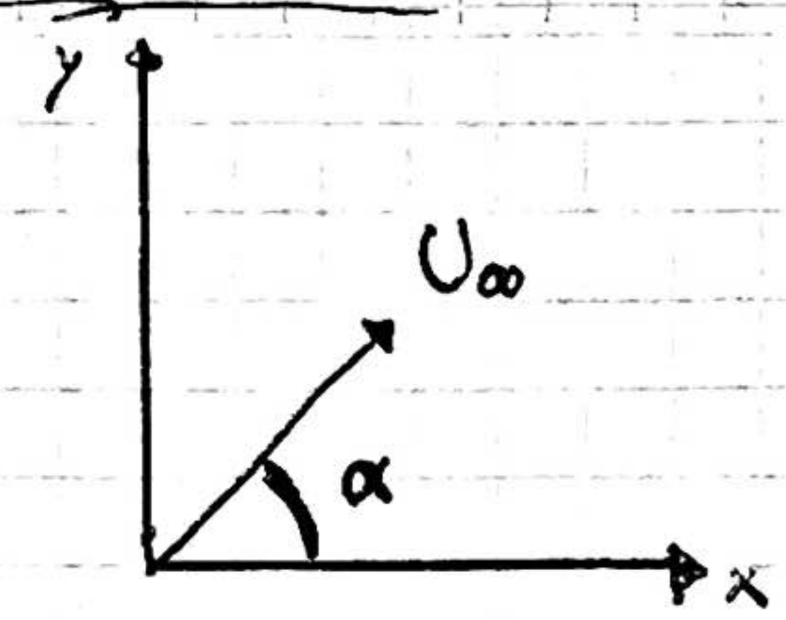
4.1a) Flujos de singularidades elementales

a) campo de velocidades $\underline{u}(x,y)$

b) $\underline{w}(x,y) = \underline{\nabla} \times \underline{u}$

c) la función de corriente $\psi(x,y)$ y el gráfico de las lineas de corriente.

i)



a) $\boxed{\underline{u} = U_\infty \cos(\alpha)\hat{x} + U_\infty \sin(\alpha)\hat{y}}$ b) $\boxed{\underline{\nabla} \times \underline{u} = 0}$

incompresible $\frac{d\rho}{dt} = 0 \Rightarrow \underline{\nabla} \cdot \underline{v} = 0$

$\Rightarrow \exists\, \underline{\psi} / \underline{\nabla} \times \underline{\psi} = \underline{u}$

ideal : no viscoso

$\underline{\omega} = \underline{\nabla} \times \underline{u}$

$$\underline{\nabla} \times \underline{\psi} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \psi_x & \psi_y & \psi_z \end{vmatrix} = \left(\frac{\partial \psi_z}{\partial y} - \frac{\partial \psi_y}{\partial z}\right)\hat{x} + \left(\frac{\partial \psi_x}{\partial z} - \frac{\partial \psi_z}{\partial x}\right)\hat{y}$$

$\Rightarrow \underline{\psi} = \psi_z(x,y)\,\hat{z}$

$\partial_y \psi = U_\infty \cos(\alpha) \quad -\partial_x \psi = U_\infty \sin(\alpha)$

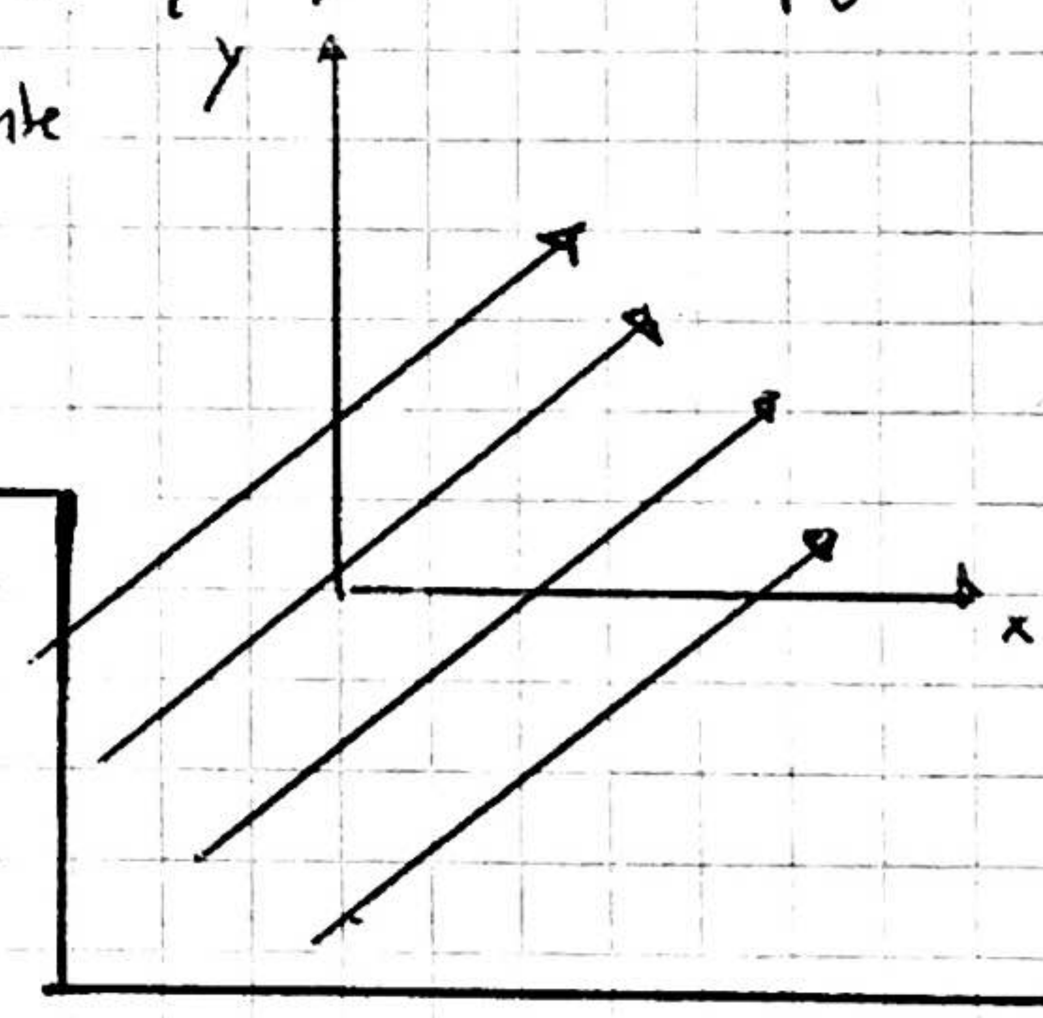
c)

$\Rightarrow \psi(x,y) = U_\infty \cos(\alpha) y - U_\infty \sin(\alpha) x$

Si pongo $\psi(x,y) = cte$ $\Rightarrow$ despejo las lineas de corriente

$\Rightarrow U_\infty \cos(\alpha) y - U_\infty \sin(\alpha) x = C'$

$\Rightarrow y = C + tg(\alpha) x$

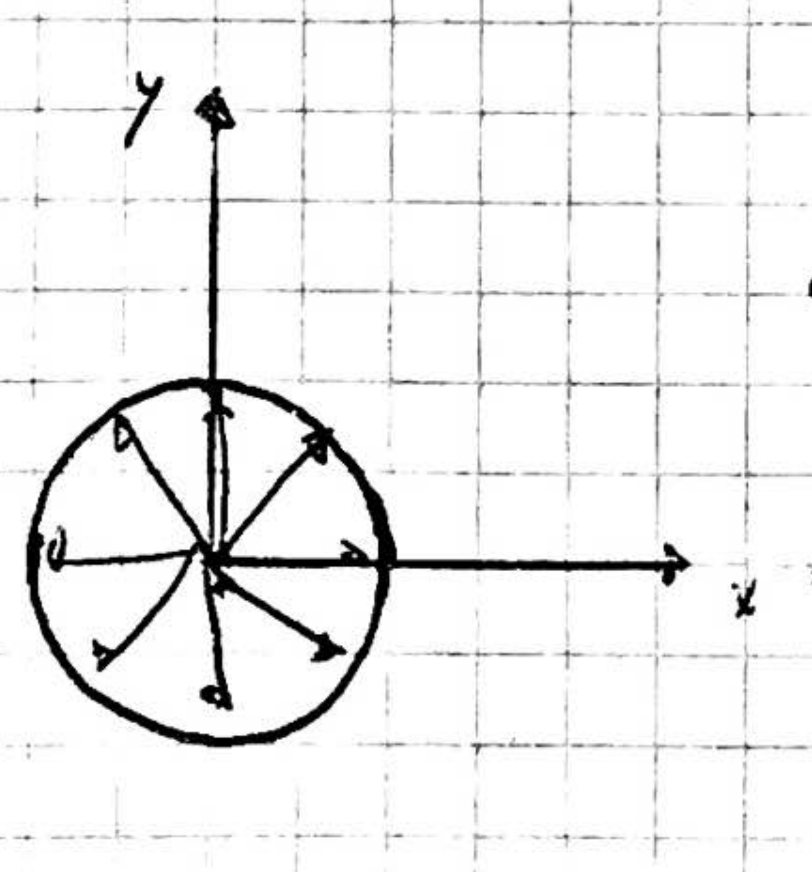


$Q = \int_1^2 \underline{v} \cdot \hat{n}\, dl$, $\underline{u} = u\hat{r}$

$\hat{n}\, dl = d\underline{l} \times \hat{z} = r d\theta \hat{r}$

$\Rightarrow Q = \int_0^{2\pi} \underline{u} \cdot r d\theta \hat{r} = u 2\pi r$

ii)



a) $\boxed{\underline{u} = \frac{Q}{2\pi r}\hat{r}}$ $\quad u_r = \frac{Q}{2\pi r} \quad u_\theta = 0$

$\underline{\nabla} \times (\psi \hat{z}) = \frac{1}{r}\partial_\theta \psi \hat{r} + \partial_r \psi \hat{\theta} = \underline{u}$

$\boxed{\underline{w} = \underline{\nabla} \times \underline{u} = 0}$

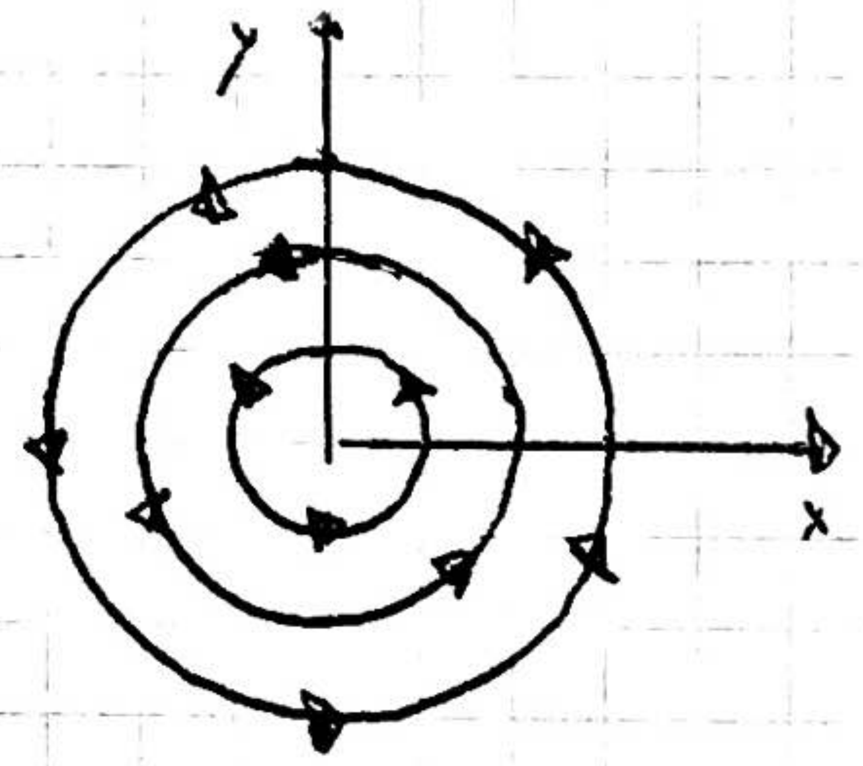
$\Rightarrow \frac{1}{r}\partial_\theta \psi = \frac{Q}{2\pi r} \Rightarrow \boxed{\psi = \frac{Q}{2\pi}\theta = \frac{Q}{2\pi} \operatorname{arctg}\left(\frac{y}{x}\right)}$

