Ondas de choque

u_1 ; u_2
ρ_1 ; ρ_2
p_1 ; p_2
T_1 ; T_2

frente de choque

discontinuidad

$\rho_1 u_1 = \rho_2 u_2$ (conservación de la masa)

$\rho_1 u_1^2 + p_1 = \rho_2 u_2^2 + p_2$ (conservación de p)

$\frac{u_1^2}{2} + h_1 = \frac{u_2^2}{2} + h_2$ (conservación de energía)

En el sistema donde el frente de ondas está quieto

$$M_2^2 = \frac{M_1^2(\gamma-1)+2}{2\gamma M_1^2-(\gamma-1)} \qquad p_2 = p_1\left[1+\frac{2\gamma}{\gamma+1}(M_1^2-1)\right]$$

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$$

$$\frac{T_2}{T_1} = \frac{(2\gamma M_1^2-\gamma)[(\gamma-1)M_1^2+2]}{(\gamma+1)^2 M_1^2}$$

Ejercicio S.S

$u_1' = 60\,m/s$ $\quad p_1 = 1\,atm$
$T_1 = 300K$

$$M = \frac{u}{c} \qquad c^2 = \left.\frac{\partial p}{\partial \rho}\right|_s = \gamma R T$$

desde el SOC $\quad U_s > 0$

$u_1' = u_1 - U_s \qquad u_1 = u_1' + U_s$

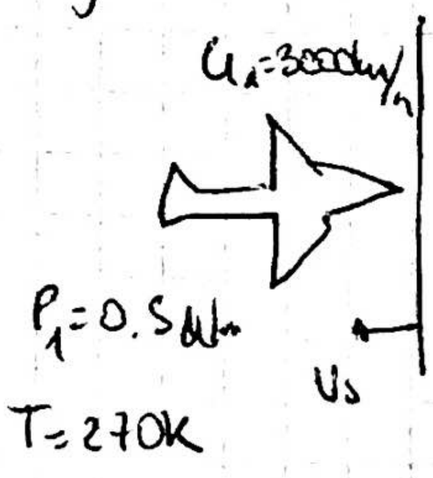
$u_2' = u_2 - U_s = 0 \Rightarrow u_2 = U_s$

$$\Rightarrow \frac{u_1'+U_s}{U_s} = \frac{(\gamma+1)\left(\frac{u_1'+U_s}{c_1}\right)^2}{2+(\gamma-1)\left(\frac{u_1'+U_s}{c_1}\right)^2} \Leftrightarrow 2(u_1'+U_s)+\frac{(\gamma-1)}{c_1^2}(u_1'+U_s)^3 = U_s\frac{(\gamma+1)}{c_1^2}(u_1'+U_s)^2$$

$$\Rightarrow \underbrace{(u_1'+U_s)}_{=0 \text{ no sirve}}\left[2 - U_s\frac{(\gamma+1)}{c_1^2}(u_1'+U_s)+\frac{(\gamma-1)}{c_1^2}(u_1'+U_s)^2\right]=0$$

$$U_s^2\frac{(\gamma-1)}{c_1^2} + \frac{2U_s u_1'(\gamma-1)}{c_1^2} + \frac{(\gamma-1)}{c_1^2}u_1'^2$$ Sale U_s de ahí.

Ejercicio 5.4



T_2, P_2 ?

$$M_1 = \frac{u_1}{c_1^2} = \frac{3000 km/h}{\sqrt{\gamma R\, 300K}}$$

$$\Rightarrow \begin{cases} u_1' = u_1 - U_s \\ u_2' = U_s \end{cases}$$

$$T_2 = 270K \cdot \frac{\left(2\gamma \frac{u_1^2}{\gamma RT} - \gamma\right)\left[(\gamma-1)\frac{u_1^2}{\gamma RT} + 2\right]}{(\gamma+1)^2 \frac{u_1^2}{\gamma RT}}$$

$c_1 = 343 m/s$

1.4

$$\Rightarrow T_2 = 270K\left(2(1.4)\frac{833.33 m/s}{343 m/s} - 1.4\right)$$

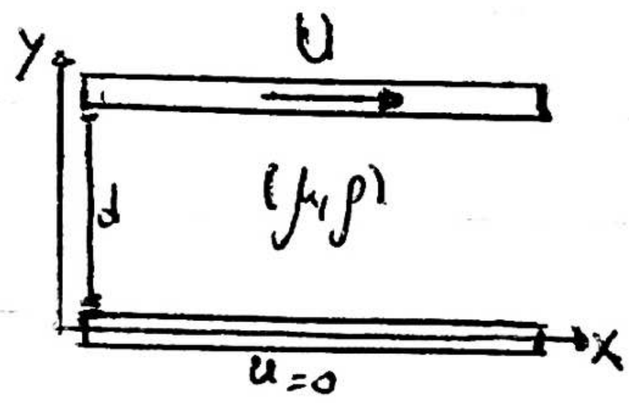
Ejercicio 6.1

Hipótesis: incompresible $\Rightarrow \underline{\nabla}\cdot\underline{u}=0$

DATOS: μ dinámico = cte

ρ = cte

$\underline{u}=\underline{u}(y)=u_x(y)\hat{x}$



a) Demuestre que en estas condiciones el término convectivo $(\underline{u}\cdot\underline{\nabla})\underline{u}\equiv 0$

$$\frac{d}{dt}(\rho\underline{u})=\underbrace{\rho\frac{\partial u}{\partial t}}_{\substack{=0\\ \text{estacionario}}}+\rho(\underline{u}\cdot\underline{\nabla})\underline{u}=\underbrace{\underline{f_v}}_{\substack{\downarrow\\ \underline{g}\\ \text{que desprecio}\\ \text{en este ej}}}+\underline{\nabla}\cdot\underline{\underline{T}}$$

$$\Rightarrow (\underline{u}\cdot\underline{\nabla})\underline{u}=\frac{1}{\rho}\underline{\nabla}\cdot\underline{\underline{T}}$$

Falta $\underline{\underline{T}}(\underline{u})$ la relación constitutiva.

$$(\underline{u}\cdot\underline{\nabla})\underline{u}=\left(u_x(y)\hat{x}\cdot\frac{\partial}{\partial x}\hat{x}\right)u_x(y)\hat{x}=0$$

$\therefore$ el término convectivo vale cero

$$\boxed{(\underline{u}\cdot\underline{\nabla})\underline{u}=0}\quad\Rightarrow\underline{\nabla}\cdot\underline{\underline{T}}=0$$

b) Halle $\underline{u}$ en los casos

$\underline{U}(y=0)=0$,

$\underline{U}(y=d)=U\hat{x}$

Para un fluído donde tengo movimiento laminar la condición de contorno es

$\underline{u}\big|_{\text{contorno}}=U_{\text{sólido}}$ (condición de adherencia)

Para un fluído newtoniano $\underline{\underline{T}}=-p\underline{\underline{I}}+2\mu\left(\underbrace{\underline{\underline{S}}}_{\substack{\downarrow\\ \underline{v}\text{ de}\\ \text{deformación}}}-\frac{1}{3}\underline{\underline{I}}(\underline{\nabla}\cdot\underline{u})\right)$

$$\therefore \underline{\nabla}\cdot\underline{\underline{T}} = 0 \Leftrightarrow \partial_i T_{ij} = -\partial_i p + \mu\Big[\underbrace{\partial_{ii}u_j}_{\underline{\nabla}^2\underline{u}} + \underbrace{\partial_{ij}u_j}_{\underline{\nabla}(\underline{\nabla}\cdot\underline{u})}\Big] = 0$$

$$\Rightarrow \mu\underline{\nabla}^2\underline{u} = \underline{\nabla}p$$

i) $\underline{\nabla}p = 0 \Rightarrow \mu\dfrac{\partial^2 u_x(y)}{\partial y^2} = 0 \Leftrightarrow u_x(y) = Ay + B$

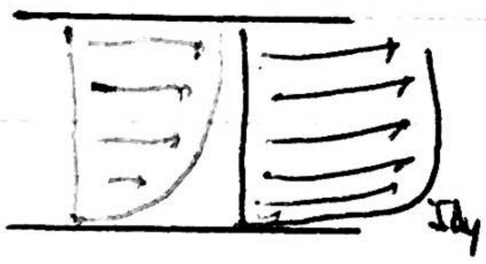
con contornos $u(0) = 0 \Rightarrow B = 0$

$u(d) = U \Rightarrow A = \dfrac{U}{d}$ $\therefore$ $\boxed{u_x(y) = \dfrac{U}{d}y}$

ii) $\underline{\nabla}p = \dfrac{\Delta p}{l} \Rightarrow u_x(y) = \dfrac{\Delta p}{\mu 2l}y^2 + By + C$

$u(0) = 0 \Leftrightarrow C = 0$

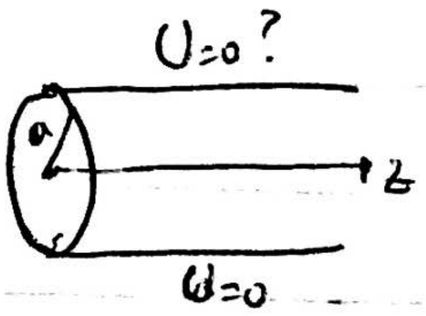
$u(d) = U \Leftrightarrow B = \dfrac{U}{d} - \dfrac{\Delta p d}{\mu 2 l}$ $\therefore$ $\boxed{u_x(y) = \left(\dfrac{U}{d} - \dfrac{\Delta p d}{\mu 2 l}\right)y + \dfrac{\Delta p}{\mu 2 l}y^2}$



caso límite $\mu \approx 0$: Tendemos el límite hacia un fluido ideal, pero no perfectamente ideal. Notamos el cambio en la capa inferior a.k.a capa límite. Esta capa límite se ubica en un Δy. Si desprecio Δy, vuelvo al fluído ideal.

