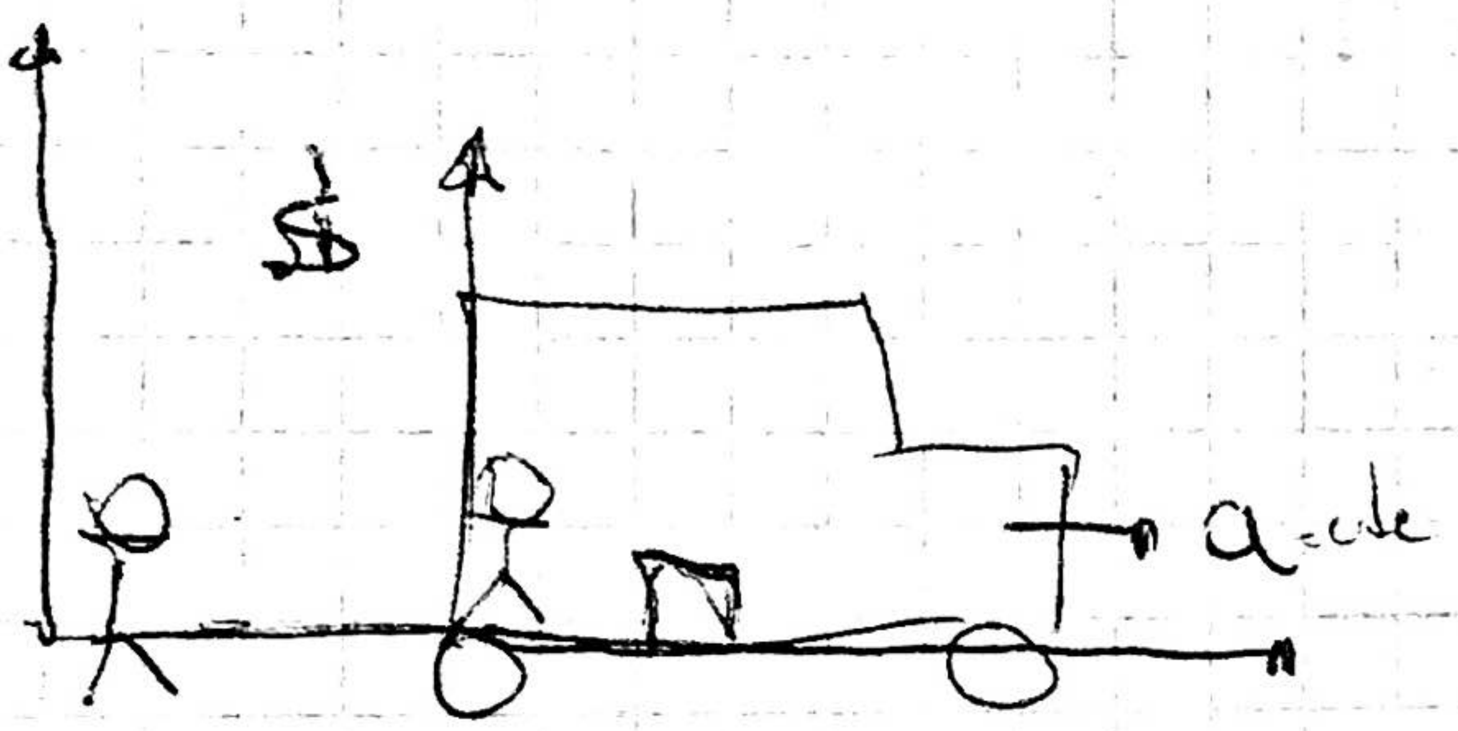
Sistemas no inerciales

1



masa pasajero = m

a) Describir mov por observador de colectivo y observador natural
Calcule la relación máxima entre aceleración del colectivo y coeficiente de roz estático

S) $(\hat{y})\ N = mg$

$(\hat{x})\ F_{RE} = m.a$

S') $(\hat{y}')\ N = mg$

$(\hat{x}')\ F_{re} - f^* = 0$

$F_{re} = f^*$

$F_{RE} = m.a$

$|F_{re}| \leq \mu_e mg$

$-\mu mg \leq ma \leq \mu_e mg$

$\boxed{a \leq \mu_e . g}$

b) No roz

S) $(\hat{y})\ N = mg$

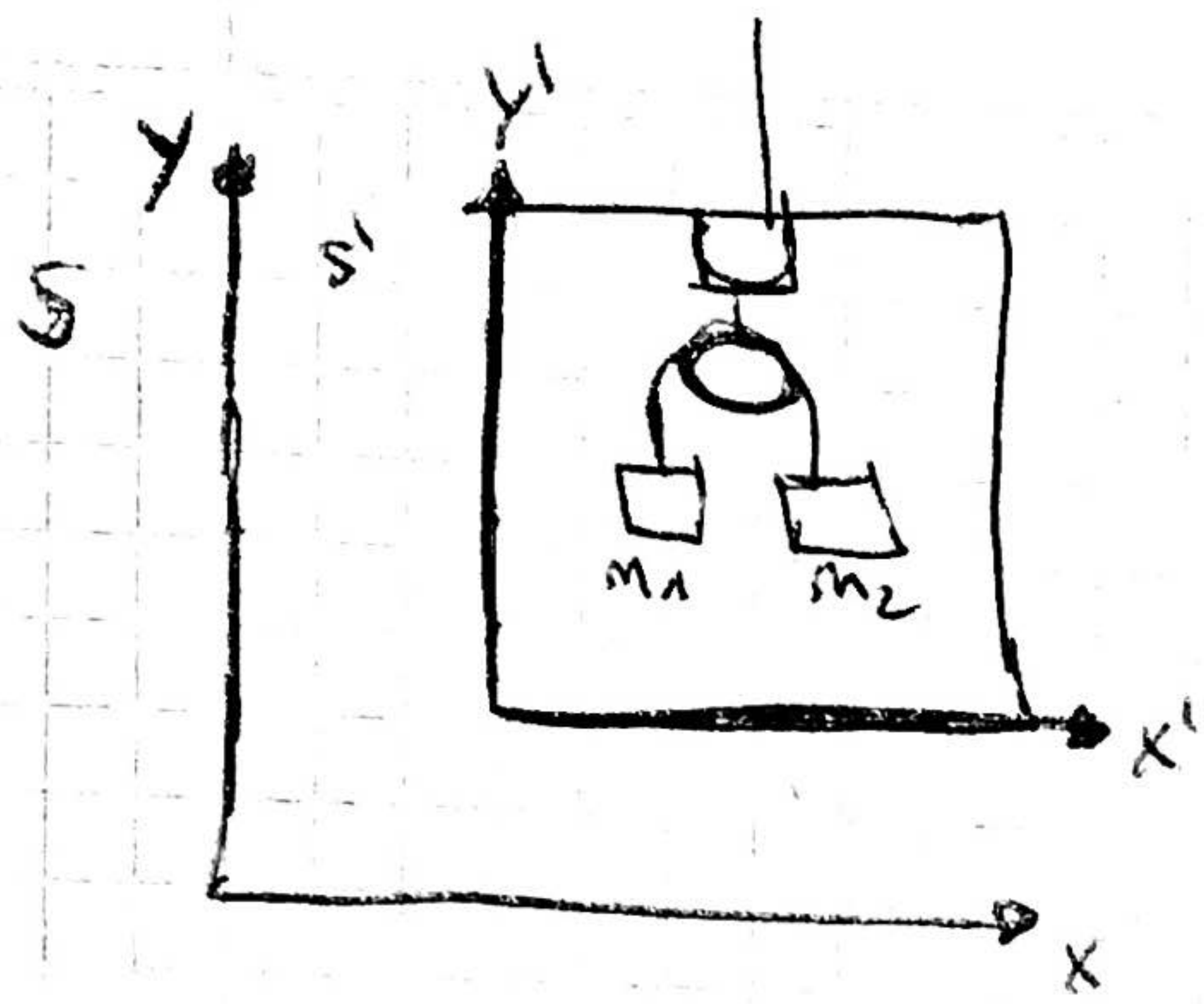
$(\hat{x})\ 0 = 0$

S') $(\hat{y}')\ N = mg$

$(\hat{x}')\ -f^* = ma$

$\boxed{f^* = -m.a}$

②



Vínculo

$l = 2y_p - y_1 - y_2$

$\ddot{y}_1 = \underbrace{2\ddot{y}_p}_{=a} - \ddot{y}_2$

a) v=cte

S') $+f^* - m_1 g + T = m_1 \ddot{y}_1$

$+f^* - m_2 g + T = m_2 \ddot{y}_2$

$\cancel{\ddot{y}_1}\ - m_1 g + m_2 g = (m_1 + m_2)\ddot{y}_1$

$\ddot{y}_1 = \dfrac{(-m_1 + m_2) g}{m_1 + m_2}$

S) $-m_1 g + T + \cancel{f^*} = m_1 \ddot{y}_1$

$-m_2 g + T = m_2 \ddot{y}_2$

$\ddot{y}_p = 0$ debido v=cte

$-m_1 g + m_2 g = m_1 \cdot m_2 \ddot{y}_1$

$\ddot{y}_1 = \dfrac{(-m_1 + m_2)}{m_1 + m_2} g$

b) v=no cte con aceleración a

S') $-m_1 g + m_2 g = \cancel{(m_1 + m_2)}\ m_1 (2\ddot{y}_p - \ddot{y}_2) - m_2 \ddot{y}_2$

$\cancel{\dfrac{-m_1 g + m_2 g}{m_1 2\ddot{y}_p - m_1 - m_2} = \ddot{y}_2}$

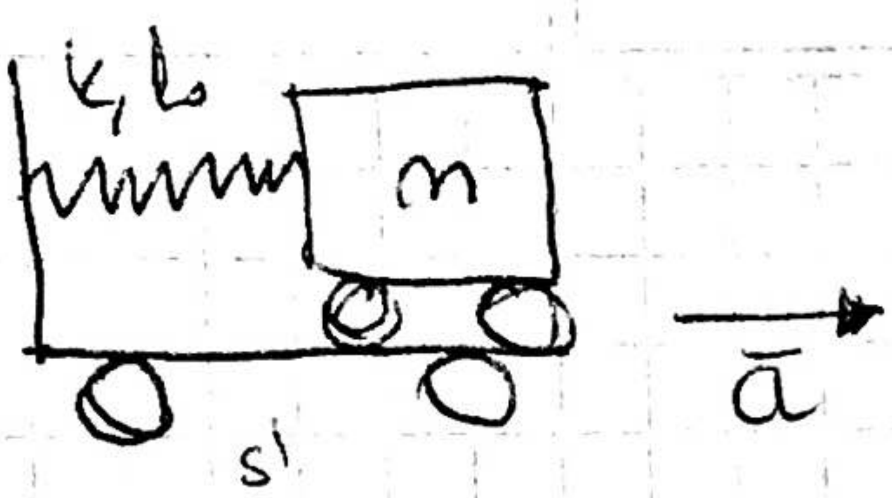
$\dfrac{-m_1 g + m_2 g - m_1 2\ddot{y}_p}{-m_1 - m_2} = \ddot{y}_2$

$\ddot{y}_1 = 2\ddot{y}_p - \left(\dfrac{-m_1 g + m_2 g - m_1 2\ddot{y}_p}{-m_1 - m_2}\right)$

$\ddot{y}_1 = \dfrac{2m_1 \ddot{y}_p + 2m_2 \ddot{y}_p - m_1 g + m_2 g - \cancel{m_1 2\ddot{y}_p}}{m_1 + m_2}$

S') $-m_1 a - m_1 g + m_2 a + m_2 g = m_1 \ddot{y}_1 + m_2 \ddot{y}_1$

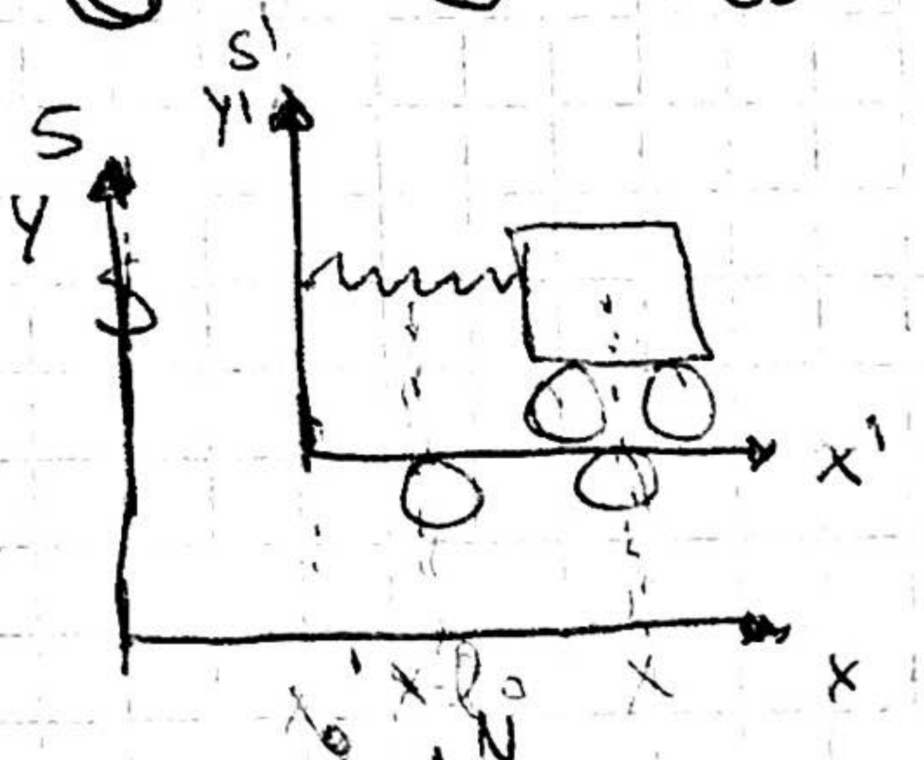
$\dfrac{-m_1 (a + g) + m_2 (a + g)}{m_1 + m_2} = \ddot{y}_1$



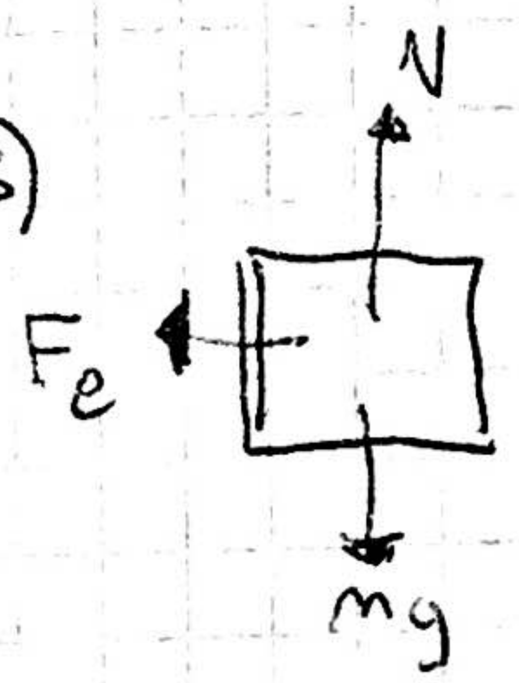
Datos

m, k, l_0, a

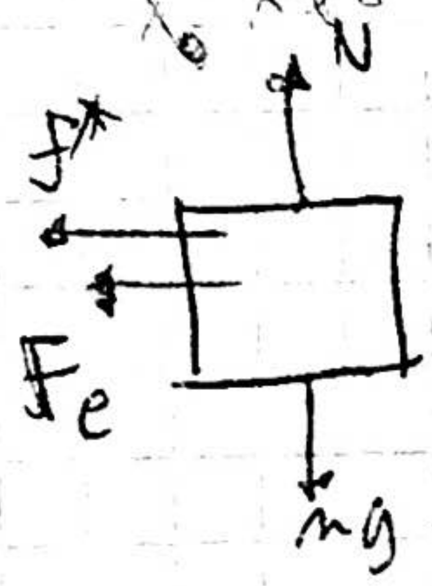
a)



