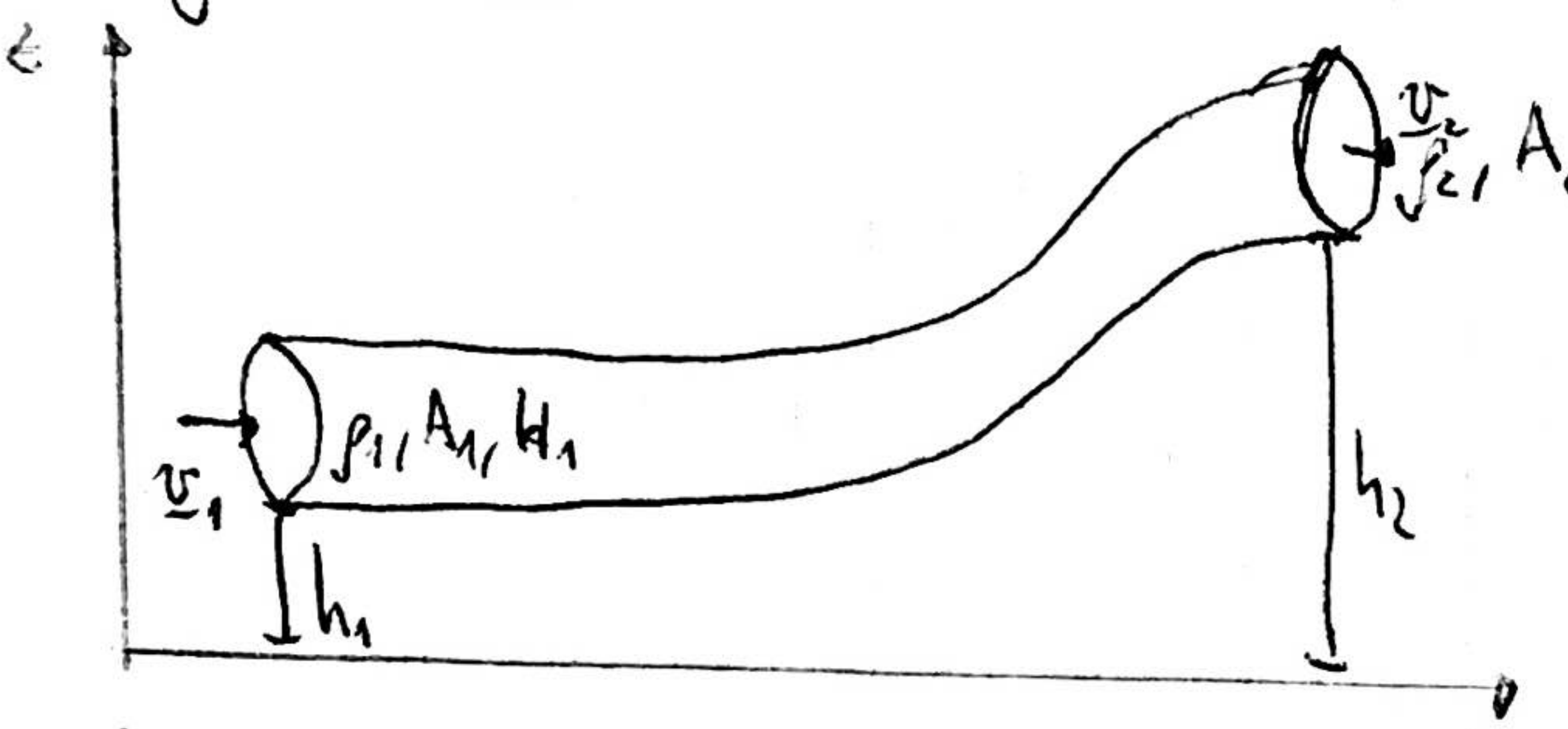
Guía 3

Ejercicio 3.1)



$\rho_0 = cte$

$A_1 > A_2$

Líquido incompresible, estacionario $\frac{\partial}{\partial t} = 0$ } conservación de la masa

Conservación de $\underline{P}$

Ecuación de Bernoulli cons. de E

Continuidad o cons de la masa

$\frac{\partial \rho}{\partial t} \underline{u} + \rho \underline{\nabla} \cdot \underline{u} = 0 \Rightarrow \rho_0 \underline{\nabla} \cdot \underline{u} = 0 \Leftrightarrow$ ~~$\int \nabla \cdot u \, dV = 0$~~

$$\Rightarrow \int_V \underline{\nabla} \cdot \underline{u} \, dV \overset{\text{gauss}}{=} \int_S \underline{u} \cdot d\underline{S} = 0 = \int_{S_1} \underline{u}_1 \cdot d\underline{S}_1 + \underbrace{\int_{S_{lat}} \underline{u} \cdot d\underline{S}_{lat}}_{=0} + \int_{S_2} \underline{u}_2 \cdot d\underline{S}_2$$

$=0$ pues consideramos $\underline{u} \parallel d\underline{l}$ y $d\underline{S}_{lat} \cdot \underline{u}_{lat} = 0$

$$\Rightarrow -v_1 A_1 + v_2 A_2 = 0 \Leftrightarrow \underbrace{v_1 A_1 = v_2 A_2}_{\text{conservación del caudal}} = Q$$

Bernoulli

$$\frac{u^2}{2} + \frac{p}{\rho} + \phi_m = \underset{\text{línea}}{C(t)} \qquad (B)$$

b) Para que haya flujo $Q > 0$

de (B) $\frac{u_1^2}{2} + \frac{p_1}{\rho_0} + g h_1 = \frac{u_2^2}{2} + \frac{p_2}{\rho_0} + g h_2$

$$\Leftrightarrow \frac{u_1^2}{2} + \frac{u_1^2 A_1^2}{2 A_2^2}(-1) = \frac{1}{\rho_0}(p_2 - p_1) + g(h_2 - h_1)$$

$$\Leftrightarrow u_1^2 \left(\frac{A_2^2 - A_1^2}{A_2^2} \right) = \frac{2}{\rho_0}(p_2 - p_1) + 2g(h_2 - h_1)$$