$\therefore$ La condición de deslizamiento en un fluído ideal es despreciar la capa límite en uno no ideal de viscosidad muy baja.

iii) (Flujo de Poiseuille). Tubo cilíndrico de radio a

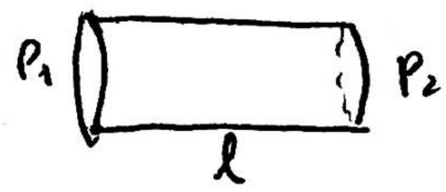


$\mu\underline{\nabla}^2 u_z($

a) $(\underline{u}\cdot\underline{\nabla})\underline{u} = 0$

$\underline{u} = u(\rho)\hat{z}$ pues tengo simetría de rotación sobre el eje $\hat{z}$

$(\underline{u}\cdot\underline{\nabla})\underline{u} = (0,0,0)$ ✓



b) $\underline{\nabla}p = \frac{\Delta p}{l}$

$\Rightarrow 0 = \underline{\nabla}\cdot\underline{\underline{T}} \Leftrightarrow \partial_i T_{ij} = -\partial_i p + \mu\left[\underbrace{\partial_{ii}u_j}_{\nabla^2\underline{u}} + \underbrace{\partial_{ij}u_j}_{\underline{\nabla}(\underbrace{\underline{\nabla}\cdot\underline{u}}_{=0})}\right] = 0$ incomp

$\Rightarrow \mu\nabla^2\underline{u} - \underline{\nabla}p = 0 \Leftrightarrow \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial u}{\partial\rho}\right) = \frac{\Delta p}{\mu l}$

$\Leftrightarrow \rho\frac{\partial u}{\partial\rho} = \frac{\Delta p}{2l\mu}\rho^2 + C \Leftrightarrow u(\rho) = \frac{\Delta p\,\rho^2}{\mu 4 l} + \underbrace{C\ln(\rho)}_{\text{esto no va}} + D$

~~bien: $\frac{1}{\rho}(\nabla\underline{u}\cdot\rho) = 0$~~

$\Rightarrow u(a) = 0 \Rightarrow D = \frac{-\Delta p\, a^2}{4\mu l}$

$$\Rightarrow \boxed{\underline{u}(\rho) = \frac{\Delta p}{4\mu l}\left[\rho^2 - a^2\right]\hat{z}}$$

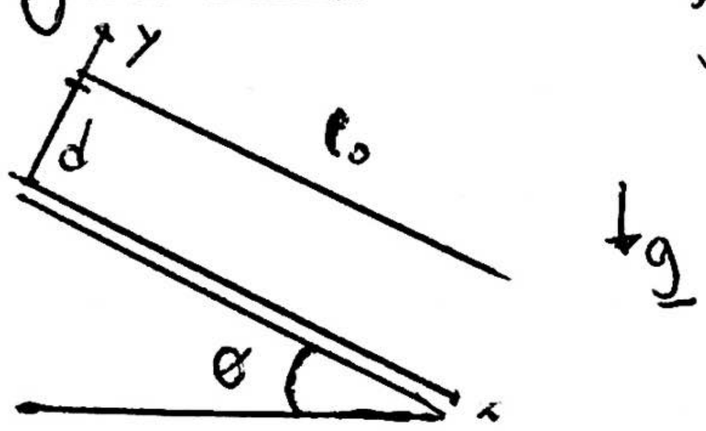
c) Hallar T en i, ii, iii)

Medio fácil, es reemplazar $T = -\begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix} + 2\mu\begin{pmatrix} \partial u & & \\ & & \\ & & \ldots \end{pmatrix}$

Ejercicio 6.2

Datos ρ=cte
μ=cte
dinám

Hipótesis: Estacionario
ρ uniforme $\Rightarrow \underline{\nabla}\cdot\underline{u}=0$



a) $\underline{u}=u_x(y)\hat{x} \Rightarrow \underbrace{\rho\frac{\partial \underline{u}}{\partial t}}_{=0 \text{ estacion.}} + \rho\underbrace{(\underline{u}\cdot\underline{\nabla})\underline{u}}_{=0} = f_{\underline{v}} + \underline{\nabla}\cdot\underline{\underline{T}}$

$\Leftrightarrow 0=\rho\underline{g}+\underline{\nabla}\cdot\underline{\underline{T}}$ $\qquad \partial_i T_{ij} = -\partial_i p + \mu\left[\underbrace{\partial_{ii}u_j}_{\underline{\nabla}^2\underline{u}} + \underbrace{\partial_{ij}u_i}_{\underline{\nabla}(\underbrace{\underline{\nabla}\cdot\underline{u}}_{=0})}\right]=0$

$\Rightarrow 0=\rho\underline{g}-\underline{\nabla}p+\mu\underline{\nabla}^2\underline{u}$

$\Leftrightarrow +\rho g(-\cos(\theta)\hat{y}+\sin(\theta)\hat{x}) - \left(\frac{\partial p}{\partial x}\hat{x}+\frac{\partial p}{\partial y}\hat{y}+\underbrace{\frac{\partial p}{\partial z}}_{=0}\hat{z}\right)+\mu\frac{\partial^2}{\partial y^2}u_x(y)\hat{x}=0$

$\therefore \frac{\partial^2}{\partial y^2}u_x(y) = \frac{1}{\mu}\frac{\partial p}{\partial x} - \frac{\rho g}{\mu}\sin(\theta)$

$0=-\rho g\cos(\theta)-\frac{\partial p}{\partial y} \Leftrightarrow p=-\rho g\cos(\theta)y+p(x)$

sé que $p(d)=p_0=-\rho g\cos(\theta)d+p(x) \quad \forall x \therefore p(x)=C$

$\Rightarrow p(y)=-\rho g\cos(\theta)(y-d)+p_0$ CAMPO DE PRESIONES

$\therefore \frac{\partial p}{\partial x}=0$

$\Rightarrow \frac{\partial^2 u_x(y)}{\partial y^2} = -\frac{\rho g}{\mu}\sin(\theta) \Leftrightarrow u_x(y)=-\frac{\rho g}{2\mu}\sin(\theta)y^2+By+D$

CONTORNOS

$u_x(0)=0 \Rightarrow D=0$

$u_x(d)=u_{aire}$

$\Leftrightarrow 0=-\frac{\rho g}{\mu}\sin(\theta)d+B$

$\Rightarrow \mu\frac{\partial u_x}{\partial y} = \underbrace{\mu_{aire}}_{\approx 0}\frac{\partial u_x}{\partial y} \Leftrightarrow \left.\frac{\partial u_x}{\partial y}\right|_d=0$

$$\Rightarrow \boxed{u_x = -\frac{\rho g}{2\mu}\sin(\theta)y^2 + \frac{\rho g}{\mu} d \sin(\theta)\, y}$$

b) $$u_{prom} = \frac{1}{d}\int_0^d u\, dy = \frac{\rho g \sin(\theta)}{d\mu}\left[-\frac{y^3}{6} + \frac{d y^2}{2}\right]_0^d$$

$$\Rightarrow u_{prom} = \frac{\rho g \sin(\theta)}{d\mu}\frac{d^3}{3} = \frac{\rho g \sin(\theta) d^2}{3\mu}$$

$$u_{máx} = u(d) = -\frac{\rho g}{2\mu}\sin(\theta)d^2 + \frac{\rho g}{\mu}d^2\sin(\theta) = \frac{1}{2}\frac{\rho g}{\mu}d^2\sin(\theta)$$

$$\Rightarrow \boxed{\frac{u_{máx}}{u_{prom}} = \frac{3}{2}}$$

c) Caudal másico por unidad ancho de plano

$$\Rightarrow \cancel{Q = \int_{\text{línea de corriente } y=0}^{\text{línea de corriente } y=d} \underline{u}\cdot\hat{n}\, dl} =$$

Rta. El caudal será:

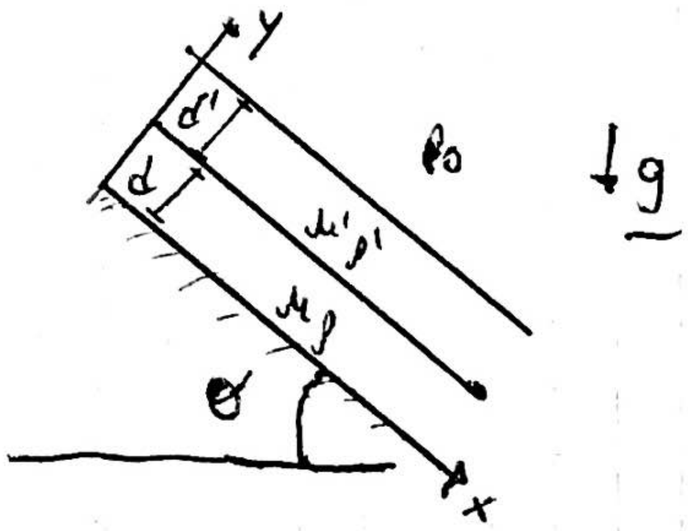
$$Q = \int_{\text{línea de corriente } y=0}^{\text{línea de corriente } y=d} \underline{u}\cdot\hat{n}\, dl$$

$$\Rightarrow Q = \int_0^d \frac{\rho g}{\mu}\sin(\theta)\left(-\frac{y^2}{2} + y d\right)dy = \frac{\rho g}{\mu}\sin(\theta)\left(-\frac{y^3}{6} + \frac{y^2 d}{2}\right)\Bigg|_0^d = \frac{\rho g}{\mu}\sin(\theta)\frac{d^3}{3}$$

caudal máximo por u de ancho

$$\boxed{Q = \frac{\rho g}{\mu}\sin(\theta)\frac{d^3}{3}}$$

Ejercicio 6.3. Opcional



- Segundo líquido inmiscible con el primero
- Estacionario $\frac{\partial u}{\partial t} = 0$
- fluídos incompresibles $\rho, \rho' = cte$
-

a) ¿$(\underline{u}.\underline{\nabla})\underline{u} = 0$? $\underline{u} = u(y)\hat{x}$ como $\underline{\nabla}.\underline{u} = 0$ por incompresible