S)



S')



$f^* = ma$

(S') $(\hat{x}')$ $-k(x'-l_0)-f^* = m\ddot{x}'$

$$\frac{kl_0}{m} - \cancel{ma}\, a = \frac{kx'}{m} + \ddot{x}'$$

$$\ddot{x}' + \omega_0^2 x' = \omega_0^2 l_0 - a$$

(S) $(\hat{x})$ $-k(x - x_0' - l_0) = m\ddot{x}$

$$\ddot{x} + \omega_0^2 x = \omega_0^2\left(\frac{at^2}{2} + l_0\right)$$

b) Mov de m respecto de la plataforma

S') $l_0 = A\cos(\cancel{\omega t} + \varphi) + l_0 - \frac{a}{\omega_0^2}$ $\qquad 0 = -A\sin\varphi$

$\frac{a}{\omega_0^2} = A\cos(\varphi)$ $\qquad \varphi = 0$

$$x'(t) = \frac{a}{\omega_0^2}\cos(\omega t) + l_0 - \frac{a}{\omega_0^2}$$

S) $l_0 = A\cos\varphi + \frac{at^2}{2} + l_0$ $\qquad 0 = -A\sin\varphi$

$\varphi = 0$

$-\frac{at^2}{2} = A\cos\varphi$

$$x(t) = -\frac{at^2}{2}\cos\omega t + l_0 - \frac{at^2}{\omega_0^2}$$

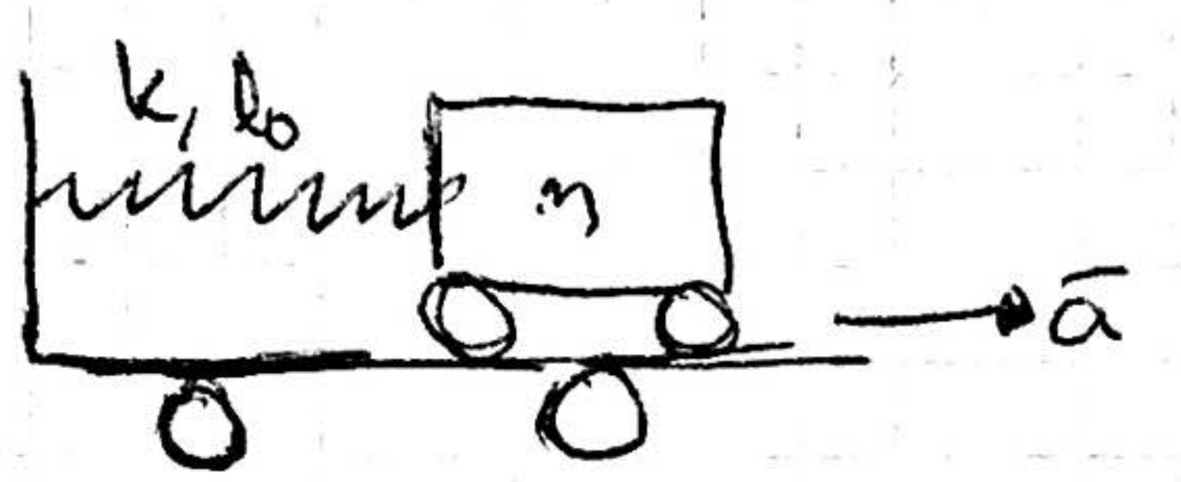
$A = -\frac{at^2}{2}$

$$x(t) = x'(t) + x_s$$

$$-\frac{at^2}{2}\cos\omega t + l_0 - \frac{at^2}{2} = \frac{a}{\omega_0^2}\cos\omega t + l_0 - \frac{a}{\omega_0^2} + \frac{at^2}{2}$$

c) $\Sigma F_s = M . a_s$

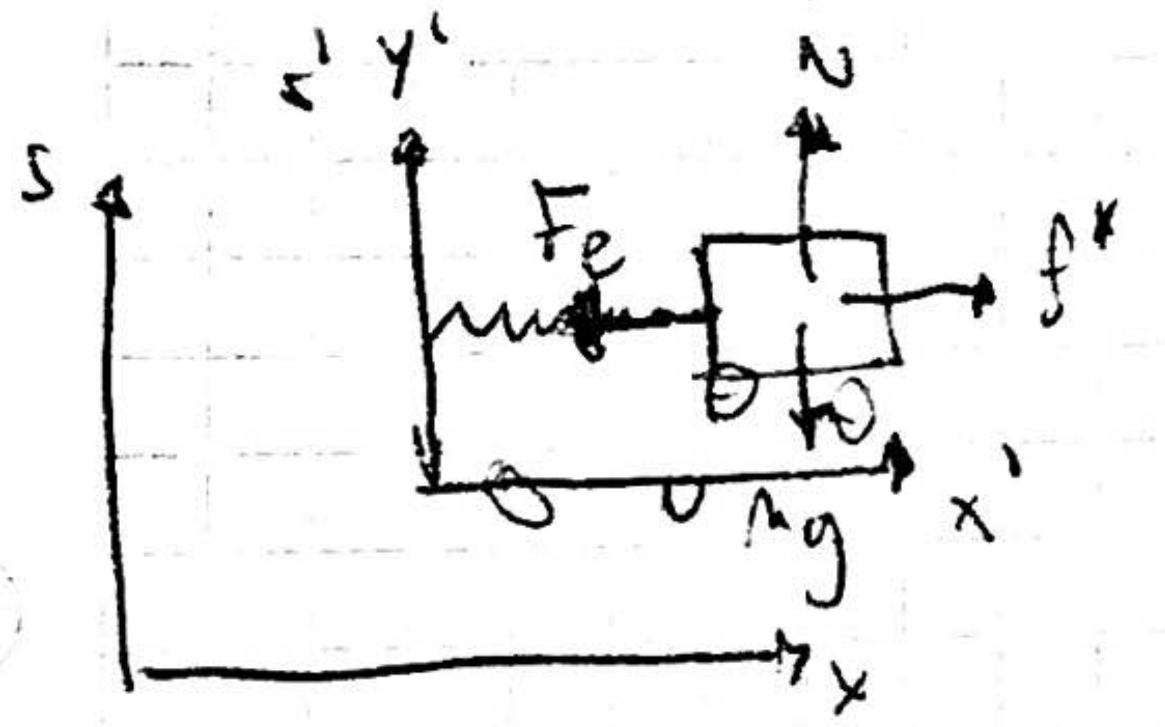
③



Datos

m, k, l_0, a

a) Dibuja las fuerzas que actúan sobre la masa en un sistema de referencia unido a la plataforma y luego en otro exterior a ella, en reposo



S

(S') $(\hat{x}')$: $-k(x'-l_0)+f^* = m\ddot{x}'$

$-kx' + kl_0 - m a = m\ddot{x}'$

$\ddot{x}' + \omega_0^2 x' = \omega_0^2 l_0 - a$

(S) $(\hat{x})$ $-k(x - x_0' - l_0) = m\ddot{x}$

$\ddot{x} + \omega_0^2 x = \omega_0^2(x_0' + l_0)$

$\ddot{x} + \omega_0^2 x = \omega_0^2\left(\frac{at^2}{2} + l_0\right)$

b) Movimiento de m respecto de la plataforma

$\ddot{x}' + \omega_0^2 x' = \omega_0^2 l_0 - a$

$x(t) = A\sin(\omega_0 t + \varphi) + C$ ~~φ = A cos(ωt~~

Para c $\ddot{C} = 0$

$\Rightarrow \omega_0^2 C = \omega_0^2 l_0 - a$

$\cancel{l_0} = A\cos(\omega t+\varphi) + \cancel{l_0} - \frac{a}{\omega_0^2}$ $\qquad 0 = A\omega\sin\varphi$

$\frac{a}{\omega_0^2} = A\cos\varphi \qquad \varphi = 0$

$A = \frac{a}{\omega_0^2}$

$$\Rightarrow \boxed{x'(t) = \frac{a}{\omega_0^2}\left(\cos(\omega t) - 1\right) + l_0}$$

c) Si la plataforma tiene masa M, determinar la fuerza necesaria para mantener constante su aceleración

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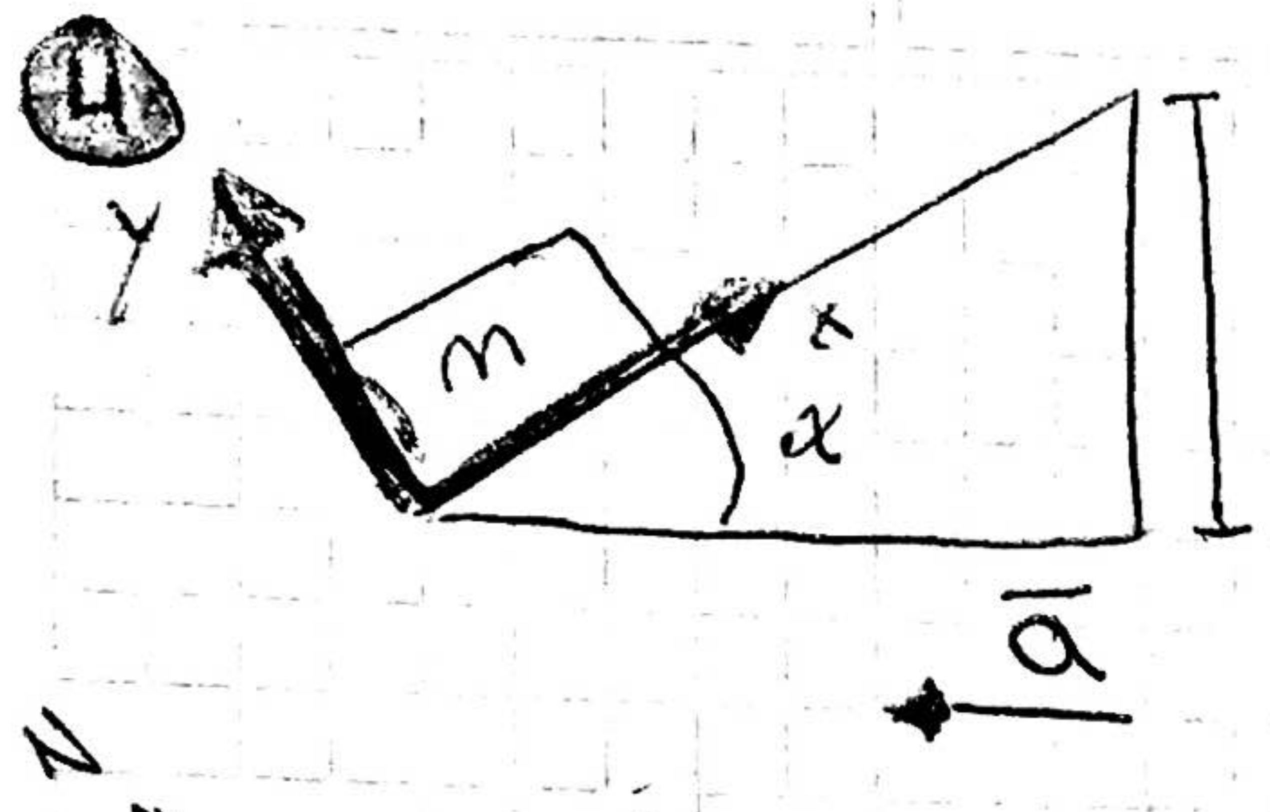
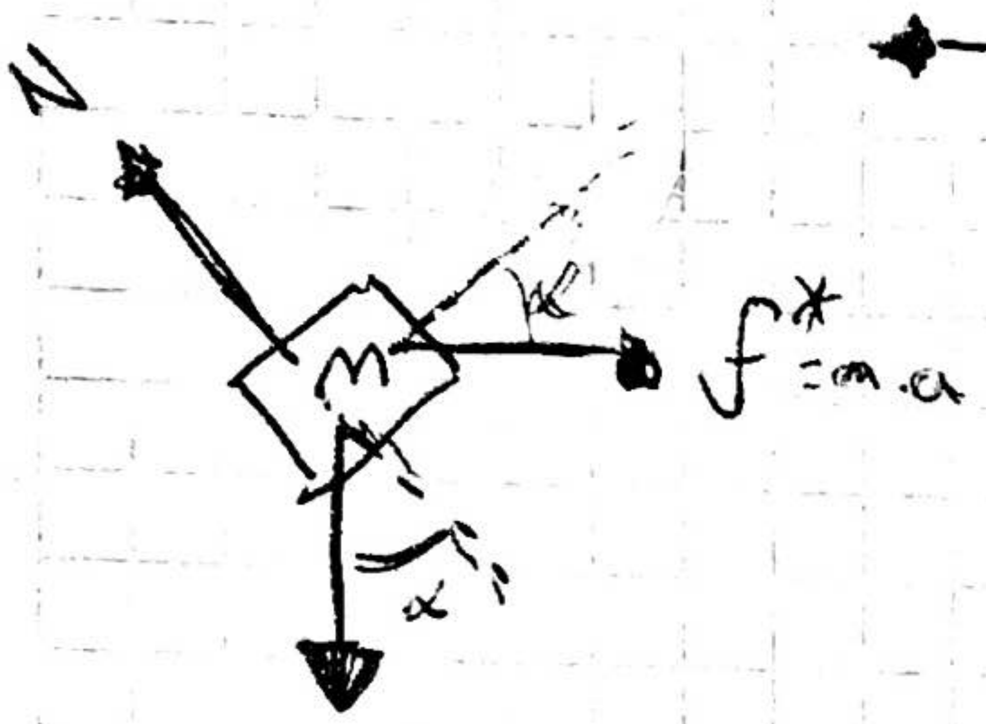


$\bar g \downarrow$

No roz

$v_0 = 0 \quad v_f = v_i$

$\ddot x = ?$



(x) $f^*_x - mg\sin\alpha = m\ddot x$

(y) $N - mg\cos\alpha - f^*_y = 0$

$\cancel{m}.a.\cos\alpha - \cancel{m}g\sin\alpha = \cancel{m}\ddot x$

$a\cos\alpha = \dfrac{v_i^2}{2h\sin\alpha} + g\sin\alpha$

$a = \dfrac{v_i^2}{2h\sin\alpha\cos\alpha} + g\,tg\,\alpha$

$$\bar a = \left(\frac{v_i^2}{2h\sin\alpha\cos\alpha} + g\,tg\,\alpha\right)(\cos\alpha\hat x + \sin\alpha\hat y)(-1)$$