$\Rightarrow Q = A_1 u_1$

$$\Leftrightarrow u_1^2 = \frac{A_2^2}{(A_2^2 - A_1^2)} \left[\frac{2}{\rho_0}(p_2 - p_1) + 2g(h_2 - h_1) \right]; \quad u_1 = \frac{A_2}{\sqrt{A_2^2 - A_1^2}} \left[\frac{2}{\rho_0}(p_2 - p_1) + 2g(h_2 - h_1) \right]^{1/2}$$

$$\Rightarrow Q = \frac{A_2 A_1}{\sqrt{A_2^2 - A_1^2}} \left[\frac{2}{\rho_0}(p_2 - p_1) + 2g(h_2 - h_1) \right]^{1/2}$$

Condición de existencia de flujo

$A_2 > A_1 \quad p_2 - p_1 > g\rho_0(h_1 - h_2)$

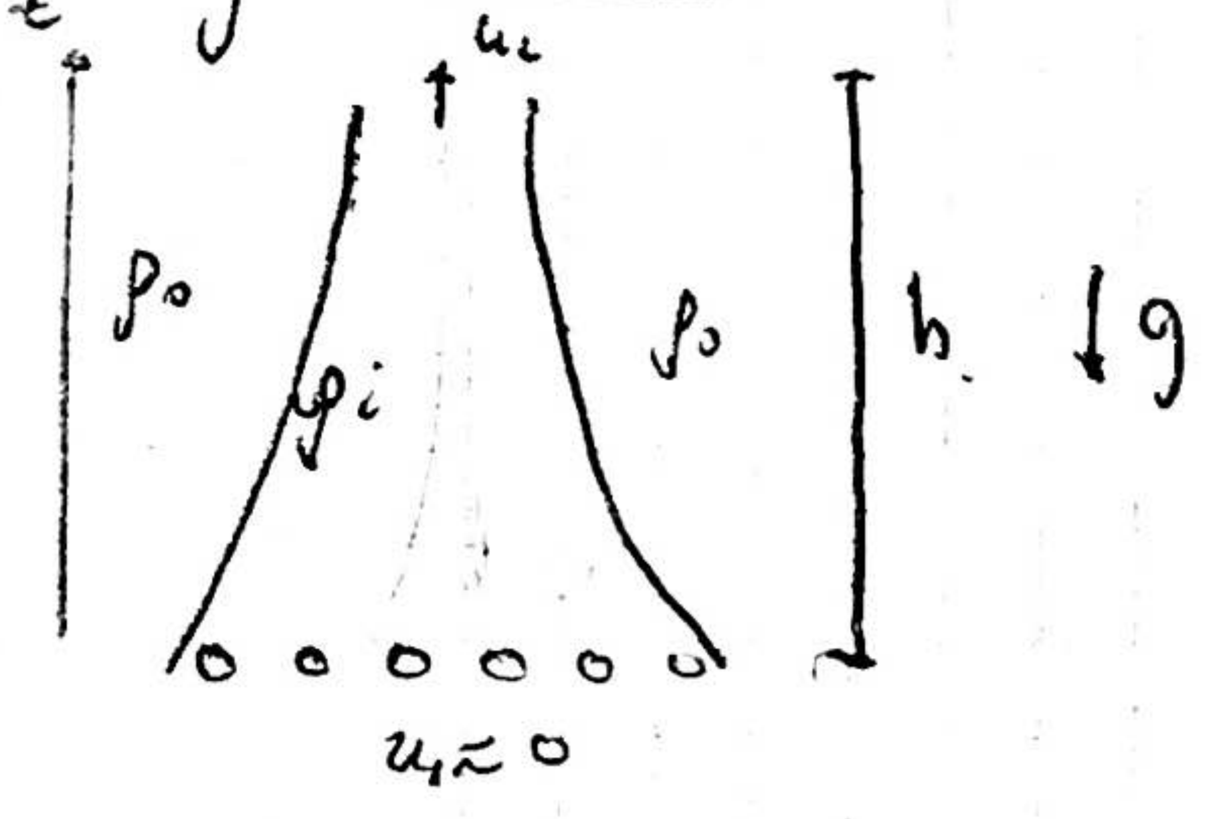
Sí o sí $A_2 > A_1$

$\Rightarrow$ Si $\frac{2}{\rho_0}(p_2 - p_1) + 2g(h_2 - h_1) > 0 \Rightarrow \Delta p > g\rho_0(h_1 - h_2)$

c) si $h_1 \neq h_2 \Rightarrow p_2 > g\rho_0(h_1 - h_2) + p_1$

d) Si $A_1 = A_2 \Rightarrow u_1 = u_2$ y tenemos flujo estacionario, osea $Q > 0$ y $Q = cte.$

Ejercicio 3.2)



si tenemos flujo estacionario $\frac{\partial}{\partial t} = 0$

~~$Q = cte$~~

~~$Q =$~~ u_2.