$\underline{u} \neq u(x)$

$\Rightarrow \left[u(y)\hat{x}.\frac{\partial}{\partial x}\hat{x}\right]u(y)\hat{x} = 0$ ✓

$(\underline{u}.\underline{\nabla})\underline{u}$

1) $0 = \frac{1}{\rho}\underline{\nabla}.\underline{\underline{T}} + \underline{f_v}$ $T_{ij} = -p\delta_{ij} + 2\mu[\partial_i u_j + \partial_j u_i]$

$\underline{\nabla}^2\underline{u}$ $\underline{\nabla}(\underline{\nabla}.\underline{u})$ $= 0$

$\Rightarrow \rho)$ $\frac{\partial p}{\partial x}\hat{x} + \frac{\partial p}{\partial y}\hat{y} + \frac{\partial p}{\partial z}\hat{z} = \mu\underline{\nabla}^2\underline{u} + \rho g\cos(\theta)\hat{x} - \rho g\sin(\theta)\hat{y}$

$\rho')$ $\frac{\partial p}{\partial x}\hat{x} + \frac{\partial p}{\partial y}\hat{y} + \frac{\partial p}{\partial z}\hat{z} = \mu'\underline{\nabla}^2\underline{u} + \rho' g\cos(\theta)\hat{x} - \rho' g\sin(\theta)\hat{y}$

$\frac{\partial p}{\partial z} = 0 \Rightarrow p = p(x,y)$

$\mu\underline{\nabla}^2\underline{u} = \mu\frac{\partial^2 u}{\partial y^2}\hat{x}$ $\hat{y})$ $\rho') \cdot \frac{\partial p}{\partial y} = -\rho' g\sin(\theta)$

$\Rightarrow p = -\rho' g\sin(\theta)y + f(x)$

$p(d+d') = P_0 = -\rho' g\sin(\theta)(d+d') + f(x)$

pero no depende de x $\Rightarrow \frac{\partial p}{\partial x} = 0$; $f(x) = C$

$\rightarrow p(d \leq x \leq d') = \rho' g\sin(\theta)(d+d'-y) + P_0$

$\rho)$ $\frac{\partial p}{\partial y} = -\rho g\sin(\theta) \Rightarrow p = -\rho g\sin(\theta)y + C$

$p(d) = -\rho g\sin(\theta)d + C = \rho' g\sin(\theta)(d') + P_0$

$\therefore p = \begin{cases} P_0 & x > d' \\ \rho' g\sin(\theta)(d+d'-y) + P_0 & d < x \leq d' \\ +\rho g\sin(\theta)(d-y) + \rho' g\sin(\theta)d' + P_0 & 0 < x \leq d \end{cases}$ $\leftarrow C = cons$

$\Rightarrow \hat{x})$ $\rho')$ $\mu'\frac{\partial^2 u}{\partial y^2} = -\rho' g\cos(\theta) \Rightarrow u' = -\frac{\rho' g\cos(\theta)}{\mu'}\frac{y^2}{2} + Cy + D$ $d < y \leq d'$

$\rho)$ $\mu\frac{\partial^2 u}{\partial y^2} = -\rho g\cos(\theta)$

$$u = -\frac{\rho}{\mu 2} g\cos(\theta) y^2 + C' y + D' \qquad 0 < y \leq d$$

$$u(0) = D' = 0 \Rightarrow u = -\frac{\rho}{\mu 2} g\cos(\theta) y^2 + C' y \qquad 0 < y \leq d$$

$$u'(d+d') = u_{max} \Rightarrow \left.\frac{\partial u}{\partial y}\right|_{d+d'} = 0 = -\frac{\rho' g}{\mu'} \cos(\theta) y + C \Rightarrow C = \frac{\rho' g}{\mu'} \cos(\theta)(d+d')$$

$$\Rightarrow u = -\frac{\rho}{2\mu} g\cos(\theta) y^2 + C' y$$

$$u' = -\frac{\rho'}{2\mu'} g\cos(\theta) y^2 + \frac{\rho' g}{\mu'} \cos(\theta)(d+d') y + D$$

en la interfaz $u(d) = u'(d)$ como la soga, contorno continuo

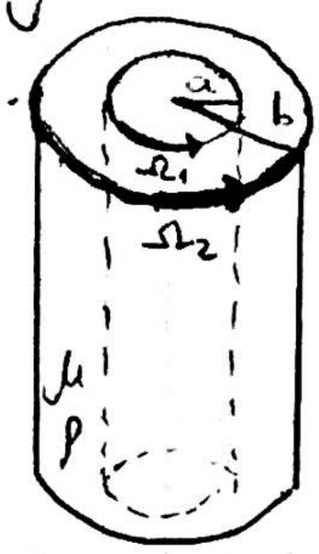
$$\mu \frac{\partial u}{\partial y}(d) = \frac{\partial u'}{\partial y}(d) \mu'$$

$$\boxed{\begin{aligned} &\Rightarrow -\frac{\rho}{2\mu} g\cos(\theta) d^2 + C' d = \frac{\rho' g}{\mu'} \cos(\theta)\left(\frac{d^2}{2} + d' d\right) + D \\ &\mu - \frac{\rho}{2\mu} g\cos(\theta)\, 2d + C' = \frac{\rho' g}{\mu'} \cos(\theta)(d')\mu' \end{aligned}}$$

El tensor de esfuerzos alta fiaca, es nada más reemplazar. ✓

Ejercicio 6.4 : Viscosímetro a) $\underline{u}$?

i)



Hipótesis: estacionario $\frac{\partial \underline{u}}{\partial t}=0$

laminar $u_z \approx 0$

fluído incompresible: $\rho = cte$
$\underline{\nabla}\cdot\underline{u}=0$

simetría de revolución $\Rightarrow \underline{u} \neq \underline{u}(\varphi)$

simetría de traslación en z $\Rightarrow \underline{u} \neq \underline{u}(z)$

no hay g

Contornos $\underline{u}(a) = \Omega_1 a \hat{\varphi}$

$\underline{u}(b) = \Omega_2 b \hat{\varphi}$

$\underline{u} = u(\varphi)(\rho)\hat{\varphi}$

Voy a tener así láminas para cada ρ_0 fijo

$$(\underline{u}\cdot\underline{\nabla})\underline{u} = \frac{1}{\rho_0}\underline{\nabla}\cdot\underline{\underline{T}} \qquad \underline{\underline{T}} = -p\underline{\underline{I}} + 2\mu\underline{\underline{S}}$$

$$\Rightarrow \begin{pmatrix} -\frac{u_\varphi^2}{\rho} \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\rho_0}\begin{pmatrix} \cdots \end{pmatrix}$$

$$\Rightarrow \hat{\rho})\ -\frac{u_\varphi^2}{\rho} = \frac{1}{\rho_0}\left[\frac{\partial T_{\rho\rho}}{\partial \rho} + \frac{T_{\rho\rho}}{\rho} + \frac{1}{\rho}\frac{\partial T_{\varphi\varphi}}{\partial \varphi} + \frac{\partial T_{z\rho}}{\partial z} - \frac{T_{\varphi\varphi}}{\rho}\right]$$

Los esfuerzos en z son nulos por que mi fluído no se mueve ahí. De forma análoga u no depende de φ, por que lo haría T?

$$\Rightarrow \begin{pmatrix} -\frac{u_\varphi^2}{\rho} \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\rho_0}\begin{pmatrix} \frac{\partial T_{\rho\rho}}{\partial \rho} + \frac{T_{\rho\rho}}{\rho} - \frac{T_{\varphi\varphi}}{\rho} \\ \frac{T_{\rho\varphi}}{\rho} + \frac{T_{\varphi\rho}}{\rho} + \frac{\partial T_{\rho\varphi}}{\partial \rho} \\ 0 \end{pmatrix}$$

Como el medio es isótropo, la presión va a ser igual en toda cara de un diferencial de fluído $\Rightarrow T_{\rho\rho} = T_{\varphi\varphi}$