$x = \dfrac{\ddot x t^2}{2}$

$\dot x = \ddot x t$

$\dfrac{h}{\sin\alpha} = \dfrac{\ddot x t_f^2}{2}$

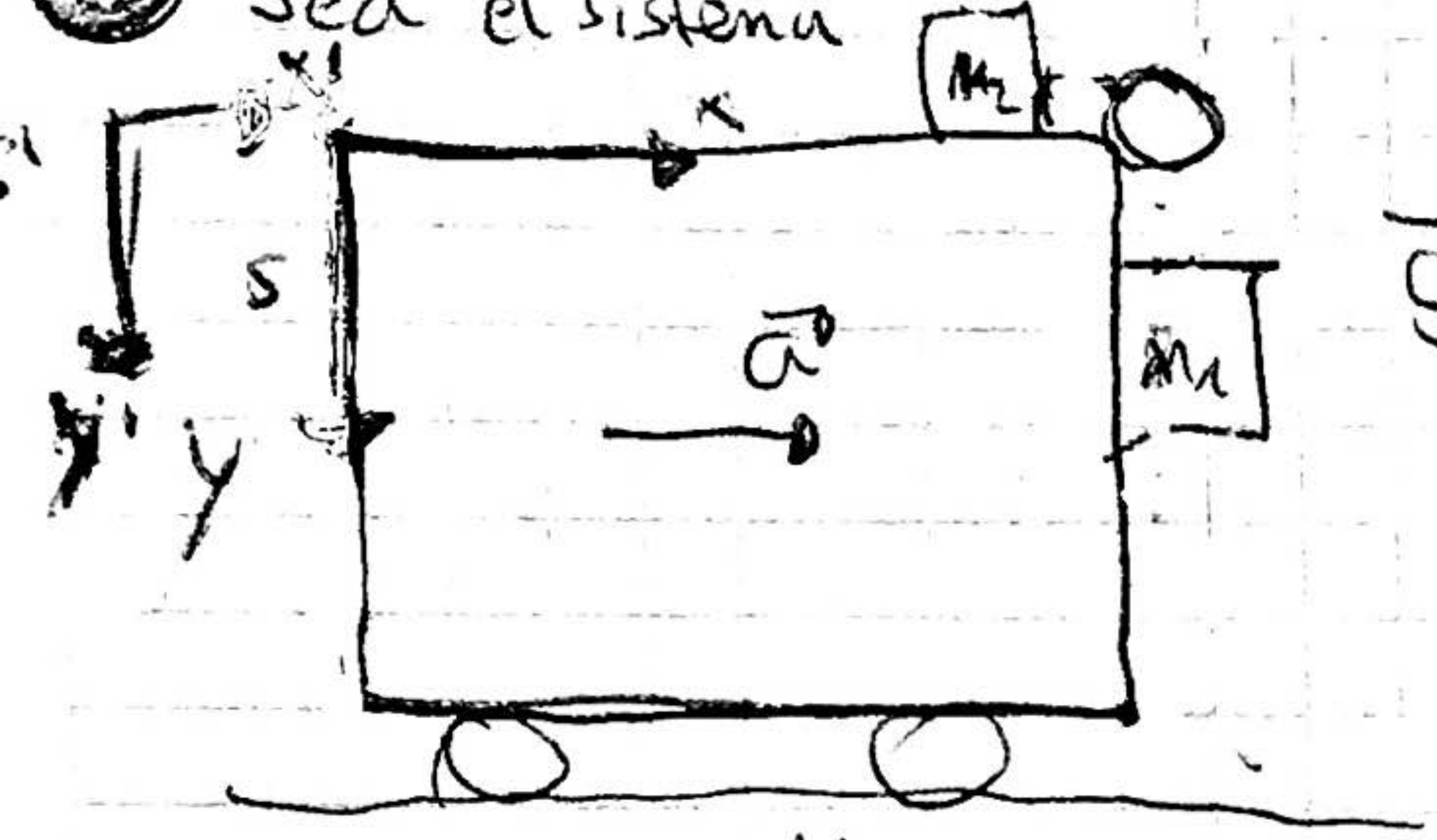
$v_i = \ddot x t_f$

$\dfrac{v_i}{\ddot x} = t_f$

$\Rightarrow \dfrac{h}{\sin\alpha} = \dfrac{\cancel{\ddot x}\, v_i^2}{2\,\ddot x^{\cancel{2}}}$

$\ddot x = \dfrac{v_i^2}{2h\sin\alpha}$

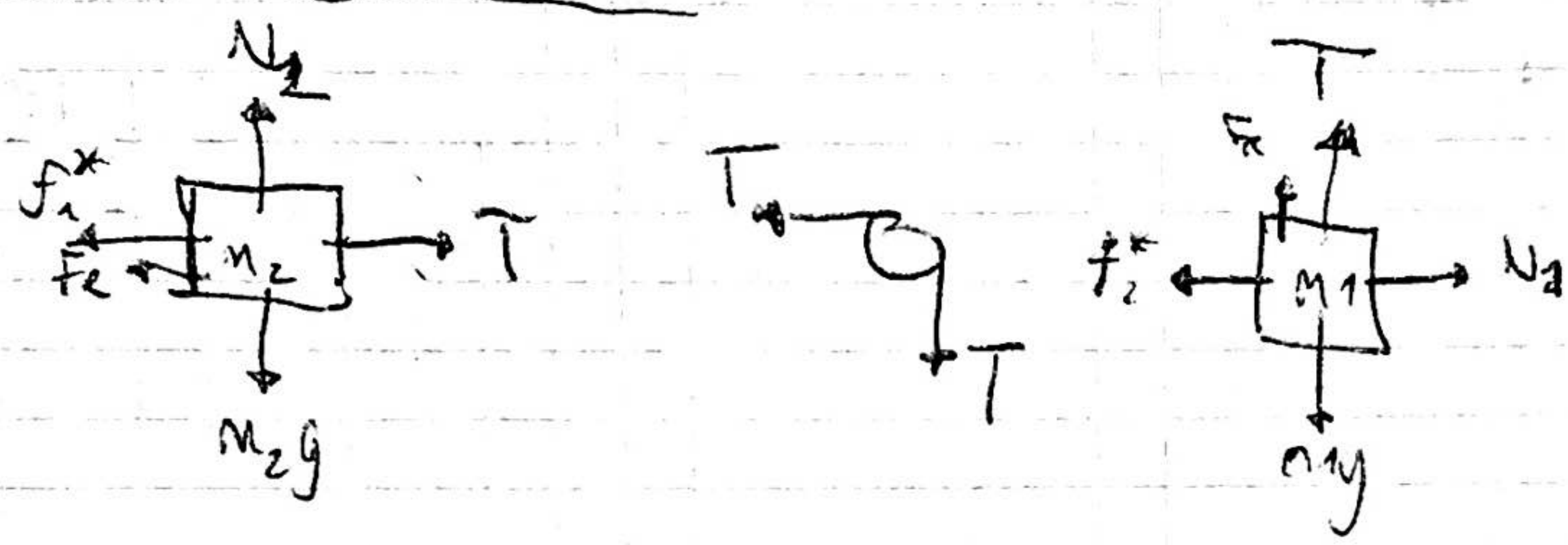
5 Sea el sistema



Dato

μ_{e_2} y μ_{e_1}

¿Para que valores de a, m_1 no sube ni baja?



vínculo $x_p - x_2 + x_1 = l$

$\ddot{x}_2 = \ddot{x}_1$

Estático

m_2

$(\hat{x}) \quad -f_1^* - F_{RE_1} + T = 0$

$(\hat{y}) \quad N_2 = m_2 g$

m_1

$(\hat{x}) \quad N_1 = f_2^*$

$(\hat{y}) \quad -T + m_1 g - F_{RE_2} = 0$

$Ec_{m_2} + E_{Em_1}$

$|F_{RE_2}| \leq \mu_{E_2} \cdot m_2 \cdot g$

$|F_{RE_1}| \leq \mu_{E_1} \cdot m_1 \cdot a$

$-f_2^* - F_{RE_2} + \cancel{T} - \cancel{T} + m_1 g - F_{RE_1} = 0$

$-m_2 a - \cancel{F_{EE_2}} + m_1 g - F_{RE_1} = 0$

$a = \dfrac{-F_{EE_2} + m_1 g - F_{EE_1}}{m_2}$

$-m_2 a + m_1 g = F_{RE_2} + F_{EE_1}$

$|F_{EE_2} \; F_{EE_1}| = |-m_2 a + m_1 g| \leq \mu_{e_2} m_2 g + \mu_{E_1} m_1 a$

$\boxed{-\mu_{e_2} m_2 g - \mu_{E_1} m_1 a \leq \boxed{-m_2 a + m_1 g} \leq \mu_{e_2} m_2 g + \mu_{e_1} m_1 a}$

1 2

(1) $-\mu_{e_2} m_2 g - \mu_{e_1} m_1 a \leq -m_2 a + m_1 g$

$-\mu_{e_1} m_1 a + m_2 a \leq m_1 g + \mu_{e_2} m_2 g$

$a(-\mu_{e_1} m_1 + m_2) \leq m_1 g + \mu_{e_2} m_2 g$

si $-\mu_{e_1} m_1 + m_2 > 0$

$\Rightarrow a \leq \dfrac{m_1 g + \mu_{e_2} m_2 g}{-\mu_{e_1} m_1 + m_2}$

si $-\mu_e m_1 + m_2 < 0$

$a \geq \dfrac{m_1 g + \mu_{e_2} m_2 g}{-\mu_{e_1} m_1 + m_2}$

si $-\mu_e m_1 + m_2 = 0$

$0 \leq m_1 g + \mu_{e_2} m_2 g$

se cumple la desigualdad, pero no dice nada aún

(2) $-m_2 a + m_1 g \leq \mu_{e_2} m_2 g + \mu_{e_1} m_1 a$

$a(-m_2 - \mu_{e_1} m_1) \leq (\mu_{e_2} m_2 - m_1) g$

$a \geq \dfrac{(\mu_{e_2} m_2 - m_1) g}{-m_2 - \mu_{e_1} m_1}$

$\Rightarrow$ si $-\mu_{e_1} m_1 + m_2 > 0$

$\Rightarrow$ Para estática

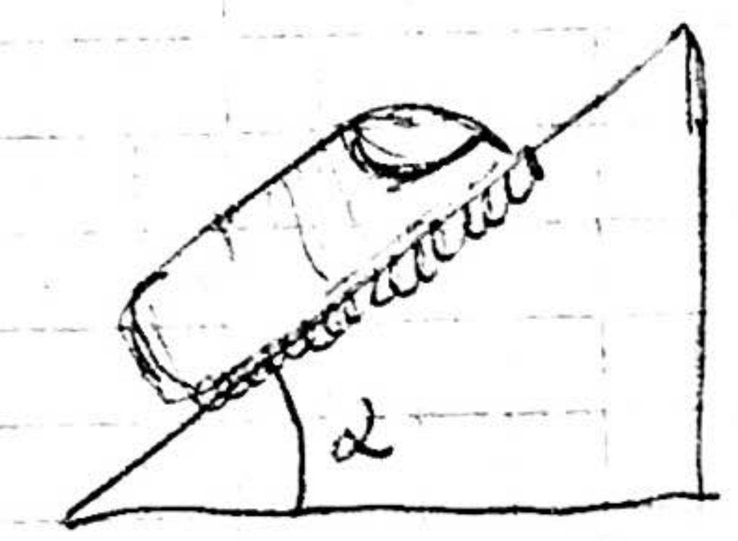
$$\boxed{\frac{(\mu_{e_2} m_2 - m_1) g}{-m_2 - \mu_{e_1} m_1} \leq a \leq \frac{m_1 g + \mu_{e_2} m_2 g}{-\mu_{e_1} m_1 + m_2}}$$

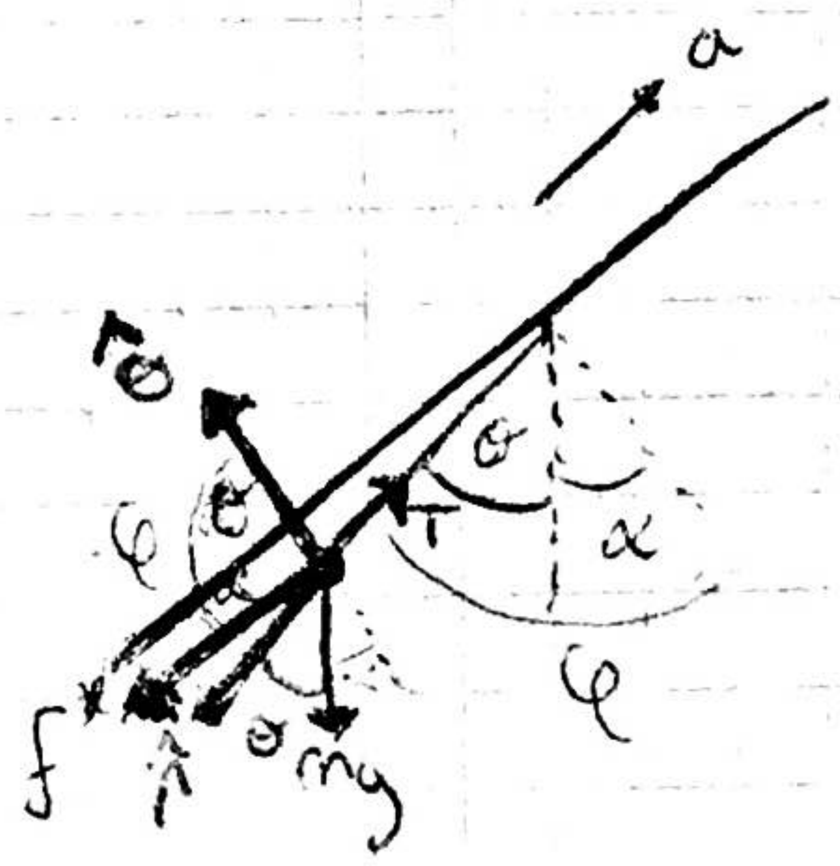
$\Rightarrow$ si $-\mu_{e_1} m_1 + m_2 < 0$

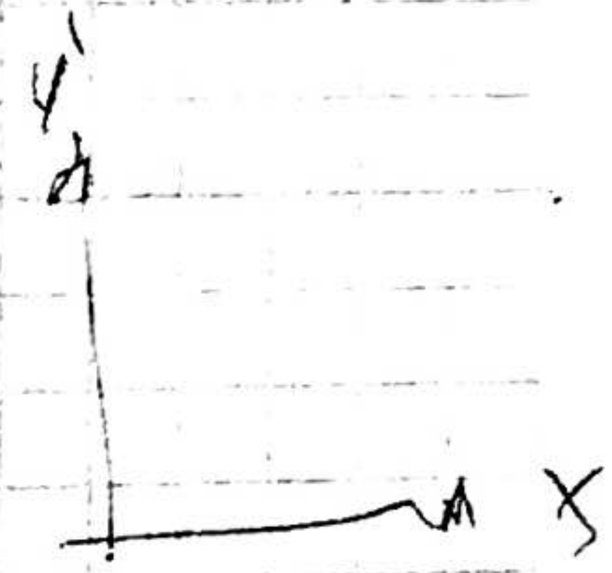
$$\boxed{a \geq \text{máx}\left\{ \frac{m_1 g + \mu_{e_2} m_2 g}{-\mu_{e} m_1 + m_2} , \frac{\mu_{e_2} m_2 g - m_1 g}{-m_2 - \mu_{e_1} m_1} \right\}}$$