Supongo un gas ~~perfecto~~ ideal $\Rightarrow p = \rho \frac{RT}{m} = \rho_i \frac{RT}{m}$

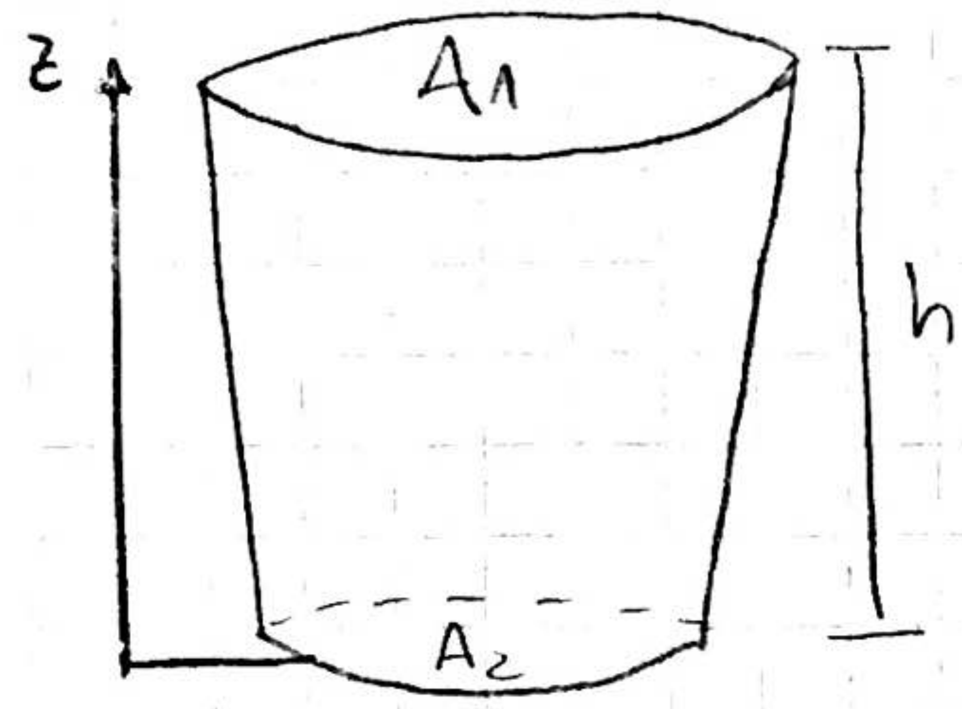
$\Rightarrow$ Bernoulli

$$\frac{|\underline{u}_1|^2}{2} + \varphi_{g_1} + P_1 = \frac{|\underline{u}_2|^2}{2} + \varphi_{g_2} + P_2 = C_{linea}$$

$$\Rightarrow 0 + g.0 + \frac{P_1}{\rho_1} = \frac{u_2^2}{2} + gh + \frac{P_2}{\rho_1} \Rightarrow u_2^2 = 2\left(\frac{P_1}{\rho_1} - \frac{P_2}{\rho_1}\right) - 2gh$$

$$\therefore u_2 = \sqrt{2\left[\left(\frac{P_1 - P_{atm}}{\rho_i}\right) - gh\right]}$$

Ejercicio 3.3



$t>0$ hay flujo

Hipótesis de la clase
- $\underline{v}$ en A_1, A_2 direccionado en z
- no hay $\underline{E}$ no conservativas
- $\underline{u} = 0$ irrotacional $\Rightarrow \underline{\omega} = 0$
  $\underline{u} = \nabla \phi$
- viscosidad despreciable

Cuando $A_1 = (1+\varepsilon)A_2$ $\varepsilon \ll 1$ y despreciando $\underline{u}_{horizontal}$

Suponga que la variación es lineal, osea, obtenga que $A(z) = (1+\varepsilon \frac{z}{h})A_2$

- Fluído incompresible $\Rightarrow \frac{d\rho}{dt} = 0$

- $t=0$ evento

$A_1 = (1+\frac{\varepsilon z}{h})A_2$

- Conservación de Q $\Rightarrow \underline{\omega} = 0 \therefore \underline{u} = \nabla\varphi$

Además supone $u_x, u_y = 0$
$\Rightarrow \underline{u} = u\hat{z} = \partial_z \varphi$

Bernoulli $\frac{\partial\varphi}{\partial t} + \frac{|\underline{u}|^2}{2} + \varphi_m + \mathcal{P} = C_{línea}(t)$

$$\therefore \frac{\partial\varphi}{\partial t} + \frac{\left(\frac{\partial\varphi}{\partial z}\right)^2}{2} + \frac{p}{\rho} + \varphi_m = C(t)$$

$Q_1(z,t) = Q_2(z,t)$

$\Rightarrow u_1 A_1 = u_2 A_2$ $\quad A = A(z)$ (no se deforma)

Como entra lo mismo que sale $u(z,t) \approx u(t)$

$\therefore$ JUNTANDO

$$(1)\; u_1(t) A_1(z) = u_2(t) A_2(z) \;,\; (2)\; u(t)\hat{z} = \frac{\partial\varphi(z,t)}{\partial z} \;,\; (3)\; \frac{\partial\varphi}{\partial t} + \frac{\left(\frac{\partial\varphi}{\partial z}\right)^2}{2} + \varphi_v + \frac{p}{\rho} = C(t)$$

$u_1 = u_2$

$$\therefore (1)(2) \Rightarrow \frac{\partial\varphi}{\partial z}(z,t) = u(t)\frac{A_2(z)}{A_1(z)} = \frac{A_2}{(1+\frac{\varepsilon z}{h})A_2} u(t)$$

$$\therefore \frac{\partial\varphi}{\partial z}(z,t) = u(t)\frac{1}{(1+\frac{\varepsilon z}{h})} \Leftrightarrow \int_{\varphi_0(z_0,t)}^{\varphi(z,t)} \partial\varphi = \int_0^z \frac{u(t)}{1+\frac{\varepsilon z}{h}} \partial z$$

$$\Rightarrow \varphi(z,t) - \varphi_0(t) = u(t)\frac{h}{\varepsilon}\ln\left(1+\frac{\varepsilon z}{h}\right)$$