Finalmente $-\frac{u_\varphi^2}{\rho} = \frac{1}{\rho_0}\frac{\partial T_{\rho\rho}}{\partial \rho} \Rightarrow u_\varphi^2 = -\frac{1}{\rho_0}\left(\rho\frac{\partial T_{\rho\rho}}{\partial \rho}\right)$

$$\frac{\partial T_{\rho\varphi}}{\partial \rho} + \frac{2T_{\rho\varphi}}{\rho} = 0 \Rightarrow \frac{\partial T_{\rho\varphi}}{2T_{\rho\varphi}} = -\frac{d\rho}{\rho}$$

$$\Rightarrow \frac{\partial T_{\rho\varphi}}{T_{\rho\varphi}} = -2\frac{d\rho}{\rho} \Rightarrow \ln(T_{\rho\varphi}) = -2\ln(\rho) + C$$

$$\Rightarrow e^{\ln(T_{\rho\varphi})} = e^{-2\ln(\rho)+C} = e^{C}e^{\ln(\rho^{-2})} = Ae^{\ln(\rho^{-2})}$$

$$\Rightarrow T_{\rho\varphi} = \frac{A}{\rho^2}$$

$$T = \begin{pmatrix} T_{\rho\rho} & T_{\rho\varphi} & \overset{=0}{T_{\rho z}} \\ T_{\varphi\rho} & T_{\varphi\varphi} & \overset{=0}{T_{\varphi z}} \\ \underset{=0}{T_{z\rho}} & \underset{=0}{T_{z\varphi}} & T_{zz} \end{pmatrix} = \begin{pmatrix} T_{\rho\rho} & T_{\rho\varphi} & 0 \\ T_{\rho\varphi} & T_{\varphi\varphi} & 0 \\ 0 & 0 & T_{zz} \end{pmatrix} = -\begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix} + 2\mu \begin{pmatrix} 0 & \frac{1}{2}\left(\frac{\partial u_\varphi}{\partial \rho} - \frac{u_\varphi}{\rho}\right) & 0 \\ \frac{1}{2}\left(\frac{\partial u_\varphi}{\partial \rho} - \frac{u_\varphi}{\rho}\right) & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T_{\rho\varphi} = \mu\left(\frac{\partial u_\varphi}{\partial \rho} - \frac{u_\varphi}{\rho}\right) = \frac{A}{\rho^2}$$

factor integrante

$$\underbrace{\frac{du_\varphi}{d\rho} e^{f(\rho)} - \frac{u_\varphi}{\rho} e^{f(\rho)}}_{\frac{d}{d\rho}\left(u_\varphi(\rho) e^{f(\rho)}\right)} = \frac{A e^{f(\rho)}}{\mu \rho^2} \qquad f(r) = \frac{c}{r}$$

$$\Rightarrow \frac{c}{\rho} u_\varphi = \frac{A'}{\mu \rho^2} + C \quad \Leftrightarrow \quad u_\varphi = \frac{\alpha}{\mu \rho} + \beta \rho$$

Contorno: $u_\varphi(a) = \Omega a . a$

$u_\varphi(b) = \Omega b . b$

$$\Leftrightarrow \begin{matrix} \Omega a\, a = \frac{\alpha}{\mu a} + \beta a \\ \Omega b\, b = \frac{\alpha}{\mu b} + \beta b \end{matrix} \quad \Leftrightarrow \quad \beta =$$

$$\Rightarrow \beta = \Omega a - \frac{\alpha}{\mu a^2} = \Omega b - \frac{\alpha}{\mu b^2} \quad \Leftrightarrow \quad \alpha = (\Omega b - \Omega a)\mu\left[\frac{b^2 - a^2}{a^2 b^2}\right]^{-1}$$

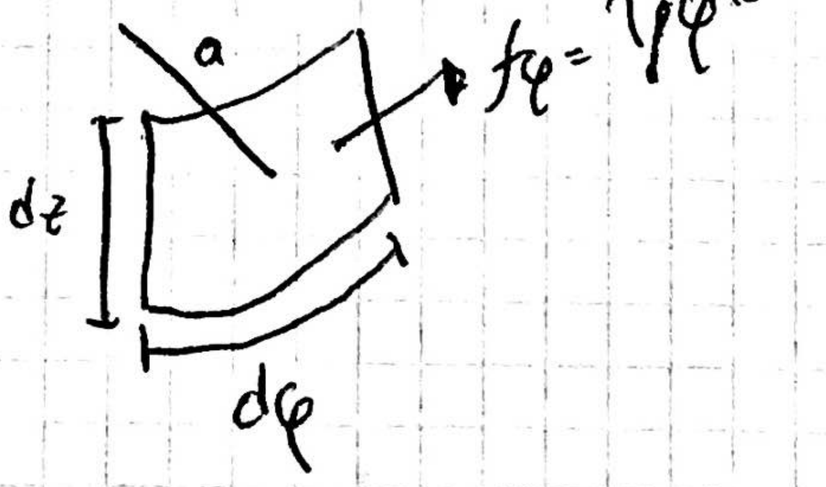
$$\alpha = \mu a^2 (\Omega a - \beta) = \mu b^2 (\Omega b - \beta) \Rightarrow \beta = \frac{b^2 \Omega b - a^2 \Omega a}{b^2 - a^2}$$

$$\boxed{\bar{u} = u_\varphi = \frac{a^2 b^2}{(b^2 - a^2)} \frac{(\Omega b - \Omega a)}{\rho} + \left[\frac{b^2 \Omega b - a^2 \Omega a}{b^2 - a^2}\right] \rho\, \hat{\varphi}}$$

b)

Torque $\underline{\tau} : \underline{r} \times \underline{f}$

Si tomo un diferencial de pared



$$T_{\rho\varphi} = \mu\left(\frac{-2a^2 b^2 (\Omega b - \Omega a)}{(b^2 - a^2)\rho^2}\right)$$

$$a\, ds = a\, 2\pi\, dz$$

$$\Rightarrow \underline{f_\varphi} = \mu\left(\frac{2a^2 b^2 (\Omega b - \Omega a)}{(b^2 - a^2)\rho^2}\right) 2\pi \frac{a}{a^2} z\, \hat{\varphi} = -\frac{4\pi \mu a^3 b^2 z (\Omega b - \Omega a)}{(b^2 - a^2)\rho^2 a^2} \hat{\varphi}$$

$$\Rightarrow \underline{\tau} = a\rho\, \hat{\rho} \times f_\varphi \hat{\varphi} = -\frac{4\pi \mu a^2 b^2 z}{(b^2 - a^2)\rho^2} (\Omega b - \Omega a) \hat{z}$$

Parámetros $(a, b, \Omega_1, \Omega_2, l, \mu, \rho_0)$ analizo τ

l : no lo quiero $\Rightarrow$ analizo τ/l

ρ_0 : no aparece, pues despreciamos g, luego solo aparece en f_0, que no afecta en el torque

$$\Rightarrow \left[\tau/l\right] = \left[\frac{F \cdot r}{l}\right] = \frac{\frac{ML}{T^2} \cdot L}{L} = \frac{ML}{T^2}$$

$[a] = [b] = L$

$[\Omega_a] = [\Omega_b] = 1/T$

$[\mu] = M/LT$

$\Rightarrow$ Parame $= 5$

Dim $= 3$

$\Rightarrow 5 - 3 = 2$ param adimensionales

$\Rightarrow \pi_1 = a/b, \quad \pi_2 = \Omega_a/\Omega_b$

$$\Rightarrow \left[\frac{\tau}{l}\right] = [a]^a [\Omega_a]^b [\mu]^\gamma$$

	τ/l	a	Ω_a	μ	a/b	Ω_a/Ω_b
$\Rightarrow$ M	1	0	0	1	0	0
L	-1	1	0	-1	0	0
T	-1	0	-1	-1	0	0

$\Rightarrow \quad 1 = \gamma$

$1 = \alpha + (-\gamma) \quad \Rightarrow \alpha = 2$

$-2 = -b + (-\gamma) \quad \Rightarrow b = 1$

$$\therefore \frac{\tau}{l} = \mu a^2 \Omega_a F_2\left(\frac{a}{b}\right) F_1\left(\frac{\Omega_a}{\Omega_b}\right)$$

Si tomo

	τ/l	$b-a$	$\Omega_b-\Omega_a$	μ
M	1	0	0	1
L	+1	1	0	-1
T	-2	0	-1	-1

$$\Rightarrow \frac{\tau}{l} = A \mu (b-a)^2 (\Omega_b - \Omega_a)$$

$$Q = \rho_0 \int_0^{2\pi} \int_{\rho_1}^{\rho_2} \underline{u} \cdot d\underline{S} = \rho_0 2\pi \int_{\rho_1}^{\rho_2} u_\varphi \hat{z} \rho$$

$$Q = \rho_0 2\pi \left[\frac{a^2 b^2}{(b^2-a^2)} (\Omega_b - \Omega_a) \ln\left(\rho_2/\rho_1\right)\right] + \frac{1}{2}\left(\frac{b^2\Omega_b - a^2\Omega_a}{b^2-a^2}\right)\left(\rho_2^2 - \rho_1^2\right)\Big] \hat{z}$$

divide multiplicar ρ

$\Rightarrow (a, b, \Lambda_a, \Lambda_b, \rho_0, \mu)$

$$[Q] = [\rho v A] = \frac{M}{L^3}\frac{L}{T} \cdot L^2 = \frac{M}{T}$$

de la viscosidad no depende

$$[\mu] = M/LT$$

$\Rightarrow 6 - 3 = 3$ $\quad \pi_1 = a/b, \ \pi_2 = \Lambda_a/\Lambda_b$ $\quad \cancel{\pi_3 = \rho_0 \Lambda_a a^2}$

$\Rightarrow$

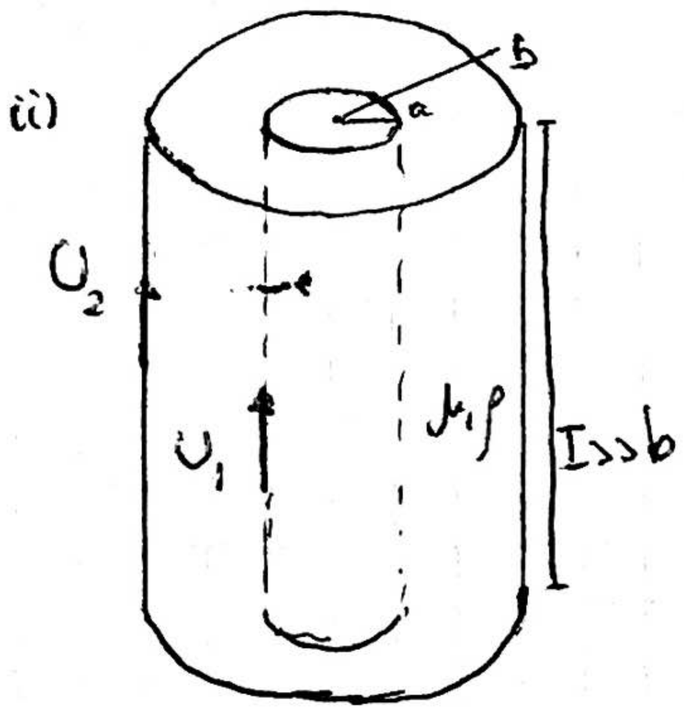
	Q	a	Λ_a	ρ_0
M	1	0	0	1
L	0	1	0	-3
T	-1	0	-1	0