$\partial_r \psi = 0 \quad \therefore \quad \psi = \psi(\theta)$

$$\underline{u} = \frac{Q}{2\pi r}\frac{r\hat{r}}{r} = \frac{Q(x\hat{x}+y\hat{y})}{2\pi(x^2+y^2)} \qquad U = \frac{Qz}{2\pi z z^*} \qquad U^* = \frac{Qz^*}{2\pi z z^*} = \frac{Q}{2\pi z}$$

iii) vórtice de circulación Γ

$$\boxed{\underline{u} = \frac{\Gamma}{2\pi r}\hat{\theta}}$$



$$\underline{u} = u\hat{\theta} \qquad \Gamma = \int_0^{2\pi} u\hat{\theta}\, r\, d\theta\, \hat{\theta} = u 2\pi r$$

$$\underline{u} = \underline{\nabla}\times(\psi \hat{z})$$

$$\frac{1}{r}\partial_\theta \psi \hat{r} = 0\hat{r}$$

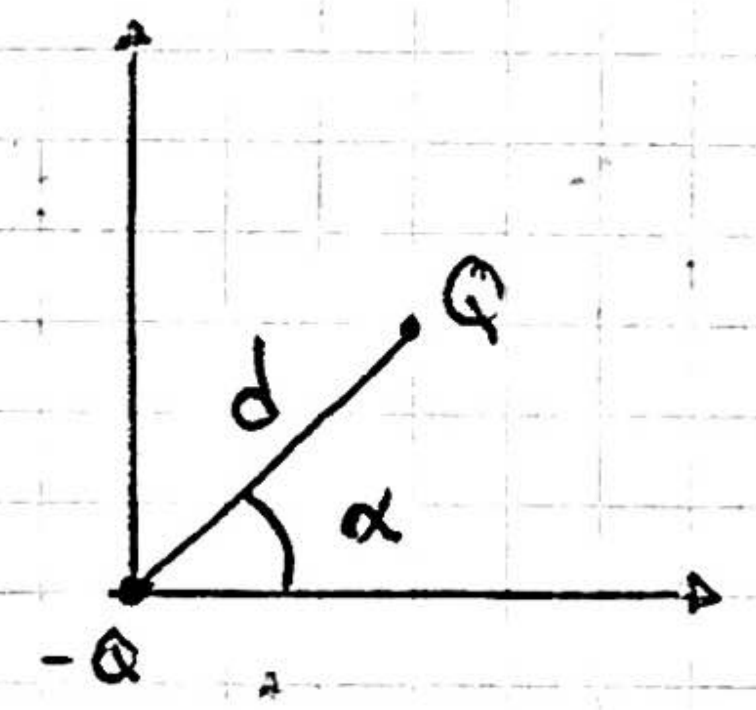
$$-\partial_r \psi \hat{\theta} = \frac{\Gamma}{2\pi r}\hat{\theta}$$

$$\boxed{\underline{\omega} = 0}$$

$$\therefore \psi = \psi(r) = \frac{-\Gamma}{2\pi}\ln(r)$$

$W(z)$

iv)



$$\underline{u}_{-Q} = \frac{-Q}{2\pi r}\hat{r} \qquad u_Q = \frac{Q}{2\pi r}\hat{r}$$ si estuvieran en el origen

$$\underline{u} = \frac{Q}{2\pi}\left(-\frac{\hat{r}}{r} + \frac{\bar{r} - d(\cos(\alpha)\hat{x} + \sin(\alpha)\hat{y})}{(x-d\cos(\alpha))^2+(y-d\sin(\alpha))^2}\right) = \frac{Q}{2\pi}\left[-\frac{(x,y)}{x^2+y^2} + \frac{(x-d\cos(\alpha),\, y-d\sin(\alpha))}{(x-d\cos(\alpha))^2+(y-d\sin(\alpha))^2}\right]$$

inmanejable

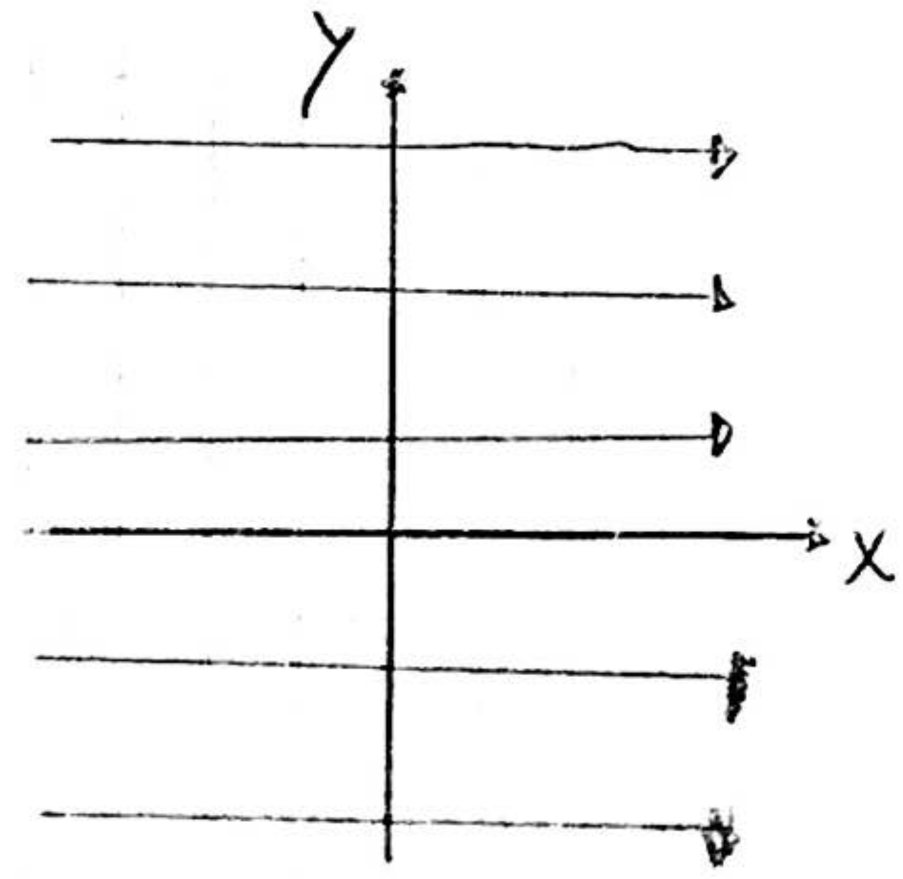
b) Calcule a) $\underline{u}(x,y)$; b) puntos de estancamiento osea $\underline{u}(x,y)=0$

c) $\underline{w}=\underline{\nabla}\times\underline{u}$, d) graficar lineas de corriente

i) $\psi(x,y)=ay$

$$-\partial_x\psi=u_y \Rightarrow \underline{u}=a\hat{x}$$
$$\partial_y\psi=u_x \qquad \text{p.d.e}=\emptyset$$

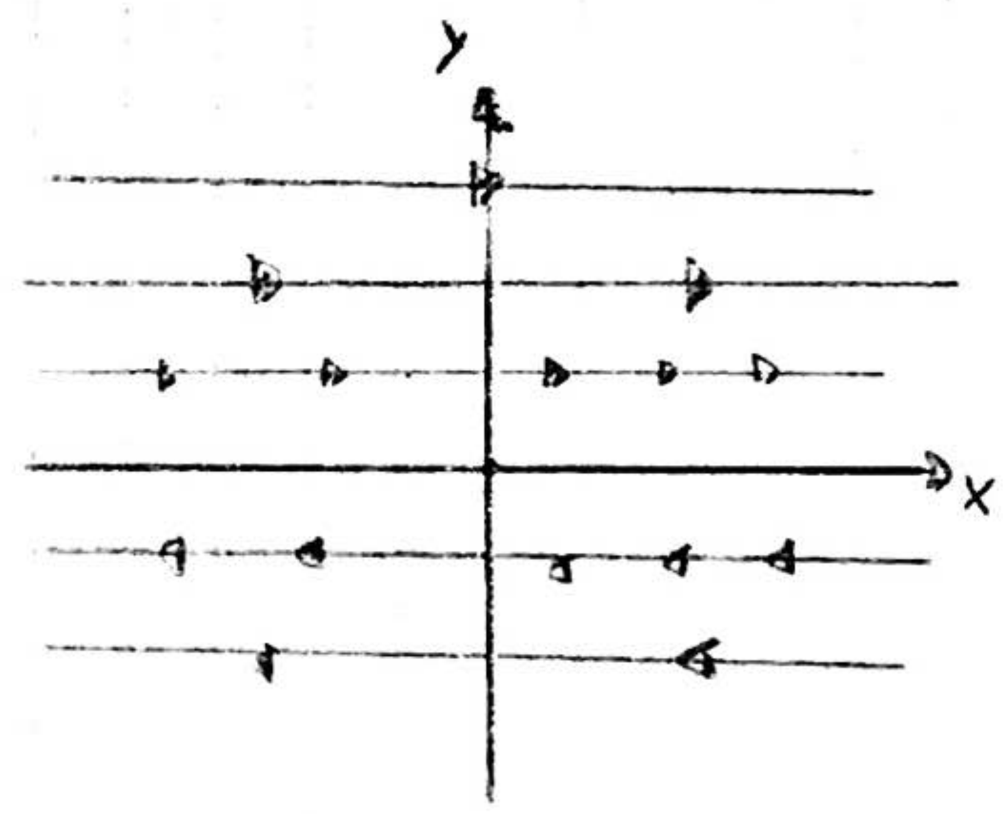
$$\underline{w}=0$$



ii) $\psi(x,y)=by^2 \Rightarrow u_x=2by \wedge \underline{u}=2by\,\hat{x}$

p.d.e : $\{y=0\}$

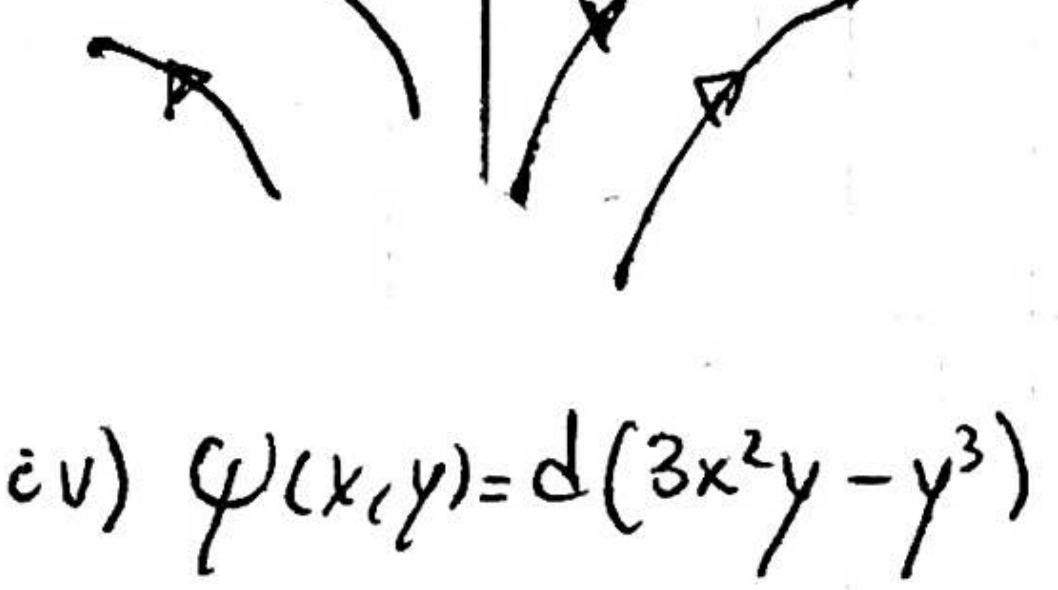
$\underline{w}=-2b\hat{z}$



iii) $\psi(x,y)=cxy$

$$u_y=-\partial_x\psi=-cy$$
$$u_x=\partial_y\psi=cx \qquad \Rightarrow \underline{u}=cx\hat{x}-cy\hat{y}$$

p.d.e : $\{(x,y)=(0,0)\}$ $\qquad \underline{\nabla}\times\underline{u}=(\partial_x,\partial_y,\partial_z)\times(cx,-cy,0)=(0,0,0)$