Reemplazando en (3) y haciendo $\varphi(0,t)=\varphi(h,t)$

$$\frac{\partial u(t)}{\partial t}\ln\left(1+\frac{\varepsilon z}{h}\right)\frac{h}{\varepsilon}+\frac{\partial \varphi_0}{\partial t}(t)+\frac{1}{2}\left(\frac{u(t)}{1+\frac{\varepsilon z}{h}}\right)^2+\frac{P_0}{\rho}+gz=C(t)$$

$0=h$

$$\Leftrightarrow \underbrace{\frac{\partial u}{\partial t}\ln(1)}_{=0}+\frac{\partial \varphi_0(t)}{\partial t}+\frac{u^2(t)}{2(1)^2}+\frac{P_{atm}}{\rho_0}+g.0=\frac{\partial u}{\partial t}(t)\ln(1+\varepsilon)\frac{h}{\varepsilon}+\frac{\partial \varphi_0}{\partial t}(t)+\frac{u^2(t)}{2(1+\varepsilon)^2}+\frac{P_{atm}}{\rho_0}+gh$$

$$\Leftrightarrow \frac{u^2(t)}{2}\left[1-\frac{1}{(1+\varepsilon)^2}\right]=\frac{\partial u}{\partial t}(t)\ln(1+\varepsilon)\frac{h}{\varepsilon}+gh$$

$$\Leftrightarrow \frac{\partial u}{\partial t}(t)-\frac{u^2(t)}{2}\left[1-\frac{1}{(1+\varepsilon)^2}\right]\cdot\frac{1}{\ln(1+\varepsilon)}\frac{\varepsilon}{h}+\frac{g\varepsilon}{\ln(1+\varepsilon)}=0$$

Usando aproximaciones $\ln(1+\varepsilon)\approx\varepsilon$

$$\frac{1}{(1+\varepsilon)^2}\approx 1-2\varepsilon$$

$$\Rightarrow \frac{\partial u}{\partial t}(t)-u^2(t)\frac{\varepsilon}{h}+g=0$$

No me gusta aproximar $\Rightarrow$ llamo $\left[1-\frac{1}{(1+\varepsilon)^2}\right]\frac{1}{2\ln(1+\varepsilon)}\cdot\frac{\varepsilon}{h}=\gamma$ $\qquad \frac{\varepsilon}{\ln(1+\varepsilon)}=\Omega$

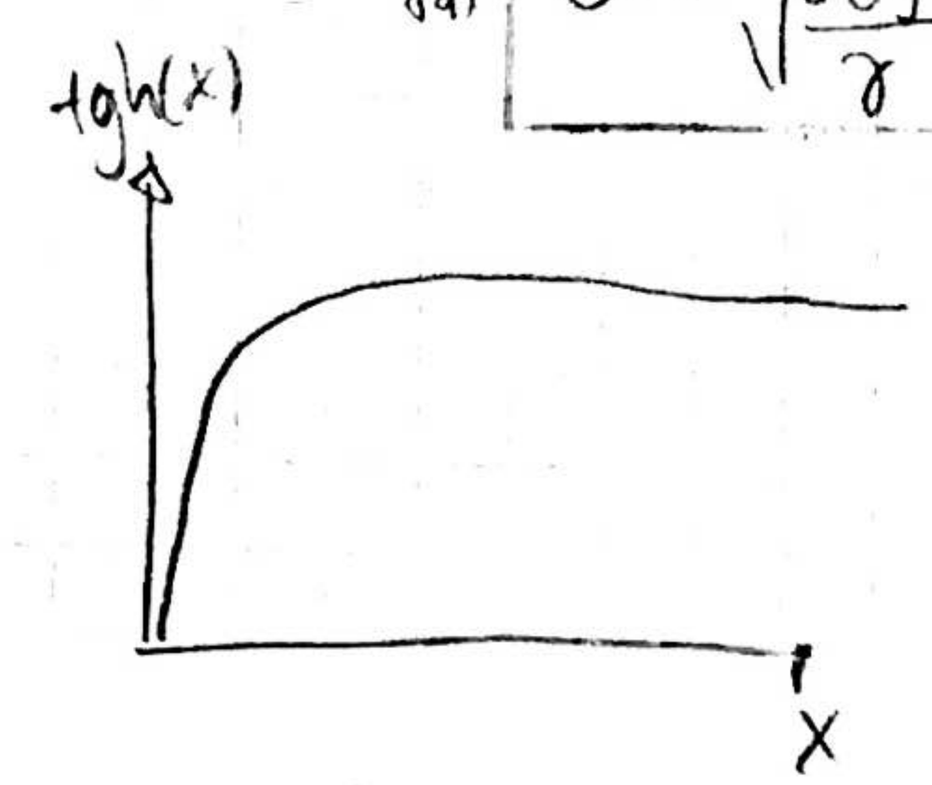
$$\Rightarrow \dot{u}-u^2\gamma+\Omega g=0 \quad\Leftrightarrow\quad \frac{\dot{u}}{\gamma u^2-\Omega g}=1$$

$$\Rightarrow \int\frac{du}{\gamma u^2-\Omega g}=\int dt \quad\Leftrightarrow\quad \int\frac{du}{\left(u^2-\frac{\Omega g}{\gamma}\right)}=\int\gamma\, dt$$