$\Rightarrow$ $1 = c$

$0 = a - 3c$ $\quad \Rightarrow \quad Q \approx a^3 \rho_0 \Lambda_a f(a/b)\, g(\Lambda_a/\Lambda_b)$

$-1 = -b$

ii)



Hipótesis: • fluido incompresible, $\rho = cte$ $\Rightarrow \underline{\nabla} \cdot \underline{u} = 0$

• régimen estacionario $\frac{\partial}{\partial t} = 0$

• cilindro "infinito" $\Rightarrow$ traslación

• régimen laminar $\Rightarrow \underline{u} = u(\rho)\hat{z}$

• no puede ser $u(\varphi)$ por simetría de rotación ni $u(z)$ por traslación en torno a $\hat{z}$ ambos

• desprecio g

CONTORNOS

$\underline{u}(a) = U_1 \hat{z}$

$\underline{u}(b) = U_2 \hat{z}$

$$\Rightarrow (\underline{u} \cdot \underline{\nabla})\underline{u} = \frac{1}{\rho_0} \underline{\nabla} \cdot \underline{\underline{T}}$$

$$\Rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\rho_0} \begin{pmatrix} 0 \\ \frac{\partial T_{zz}}{\partial z} + \frac{T_{\rho z}}{\rho} \\ \frac{\partial T_{zz}}{\partial z} + \frac{\partial T_{z\rho}}{\partial \rho} + \frac{T_{\rho z}}{\rho} + \frac{1}{\rho}\frac{\partial T_{\varphi z}}{\partial \varphi} \end{pmatrix}$$

no aporta mucho

$$0 = -\underline{\nabla} \begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix} + 2\mu \underline{\nabla} \cdot \begin{pmatrix} 0 & 0 & \frac{1}{2}\frac{\partial u_z}{\partial \rho} \\ 0 & 0 & 0 \\ \frac{1}{2}\frac{\partial u_z}{\partial \rho} & 0 & 0 \end{pmatrix}$$

$$\Rightarrow -\frac{\partial p}{\partial \rho} = 0 \qquad -\frac{\partial p}{\partial \varphi} = 0 \qquad -\frac{\partial p}{\partial z} + 2\mu \frac{\partial}{\partial \rho}\left[\frac{1}{2}\frac{\partial u_z}{\partial \rho}\right] + 2\mu \frac{1}{2\rho}\frac{\partial u_z}{\partial \rho} = 0$$

$$\Rightarrow p = p(z) + C \qquad \Rightarrow \frac{dp}{dz} = \mu\left[\frac{\partial^2 u_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial u_z}{\partial \rho}\right]$$

Pero $\underbrace{\frac{dp(z)}{dz}}_{f(z)} = \underbrace{\mu \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial u_z(\rho)}{\partial \rho}\right)}_{f(\rho)} \Rightarrow p = cte \neq p(z)$

$$\Rightarrow 0 = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial u_z}{\partial \rho}\right) \Rightarrow \rho \frac{\partial u_z}{\partial \rho} = C \Rightarrow u_z(\rho) = A\ln(\rho) + B$$

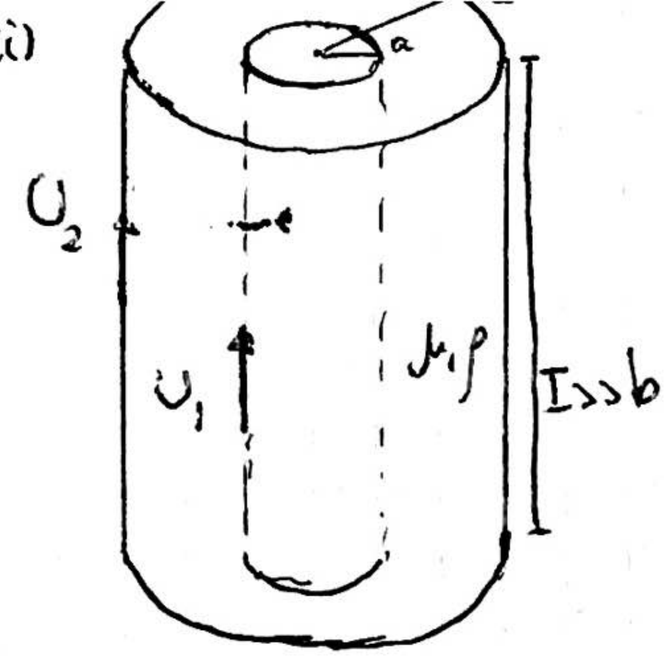
contorno

$$\Rightarrow \begin{aligned} u(a) &= U_1 = A\ln(a) + B \\ u(b) &= U_2 = A\ln(b) + B \end{aligned} \Rightarrow A = \frac{U_2 - U_1}{\ln(b/a)} \qquad B = U_1 - \frac{(U_2 - U_1)}{\ln(b/a)}\ln(a)$$

$$\Rightarrow B = \frac{U_1(\ln(b) - \ln(a)) - U_2\ln(a) + U_1\ln(a)}{\ln(b/a)}$$

$$\Rightarrow B = \frac{U_1\ln(b) - U_2\ln(a)}{\ln(b/a)}$$

ii)



$\Rightarrow \underline{\nabla} \cdot \underline{u} = 0$

- régimen estacionario $\frac{\partial}{\partial t} = 0$
- cilindro "infinito" $\Rightarrow$ traslación
- régimen laminar $\Rightarrow \underline{u} = u(\rho)\hat{z}$
- no puede ser $u(\varphi)$ por simetría de rotación ni $u(z)$ por traslación en torno a $\hat{z}$ ambos

CONTORNOS

$\underline{u}(a) = U_1 \hat{z}$

$\underline{u}(b) = U_2 \hat{z}$

- despreciamos g

$$\Rightarrow (\underline{u} \cdot \underline{\nabla})\underline{u} = \frac{1}{\rho_0} \underline{\nabla} \cdot \underline{\underline{T}}$$

$$\Leftrightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \frac{1}{\rho_0} \begin{pmatrix} 0 \\ \frac{\partial T_{zz}}{\partial z} + \frac{T_{\rho z}}{\rho} \\ \frac{\partial T_{zz}}{\partial z} + \frac{\partial T_{z\rho}}{\partial \rho} + \frac{T_{\rho z}}{\rho} + \frac{1}{\rho}\frac{\partial T_{\varphi z}}{\partial \varphi} \end{pmatrix}$$

no aporta mucho

$$0 = -\underline{\nabla} \begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix} + 2\mu \underline{\nabla} \cdot \begin{pmatrix} 0 & 0 & \frac{1}{2}\frac{\partial u_z}{\partial \rho} \\ 0 & 0 & 0 \\ \frac{1}{2}\frac{\partial u_z}{\partial \rho} & 0 & 0 \end{pmatrix}$$

$$\Leftrightarrow -\frac{\partial p}{\partial \rho} = 0 \qquad -\frac{\partial p}{\partial \varphi} = 0 \qquad -\frac{\partial p}{\partial z} + 2\mu \frac{\partial}{\partial \rho}\left[\frac{1}{2}\frac{\partial u_z}{\partial \rho}\right] + 2\mu \frac{1}{2\rho}\frac{\partial u_z}{\partial \rho} = 0$$

$$\Rightarrow p = p(z) + C \qquad \Rightarrow \frac{dp}{dz} = \mu\left[\frac{\partial^2 u_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial u_z}{\partial \rho}\right]$$

Pero $\underbrace{\frac{dp(z)}{dz}}_{f(z)} = \mu \frac{1}{\rho} \underbrace{\frac{\partial}{\partial \rho}\left(\rho \frac{\partial u_z(\rho)}{\partial \rho}\right)}_{f(\rho)} \Rightarrow p = cte \neq p(z)$

$$\Rightarrow 0 = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial u_z}{\partial \rho}\right) \Leftrightarrow \rho \frac{\partial u_z}{\partial \rho} = C \Leftrightarrow u_z(\rho) = A\ln(\rho) + B$$

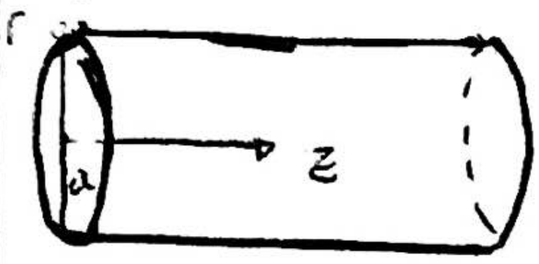
contorno

$$\Rightarrow \begin{aligned} u(a) &= U_1 = A\ln(a) + B \\ u(b) &= U_2 = A\ln(b) + B \end{aligned} \Leftrightarrow A = \frac{U_2 - U_1}{\ln(b/a)} \qquad B = U_1 - \frac{(U_2 - U_1)}{\ln(b/a)}\ln(a)$$

$$\Rightarrow B = \frac{U_1(\ln(b) - \ln(a)) - U_2\ln(a) + U_1\ln(a)}{\ln(b/a)}$$

$$\Rightarrow B = \frac{U_1\ln(b) - U_2\ln(a)}{\ln(b/a)}$$

Ejercicio 6.5



$\underline{u} = u(r)\hat{z}$

Hipótesis

- estacionario $\frac{\partial}{\partial t} = 0$
- incompresible $\rho = cte$ $(\underline{\nabla} \cdot \underline{u}) = 0$