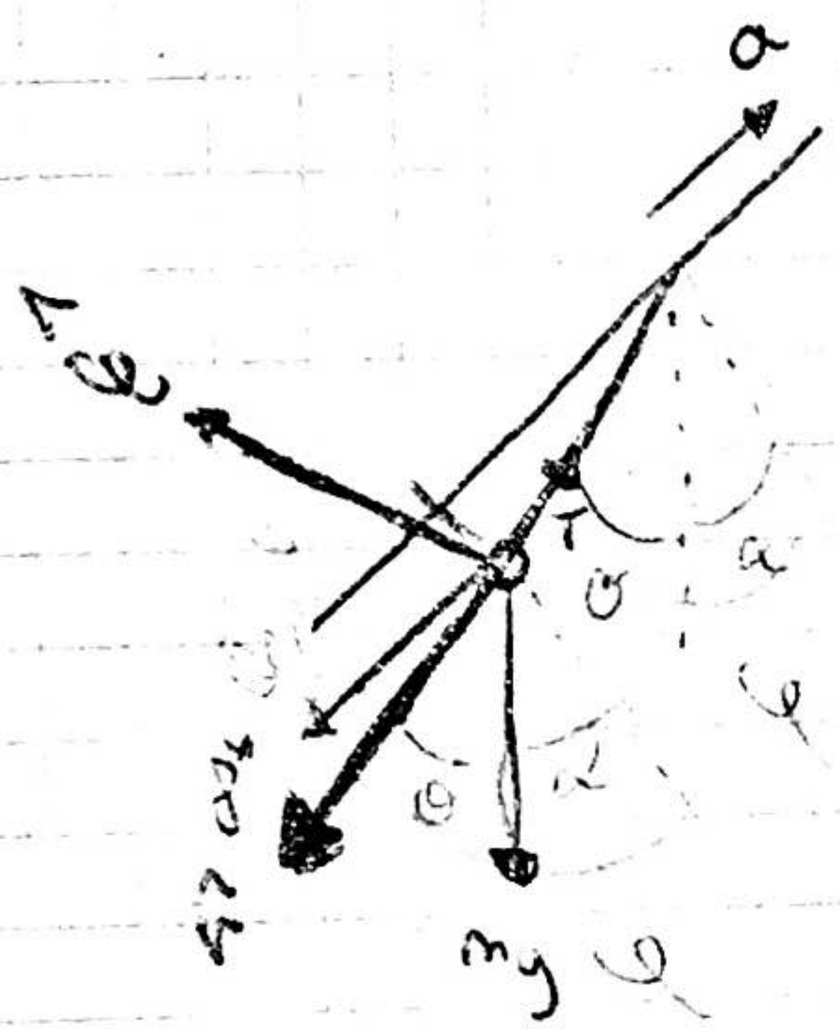
Si $-\mu_e g m_1 + m_2 = 0$

$$\boxed{a \geq \frac{(\mu_{e} g m_2 - m_1) g}{-\mu_{e} g m_1 - m_2}}$$









$\cancel{(\hat r) - mg\sin\alpha + T\sin\varphi = m\ddot x}$ $\qquad \varphi = \theta + \alpha$

$\cancel{(\hat\varphi) - mg\cos\alpha \pm T\cos\varphi = m\ddot y}$ $\qquad \theta = \varphi - \alpha$

Si $(\hat\varphi) \to f^* \cos\varphi - mg\sin(\varphi - \alpha) = m l \ddot\varphi$

$(\hat r) \to f^* \sin\varphi + mg\cos(\varphi - \alpha) - T = -m l \dot\varphi^2$

Para hallar equilibrios

$0 = ma\cos\varphi - mg\sin(\varphi - \alpha)$

$ma\cos\varphi = mg\sin(\varphi - \alpha)$

$a\cos\varphi = \sin\varphi\cos\alpha - \sin\alpha\cos\varphi$

$a = tg\varphi\cos\alpha - \sin\alpha$

$\frac{a}{g\cos\alpha} + tg\alpha = tg\varphi_{eq}$ $\qquad$ i) $\alpha = 0$, $a \neq 0$

$tg\varphi_{eq} = \frac{a}{g}$

ii) $\alpha \neq 0$, $a = 0$

$\dot{\alpha} = \varphi_{eq}$

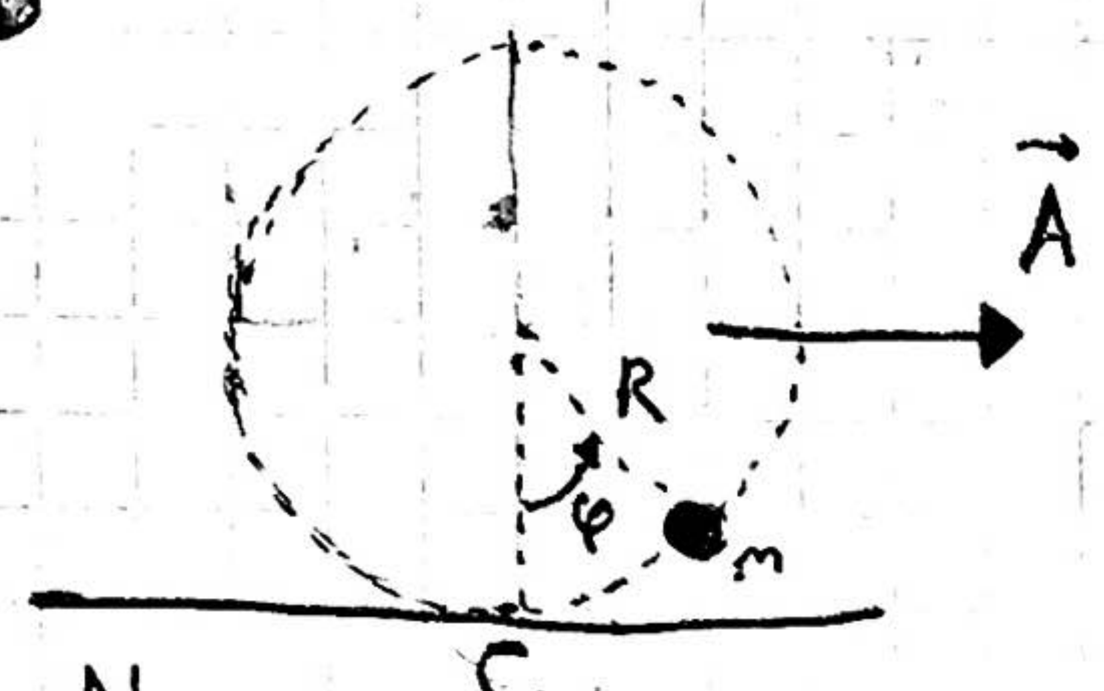
iii) $\alpha \neq 0$, $a = -g\sin\alpha$

$$tg\varphi_{eq} = \frac{-g\sin\alpha}{g\cos\alpha} + tg\alpha$$

$tg\varphi_{eq} = 0$

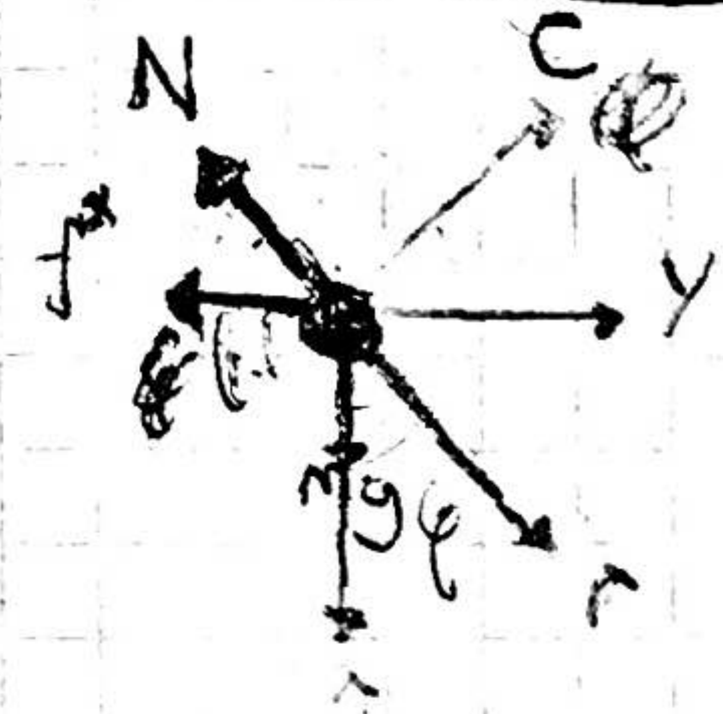
iv) $\alpha \neq 0$, $a \neq 0$

$$tg\varphi_{eq} = \frac{a}{g\cos\alpha} + tg\alpha$$



Datos

m, R, no roz, reposo en C



$\hat{\varphi})\ -f^*\cos\varphi + mg\sin\varphi = mR\ddot{\varphi}$

$\hat{r})\ -f^*\sin\varphi + mg\cos\varphi - N = -mR\dot{\varphi}^2$

$$\ddot{\varphi} + \frac{g}{R}\sin\varphi + \frac{f^*}{R}\cos\varphi = 0$$

$$\boxed{\ddot{\varphi} = -\frac{g}{R}\sin\varphi - \frac{f^*}{Rm}\cos\varphi}$$

$$\ddot{\varphi} = -\frac{g}{R}\sin\varphi - \frac{\cancel{m}A\cos\varphi}{R\cancel{m}}$$

$$\boxed{\ddot{\varphi} = -\frac{g}{R}\sin\varphi - \frac{A}{R}\cos\varphi}$$

b) Halle el valor de N en función de φ

$N = -mA\sin\varphi + mg\cos\varphi + mR\dot{\varphi}^2$

$$\frac{d\dot{\varphi}}{d\varphi}\frac{d\varphi}{dt} = \ddot{\varphi}$$

$$\Rightarrow \int_0^{\dot{\varphi}(\varphi)} \dot{\varphi}\, d\dot{\varphi} = \int_0^{\varphi} -\frac{g}{R}\sin\varphi - \frac{A}{R}\cos\varphi \, d\varphi$$

$$\left.\frac{\dot{\varphi}^2}{2}\right|_0^{\dot{\varphi}(\varphi)} = \left. +\frac{g}{R}\cos\varphi - \frac{A}{R}\sin\varphi \right|_0^{\varphi}$$

$$\dot{\varphi}^2 = \frac{2}{R}\left(g\cos\varphi - g\cos 0 - \left(A\sin\varphi - A\sin 0\right)\right)$$

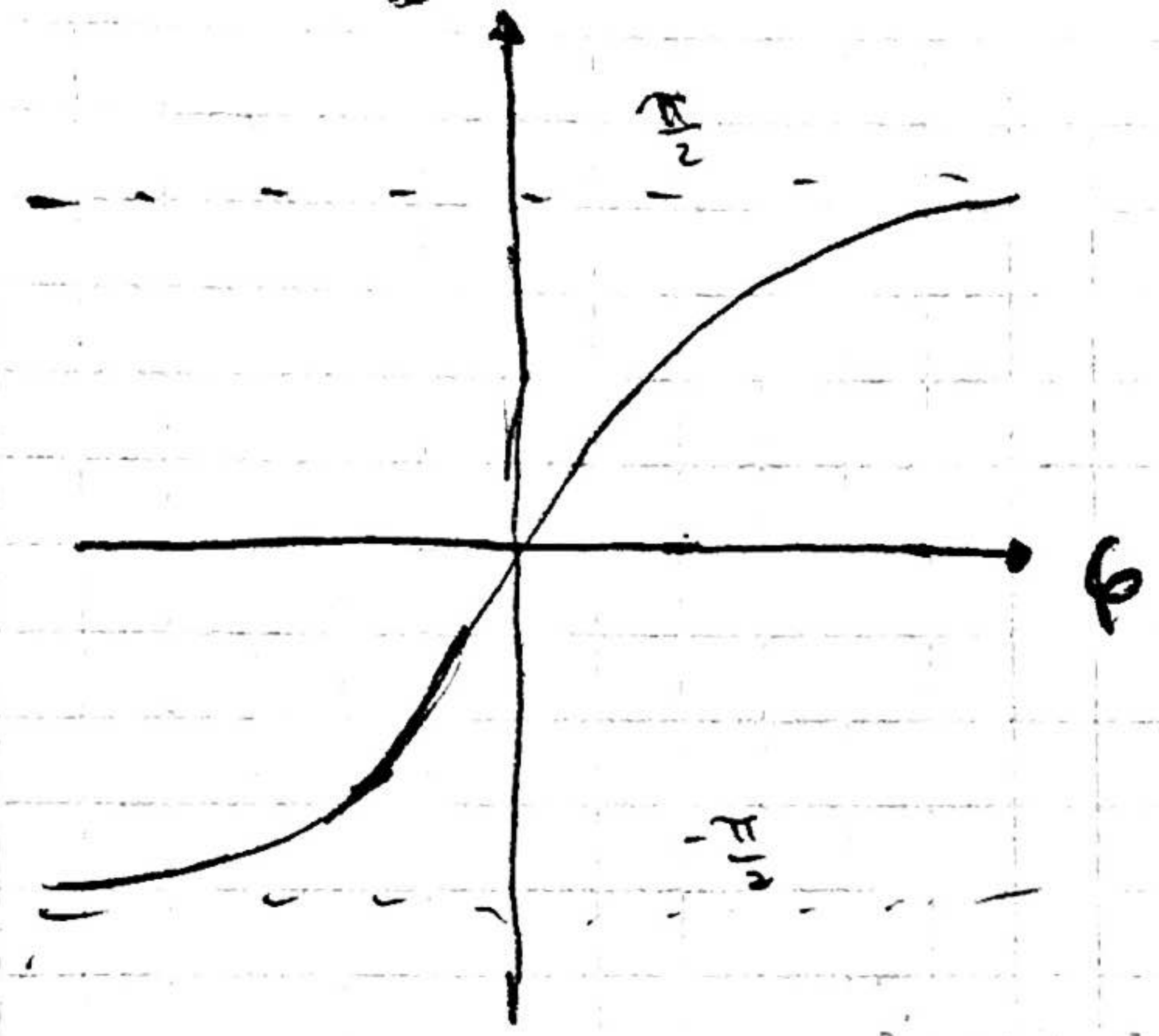
$N = -mA\sin\varphi + mg\cos\varphi + m\left(2g\cos\varphi - 2g - 2A\sin\varphi\right)$