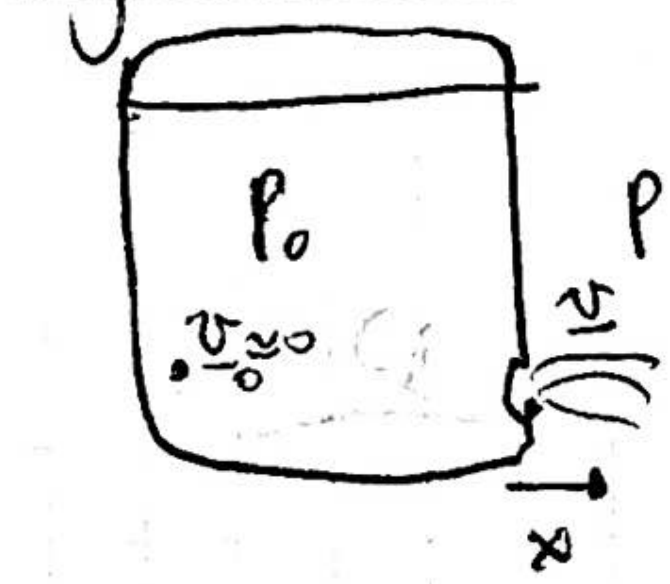
$$\Leftrightarrow -\frac{\operatorname{arctgh}\left(\frac{u}{\sqrt{\frac{\Omega g}{\gamma}}}\right)}{\sqrt{\frac{\Omega g}{\gamma}}}=\gamma(t-t_0)+C \quad\Leftrightarrow\quad \operatorname{arctgh}\left(\frac{u}{\sqrt{\frac{\Omega g}{\gamma}}}\right)=-\sqrt{\gamma\Omega g}\,(t-t_0)+C$$

$$\Leftrightarrow \boxed{u=\sqrt{\frac{\Omega g}{\gamma}}\cdot\operatorname{tgh}\left(-\sqrt{\gamma\Omega g}\,(t-t_0)+C\right)}$$

sí, pues llega a un estacionario



Ejercicio 3.4



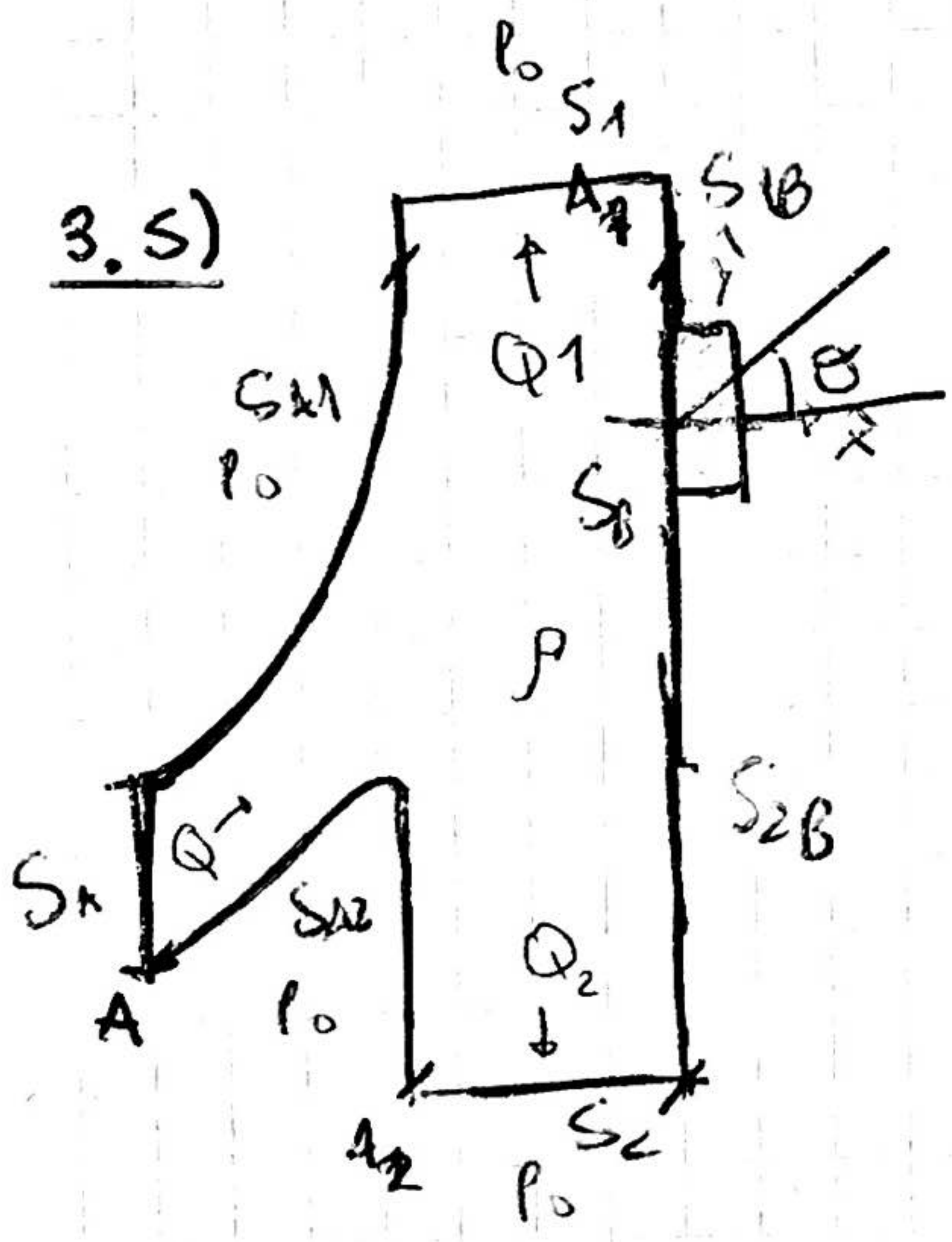
$\underline{v} = v\hat{x}$

$\Rightarrow$ Bernoulli

$$\frac{|\underline{v}_0|^2}{\underset{\approx 0}{2}} + \frac{P_0}{\rho} + gz = \frac{|\underline{v}|^2}{2} + \frac{P}{\rho} + gz_{sal}$$

$$\Rightarrow |\underline{v}| = \sqrt{2\left(\frac{P_0 - P}{\rho}\right) + 2g(z - z_{sal})}$$