$\Rightarrow$ N-S $\underbrace{\frac{\partial \underline{u}}{\partial t}}_{=0} + \underbrace{(\underline{u} \cdot \underline{\nabla})\underline{u}}_{=0 \text{ cortín}} = \frac{1}{\rho} \underline{\nabla} \cdot \underline{\underline{T}} + f_{\underline{m}}$

$T_{ij} = -p\delta_{ij} + 2\mu S_{ij}$ (fluido newt)

- $\underline{\nabla} \cdot \underline{\underline{T}} = 0 \Rightarrow \underline{\nabla} p = \mu \underline{\nabla}^2 \underline{u}$

$\Rightarrow \underline{\nabla}^2 u = \frac{\underline{\nabla} p}{\mu}$

$[\underline{\nabla} p] = [\rho_0 (\underline{u} \cdot \underline{\nabla}) \underline{u}] = \frac{M}{L^3} \cdot \frac{L}{T} \cdot \frac{1}{T} = \frac{M}{L^2 T^2}$

$\Rightarrow \frac{[\underline{\nabla} p]}{[\mu]} = \frac{M}{L^2 T^2} \cdot \frac{LT}{M} = \frac{1}{LT}$

Parámetros $\left[\frac{\underline{\nabla} p}{\mu}, a\right]$ Dim $[L, T]$

$P - D = 0$

$\Rightarrow [Q] = \frac{M}{T} \Rightarrow [Q] = [\rho_0] \alpha \left[\frac{\underline{\nabla} p}{\mu}\right]^{\gamma} [a]^{\beta}$

$\Rightarrow$

	Q	ρ_0	$\frac{\nabla p}{\mu}$	a
M	1	1	0	0
T	-1	0	-1	0
L	0	-3	-1	1

$\Rightarrow 1 = 1$ ✓

$-1 = -\gamma \Rightarrow \gamma = 1$

$0 = -3 - \gamma + \beta \Rightarrow \beta = 4$

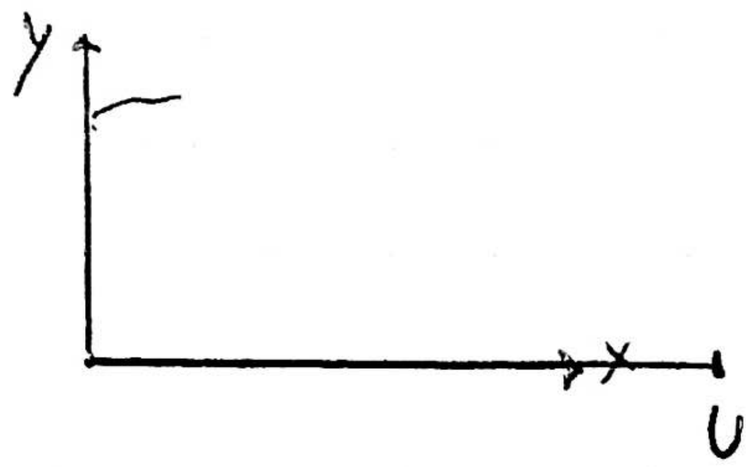
$\Rightarrow Q = \alpha \rho_0 \frac{\underline{\nabla} p}{\mu} \cdot a^4$

La cuenta de verdad $\quad Q = \int_0^{2\pi}\int_0^{a} \rho_0 \frac{\Delta p}{4\mu} [\rho^2 - a^2]\, \rho\, d\rho\, d\varphi\ \hat{z}$

$$= \frac{2\pi \rho_0}{4\mu} \Delta p \left[\frac{a^4}{4} - \frac{a^4}{2}\right]\hat{z} = \frac{1}{4}\frac{\pi \rho_0}{\mu}\Delta p \left(-\frac{a^4}{2}\right)\hat{z} = -\frac{\pi}{8}\frac{\rho_0}{\mu}\Delta p\, a^4\, \hat{z}$$

$$\Rightarrow \boxed{\alpha = \frac{\pi}{8}}$$ ✓

Problema 6.7



$\underline{u} = u(y,t)\hat{x} \qquad 0 \leq y < \infty$

A $t=0$, se pone en velocidad el suelo con U en $\hat{x}$

$\Rightarrow u_x(y=0,t) = U \qquad t \geq 0$

$\underline{f}_V = 0, \ p = 0 \Rightarrow \partial_t u = \nu \overset{\nabla^2 \underline{u}}{\partial_y^2 u}$

$\Rightarrow [\partial_t u] = \frac{U}{T} \qquad [\partial_y^2 u] = \frac{U}{L^2} \qquad [\nu] = \frac{M}{LT}$

$\Rightarrow \frac{U}{T} = \nu \frac{U}{L^2}$ analizando unidades $\Rightarrow u = f\left(\frac{y}{\sqrt{\nu t}}\right) U$

$\Rightarrow U\left(-\frac{1}{2}\right)\left(\frac{y}{\sqrt{\nu t}}\right) f'\left(\frac{y}{\sqrt{\nu t}}\right) = U\left[f'\left(\frac{y}{\sqrt{\nu t}}\right)\left(\frac{1}{\sqrt{\nu t}}\right)\right]'$

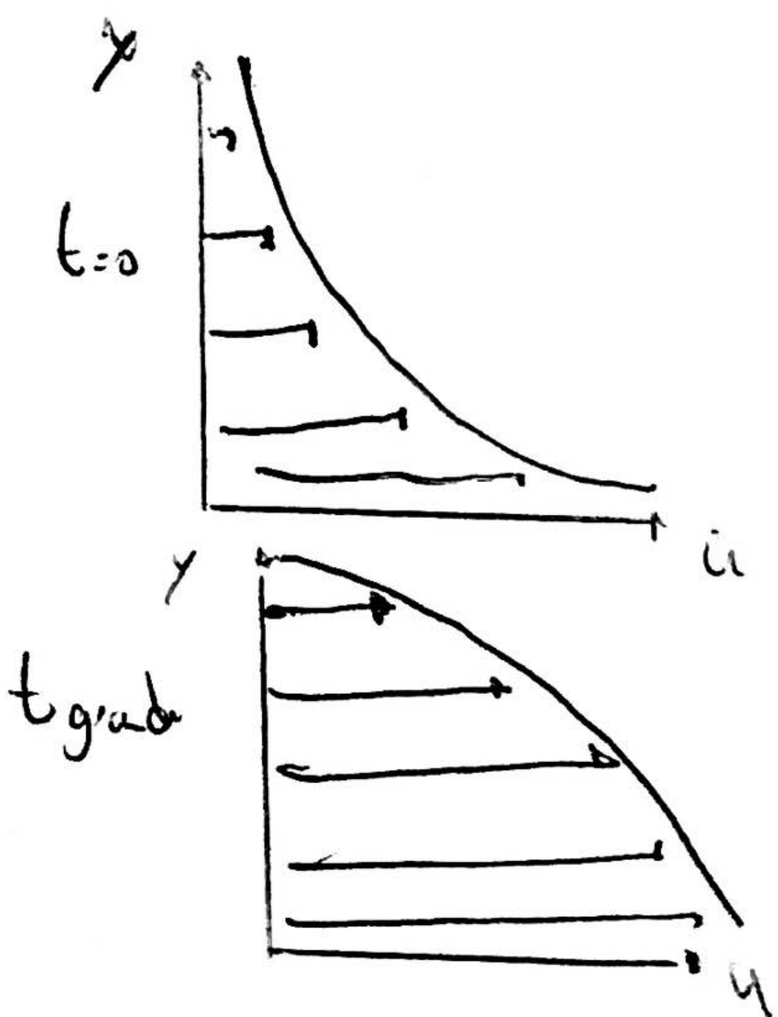
$= U\left[f''\left(\frac{y}{\sqrt{\nu t}}\right)\left(\frac{1}{\nu t}\right)\right]$

$\Rightarrow -\frac{1}{2}\frac{y}{\sqrt{\nu t}} f'\left(\frac{y}{\sqrt{\nu t}}\right) = f''\left(\frac{y}{\sqrt{\nu t}}\right)$

wolfram $\quad f = A \operatorname{erf}\left(\frac{1}{2}\frac{y}{\sqrt{\nu t}}\right) + B$

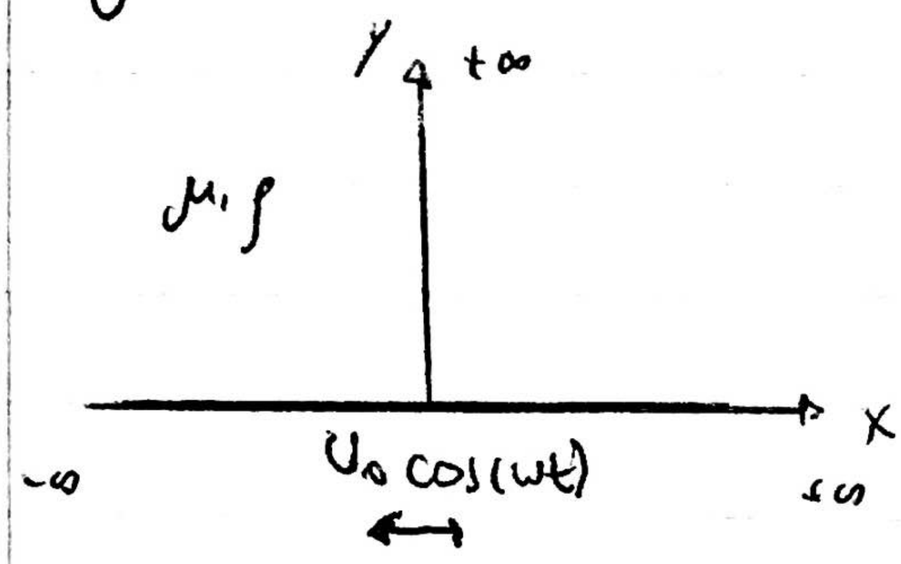
$u \xrightarrow[y \to \infty]{} 0 \qquad u(0,t) = U$