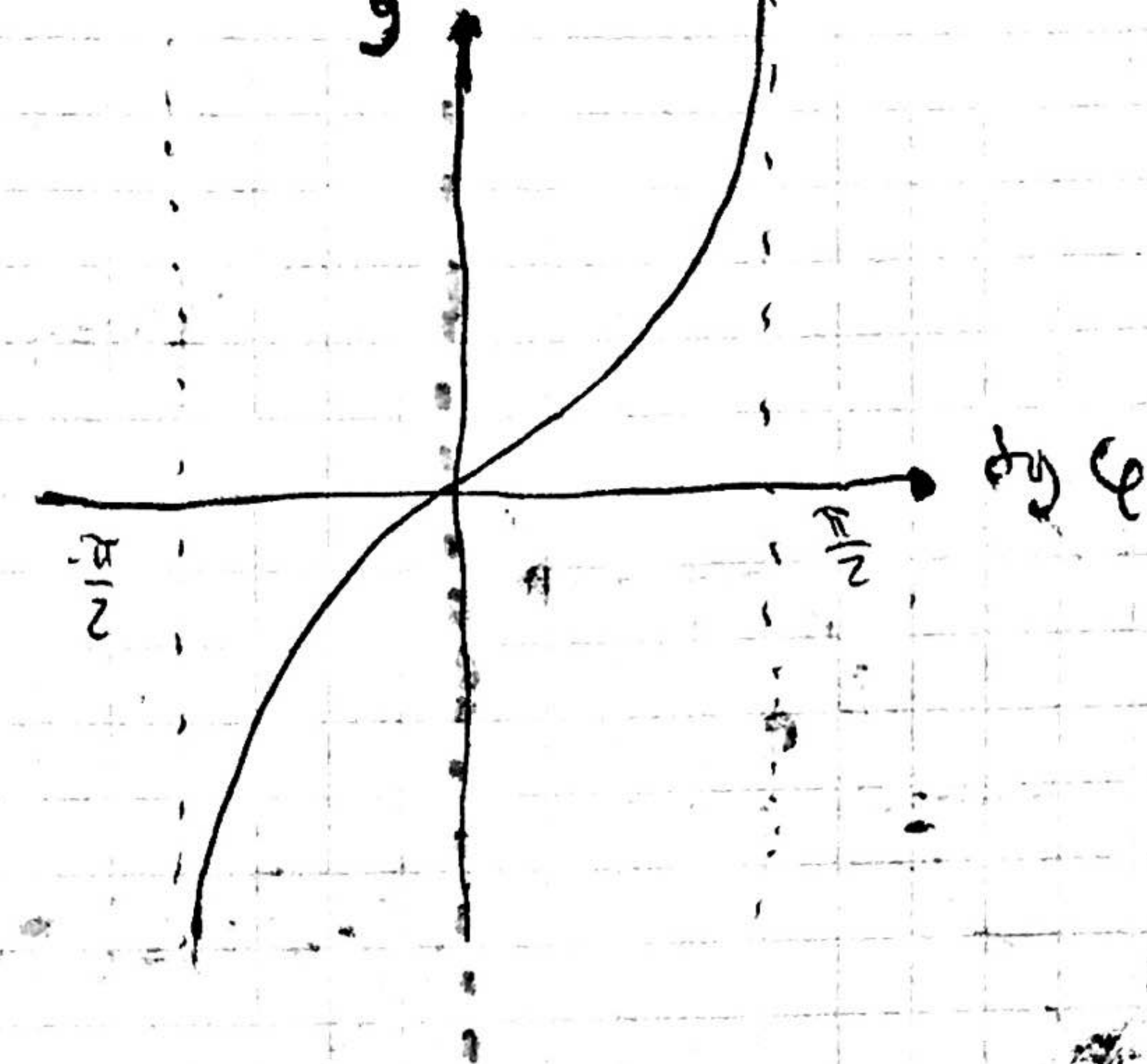
$$\boxed{N = -3mA\sin\varphi + 3mg\cos\varphi - 2mg}$$

c) Encuentre la posición de eq. det si estable o inest

Para eq $\ddot{\varphi} = 0$

$$\frac{g}{\cancel{R}}\,tg\,\varphi = -\frac{A}{\cancel{R}} \qquad tg\,\varphi = -\frac{A}{g} \qquad \varphi = arctg\left(-\frac{A}{g}\right).$$



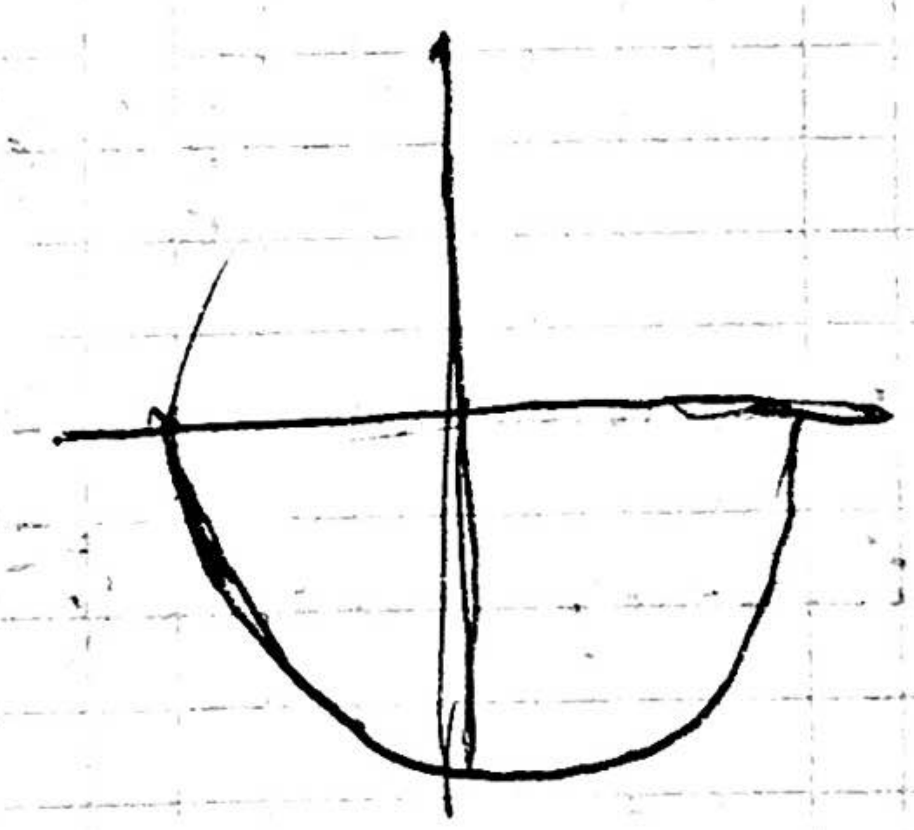


eq inestable

$$\ddot{\varphi} = -\frac{A}{R}\cos\varphi + \frac{g}{R}\sin\varphi \qquad \operatorname{arctg}\left(-\frac{A}{g}\right) = \varphi_{eq}$$

$$0 = -\frac{A}{R}\cos\varphi + \frac{g}{R}\sin\varphi$$

$$\left.\frac{dF_\varphi}{d\varphi}\right|_{\varphi_{eq}} = \frac{A}{R}\sin\varphi + \frac{g}{R}\cos\varphi$$



$$= \cos\varphi\left(\frac{A}{R}\,tg\,\varphi + \frac{g}{R}\right)$$

$$= \underset{>0}{\cos\varphi}\underset{<0}{\left(-\frac{A^2}{Rg} + \frac{g}{R}\right)}$$

$\therefore$ eq estable

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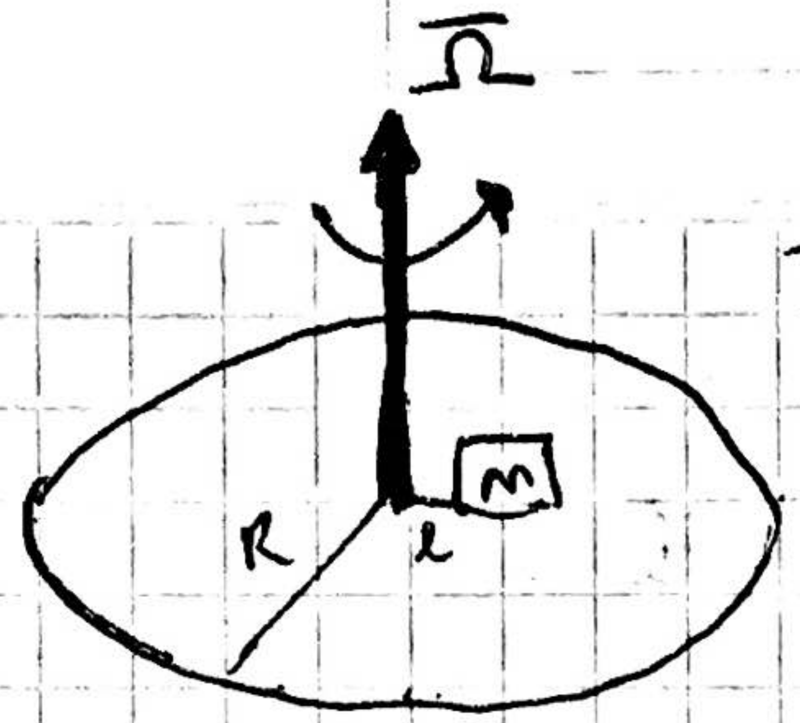
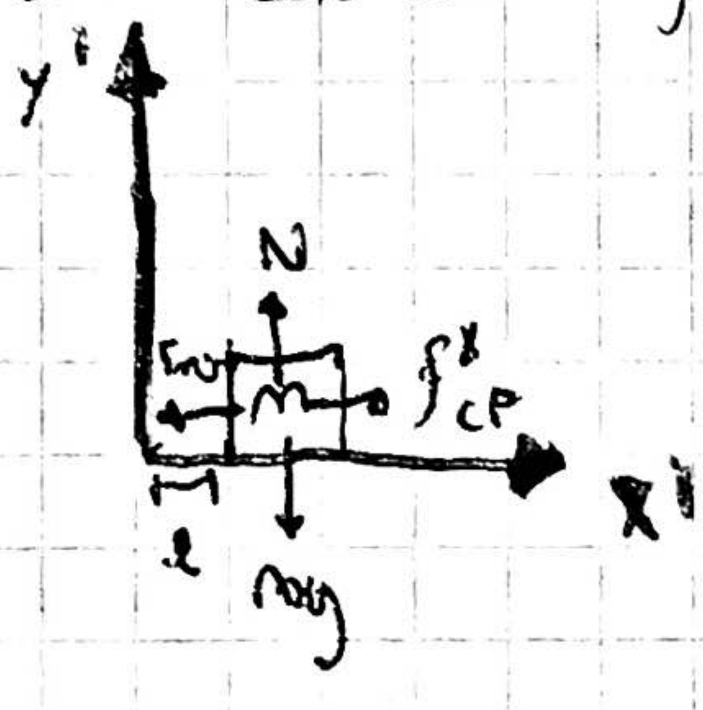


$g\downarrow$ Datos $r=R$, $\Omega=cte$ $\therefore \dot\Omega=0$

m, μ_e, μ_d

a) Ec Newton con sist fijo a plataforma Estático



$$m\bar a' = m\bar a + 2m\bar\Omega\times\bar v' - m\bar\Omega\times(\bar\Omega\times\bar r') - m\dot{\bar\Omega}\times\bar r'$$

$\bar\Omega = \Omega\hat y'$

$\bar r' = l\hat x'$

$\bar v' = \bar v - \bar\Omega\times\bar r'$

$= \cancel{\bar\Omega\times r - \bar\Omega\times r}$

$-m\Omega\hat y'\times(\Omega\hat y'\times l\hat x')$

$\bar v' = 0$

$+m\Omega^2 l\hat x'$

$\dot\Omega = 0$

$$\boxed{m\ddot x' = -F_{roz}\hat x' + m\Omega^2 l\hat x'}$$

$0 = -F_{roz}\hat x' + m\Omega^2 l\hat x'$

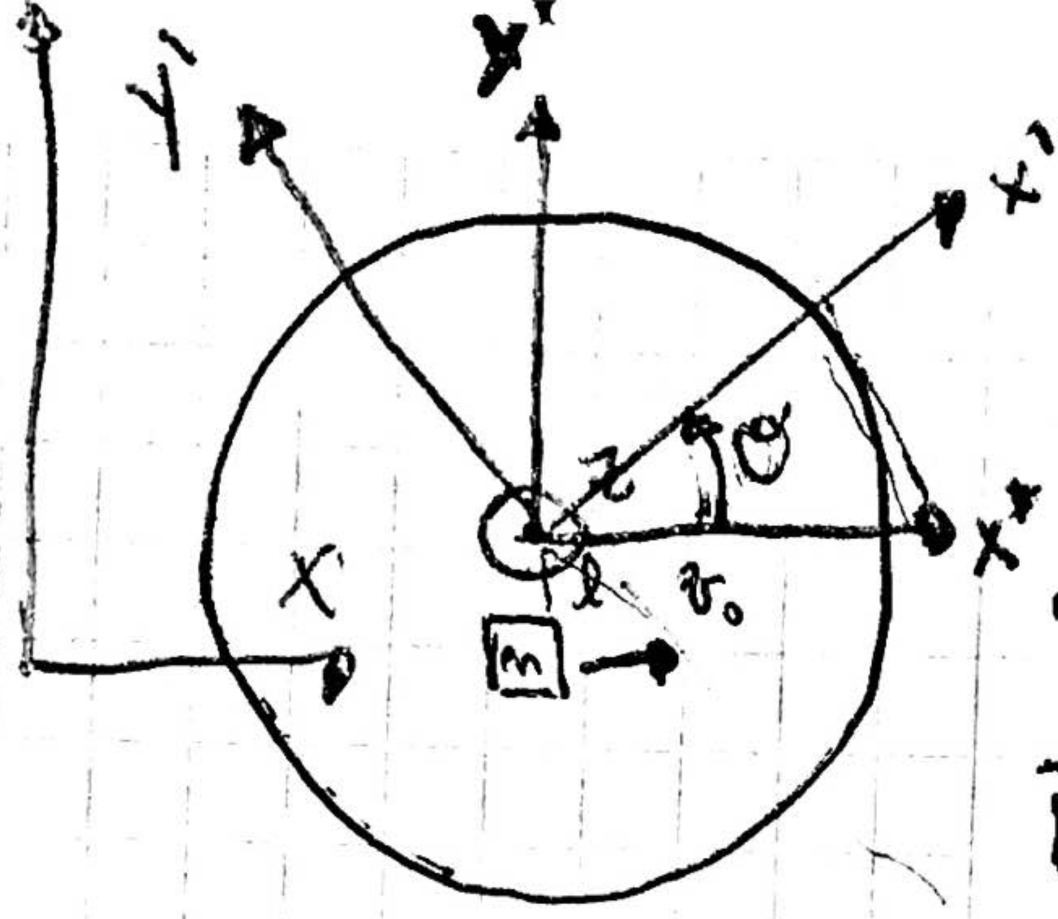
$F_{roz} = m\Omega^2 l$

$|F_{roz\,m\acute{a}x}| = \mu_e\, mg$

$\mu_e \cancel{m} g = \cancel{m}\Omega^2 l$

$$\boxed{\Omega_{m\acute{a}x} = \sqrt{\frac{\mu_e g}{l}}}$$

c) Si desaparece roz, halle v como función de distancia del paquete en S'