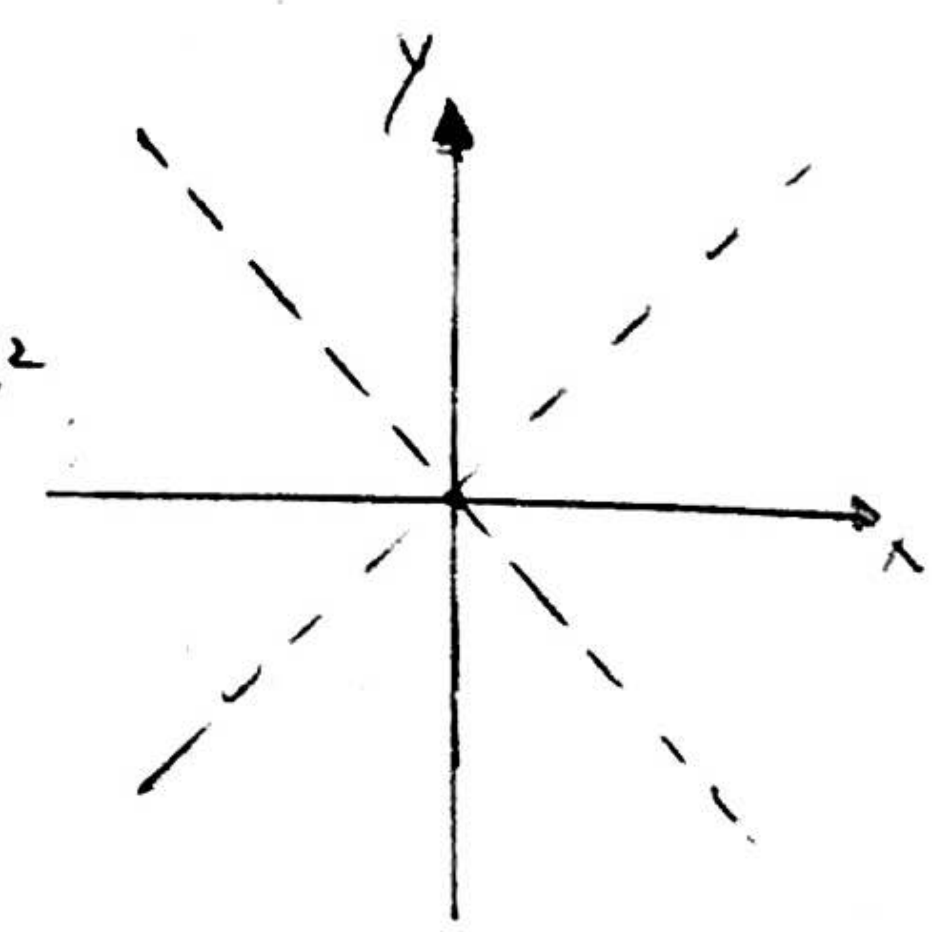


$$\psi(x,y)=\text{cte} \rightarrow \cancel{y}=\frac{C}{x}$$

iv) $\psi(x,y)=d(3x^2y-y^3)$

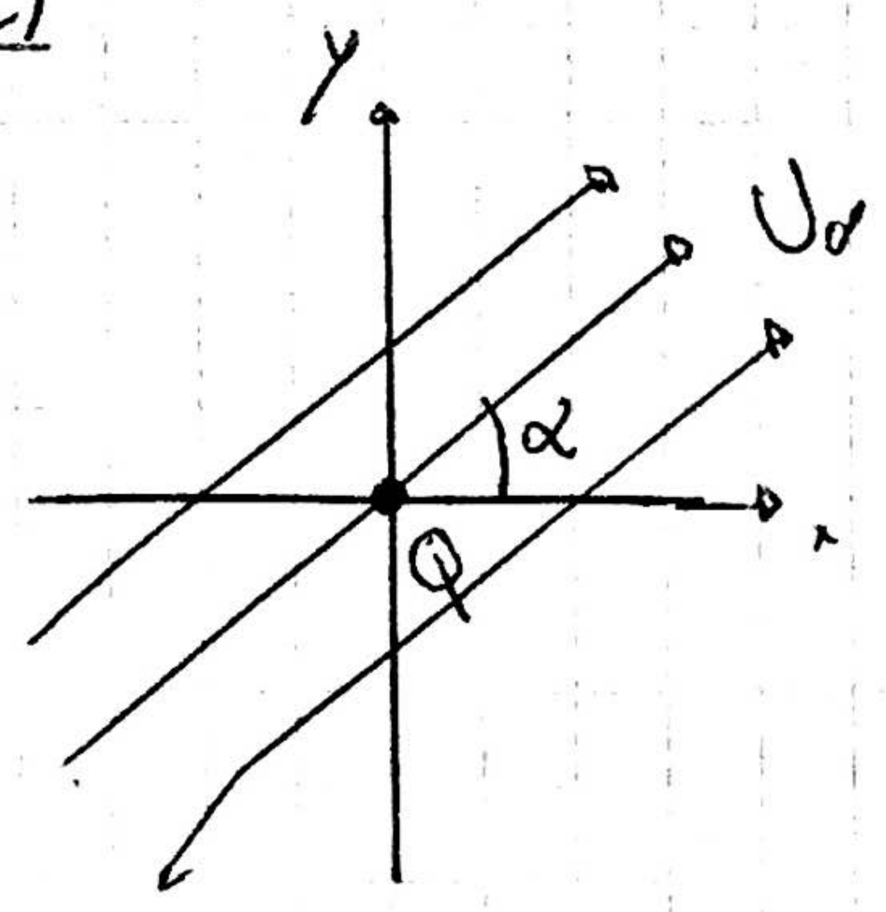
$$u_y=-\partial_x\psi=-6dxy$$
$$u_x=\partial_y\psi=3dx^2-3dy^2$$

$$\Rightarrow \underline{u}=3d(x^2-y^2)\hat{x}-6dxy\hat{y}$$



$$\text{p.d.e}=\begin{cases}x^2=y^2\\ x,y=0\end{cases} \Leftrightarrow x,y=0$$

6.2)



$$\psi = \psi_{U_\infty} + \psi_Q$$

$$\underline{u}_Q = \frac{Q}{2\pi r}\hat{r} \Rightarrow \frac{1}{r}\partial_\theta \psi \hat{r} - \partial_r \psi \hat{\theta} = \underline{u}$$

$$\Rightarrow \psi = \frac{Q\theta}{2\pi}$$

$$u_\infty = U_\infty \cos(\alpha)\hat{x} + U_\infty \sin(\alpha)\hat{y}$$

$$\Rightarrow \partial_y \psi = u_x$$
$$-\partial_x \psi = u_y \quad \Rightarrow \psi = U_\infty \cos(\alpha) y - U_\infty \sin(\alpha) x$$

$$\therefore \boxed{\psi = U_\infty \cos(\alpha) y - U_\infty \sin(\alpha) x + \frac{Q}{2\pi} \operatorname{arctg}\left(\frac{y}{x}\right)}$$

p.d.e. $\underline{u} = \frac{Q}{2\pi(x^2+y^2)}(x\hat{x}+y\hat{y}) + U_\infty \cos(\alpha)\hat{x} + U_\infty \sin(\alpha)\hat{y}$

$$\underline{u} = 0 \Rightarrow \frac{Q}{2\pi(x^2+y^2)}x + U_\infty \cos(\alpha) = 0 \Rightarrow \frac{Q}{2\pi(x^2+y^2)}x = -U_\infty \cos(\alpha)$$

$$\frac{Q}{2\pi(x^2+y^2)}y + U_\infty \sin(\alpha) = 0 \qquad \frac{Q}{2\pi(x^2+y^2)}y = -U_\infty \sin(\alpha)$$

$$\Rightarrow \frac{y}{x} = tg(\alpha) \quad \text{p.d.e.} \quad \left\{(x,y)\in\mathbb{R}^2 / y = tg(\alpha)x, \ (x,y)\neq(0,0)\right\}$$

6.4) i) $\underline{u} = U_\infty \cos(\alpha)\hat{x} + U_\infty \sin(\alpha)\hat{y}$

$$\psi = U_\infty \cos(\alpha) y - U_\infty \sin(\alpha) x$$

$\nabla \times \underline{u} = 0 \Rightarrow \exists\ \phi \quad \underline{u} = \nabla\phi$

$\phi = U_\infty \cos(\alpha) x + U_\infty \sin(\alpha) y$

$\Rightarrow W(z) = \phi + i\psi = U_\infty \cos(\alpha)(x+iy) + U_\infty \sin(\alpha)(y - ix)$

$= U_\infty \cos(\alpha) z - i U_\infty \sin(\alpha)(x+iy) \Rightarrow \boxed{W(z) = U_\infty \cos(\alpha) z - i U_\infty \sin(\alpha) z}$

$\Rightarrow W = U_\infty e^{-i\alpha} z$

ii) $\underline{u} = \frac{Q}{2\pi r}\hat{r} \qquad \psi = \frac{Q}{2\pi}\theta = \frac{Q}{2\pi}\operatorname{arctg}\left(\frac{y}{x}\right)$

$\nabla \times \underline{u} = 0 \Rightarrow \underline{u} = \nabla\phi \qquad \nabla\phi = \frac{\partial\phi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{\theta}$

$\Rightarrow \frac{Q}{2\pi r} = \frac{\partial\phi}{\partial r} \Rightarrow \phi = \frac{Q}{2\pi}\ln(r)$

$\frac{\partial\phi}{\partial\theta} = 0 \Rightarrow \phi = \phi(r) \qquad \therefore W = \phi + i\psi = \frac{Q}{2\pi}\ln(r) + i\frac{Q}{2\pi}\theta$

$\ln(z) = \ln(|z|) + i\theta$

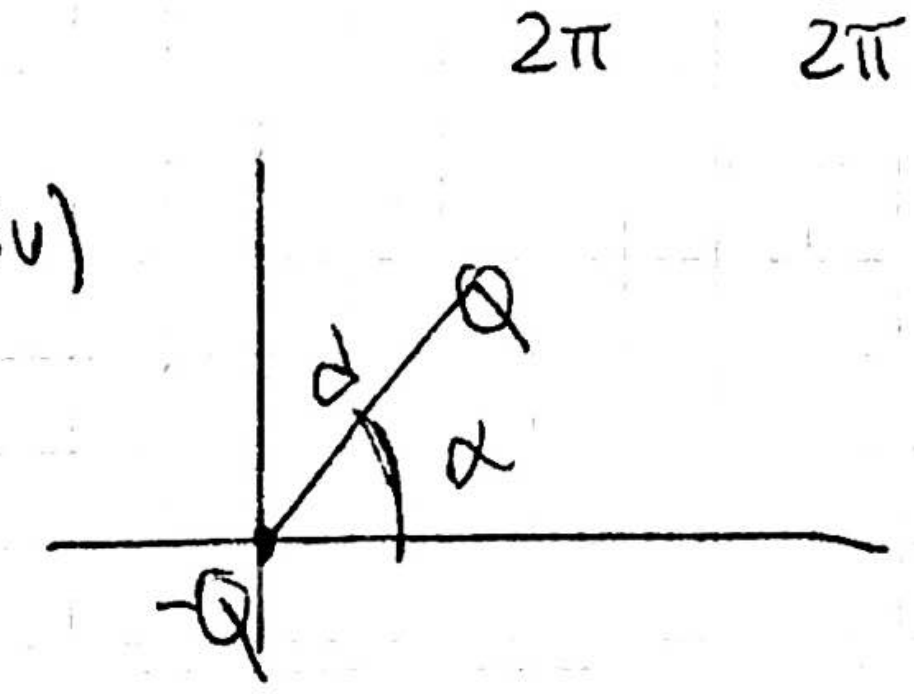
$z = x + iy \qquad r = \sqrt{x^2+y^2} = |z| \Rightarrow \boxed{W(z) = \frac{Q}{2\pi}\ln(z)}$

iii) $\underline{u} = \frac{\Gamma}{2\pi r}\hat{\theta} \qquad \psi = -\frac{\Gamma}{2\pi}\ln(r)$