$\Rightarrow u(y,t) = \left[1 - \operatorname{erf}\left(\frac{y}{2\sqrt{\nu t}}\right)\right] U$



Ejercicio 6.9

δ: long de penetración



$$\underline{u}(y,t) = \mathrm{Re}\{\hat{\underline{U}}(y)e^{i\omega t}\} \qquad \underline{u} = u(y,t)\hat{x}$$

N-S
$$\begin{cases} \partial_t u_x = -\frac{1}{\rho}\overbrace{\partial_x P}^{=0 \text{ por simetría de tubos}} + \nu\,\partial^2_{yy} U_x \\ 0 = -\frac{1}{\rho}\partial_y P \Rightarrow P = cte \end{cases}$$

$$\Rightarrow \partial_t\left(\hat{U}(y)e^{i\omega t}\right) = \partial^2_y\left(\hat{U}(y)e^{i\omega t}\right)\nu$$

$$\Leftrightarrow \hat{U}(y)\cancel{e^{i\omega t}}(i\omega) = \nu\,\ddot{\hat{U}}(y)\cancel{e^{i\omega t}} \qquad \forall t$$

$$\Leftrightarrow \ddot{\hat{U}}(y) - \frac{i\omega}{\nu}\hat{U}(y) = 0$$

Propongo $\hat{U} = e^{\lambda y} \Rightarrow \lambda^2 = \frac{i\omega}{\nu}$

$$= \pm\sqrt{\frac{i\omega}{\nu}} \qquad \sqrt{i} = \sqrt{\frac{(1+i)^2}{2}} \Rightarrow \lambda = \pm\frac{(1+i)}{\sqrt{2}}\sqrt{\frac{\omega}{\nu}} \qquad \left[\sqrt{\frac{\omega}{\nu}}\right] = \frac{1}{L}$$

$$\delta = \sqrt{\frac{2\nu}{\omega}} \quad \text{CDC} \Rightarrow \hat{U} = a e^{\frac{1+i}{\delta}y} + b e^{-\frac{1+i}{\delta}y}$$

CONTORNO

$$\underline{u} = \mathrm{Re}\{[A e^{\frac{1+i}{\delta}y} + B e^{-\frac{1+i}{\delta}y}]e^{i\omega t}\}$$

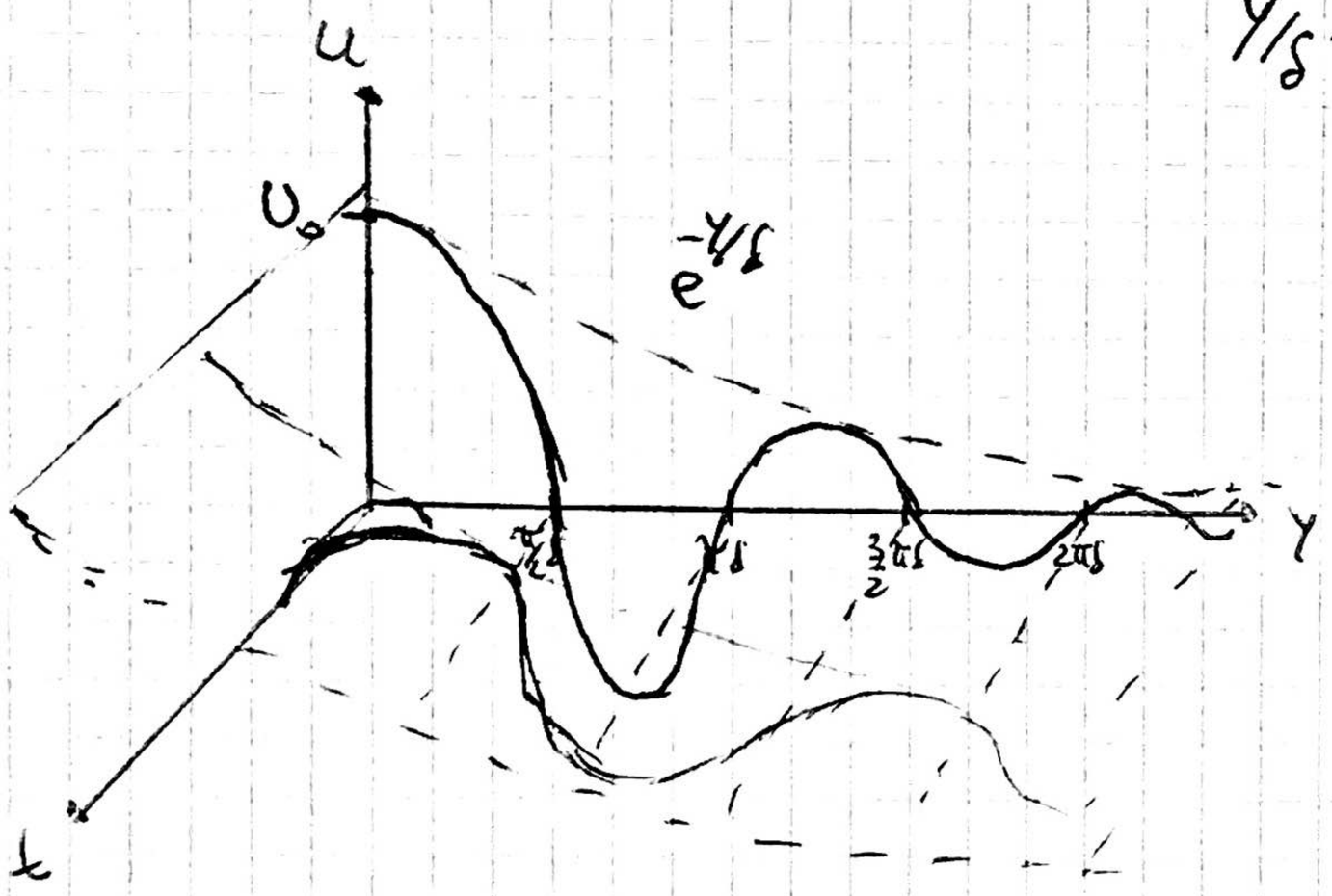
$\forall t$ quiero $\underline{u} \xrightarrow[y\to\infty]{} 0 \Rightarrow A = 0 \qquad u(0,t) = U_0\cos(\omega t)$

$$\Rightarrow \underline{u} = \mathrm{Re}\left\{ U_0 e^{-\frac{1-i}{\delta}y} e^{i\omega t} \right\} = U_0 \mathrm{Re}\left\{ e^{-y/\delta}\left[\cos(y/\delta) - i\sin(y/\delta)\right]\left[\cos(\omega t) + i\sin \right.\right.$$

$$\Rightarrow \underline{u} = U_0 e^{-y/\delta}\left[\cos(y/\delta)\cos(\omega t) + \sin(y/\delta)\sin(\omega t)\right]$$

$$\Rightarrow \boxed{\underline{u} = U_0 e^{-y/\delta}\cos(y/\delta - \omega t)}$$

$$y/\delta - \omega t = -\frac{\pi}{2} \Rightarrow t = \frac{\pi}{2\omega}$$



b) $\dfrac{U}{T} = \dfrac{U}{L^2}$ $\qquad [u] = \dfrac{[L]}{[T]}$

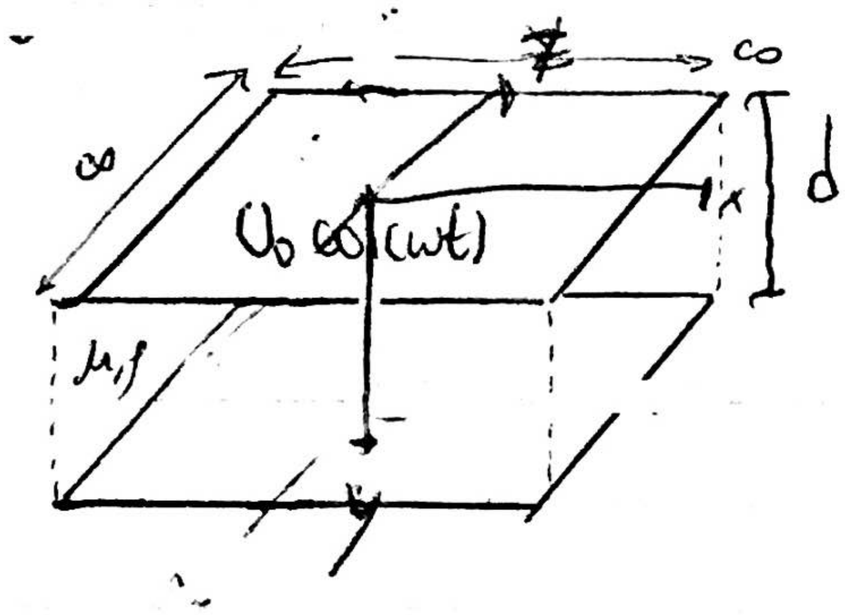
Parámetros $\left[\omega t, \frac{x}{\delta}\right]$ $\qquad [x] = L \quad [\delta] = L \qquad [\omega] = \frac{1}{T} \quad [t] = T$

La idea es obtener la dep funcional ej $\underline{u} = U_0 f(\omega, t, x, \delta)$

$\Rightarrow P = [\omega, t, x, \delta] \quad D = [L, T] \Rightarrow 4 - 2 = 2$

$\Rightarrow \omega t = \Pi_1, \ x/\delta = \Pi_2 \Rightarrow \boxed{f \sim f(\omega t, x/\delta)}$ ✓