$\cancel{v_0 = \Omega \ell}$

$v_0 = \Omega \ell \hat{x}$

$\cancel{\bar{r} = \Omega \ell t \hat{x} + v_0 = \Omega \ell t - \ell}$

$\bar{r} = \Omega \ell t \hat{x} - \ell \hat{y}$

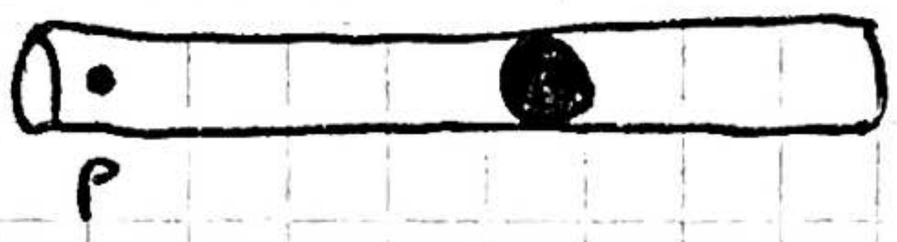
$\hat{x} = \cos\theta \, \hat{x}' - \sin\theta \, \hat{y}'$

$\hat{y} = \cos\theta \, \hat{y}' + \sin\theta \, \hat{x}'$

$\bar{r}' = \Omega \ell \cos\theta \, \hat{x}' - \Omega \ell \sin\theta \, \hat{y}' - \ell \cos\theta \, \hat{y}' - \ell \sin\theta \, \hat{x}'$

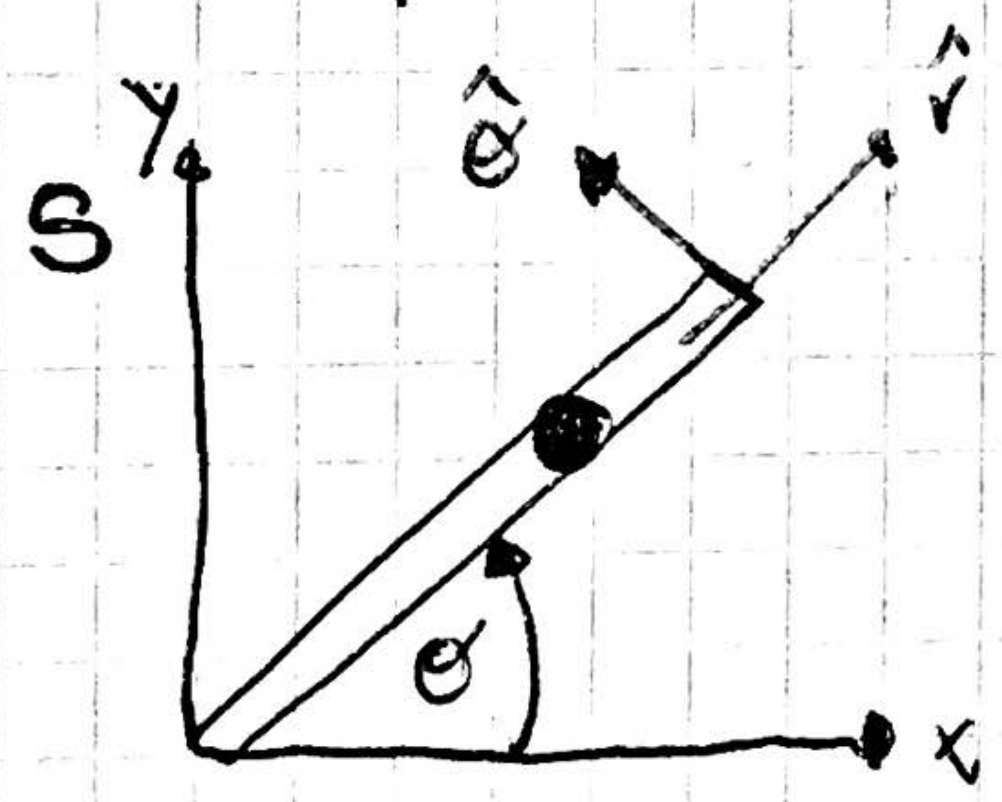
$\theta = \Omega t$ $\quad \boxed{\bar{r}' = \ell\left(\Omega \cos\Omega t \, \hat{x}' - \Omega \sin\Omega t \, \hat{y}' - \cos\Omega t \, \hat{y}' - \sin\Omega t \, \hat{x}'\right)}$

$\omega)$



m, ω cte

a) a bolita respecto S y S'



$(\hat{r})$ $\quad m\ddot{r} = 0$

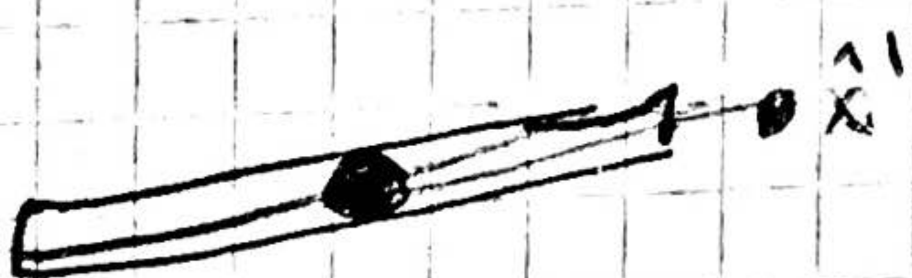
$(\hat{\theta}) \cdot m\ddot{\theta} = F_v$

$m(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = F_v$

$m\dot{r}\dot{\theta} = F_v$

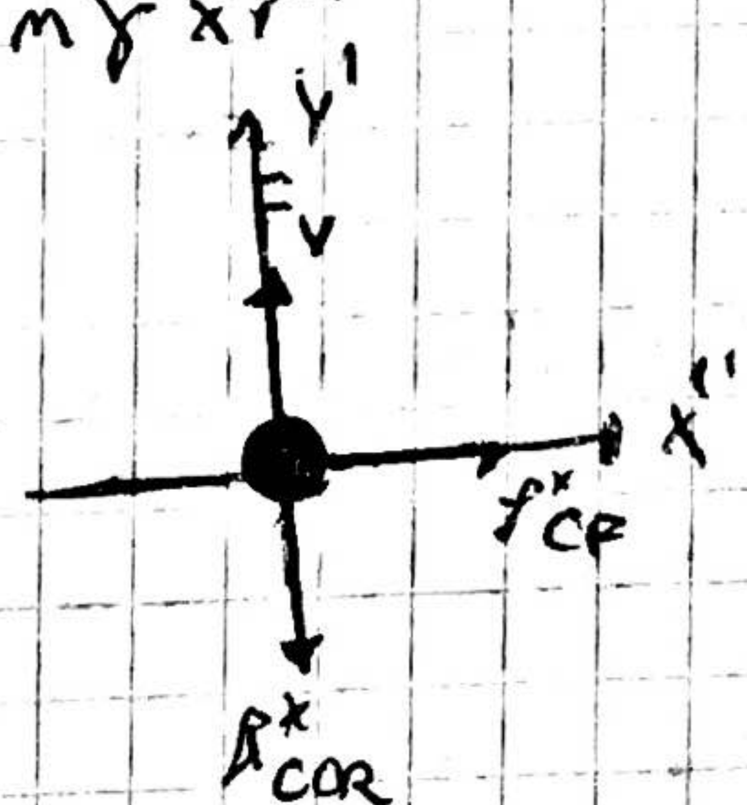
$m\dot{r}\omega = F_v$

$S')$



$$m\bar{a}' = m\bar{a} + 2m\bar{\Omega}\times\bar{v}' - m\bar{\Omega}\times(\bar{\Omega}\times\bar{r}') - m\bar{\gamma}\times\bar{r}'$$

$$m\bar{a}' = F_v - 2\omega\dot{x}'\hat{y}' + m\omega^2 x'\hat{x}'$$

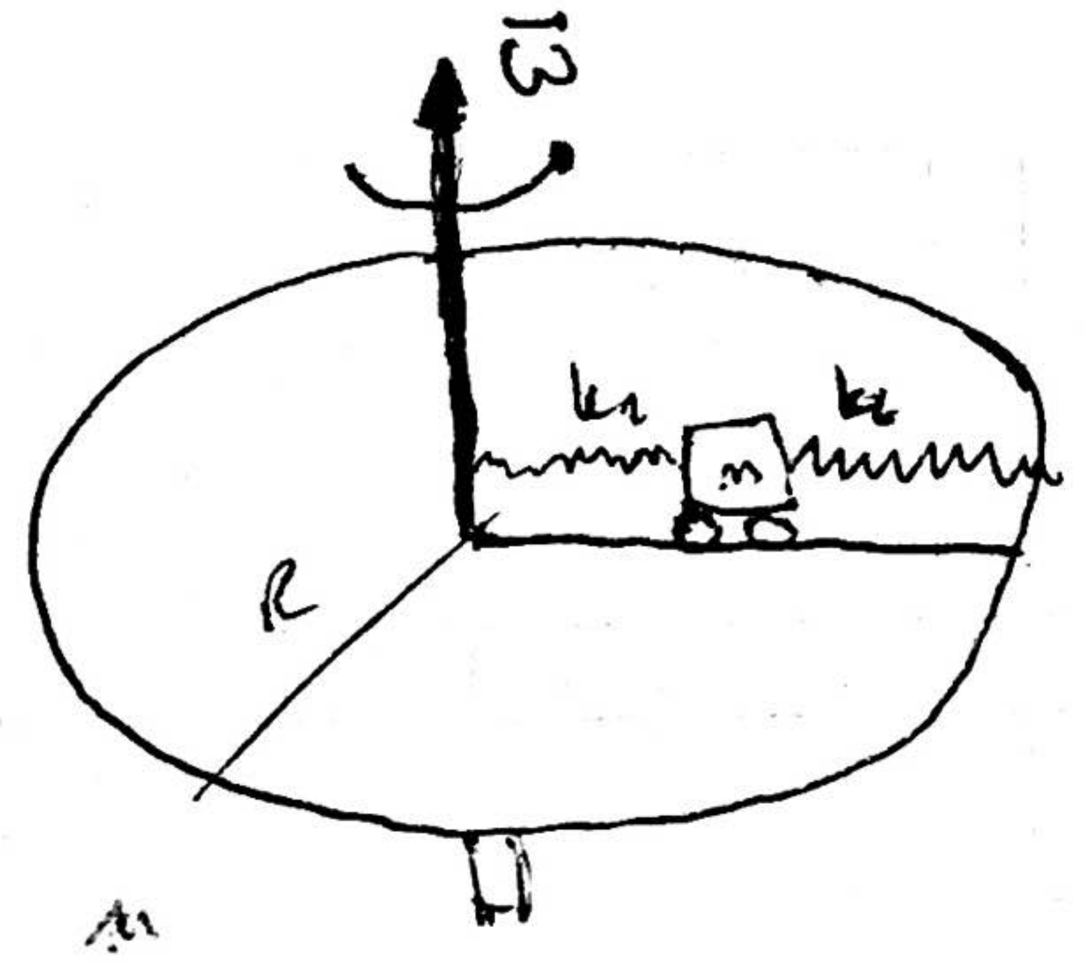


$\bar{\Omega} = \omega\hat{z}'$

$\bar{v}' = \dot{x}'\hat{x}'$

$\bar{r}' = x'\hat{x}'$

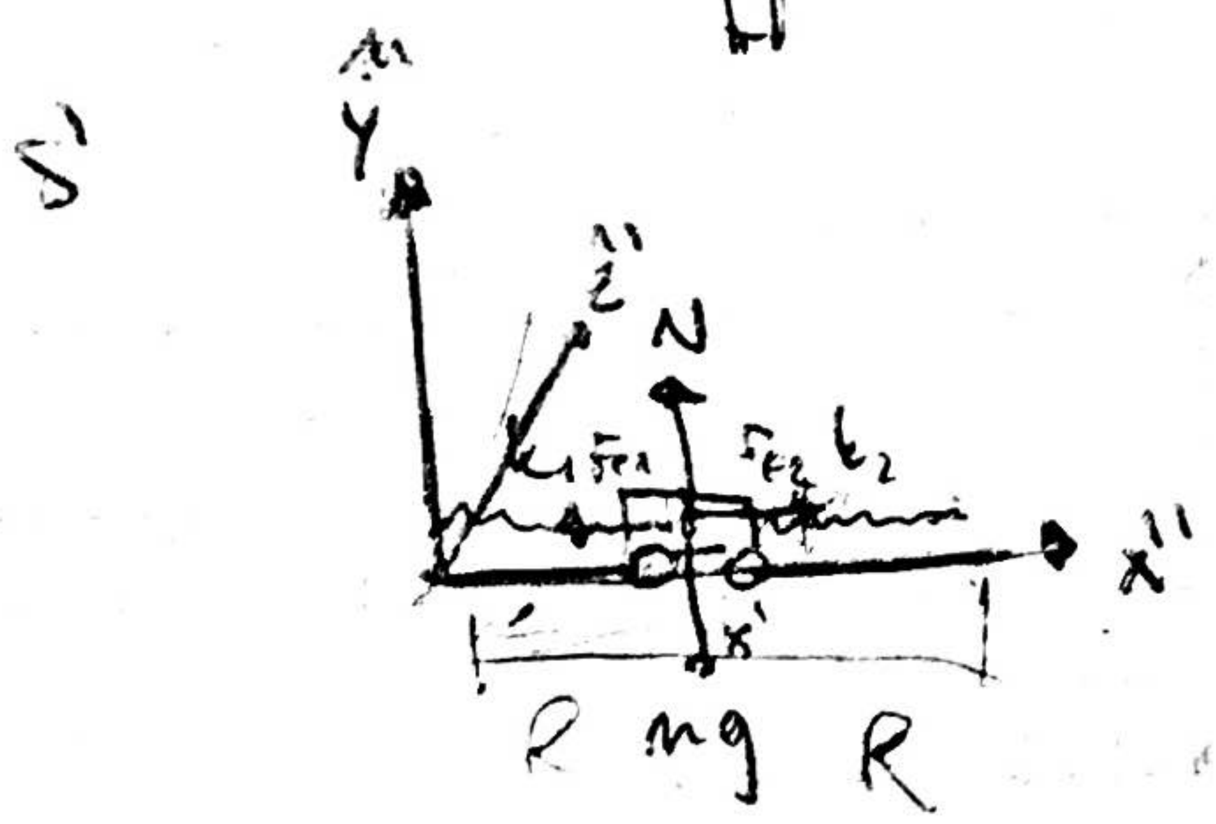
$\bar{\gamma} = 0$



Datos

$m, k_1, k_2, l_{o1}, l_{o2}$

Ec Newton sist fijo a la mesa



$$\vec{\Omega} = \omega\,\hat{y}'$$
$$\vec{v}' = \dot{x}'\,\hat{x}'$$

$$\cancel{-2m\vec{\Omega}\times\vec{v}'}$$
$$\cancel{\vec{v} = \vec{v}' + \vec{\Omega}\times\vec{r}}$$
$$\cancel{\vec{v}' = \dot{x}'\hat{x}' + (x'\hat{x}' \times \omega\hat{y}')}$$
$$\cancel{\vec{v}' = \dot{x}'\hat{x}' + \omega x'\hat{z}'}$$
$$\cancel{-2m\omega\hat{y}'(\dot{x}'\hat{x}' + \omega x'\hat{z}')}$$