$\phi / \ \nabla\phi = \underline{u} \Rightarrow \frac{\partial\phi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\phi}{\partial\theta}\hat{\theta} = \frac{\Gamma}{2\pi r}\hat{\theta} \Rightarrow \phi = \frac{\Gamma}{2\pi}\theta$

$\therefore W = \frac{\Gamma}{2\pi}\theta - i\frac{\Gamma}{2\pi}\ln(r) = \frac{i}{i}\frac{\Gamma}{2\pi}\theta + \frac{\Gamma}{2\pi i}\ln(r) = \boxed{\frac{\Gamma}{2\pi i}\ln(z) = W(z)}$

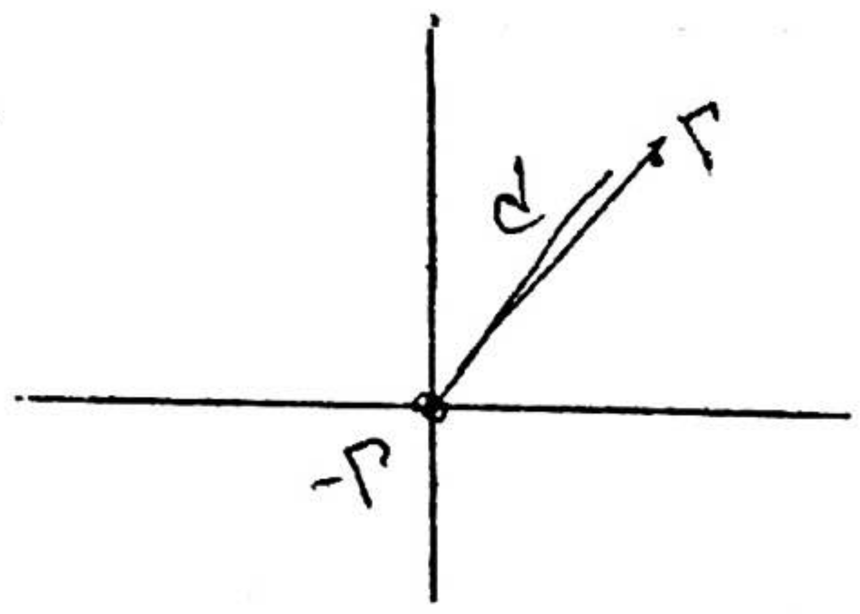
iv)



$\lim_{\substack{d\to 0\\ Q\to\infty}} Qd = \mu_a$

$$\boxed{W(z) = -\frac{\mu_a}{2\pi z}}$$

v)



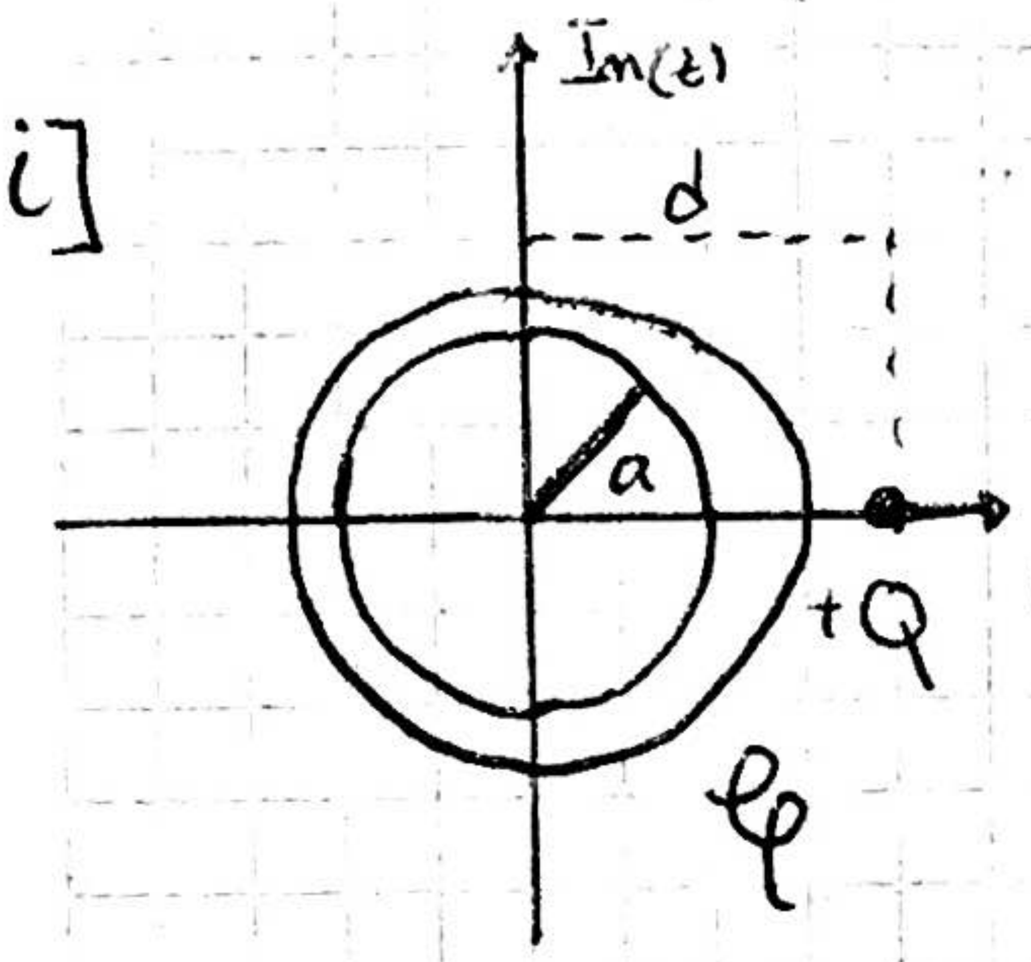
$$\lim_{\substack{d \to 0 \\ \Gamma \to \infty}} \Gamma d = \mu_\Gamma$$

$$\boxed{W = -\frac{\mu_\Gamma}{2\pi i z} e^{i\alpha}}$$

6.7) fluídos ideales, incompresibles, irrotacionales

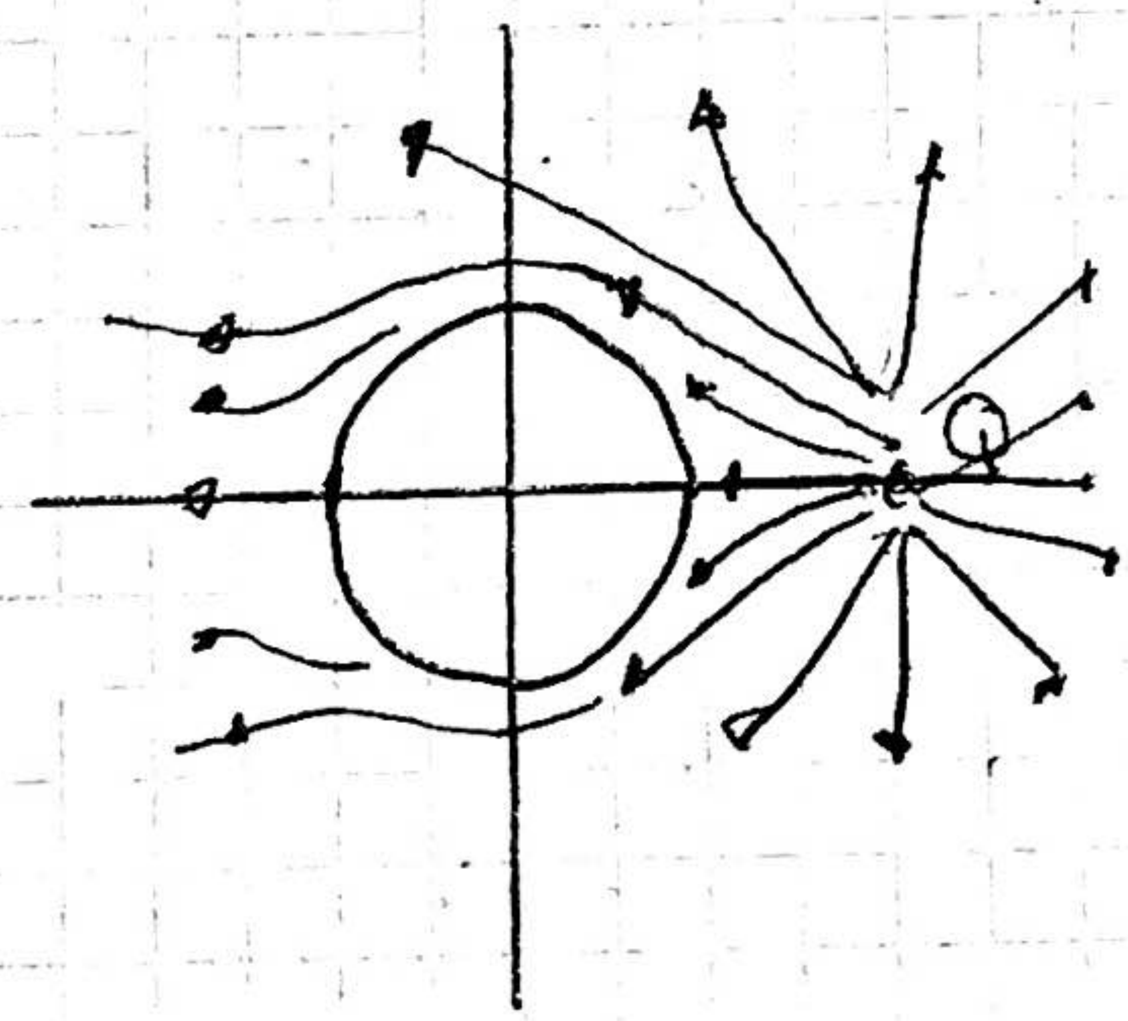
a) líneas de corriente b) W c) p.d.e d) $p(\mathcal{C})$ puntos del contorno sólido

i]



$$W = \underset{\sim}{W_0} + \overline{W_0\left(a^2/\bar{z}\right)}$$

pot $\in \mathbb{C}$ cuyas sing están fuera del círculo



$$W = \frac{Q}{2\pi}\ln(z-d) + \frac{Q}{2\pi}\ln\left(\frac{a^2}{z}-d\right)$$

$$\frac{a^2-dz}{z} = \frac{d}{z}\left(\frac{a^2}{d}-z\right)$$

$$\Rightarrow W = \frac{Q}{2\pi}\ln(z-d) + \frac{Q}{2\pi}\left[\ln(+d) - \ln(z) + \ln\left(\frac{a^2}{d}-z\right)\right]$$

$$\boxed{W = \frac{Q}{2\pi}\ln(z-d) + \frac{Q}{2\pi}\ln\left(z - \frac{a^2}{d}\right) + \frac{Q}{2\pi}\left[\ln(+d) - \ln(z)\right]}$$

c) p.d.e $\quad \frac{dW}{dz} = 0 \quad \Rightarrow \frac{dW}{dz} = \frac{Q}{2\pi(z-d)} + \frac{Q}{2\pi(z-a^2/d)} - \frac{Q}{2\pi z} = 0$

$$\frac{(z-a^2/d)+(z-d)}{(z-d)(z-a^2/d)} - \frac{1}{z} = 2z^2 - z(a^2/d - d) \ldots$$