Ejercicio 10



Hipótesis: mov laminar $u = u(y)$

- ~~incompresible pared, $\nabla \cdot \underline{u} = 0$~~

fluído muy viscoso $\Rightarrow \nabla \cdot \underline{u} \sim 0$

$$\underline{u} = u(y,z,t)\hat{x} = Re\left\{U(y,z)e^{i\omega t}\right\}\hat{x} \overset{\text{laminar}}{=} Re\left\{U(y)e^{i\omega t}\right\}\hat{x}$$

N-S

$$\frac{\partial \underline{u}}{\partial t} = \frac{1}{\rho_0}\underline{\nabla} \cdot \underline{\underline{T}} = -\frac{\underline{\nabla} p}{\rho_0} + \overset{\nu}{\frac{\mu}{\rho_0}} \underline{\nabla}^2 \underline{u}$$

$$\hat{x}) \quad \frac{\partial u_x}{\partial t} = \frac{-\partial p}{\rho_0 \partial x} + \nu \frac{\partial^2 u_x}{\partial y^2}$$

$$\hat{y}) \quad \frac{\partial p}{\partial y} = 0$$

$$\hat{z}) \quad \frac{\partial p}{\partial y} = 0$$

$\Rightarrow p \cong p(x)$ pero si me paro en una lámina $p = cte \ \forall x / x \in \pi$

$\Rightarrow p = cte$

$$\therefore \quad \frac{\partial u_x}{\partial t} = \nu \frac{\partial^2 u_x}{\partial y^2} \Leftrightarrow U(y)(i\omega)\cancel{e^{i\omega t}} = \nu \frac{\partial^2 U(y)}{\partial y^2}\cancel{e^{i\omega t}} \quad \underline{\forall t}$$

$$\Rightarrow \frac{\partial^2 U(y)}{\partial y^2} - \frac{i\omega}{\nu}U(y) = 0$$

Propongo $U(y) = e^{\lambda y}$

$$\Rightarrow \lambda^2 - \frac{i\omega}{\nu} = 0 \Leftrightarrow \lambda = \pm \frac{(1+i)}{\sqrt{2}}\sqrt{\frac{\omega}{\nu}} \qquad \sqrt{\frac{2\nu}{\omega}} = \delta$$

$$\Rightarrow \underline{u} = Re\left\{\left(Ae^{\frac{(1+i)}{\delta}y} + Be^{-\frac{(1+i)}{\delta}y}\right)e^{i\omega t}\right\}$$

$$\underline{u}(0) = U_0 \cos(\omega t) = (A+B)\cos(\omega t) \Leftrightarrow A+B = U_0$$

$$U(d) = 0 = A e^{d/\delta} e^{id/\delta} + B e^{-d/\delta} e^{-id/\delta}$$

$$\Rightarrow A e^{d/\delta} e^{id/\delta} + (U_0 - A) e^{-d/\delta} e^{-id/\delta}$$

$$\Leftrightarrow A \left(e^{d/\delta} e^{id/\delta} - e^{-d/\delta} e^{-id/\delta} \right) = -U_0 e^{-d/\delta} e^{-id/\delta}$$

$$\Leftrightarrow A \underbrace{\left(1 - e^{2d/\delta} e^{i2d/\delta} \right)}_{\alpha} = U_0 \qquad B = U_0 - A$$

$$\underline{u} = \mathrm{Re}\left\{ \left[\frac{U_0}{\alpha} e^{\frac{(1+i)}{\delta} y} + U_0 \left(1 - \frac{1}{\alpha}\right) e^{-\frac{(1+i)}{\delta} y} \right] e^{i\omega t} \right\}$$

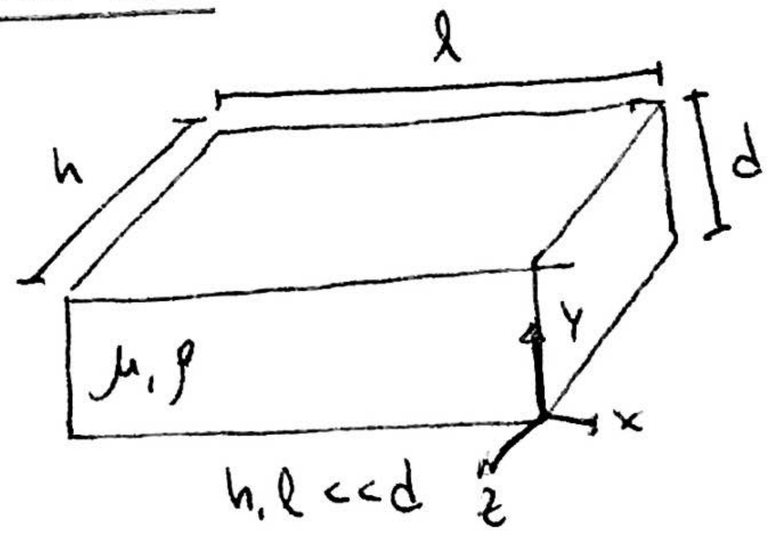
si $\delta \ll d \Rightarrow d/\delta \gg 1 \Rightarrow e^{d/\delta} \to 1$

$e^{-d/\delta} \to 1$

$\Rightarrow$

Ejercicio 6.11

incompresible ; $\rho = cte$; $\underline{\nabla} \cdot \underline{u} = 0$



⇒ La ecuación dif por Navier-Stokes

$$\frac{\partial \underline{u}}{\partial t} = -\frac{\Delta p(t)}{\rho} + \nu \nabla^2 \underline{u}$$

Separación de variables

$$\underline{u} = Re\{\mathbb{U}(y,t)\} = Re\{U(y)\, e^{i\omega t}\}$$

$$\Leftrightarrow \quad i\omega U e^{i\omega t} = \frac{\overbrace{p_2 - p_1}^{-\frac{b}{\rho}\cos(\omega t)}}{\rho} + \nu \frac{\partial^2 U}{\partial y^2} e^{i\omega t}$$

$$\Leftrightarrow \quad \left[\frac{\partial^2 U}{\partial y^2} - \frac{i\omega}{\nu} U\right] e^{i\omega t} = \frac{b}{\rho \nu} \cos(\omega t)$$ Ecuación no homogenea

Será más comodo proponer $\mathbb{U}(y,t) = U(y)\left[A\cos(\omega t) + B\sin(\omega t)\right]$

$$-U\frac{\omega}{\nu}\left[-A s(\omega t) + B c(\omega t)\right] + \frac{\partial^2 U}{\partial y^2}\left[A c(\omega t) + B s(\omega t)\right] = \frac{b}{\rho \nu} c(\omega t)$$

$$\Leftrightarrow \quad \frac{\partial^2 U}{\partial y^2} A - \frac{U\omega}{\nu} B = \frac{b}{\rho \nu} \quad \Leftrightarrow \quad A = -\frac{\nu}{\omega U}\frac{\partial^2 U}{\partial y^2} B$$

$$\frac{\partial^2 U}{\partial y^2} B + \frac{U\omega}{\nu} A = 0$$

$$\Rightarrow -\left[\left(\frac{\partial^2 U}{\partial y^2}\right)^2 \frac{\nu}{\omega U} + \frac{U\omega}{\nu}\right] B = \frac{b}{\rho \nu}$$

$$\frac{b}{\rho \nu}\cos(\omega t) = \frac{b}{\rho \omega}\left(\frac{e^{i\omega t} + e^{-i\omega t}}{2}\right)$$

Mal, la idea será ⇒ $\underline{u} = Re\{U_1(y)$

$$\Rightarrow \quad i\omega U_1 e^{i\omega t} - i\omega U_2 e^{-i\omega t} = -\frac{b}{2\rho \nu}\left(e^{i\omega t} + e^{-i\omega t}\right) + \nu \frac{\partial^2 U_1}{\partial y^2} e^{i\omega t} + \nu \frac{\partial^2 U_2}{\partial y^2} e^{-i\omega t}$$

$$\Leftrightarrow \quad \begin{cases} \dfrac{\partial^2 U_1}{\partial y^2} - \dfrac{i\omega}{\nu} U_1 = \dfrac{b}{2\rho \nu} \\ \dfrac{\partial^2 U_2}{\partial y^2} + \dfrac{i\omega}{\nu} U_2 = \dfrac{b}{2\rho \nu} \end{cases}$$

$U_{1h} + U_{1p}$ $\quad U_{1p} = C_1 \Rightarrow -\frac{i\omega}{\nu} C = \frac{b}{2\rho \nu}$

$$\Leftrightarrow C_1 = \frac{ib}{2\omega \rho}$$

$U_{1h} = Ae^{\lambda y}$

$$\Rightarrow \lambda^2 = \frac{i\omega}{\nu} \Leftrightarrow \lambda = \pm \frac{\sqrt{(1+i)^2}}{\sqrt{2}}\sqrt{\frac{\omega}{\nu}} \qquad \sqrt{\frac{2\nu}{\omega}} = \delta$$

$$\Rightarrow U_1(y) = A_1 e^{\frac{(1+i)}{\delta} y} + B_1 e^{-\frac{(1+i)}{\delta} y} + \frac{ib}{2\omega\rho}$$

$$\underline{u}(d,t) = 0$$

$$\Rightarrow \mathrm{Re}\left\{\left(A_1 + B_1 + \frac{ib}{2\omega\rho}\right) e^{i\omega t}\right\} = 0 \quad \forall t$$

$$0 = A_1 + B_1 + \frac{ib}{2\omega\rho}$$

$$0 = A_1 e^{d/\delta} e^{id/\delta} + B_1 e^{-d/\delta} e^{-id/\delta} + \frac{ib}{2\omega\rho}$$