3.5)



Hipotesis chart

- líquido incompresible $\frac{d\rho}{dt} = 0$
- fluido ideal $\Rightarrow$ no $\exists$ viscosidad
- no $\exists$ $\underline{f}$ externas
- flujo estacionario

Datos: A, Q, ρ

Suponemos que en zonas A, A_1, A_2 la velocidad del fluido es cuasi uniforme y

$\Rightarrow$ por conservación de caudal $\quad Q = Q_1 + Q_2 \Leftrightarrow uA = u_1A_1 + u_2A_2$

Por otro lado, como ρ = uniforme y estamos en estacionario podemos aplicar Bernoulli:

$\Rightarrow \exists$ $\quad \frac{u^2}{2} + \frac{P_0}{\rho} = \frac{u_1^2}{2} + \frac{P_0}{\rho} = C_1$ $\quad$ ☆ (en la guía)

$\frac{u^2}{2} + \frac{P_0}{\rho} = \frac{u_2^2}{2} + \frac{P_0}{\rho} = C_2$

$\Leftrightarrow u = u_1$
$\quad u = u_2$ $\quad \Rightarrow A_1 + A_2 = A$

$\Rightarrow u(A) = u(A_1 + A_2)$

$\underline{F}$ sobre la placa / eq $\quad$ Para que permanezca en equilibrio la fuerza que aplica la placa debe ser = a la que aplica el fluido sobre la placa en | | y $\neq$ signo

De la conservación del impulso $\qquad T_{ij} = -p\delta_{ij} - \rho u_i u_j$

$$\underbrace{\frac{dP}{dt} = 0}_{\text{estacionario}} \Rightarrow \int_V \frac{dP}{dt} = \underbrace{\int_V f\underline{v}\, dV}_{=0} + \oint_{S(V)} \underline{\underline{T}} \cdot d\underline{S}$$

$$\Rightarrow 0 = \oint_{S(V)} -\rho \underline{u}(\underline{u}\cdot\hat{n})dS - \oint_{S(V)} p\cdot\hat{n}\, dS$$

observación $\quad -\oint_S p_0 \hat{n}\, dS = -\oint_S (p-p_0)\hat{n}\, dS \qquad$ Dem $\quad \oint_S p_0\hat{n}\, dS = 0 = \oint_V \nabla p_0 \hat{n}\, dV = 0$

$$\Rightarrow 0 = \oint_{S(V)} -\rho \underline{u}(\underline{u}\cdot\hat{n})dS - \oint_{S(V)} (p-p_0)\cdot\hat{n}\, dS$$

$S = S_A \cup S_{A1} \cup S_1 \cup S_{1B} \cup S_B \cup S_{2B} \cup S_{A2} \cup S_{A_2}$

Sobre $S_{A_1}, S_{A_2}, S_{1B}, S_{2B}, S_B \quad \underline{u} \perp \hat{n}$

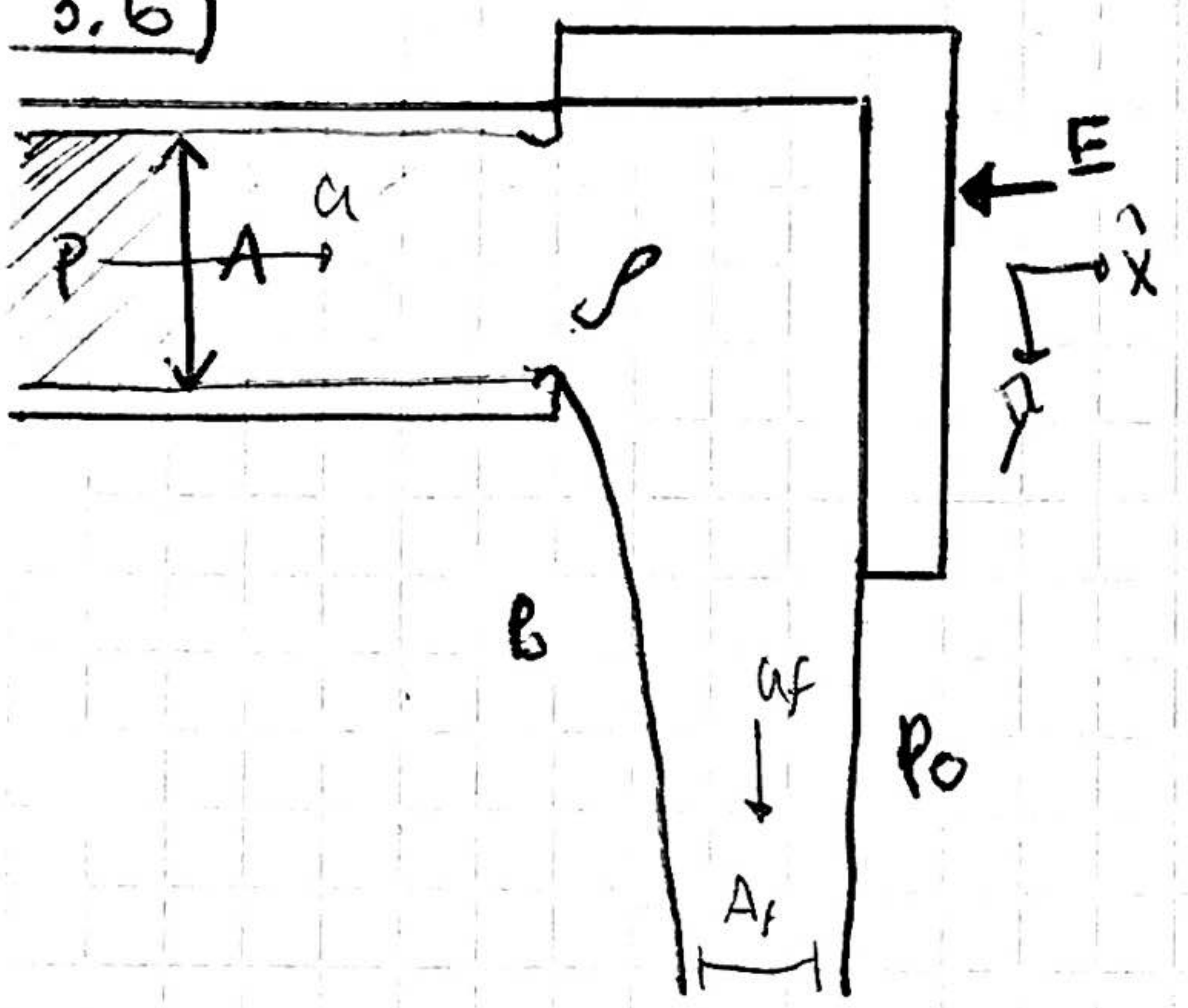
Sobre $S_{A_1}, S_{A_2}, S_{1B}, S_{2B} \quad p = p_0$; También sobre S_A, S_1, S_2