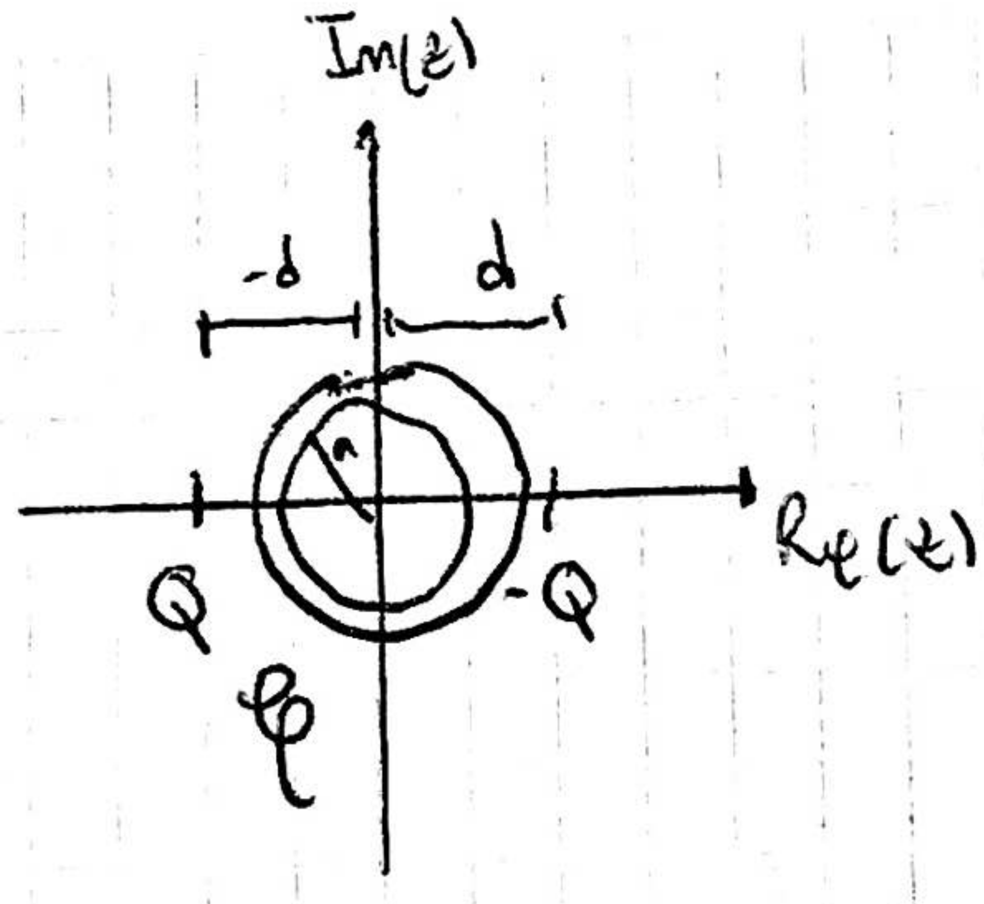
Wolfram dice $z=a$, $z=-a$

d) $p = -\rho\frac{v^2}{2} + C = -\rho\cdot\frac{1}{2}\left|\frac{dW}{dz}\right|^2 + K$ en el contorno sólido $z = ae^{i\theta}$ $\theta\in[0,2\pi)$

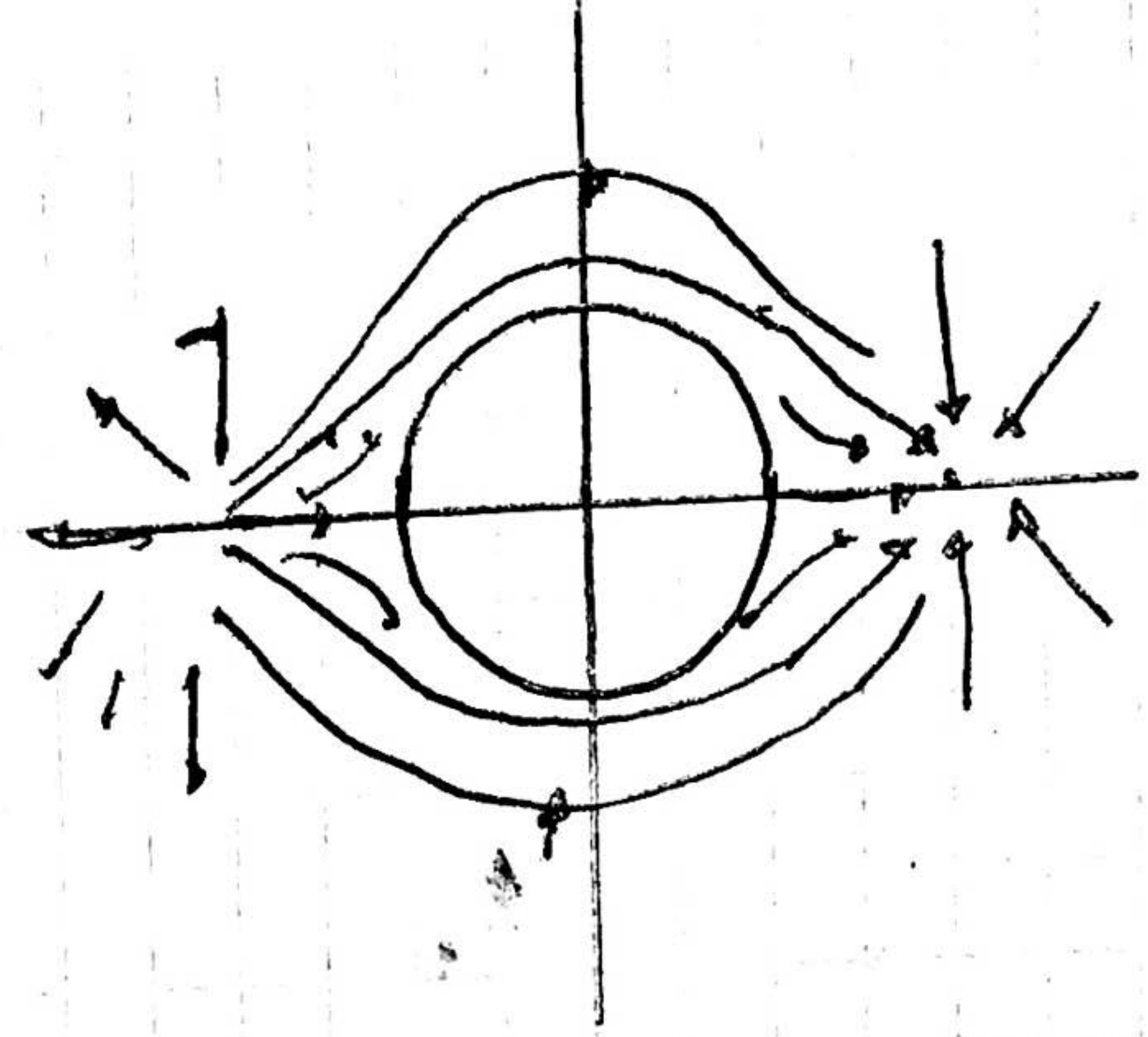
$$\Rightarrow p = -\frac{\rho Q^2}{2\,4\pi^2}\left[\frac{1}{ae^{i\theta}-d} + \frac{1}{ae^{i\theta}-a^2/d} - \frac{1}{ae^{i\theta}}\right]^2 + K$$

Graficar con wolfram...

ii]



a)



b) $$W(z) = \frac{Q}{2\pi}\ln(z+d) - \frac{Q}{2\pi}\ln(z-d) + \frac{Q}{2\pi}\ln\left(\frac{a^2}{z}+d\right) - \frac{Q}{2\pi}\ln\left(\frac{a^2}{z}-d\right)$$

$$\frac{a^2}{z}+d = \frac{a^2+dz}{z} = \frac{d}{z}\left(\frac{a^2}{d}+z\right)$$

$$\frac{a^2}{z}-d = +\frac{d}{z}\left(z+\frac{a^2}{d}\right)$$

$$\Rightarrow W(z) = \frac{Q}{2\pi}\left[\ln(z+d)-\ln(z-d)\right] + \frac{Q}{2\pi}\left[\ln(d)-\ln(z)+\ln\left(z+\frac{a^2}{d}\right)\right] - \frac{Q}{2\pi}\left[\ln(+d)-\ln(z)+\ln\left(z+\frac{a^2}{d}\right)\right]$$

$$\Rightarrow \boxed{W(z) = \frac{Q}{2\pi}\left[\ln(z+d)-\ln(z-d)\right] + \frac{Q}{2\pi}\left[\ln\left(z+\frac{a^2}{d}\right)-\ln\left(\frac{a^2}{d}-z\right)\right]}$$

p.d.e $$\frac{\partial W}{\partial z} = 0 = \frac{Q}{2\pi}\left[\frac{1}{z+d}-\frac{1}{z-d}\right] + \frac{Q}{2\pi}\left[\frac{1}{z+a^2/d}-\frac{1}{z-a^2/d}\right] = 0$$

$$\Leftrightarrow \frac{z-d-(\cancel{z}+d)}{(z^2-d^2)} + \frac{z-a^2/d-(z+a^2/d)}{(z^2-a^4/d^2)} = 0$$

$$\Leftrightarrow \frac{-2d}{(z^2-d^2)} - \frac{2a^2/d}{(z^2-a^4/d^2)} = 0 \Leftrightarrow d^2\left(z^2-a^4/d^2\right) = -a^2(z^2-d^2)$$

$$\Leftrightarrow z^2(d^2+a^2) = a^2d^2+a^4$$

$$\Leftrightarrow z^2 = \frac{a^2d^2+a^4}{d^2+a^2} = \frac{(d^2+a^2)a^2}{d^2+a^2}$$

$$\Leftrightarrow \boxed{z = \pm a}$$

$$p = -\frac{\rho}{2}\left|\frac{\partial W}{\partial z}\right|^2 + K$$

Pero obviamente los únicos con significado físico para este problema serán

$$\boxed{z=a \quad , \quad z=-a}$$

1d)

$$P = -\rho\frac{v^2}{2} + K = -\frac{\rho}{2}\left|\frac{dW}{dz}\right|^2 + K \; ; \text{ pero evaluado en } z = a\,e^{i\theta}$$

$$\left.\frac{dW}{dz}\right|_{z=ae^{i\theta}} = \frac{Q}{2\pi}\left[\frac{ae^{-i\theta}-d}{\underbrace{|ae^{i\theta}-d|^2}_{a^2\left|e^{i\theta}-\frac{d}{a}\right|^2}} + \frac{ae^{-i\theta}-a^2/d}{\underbrace{\left|ae^{i\theta}-\frac{a^2}{d}\right|^2}_{a^2\left|e^{i\theta}-\frac{a}{d}\right|^2}} - \frac{\lambda e^{-i\theta}}{a^2}\right]$$

$$(\cos\theta - \#)^2 + \text{sen}^2\theta$$
$$1 - 2\#\cos\theta + \#^2$$

$$\left(\frac{e^{-i\theta}-d/a}{\underbrace{a\left(1-2\frac{d}{a}\cos\theta+\frac{d^2}{a^2}\right)}_{a\cdot\frac{d}{a}\left(\frac{a}{d}-2\cos\theta+\frac{d}{a}\right)}} + \frac{e^{-i\theta}-a/d}{\underbrace{a\left(1-2\frac{a}{d}\cos\theta+\frac{a^2}{d^2}\right)}_{a\cdot\frac{a}{d}\left(\frac{d}{a}-2\cos\theta+\frac{a}{d}\right)}} - \frac{e^{-i\theta}}{a}\right)$$

$$= \frac{\cancel{\frac{a}{d}e^{-i\theta}} - \frac{1}{a}\cdot a \quad \cancel{+\frac{d}{a}e^{-i\theta}} - \frac{a}{d}\cdot\frac{d}{a} \quad \cancel{-e^{-i\theta}\frac{a}{d}} + 2\cos\theta\, e^{-i\theta} - \cancel{\frac{d}{a}e^{-i\theta}}}{\left(\frac{a}{d}-2\cos\theta+\frac{d}{a}\right)a}$$