$$F_{e1} = -k_1(x' - l_{o1})$$
$$F_{e2} = -k_2(R - x' - l_{o2})$$

$$m\vec{a}' = m\vec{a} - 2m\vec{\Omega}\times\vec{v}' - m\vec{\Omega}\times(\vec{\Omega}\times\vec{r}') - m\vec{\gamma}\times\vec{r}'$$

$$m\vec{a}' = -k_1x' + k_1l_{o1} + k_2R - k_2x' - k_2l_{o2}$$

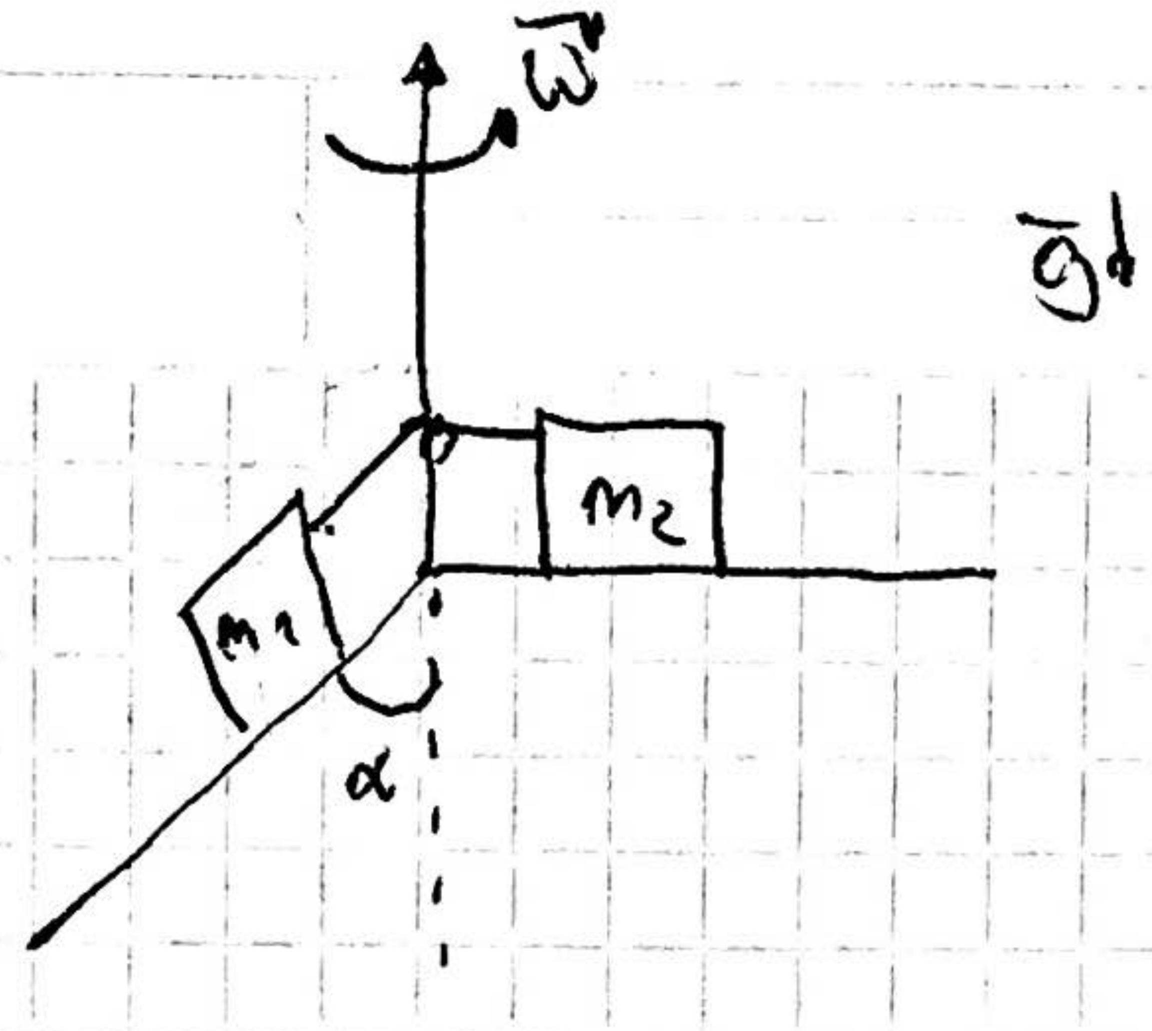
$$-2m\omega\dot{x}'\hat{z}'$$

$$+m\omega^2x'\hat{x}'$$

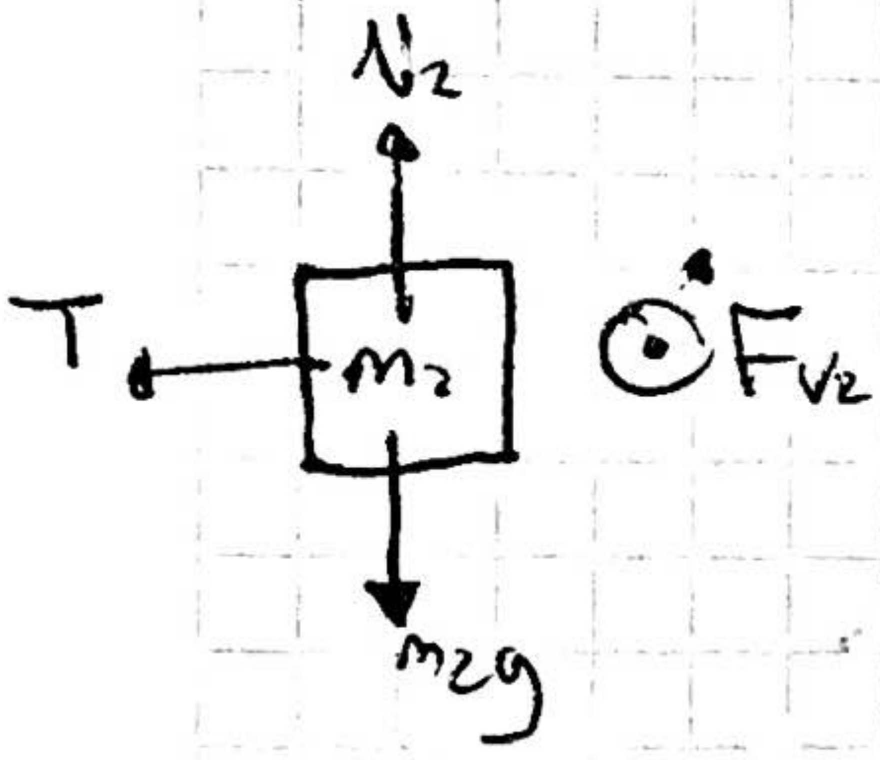
$$m\vec{a}' = -k_1x' + k_1l_{o1} + k_2R - k_2x' - k_2l_{o2} - 2m\omega\dot{x}'\hat{z}' + m\omega^2x'\hat{x}' + F_v\hat{z}'$$

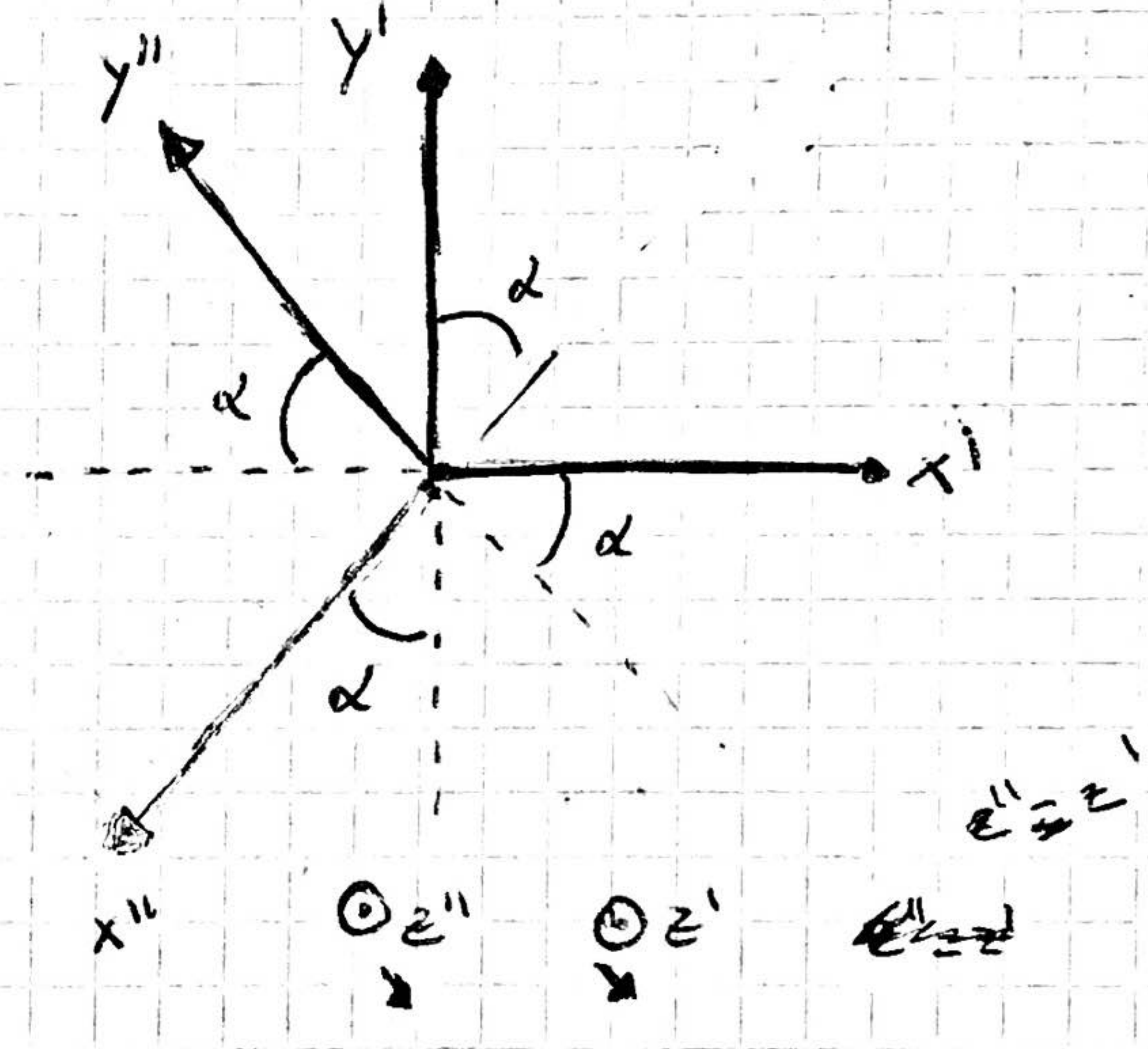
$$\boxed{\begin{aligned} m\ddot{x}' &= -k_1x' + k_1l_{o1} + k_2R - k_2x' - k_2l_{o2} + m\omega^2x' \\ m\ddot{z}' &= 0 = F_v = 2m\omega\dot{x}' \end{aligned}}$$

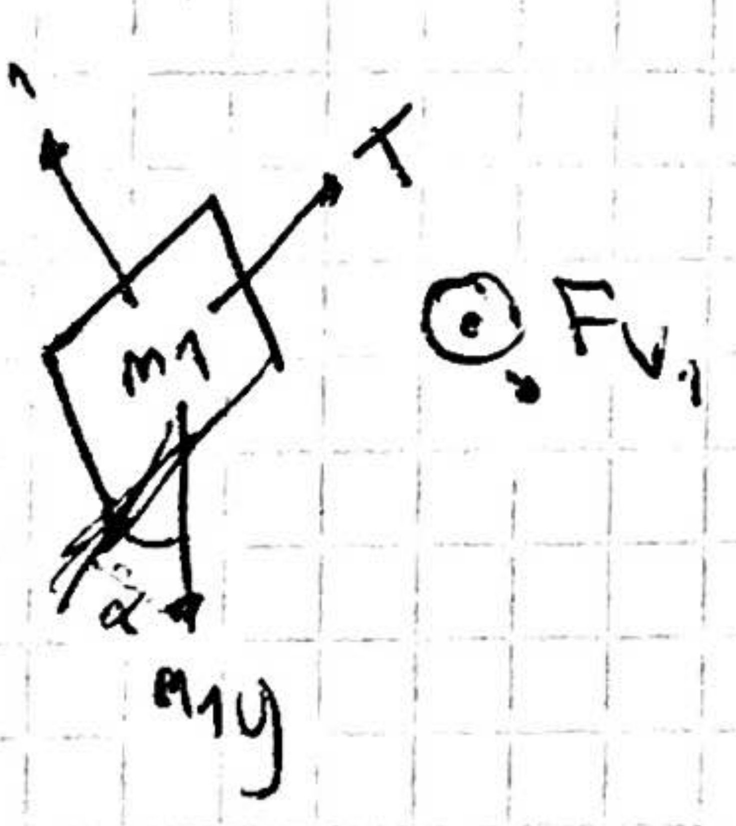
11



Datos
m_1, m_2, L, sayn inert
en $t_0 = 0$ m_1 se encuentra en A
con $v_0 = 0$

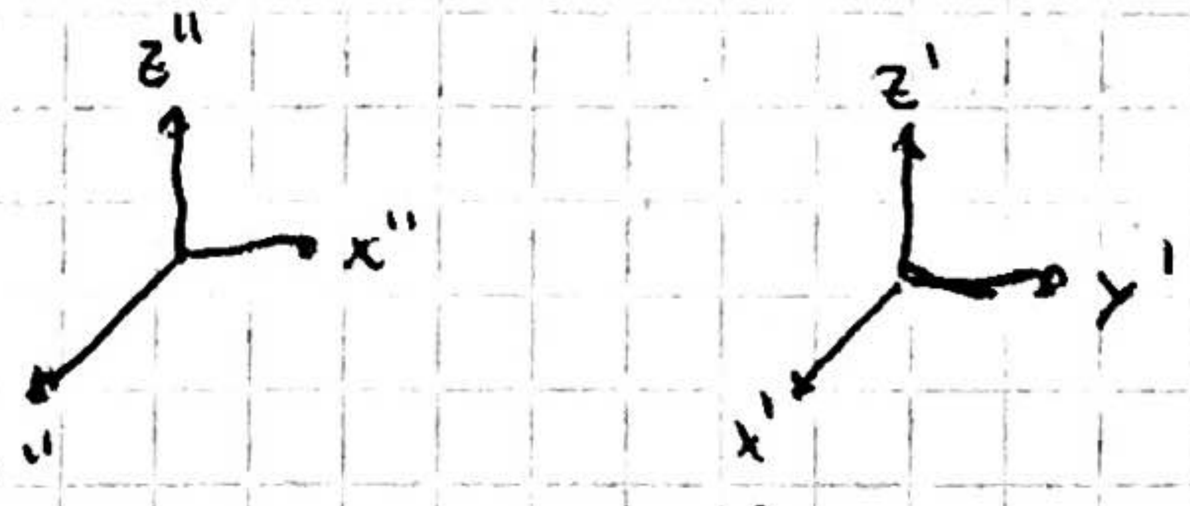






$L = x'' + x'$

$\ddot{x}'' = \ddot{x}'$



(m_1, S'')

$\bar{\omega}'' = \omega \cdot (-\cos\alpha\, \hat{x}'' + \sin\alpha\, \hat{y}'')$