$$\Rightarrow 0 = \int_{SA} u^2 \rho\, dS\, (\cos(\theta)\hat{x} + \sin(\theta)\hat{y}) + \int_{S_1} \rho u_1^2 \hat{y}\, dS_1 - \int_{S_2} \rho u_2^2 \hat{y}\, dS_2 + \oint (p-p_0)\hat{n}\, dS$$

$$\Rightarrow \rho u^2 A(\cos(\theta)\hat{x} + \sin(\theta)\hat{y}) + \rho u_1^2 A_1 \hat{y} - \rho u_2^2 A_2 \hat{y} + \underbrace{\int_{S_B} (p-p_0)\cdot\hat{n}\, dS}_{\underline{E}} = 0$$

$$\boxed{\Rightarrow \begin{cases} \underline{E} = -\rho u^2 A \cos(\theta)\hat{x} \\ 0 = \dfrac{Q^2}{A}\sin(\theta) + \dfrac{Q_1^2}{A_1} - \dfrac{Q_2^2}{A_2} \\ A = A_1 + A_2 \\ Q = Q_1 + Q_2 \end{cases}}$$

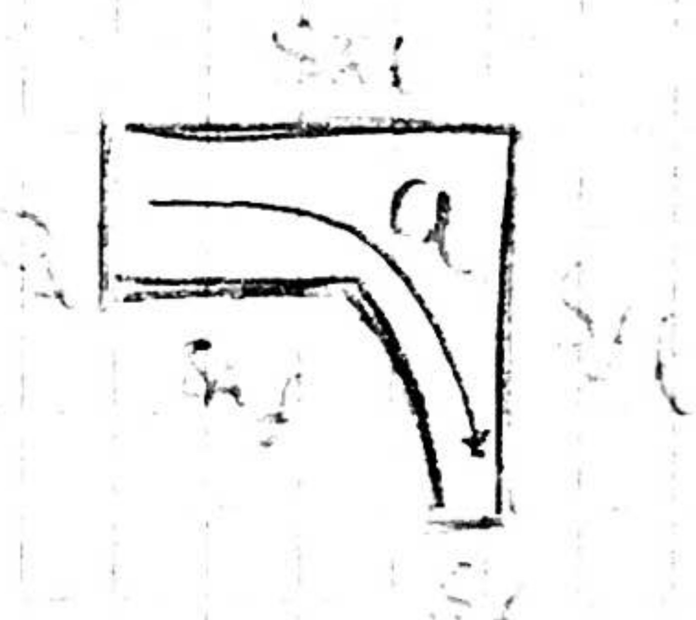
3.6)



Hipótesis chart
- líquido ideal $\Rightarrow T_{ij} = -\rho u_i u_j - p\delta_{ij}$
- incompresible $\Rightarrow \frac{d\rho}{dt} = 0$
- flujo estacionario
- la cañería se angosta lentamente de forma que $\underline{u}$ = uniforme

$$uA = A_f u_f$$

flujo estacionario $\frac{dP}{dt} = 0 \Rightarrow \int_V \underbrace{\rho \underline{f}}_{=0} dV + \oint_{S(V)} \underline{T} \cdot d\underline{s}$



$$\Leftrightarrow -\left[\oint_S \rho \underline{u}(\underline{u}\cdot\hat{n})dS + \underbrace{\oint_S p\cdot\hat{n}dS}_{E_{caño}}\right] = 0$$

$$\Rightarrow E_{caño} + \oint_S \rho\underline{u}(\underline{u}\cdot\hat{n})dS = 0$$

en S_A $\underline{u} // \hat{n}$

S_f $\underline{u}_f // \hat{n}$

S_{Af}, S_{fl}, S_{Al} $\underline{u} \perp \hat{n}$ (suponiendo $\underline{u}$ "unidireccional")

$$\Rightarrow E_{caño} + \rho u^2 A\hat{x} + \rho u_f^2 A_f \hat{y} = 0$$

$$\underline{E} = -\rho u^2 A\hat{x} - \rho u_f^2 A_f \hat{y} \qquad u_f = \frac{uA}{A_f}$$

$$uA = A_f u_f = Q = \text{cte}$$

$$\Rightarrow \underline{E} = -\rho u^2 A\hat{x} - \rho\frac{u^2A^2}{A_f}\hat{y}$$