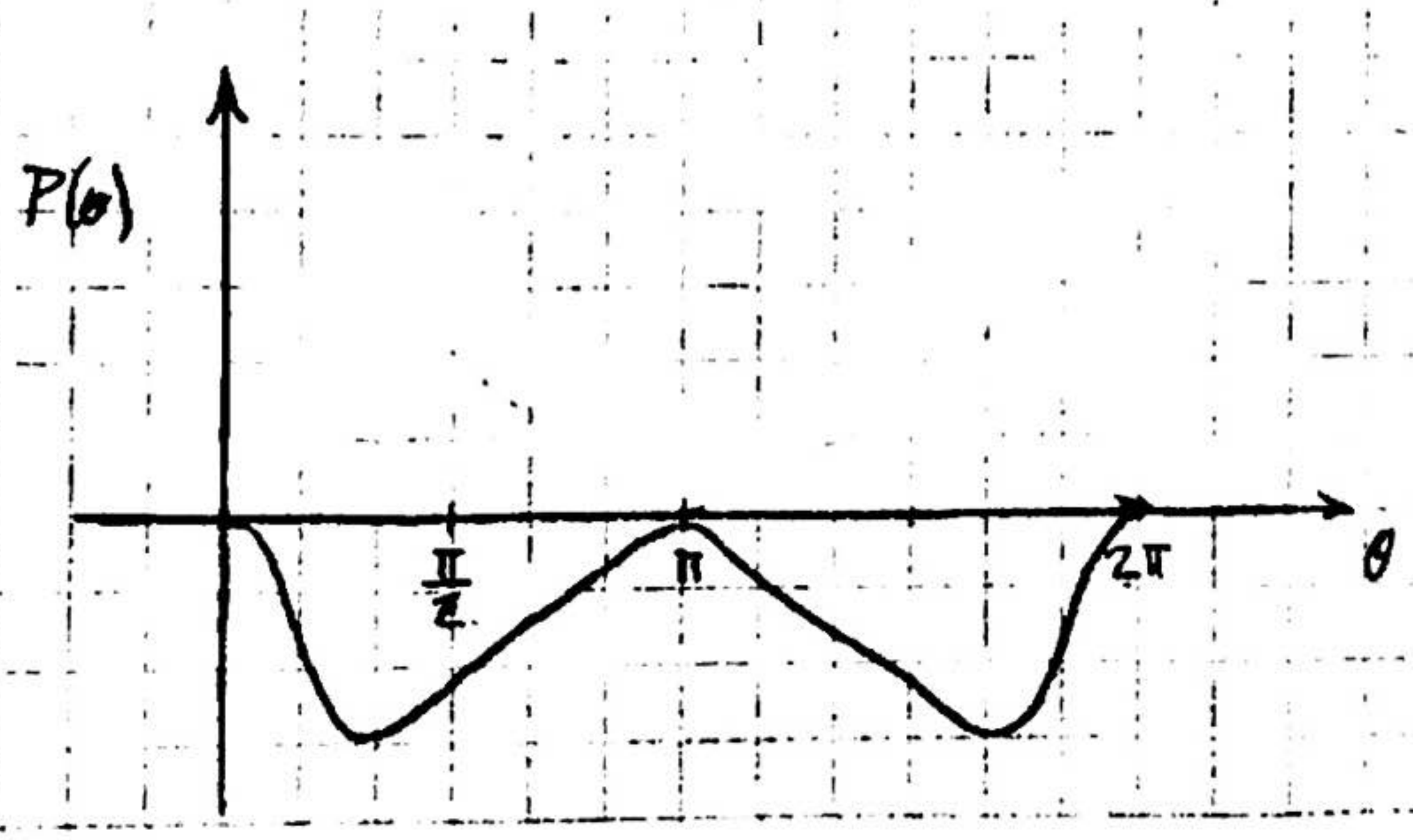
$$\frac{-2\left(1-\cos\theta\, e^{-i\theta}\right)}{} = \frac{-2\left(1-\cos^2\theta + i\cos\theta\,\text{sen}\theta\right)}{a\left(\frac{a}{d}-2\cos\theta+\frac{d}{a}\right)} = \frac{\left(\overbrace{1-2\cos^2\theta+\cos^4\theta}^{-\cos(2\theta)} + \overbrace{\cos^2\theta\cdot\text{sen}^2\theta}^{\frac{\text{sen}^2(2\theta)}{4}}\right)}{\left|-\frac{a}{2}\right|\left(\qquad\right)^2}$$

$$\left|\left.\frac{dW}{dz}\right|_{z=ae^{i\theta}}\right|^2 = \frac{}{\frac{a^2}{4}\left(\frac{a}{d}-2\cos\theta+\frac{d}{a}\right)^2} \qquad = \left|\frac{2/a}{\left(\frac{a}{d}-2\cos\theta+\frac{d}{a}\right)}\cdot\left(-\text{sen}\,\theta - i\,\text{sen}\theta\cdot\cos\theta\right)\right|^2$$

$$\left(\frac{+2/a\cdot\text{sen}\,\theta}{\left|\frac{a}{d}-2\cos\theta+\frac{d}{a}\right|}\right)^2 \underbrace{\left|\left[1+i\cos\theta\right]\right|^2}_{(1+\cos^2\theta)}$$

$$\frac{(4/a^2)\cdot\text{sen}^2\theta\cdot(1+\cos^2\theta)}{\left(\frac{a}{d}+\frac{d}{a}-2\cos\theta\right)^2}$$

Presión sobre el contorno sólido $\longrightarrow$ $$P(\theta) = -\frac{\rho}{2}\cdot\frac{4}{a^2}\cdot\frac{\text{sen}^2\theta\cdot(1+\cos^2\theta)}{\left(\frac{a}{d}+\frac{d}{a}-2\cos\theta\right)^2} + K$$



Podríamos hacer $P(\theta)$ positiva sumándole la constante K

$$z^2d^2 - a^4 = -z^2a^2 + a^2d^2$$
$$z^2(d^2+a^2) = +a^2d^2 + a^4$$
$$z^2 = \frac{a^2(\cancel{d^2+a^2})}{\cancel{a^2+d^2}} \rightarrow z^2 = a^2$$
$$z = a.e^{ik\pi} \quad k=0,1 \rightarrow$$

Puntos de estancamiento $\boxed{z = \pm a}$

2d)

$$p = -\frac{\rho}{2}\left|\frac{dW}{dz}\right|^2 + K \quad \text{evaluando en } z = a.e^{i\theta}$$

$$p(\theta) = -\frac{\rho}{2}\left|\frac{Q}{\cancel{2}\pi}\left(\frac{-\cancel{2}d}{a^2e^{i2\theta}-d^2} + \frac{-\cancel{2}a^2/d}{a^2e^{i2\theta}-\frac{a^4}{d^2}}\right)\right|^2$$

$$-\frac{\rho}{2}\cdot\frac{Q^2}{\pi^2}\left|\frac{-d}{a^2\left(e^{i2\theta}-\frac{d^2}{a^2}\right)} - \frac{a^2/d}{a^2\left(e^{i2\theta}-\frac{a^2}{d^2}\right)}\right|^2$$

$$p(\theta) = -\frac{\rho}{2}\frac{Q^2}{\pi^2}\left(\frac{1}{a^2}\right)^2\left|\underbrace{-\frac{d}{\left(e^{i2\theta}-\frac{d^2}{a^2}\right)} - \frac{a^2}{d\left(e^{i2\theta}-\frac{a^2}{d^2}\right)}}_{\equiv u}\right|^2$$

$$u \equiv -\frac{d\left(e^{-i2\theta}-d^2/a^2\right)}{\left|e^{i2\theta}-\frac{d^2}{a^2}\right|^2} - \frac{a^2\left(e^{-i2\theta}-a^2/d^2\right)}{d\left|e^{i2\theta}-\frac{a^2}{d^2}\right|^2} = -\frac{d(\qquad)}{\frac{d^2}{a^2}\left(\frac{a^2}{d^2}-2\cos(2\theta)+\frac{d^2}{a^2}\right)} + \frac{\frac{a^2}{d}(\qquad)}{\frac{a^2}{d^2}\left(\frac{d^2}{a^2}-2\cos(2\theta)+\frac{a^2}{d^2}\right)}$$

$$\left(\cos(2\theta)-\frac{d^2}{a^2}\right)^2 + \operatorname{sen}^2(2\theta)$$
$$1 - 2\frac{d^2}{a^2}\cos(2\theta) + \frac{d^4}{a^4}$$

$$= -\frac{\frac{\cancel{d}a^2}{d^{\cancel{2}}}\left(e^{-i2\theta}-d^2/a^2\right) + \frac{\cancel{d}d^{\cancel{2}}}{\cancel{d}\,\cancel{d}}\left(e^{-i2\theta}-a^2/d^2\right)}{\left(\frac{a^2}{d^2}-2\cos(2\theta)+\frac{d^2}{a^2}\right)}$$

$$= \frac{\frac{a^2}{d}e^{-i2\theta} - d + d.e^{-i2\theta} - \frac{a^2}{d}}{}$$

$$= \frac{\left(e^{-i2\theta}-1\right)\left(\frac{a^2}{d}-d\right)}{\left(\frac{a^2}{d^2}-2\cos(2\theta)+\frac{d^2}{a^2}\right)} \Longrightarrow$$

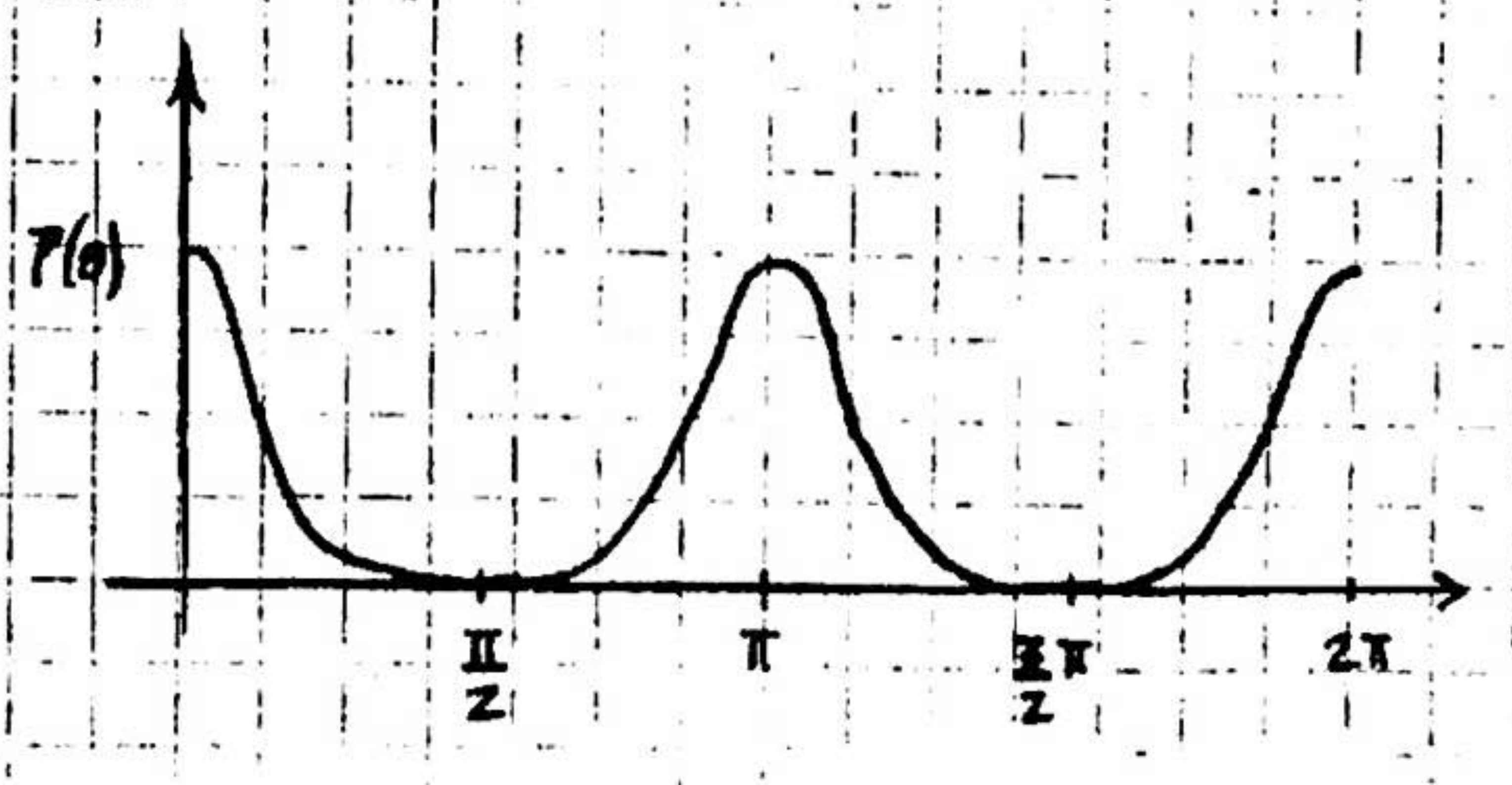
$$\cos^2(2\theta) + \operatorname{sen}^2(2\theta) + 2\cos(2\theta) + 1$$

$$p(\theta) = -\frac{\rho}{2}\frac{Q^2}{\pi^2a^4}\frac{(a^2-d^2)^2}{d^2}\frac{1}{\left(\frac{a^2}{d^2}-2\cos(2\theta)+\frac{d^2}{a^2}\right)^2}\left(\left(\cos(2\theta)+1\right)^2+\operatorname{sen}^2(2\theta)\right)$$

$$p(\theta) = -\frac{\rho}{\cancel{2}}\cdot\frac{Q^2}{\pi^2}\,(a^2$$

$$p(\theta) = -\frac{\rho Q^2}{\pi^2}.$$

Podemos graficar esta función



6.10)

i) $W = \frac{Q}{2\pi}\ln(z-d) + \frac{Q}{2\pi}\ln\left(-z+a^2/d\right) + \frac{Q}{2\pi}\left[\ln(d) - \ln(z)\right]$

$$F_x - iF_y = \frac{\rho i}{2}\oint_{\mathcal{C}}\left(\frac{\partial W}{\partial z}\right)^2 dz \underset{\text{teo residuo}}{=} \frac{\rho i}{2}(2\pi i)\sum_k^n \mathrm{Res}\left\{\left(\frac{\partial W}{\partial z}\right)^2, z_k\right\}$$ interiores a $\mathcal{C}$