$\bar{v}'' = \dot{x}''\, \hat{x}''$

$\bar{r}'' = x''\, \hat{x}''$

$\bar{\gamma} = 0$

$F_C^* = -2m\bar{\omega}\times\bar{v}'' = -2m\omega\dot{x}''(-\cos\alpha\, \hat{x}''\times\hat{x}'' + \sin\alpha\, \hat{y}''\times\hat{x}'') =$

$= -2m\omega\dot{x}''\sin\alpha\,(+\hat{z}'')$

$= -2m\omega\dot{x}''\sin\alpha\, \hat{z}''$

$f_{CF}^* = -m\omega^2 x''(-\cos\alpha\, \hat{x}'' + \sin\alpha\, \hat{y}'') \times (-\cos\alpha\, \hat{x}''\times\hat{x}'' + \sin\alpha\, \hat{y}''\times\hat{x}'') =$

$= -m\omega^2 x''(-\cos\alpha\, \hat{x}'' + \sin\alpha\, \hat{y}'') \times (+\sin\alpha\, \hat{z}'') =$

$-m\omega^2 x''(-\cos\alpha\sin\alpha\, \hat{y}'' - \sin^2\alpha\, \hat{x}'') = m\omega^2 x''(\cos\sin\alpha\, \hat{y}'' + \sin^2\alpha\, \hat{x}'')$

$m\bar{a} = F_V\hat{z}'' + m_1 g(\cos\alpha\,\hat{x}'' - \sin\alpha\,\hat{y}'') + N_1\hat{y}'' - T\hat{x}''$

$$\boxed{\begin{aligned}
&m_1\ddot{x}'' = m_1 g\cos\alpha\,\hat{x}'' - T\hat{x}'' + m_1\omega^2 x''\sin^2\alpha\,\hat{x}''\\
&m_1\ddot{y}'' = 0 = -m_1 g\sin\alpha\,\hat{y}'' + N_1\hat{y}'' + m_1\omega^2 x''\sin\alpha\cos\alpha\,\hat{y}''\\
&\quad m_1 g\sin\alpha = N_1 + m_1\omega^2 x''\sin\alpha\cos\alpha\\
&m_1\ddot{z}'' = 0 = F_V = 2m_1\omega\,\dot{x}''\sin\alpha
\end{aligned}}$$

(m_2, S')

$\bar{\omega}' = \omega\hat{y}'$

$\bar{v}' = \dot{x}'\hat{x}'$

$\bar{r}' = x'\hat{x}'$

$\bar{a}' = \bar{0}$

$f^*_C = -2m_2\bar{\Omega}\times\bar{v}'$

$= +2m_2\omega\dot{x}'\hat{z}'$

$f^*_{CF} = -m_2\bar{\Omega}\times(\bar{\Omega}\times\bar{r}')$

$f^*_{CF} = -m\omega^2 x'\hat{y}'\times(\hat{y}'\times\hat{x}')$

$= -m\omega^2 x'(\hat{y}'\times -\hat{z}')$

$= m\omega^2 x'\hat{x}'$

$$\boxed{\begin{aligned}
&m\bar{a}' = -F_V\hat{z}' + N_2\hat{y}' - m_2 g\hat{y}' - T\hat{x}' + 2m_2\omega\dot{x}'\hat{z}' + m\omega^2 x'\hat{x}'\\
&m\ddot{x}' = -T + m_2\omega^2 x'\hat{x}'\\
&m\ddot{y}' = 0 = N_2\hat{y}' - m_2 g\hat{y}'\\
&m\ddot{z}' = 0 = -F_V + 2m_2\omega\dot{x}' \Rightarrow F_V = 2m_2\omega\dot{x}'
\end{aligned}}$$

c) $\cancel{m_2\ddot{x}' = -2m_2\omega^2 x'}$

$m_2\ddot{x}' = -T + m_2\omega^2 x'$

$m_1\ddot{x}'' = -T + m_1 g\cos\alpha + m_1\omega^2 x''\sin^2\alpha$

$-\ddot{x}'' = \ddot{x}'$ $\qquad\qquad$ $x'' = L - x'$

$-m_1\ddot{x}' = -T + m_1 g\cos\alpha + m_1\omega^2 x''\sin^2\alpha$

$m_1\ddot{x}' = -T + m_1 g\cos\alpha + (m_1\omega^2 L - m_1\omega^2 x')\sin^2\alpha$

$E_{C\,mov}' - E_{C\,mov}''$

$m_2\ddot{x}' + m_1\ddot{x}' = \cancel{-T} + m_2\omega^2 x' \cancel{+T} - m_1 g\cos\alpha - m_1\omega^2 L\sin^2\alpha + m_1\omega^2 x'\sin^2\alpha$

$\ddot{x}' = \underline{m_2\omega^2 x' - m_1 g\cos\alpha - m_1\omega^2 L\sin^2\alpha + m_1\omega^2 x'\sin^2\alpha}$

$\cancel{\ddot{x}' - m_2\omega^2}$

$$\ddot{x}' - x'\underbrace{\left(\frac{m_2\omega^2 + m_1\omega^2\sin^2\alpha}{m_1+m_2}\right)}_{\omega_0^2} = \frac{-m_1 g\cos\alpha - m_1\omega^2 L\sin^2\alpha}{m_1+m_2}$$

$$\ddot{x}' - \omega_0^2 x' = \frac{-m_1 g\cos\alpha - m_1\omega^2 L\sin^2\alpha}{m_1+m_2}$$

$$x'(t) = A'\cos(\omega t + \varphi) + x'_p$$

$$x'_p = \frac{m_1 g\cos\alpha + m_1\omega^2 L\sin^2\alpha}{m_2\omega^2 + m_1\omega^2\sin^2\alpha}$$

$$x'(0) = A = A'\cos\varphi + x'_p$$

$$x'(t)' = -A'\omega\sin(\omega t + \varphi)$$

$$0 = -A'\omega\sin\varphi$$

$$\sin\varphi = 0$$

$$\varphi = 0$$

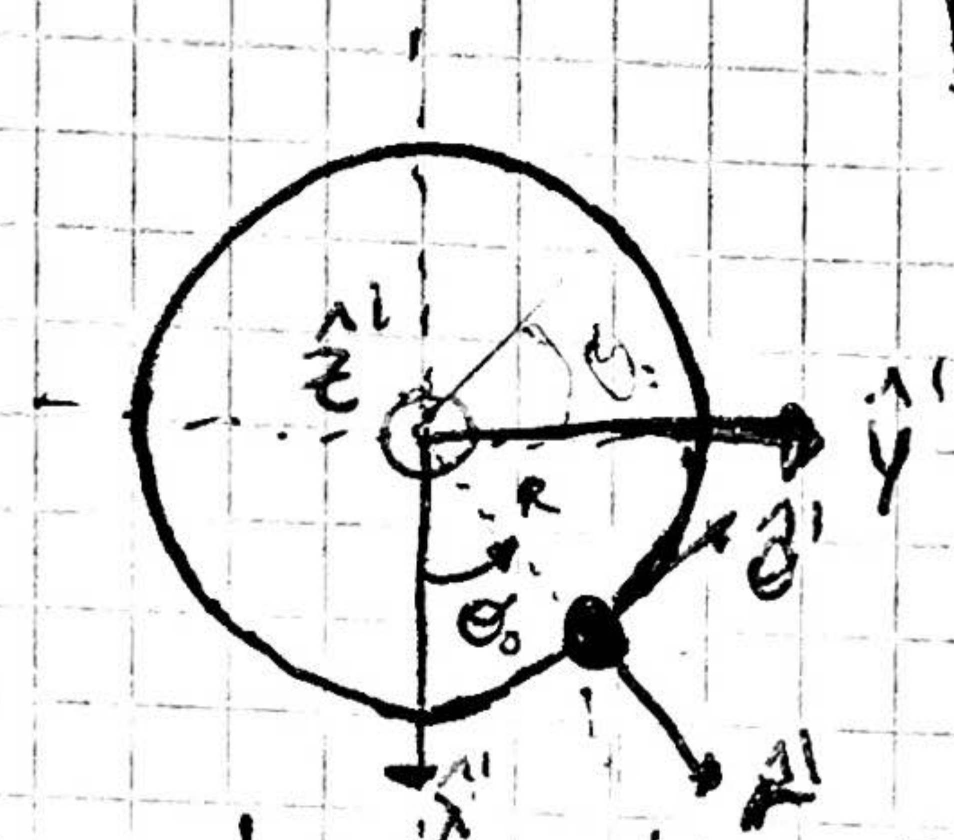
$$\Rightarrow A = A' + x'_p$$

$$A' = A + x'_p$$

$$\boxed{x'(t) = (A + x'_p)\cos\omega t + x'_p}$$

Datos

m, R $\quad \omega_0$ Cte . Resulta a θ_{eq} θ_0



a) Fuerzas de interacción, y Ec Newton fijo al aro

$$m\bar{a}' = m\bar{a} - 2m\bar{\Omega}\times\bar{v}' - m\bar{\Omega}\times(\bar{\Omega}\times\bar{r}') - \underbrace{m\dot{\bar{\Omega}}\times\bar{r}'}_{=0}$$

$$\bar{v}' = \bar{v} - \bar{\Omega}\times\bar{r} \qquad \bar{r}' = r'\hat{r}' = R\hat{r}'$$

$$\bar{v}' = \overbrace{\dot{R}\hat{r}'}^{=0} + R\dot{\theta}\hat{\theta}'$$

$$\bar{v}' = R\dot{\theta}\hat{\theta}'$$

$$\bar{\Omega} = -\omega\hat{x}'$$

$$\hat{r}' = \cos\theta\hat{x}' + \sin\theta\hat{y}'$$

$$\hat{\theta}' = \cos\theta\hat{y}' - \sin\theta\hat{x}'$$

$$\hat{y}' = \cos\theta\hat{\theta}' + \sin\theta\hat{r}'$$

$$+2m\omega R\dot{\theta}(\hat{x}'\times\hat{\theta}')$$

$$2m\omega R\dot{\theta}(\hat{x}'\times\cos\theta\hat{y}' - \hat{x}'\times\sin\theta\hat{x}')$$

$$2m\omega R\dot{\theta}\cos\theta\hat{z}'$$

$$-m\omega^2 R\hat{x}'(\hat{x}'\times\hat{r}')$$

$$-m\omega^2 R\hat{x}'(\hat{x}'\times\cos\theta\hat{x}' + \sin\theta\hat{y}')$$

$$+m\omega^2 R\sin\theta\hat{y}' = m\omega^2 R\sin^2\theta\hat{r}' + m\omega^2 R\sin\theta\cos\theta\hat{\theta}'$$

$$m\bar{a}' = mg\cos\theta\hat{r}' - mg\sin\theta\hat{\theta}' + F_V + 2m\omega R\dot{\theta}\cos\theta\hat{z}' + m\omega^2 R\sin^2\theta\hat{r}' + m\omega^2 R \ldots$$

$m\ddot{r}' = mg\cos\theta\,\hat{r}' + m\omega^2 R\sin^2\theta\,\hat{r}' - F_v\hat{r}'$

$F_v\hat{r}' = mg\cos\theta\,\hat{r}' + m\omega^2 R\sin^2\theta\,\hat{r}'$

$m\ddot{\theta}' = m\omega^2 R\sin\theta\cos\theta\,\hat{\theta}' - mg\sin\theta\,\hat{\theta}'$

$\cancel{mg\sin\theta}$ $\sin\theta(m\omega^2 R\cos\theta - mg) = 0$

$\sin\theta = 0 = (\pi, 0)$

$\cancel{m}\omega^2 R\cos\theta = \cancel{m}g$

$\cos\theta = \frac{g}{\omega^2 R}$ $\qquad \frac{g}{\omega^2 R} < 1$

$g < \omega^2 R$

$\frac{\partial F_\theta}{\partial\theta}\Big|_{\theta_0} = m\omega^2 R\cos^2\theta_0 - m\omega^2 R\sin^2\theta_0 - mg\cos\theta_0$ $\qquad \cancel{\cos^2\theta = 1 - \sin\theta^2}$

$m\omega^2 R(\cos^2\theta - 1 + \cos^2\theta) - mg\cos\theta$ $\qquad \sin^2\theta = 1 - \cos^2\theta$

$m\omega^2 R(2\cos^2\theta - 1) - mg\cos\theta$

$\frac{m\cancel{\omega^2}\cancel{R}\,2g^2}{\omega^{\cancel{4}}_2 R^{\cancel{2}}} - m\omega^2 R - \frac{mg^2}{\omega^2 R}$

$- m\omega^2 R + \frac{mg^2}{\omega^2 R}$

$= m\left(\frac{g^2}{\omega^2 R} - \omega^2 R\right)$ $\qquad \frac{g^2}{\omega^2 R} - \omega^2 R < 0$

$\frac{g^2}{\omega^2 R} < \omega^2 R$

$\frac{g^2}{\omega^4 R^2} < 1$

$g < \omega^2 R$

si existen θ_0 son siempre estables

$$\left.\frac{\partial F_\theta}{\partial\theta}\right|_0 = \left(m\omega^2 R - mg\right) = m(\omega^2 R - g)$$

$$\omega^2 R - g < 0$$

$$\omega^2 R < g$$

$$\frac{g}{\omega^2 R} > 1$$

si $\frac{g}{\omega^2 R} > 1$ 0 es estable, sino inest

$$\left.\frac{\partial F_\theta}{\partial\theta}\right|_\pi = m(-\omega^2 R - g) < 0 \;\therefore\; \text{est}$$

$$m\omega^2 R + g > 0$$

$F_v \hat{z}' = 2m\omega R\dot\theta' \cos\theta\, \hat{z}'$ $\qquad F_v \hat{r} = mg\cos\theta\, \hat{r}' + m\omega^2 R\sin^2\theta\, \hat{r}'$

$$R\ddot\theta' = \omega^2 R\sin\cos\theta\,\hat\theta' - mg\sin\theta\,\hat\theta'$$

$$\ddot\theta' - \omega^2\sin\theta\cos\theta + \frac{g}{R}\sin\theta = 0$$

$$\frac{d\dot\theta'}{d\theta}\dot\theta' = \omega^2\sin\theta\cos\theta - \frac{g}{R}\sin\theta$$

$$\int_0^{\dot\theta'} \dot\theta'\, d\dot\theta' = \int_{\theta_0}^{\theta} \omega^2\sin\theta\cos\theta - \frac{g}{R}\sin\theta\, d\theta$$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$$\frac{\dot\theta'^2}{2} - \frac{\dot\theta_0'^2}{2} = \int_{\sin\theta_0}^{\sin\theta} \omega^2 u \, du + \int_{\theta_0}^{\theta} \frac{g}{R}\sin\theta\, d\theta$$

$$\frac{\dot\theta'^2}{2} = \frac{\omega^2\sin^2\theta}{2} - \frac{\omega^2\sin^2\theta_0}{2} - \frac{g}{R}\cos\theta + \frac{g}{R}\cos\theta_0 + \frac{\dot\theta_0'^2}{2}$$

$$\boxed{\dot\theta' = \sqrt{\omega^2(\sin^2\theta - \sin^2\theta_0) + \frac{g}{R}2(-\cos\theta + \cos\theta_0) + \dot\theta_0'^2}}$$