$\Rightarrow \frac{\partial W}{\partial z} = \frac{Q}{2\pi}\left[\frac{1}{z-d} + \frac{1}{z-a^2/d} - \frac{1}{z}\right]$ $z=d$ no es interior

a^2/d y 0 sí

$$\Rightarrow \left(\frac{\partial W}{\partial z}\right)^2 = \frac{Q^2}{4\pi^2}\Bigg[\underbrace{\frac{1}{(z-d)^2}}_{\text{externo}} + \underbrace{\frac{1}{(z-a^2/d)^2}}_{\text{polo de orden 2}} + \frac{1}{z^2} \underbrace{- \frac{2}{(z-d)z}}_{\text{polo de orden 1 en 0}} \underbrace{- \frac{2}{(z-a^2/d)z}}_{\text{polo de orden 1 en 0 y } a^2/d} \underbrace{+ \frac{2}{(z-d)(z-a^2/d)}}_{\text{polo de orden 1 en } a^2/d}\Bigg]$$

Res = 0 pues = análogo es 1/z
el 1/z
(tipo del de Laurent)

$$\Rightarrow \mathrm{Res}\left\{\left(\frac{\partial W}{\partial z}\right)^2, z_k\right\}_{int} = \frac{Q^2}{4\pi^2}\left[\mathrm{Res}\left(\frac{-2}{(z-d)\cdot z}, 0\right) + \mathrm{Res}\left(\frac{-2}{(z-a^2/d)z}, 0, a^2/d\right) + \mathrm{Res}\left(\frac{2}{(z-d)(z-a^2/d)}, a^2/d\right)\right]$$

a ojo: $\frac{2}{d}$; a ojo: $\frac{2d}{a^2}$; a ojo: $-\frac{2d}{a^2}$; a ojo: $\frac{2}{\frac{a^2}{d}-d}$

$$\Rightarrow F_x - iF_y = -\pi\rho\frac{Q^2}{4\pi^2}\left[\frac{2}{d} + \frac{2d}{a^2} - \frac{2d}{a^2} + \frac{2d}{a^2-d^2}\right]$$

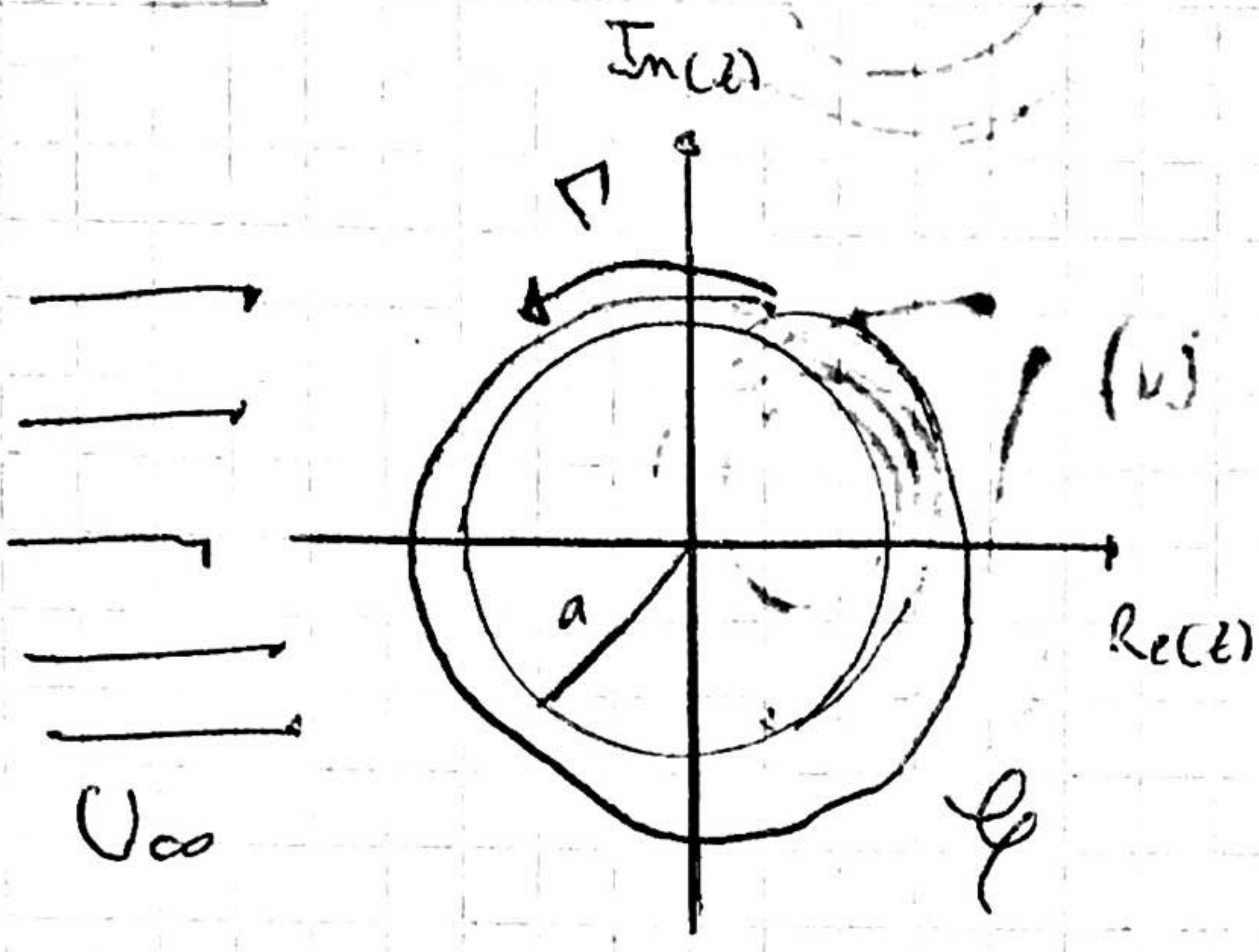
$$\Rightarrow F_x = -\frac{\rho Q^2}{4\pi}\,\frac{2(a^2-d^2)+2d^2}{d(a^2-d^2)} = \boxed{\frac{-\rho Q^2 a^2}{2\pi d(a^2-d^2)} = F_x}$$

ii) $W = \frac{Q}{2\pi}\left[\ln(z+d) - \ln(z-d)\right] + \frac{Q}{2\pi}\left[\ln\left(z+\frac{a^2}{d}\right) - \ln\left(\frac{a^2}{d} - z\right)\right]$

$$\frac{\partial W}{\partial z} = \frac{Q}{2\pi}\left[\frac{1}{z+d} - \frac{1}{z-d} + \frac{1}{z+a^2/d} + \frac{1}{z-a^2/d}\right]$$

fiaca...

Problema 8



$$W_{circ} = U_\infty e^{-i\alpha} z \quad \alpha = 0 \text{ aquí}$$

$$\Rightarrow W(z) = W_{circ}(z) + \overline{W_{circ}\left(\overline{\frac{a^2}{z}}\right)} + W_\Gamma$$

$$= U_\infty z + \frac{U_\infty a^2}{z} - \frac{i\Gamma}{2\pi}\ln(z)$$

$$\Rightarrow W(z) = U_\infty \cancel{z}\left(1 + \frac{a^2}{z^2}\right) - \frac{i\Gamma}{2\pi}\ln(z)$$

p.d.e $\Leftrightarrow \dfrac{dW}{dz} = U_\infty - \dfrac{U_\infty a^2}{z^2} - \dfrac{i\Gamma}{2\pi z} = 0$

$$\Leftrightarrow U_\infty z^2 - \frac{i\Gamma}{2\pi} z - U_\infty a^2 = 0 \qquad z \neq 0$$

$$z = \frac{\frac{i\Gamma}{2\pi} \pm \sqrt{\frac{-\Gamma^2}{4\pi^2} + 4U_\infty^2 a^2}}{2U_\infty}$$

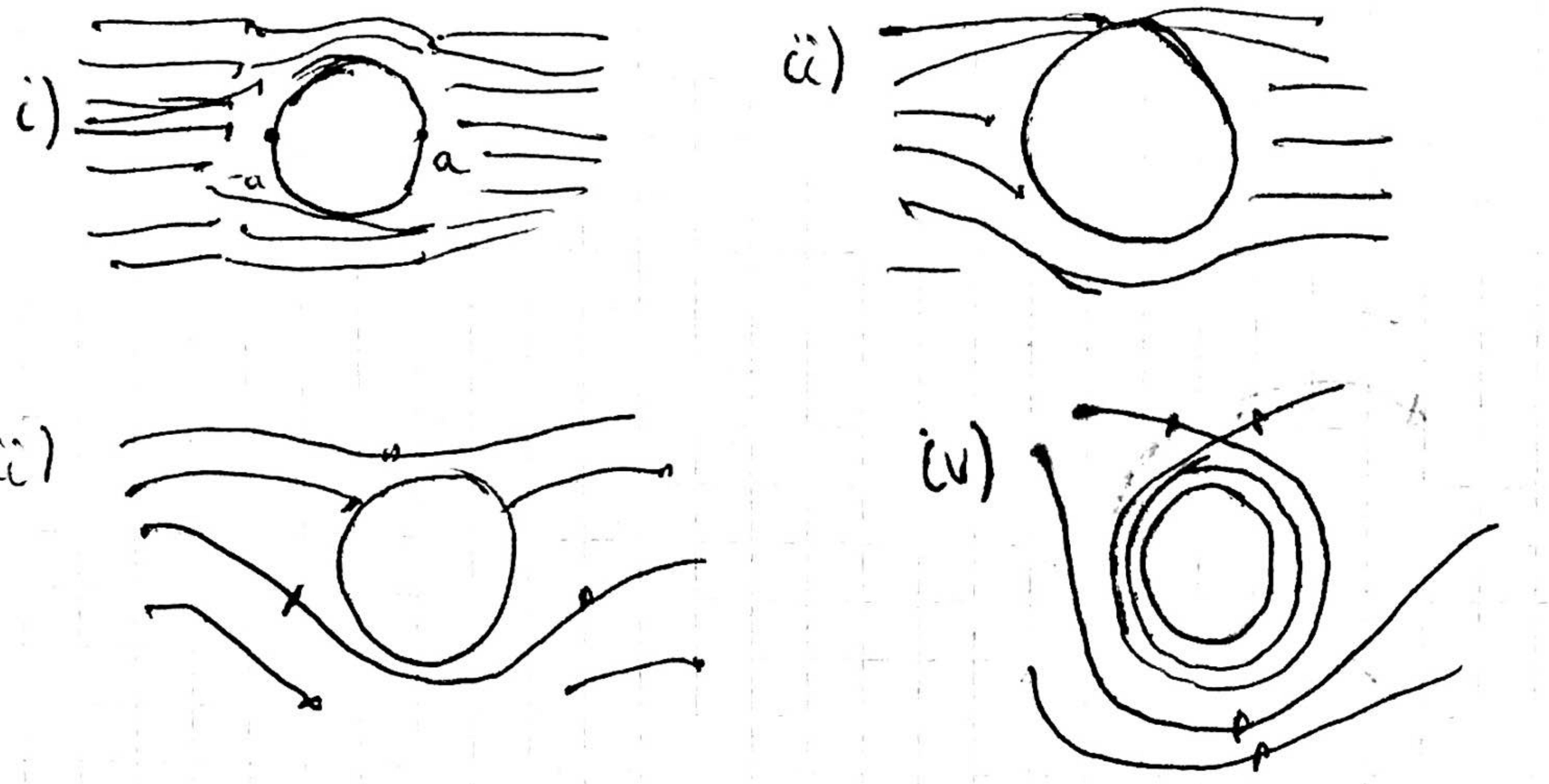
$$\Rightarrow z = \frac{i\Gamma}{4\pi U_\infty} \pm \sqrt{\frac{-\Gamma^2}{16\pi^4 U_\infty^2} + a^2}$$

i) • $\Gamma = 0 \Rightarrow z = \pm a$ puntos de estancamiento

ii) • $\dfrac{-\Gamma^2}{16\pi^4 U_\infty^2} + a^2 = 0 \Leftrightarrow \Gamma^2 = 16\pi^2 U_\infty^2 a^2 \Rightarrow z = ia$ único punto de estancamiento

iii) • $\dfrac{\Gamma^2}{16\pi^2 U_\infty^2} > a^2 \Rightarrow z = \dfrac{i\Gamma}{4\pi U_\infty} \pm i\sqrt{-a^2 + \dfrac{\Gamma^2}{16\pi^4 U_\infty^2}}$ dos p.d.e sobre el eje Im

iv) • $\dfrac{\Gamma^2}{16\pi^2 U_\infty^2} < a^2 \Rightarrow z = \dfrac{i\Gamma}{4\pi U_\infty} \pm \sqrt{a^2 - \dfrac{\Gamma^2}{16\pi^4 U_\infty^2}}$ dos puntos dee al costado del eje Im



c) $F_x - iF_y = \frac{i\rho}{2} \oint_{\mathcal{C}} \left(\frac{\partial W}{\partial z}\right)^2 dz = \frac{i\rho}{2} \oint_{\mathcal{C}} \underbrace{\left[U_\infty - \frac{U_\infty a^2}{z^2} - \frac{i\Gamma}{2\pi z}\right]^2}_{f(z)} dz$

$$= \frac{i\rho}{2} \oint_{\mathcal{C}} \left[U_\infty^2 + \underbrace{\frac{U_\infty^2 a^4}{z^4}}_{\text{polo orden 4}} - \underbrace{\frac{\Gamma^2}{4\pi^2 z^2}}_{\text{polo orden 2}} + \underbrace{\frac{2U_\infty^2 a^2}{z^2}}_{\text{polo orden 2}} + \underbrace{\frac{2U_\infty i\Gamma}{2\pi z}}_{\text{polo orden 1}} - \underbrace{\frac{2U_\infty a^2}{2\pi z^3}}_{\text{polo orden 3}}\right] dz$$

$$\underset{\text{Residuos}}{=} \frac{i\rho}{2} \cdot (2\pi i) \sum_{k=1}^{n} \text{Res}\left\{f(z), z_k\right\}$$ sing interior al contorno

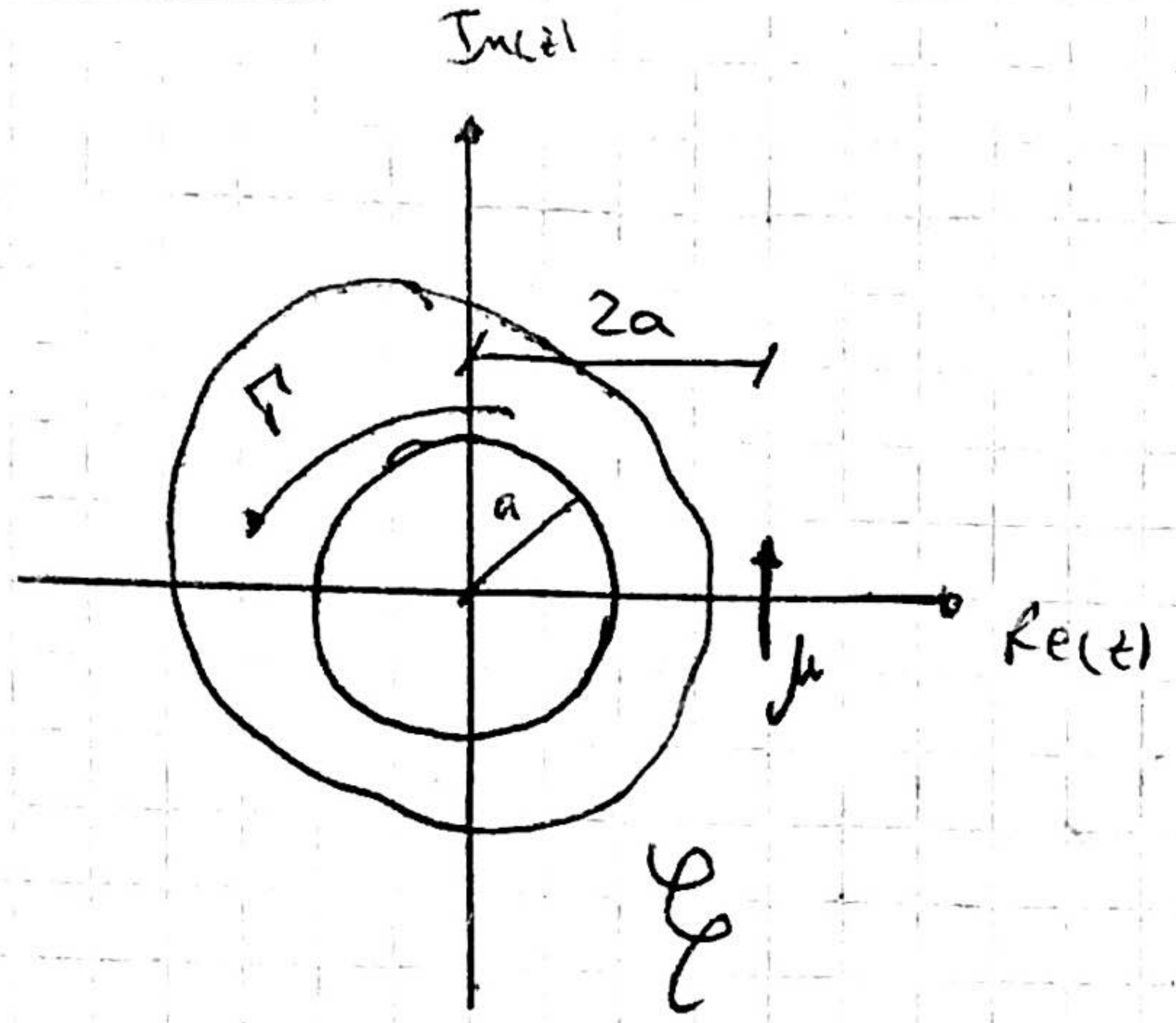
sing $z=0$ interior al contorno

Solo tengo residuo para el polo de orden 1

y vale simplemente $-\frac{U_\infty i\Gamma}{\pi}$ $\Rightarrow F_x + iF_y = \frac{i\rho}{2} 2\pi i \left(-\frac{U_\infty i\Gamma}{\pi}\right)$

$\Rightarrow F_x - iF_y = \rho i U_\infty \Gamma \Rightarrow \boxed{F_y = -\rho U_\infty \Gamma}$

Problema 6.9



Fluido incompresible irrotacional de densidad ρ. μ_0

Hallar μ_0 / $\underline{F}$ sobre cil = 0

$$W = \frac{\mu_0 e^{i\alpha}}{2\pi(z-2a)} + \frac{\mu_0 e^{-i\alpha}}{2\pi(\frac{a^2}{z}-2a)} - \frac{i\Gamma}{2\pi}\ln(z) \qquad \text{si } \alpha = \frac{\pi}{2}$$

$$\Rightarrow W(z) = \frac{\mu_0 i}{2\pi(z-2a)} - \frac{\mu_0 i}{2\pi(\frac{a^2}{z}-2a)} - \frac{i\Gamma}{2\pi}\ln(z)$$

$$F_x - iF_y = \frac{i\rho}{2}\oint_{\mathcal{C}}\left(\frac{\partial W}{\partial z}\right)^2 dz \underset{\text{res}}{=} \frac{i\rho}{2}(2\pi i)\sum_{k}^{n} \text{Res}\left\{\left(\frac{\partial W}{\partial z}\right)^2, z_k\right\}$$ z sing interiores a $\mathcal{C}$

$$\frac{\partial W}{\partial z} = -\frac{\mu_0 i}{2\pi(z-2a)^2} - \frac{\mu_0 i}{2\pi(\frac{a^2}{z}-2a)^2}\frac{a^2}{z^2} - \frac{i\Gamma}{2\pi z}$$

$$A^2+B^2+C^2+2AB+2AC+2BC = (A+B+C)^2$$

$$\Rightarrow F_x - iF_y = \frac{\rho i}{2}\oint_{\mathcal{C}}\Bigg[\underbrace{\frac{-\mu_0^2}{4\pi^2(z-2a)^4}}_{1} - \frac{\mu_0^2}{4\pi^2\underbrace{(\frac{a^2}{z}-2a)^4 z^4}_{(a^2-2az)^4}}a^4 - \overbrace{\frac{\Gamma^2}{4\pi^2 z^2}}^{3} \overbrace{-\frac{2\mu_0^2 a^2}{4\pi^2(z-2a)^2\underbrace{(\frac{a^2}{z}-2a)^2 z^2}_{(a^2-2az)^2}}}^{g(z)}$$

$$\overbrace{-\frac{\mu_0\Gamma}{4\pi^2(z-2a)^2 z}}^{f(z)} - \overbrace{\frac{\mu_0 a^2\Gamma}{4\pi^2\underbrace{(\frac{a^2}{z}-2a)^2 z^3}_{(a^2-2az)^2 z}}}^{h(z)}\Bigg]dz$$

Singularidades
- $z = 2a$ → exterior
- $z = 0$ → interior
- $z = a/2$ → interior

(1) exterior

(2) polo de orden 4 con residuo nulo pues es su propio desarrollo de Laurent

(3) polo de orden 2 también residuo nulo es su propio desarrollo de Laurent

Sólo me queda $\text{Res}(t(z), 0)$ polo orden 1

$\text{Res}(g(z), a/2)$ polo orden 2

$\text{Res}(h(z), 0)$ polo orden 1

$\text{Res}(h(z), a/2)$ polo orden 2

$$\text{Res}(t(z), 0) = \lim_{z \to 0} -\frac{\mu_0 \Gamma}{4\pi^2 (z-2a)^2} = -\frac{\mu_0 \Gamma}{16\pi^2 a^2}$$

$$\text{Res}(g(z), a/2) = \lim_{z \to a/2} \frac{d}{dz}\left(\frac{-2\mu_0^2 a^2}{4\pi^2 (z-2a)^2 (a^2 - 2az)^2} \cdot (z - a/2)^2\right)$$

$$= \lim_{z \to a/2} -\frac{2\mu_0^2 a^2}{4\pi^2} \frac{d}{dz}\left[\frac{1}{(z-2a)^2 4a^2}\right] = \lim_{z \to a/2} -\frac{\mu_0^2}{8\pi^2} \left(\frac{-2}{(z-2a)^3}\right)$$

$$= +\frac{\mu_0^2}{4\pi^2} \frac{8}{27a^3}$$

Los otros dos residuos calculados en la práctica

$$\text{Res}(h(z), 0) = 1/a^2$$

$$\text{Res}(h(z), a/2) = -1/a^2$$

$$\therefore \quad F_x - iF_y = \frac{\rho i}{2} (2\pi i) \left[-\frac{\mu_0 \Gamma}{16\pi^2 a^2} + \frac{\mu_0^2 8}{4\pi^2 27 a^3}\right]$$

$$F_x = \pi \rho \left[\frac{\mu_0 \Gamma}{16\pi^2 a^2} - \frac{\mu_0^2 8}{4\pi^2 27 a^3}\right] = 0$$

$$\Leftrightarrow \frac{\Gamma}{16} - \frac{\mu_0 8}{4 \cdot 27 a} = 0$$

$$\Leftrightarrow \boxed{\mu_0 = \frac{27 \Gamma a}{32}}$$