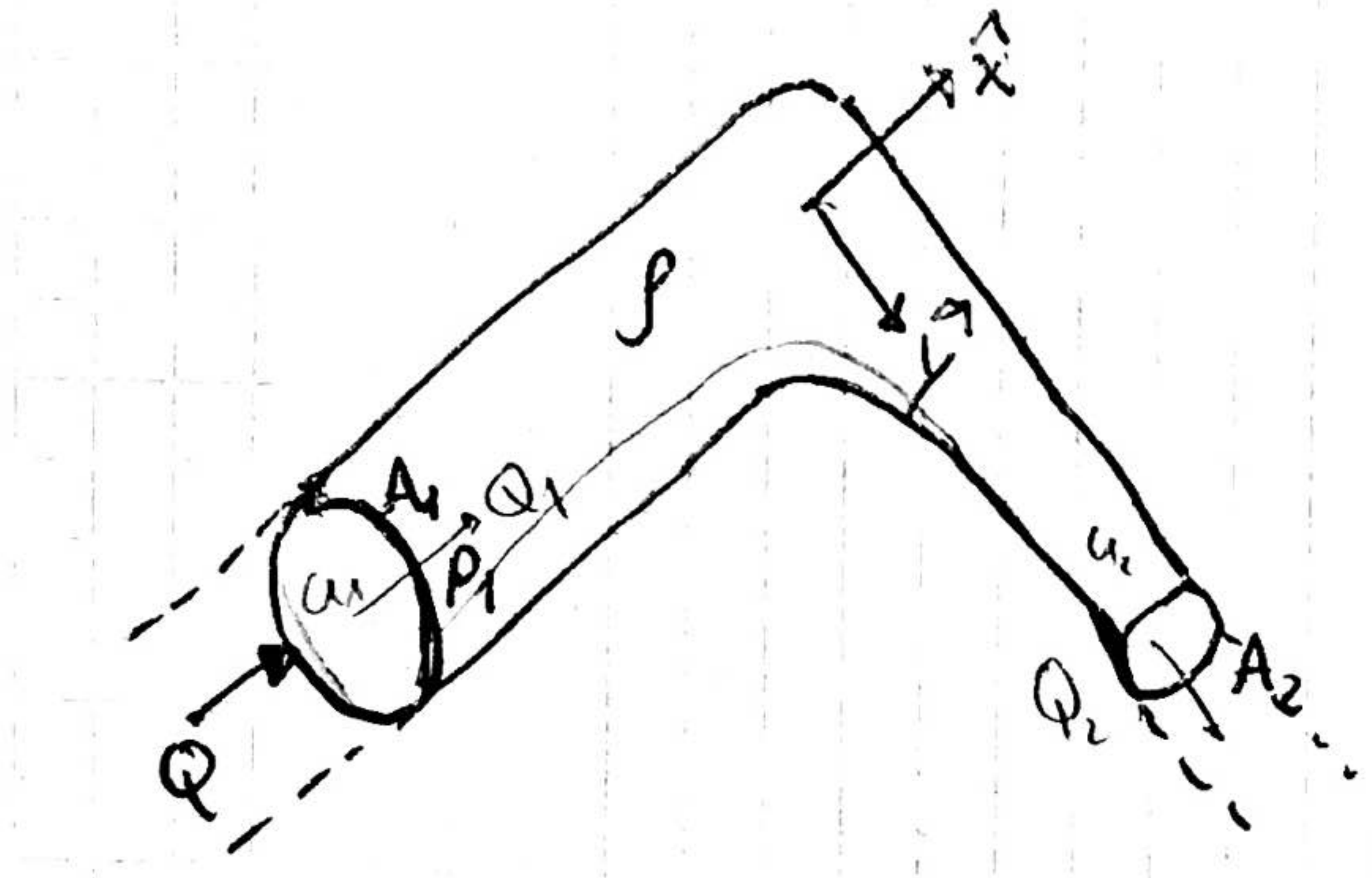
$$uA = Q$$

+ Bernoulli

$$\frac{u^2}{2} + \frac{p}{\rho} = \frac{u_f^2}{2} + \frac{P_0}{\rho} = c(t)$$

$\Rightarrow$ 3 ec, 3 incógnitas ✓

3,7)



Hipótesis chat

- líquido ideal $\Rightarrow$ no $\exists$ viscosidad
- tubería en ángulo recto
- u_1, u_2 sobre las tapas es prácticamente uniforme en superficie
- flujo estacionario

- Conservación de caudal

$Q = cte \Rightarrow Q_1 = Q_2 \Rightarrow u_1 A_1 = u_2 A_2$

- ~~Bernoulli en flujo estacionario~~ no se usa, varía ρ

$$\cancel{\frac{u_1^2}{2} + \frac{p_1}{\rho} = \frac{u_2^2}{2} + \frac{p_2}{\rho} = C(t) = C_l}$$

- Cant de mov $\frac{dP}{dt} = 0 \Rightarrow -\Big[\underbrace{\int_V \frac{\partial u}{\partial t} dV}_{=0} - \oint_S \underline{T}\cdot d\underline{S}\Big] = 0$ $\qquad T_{ij} = -p\delta_{ij} - \rho u_i u_j$

$$\Rightarrow \oint_S -\rho \underline{u}(\underline{u}\cdot\hat{n})dS - \underbrace{\oint_S p\hat{n}dS}_{\underline{F}} = 0$$

$\underline{u} \perp \hat{n}$ en S_{lat}, S_{lat}

$$\Rightarrow \underline{F} + \int_{S_1} \rho \underline{u}_1(\underline{u}_1\cdot\hat{x})dS + \int_{S_2} \rho \underline{u}_2(\underline{u}_2(+\hat{y})) = 0$$

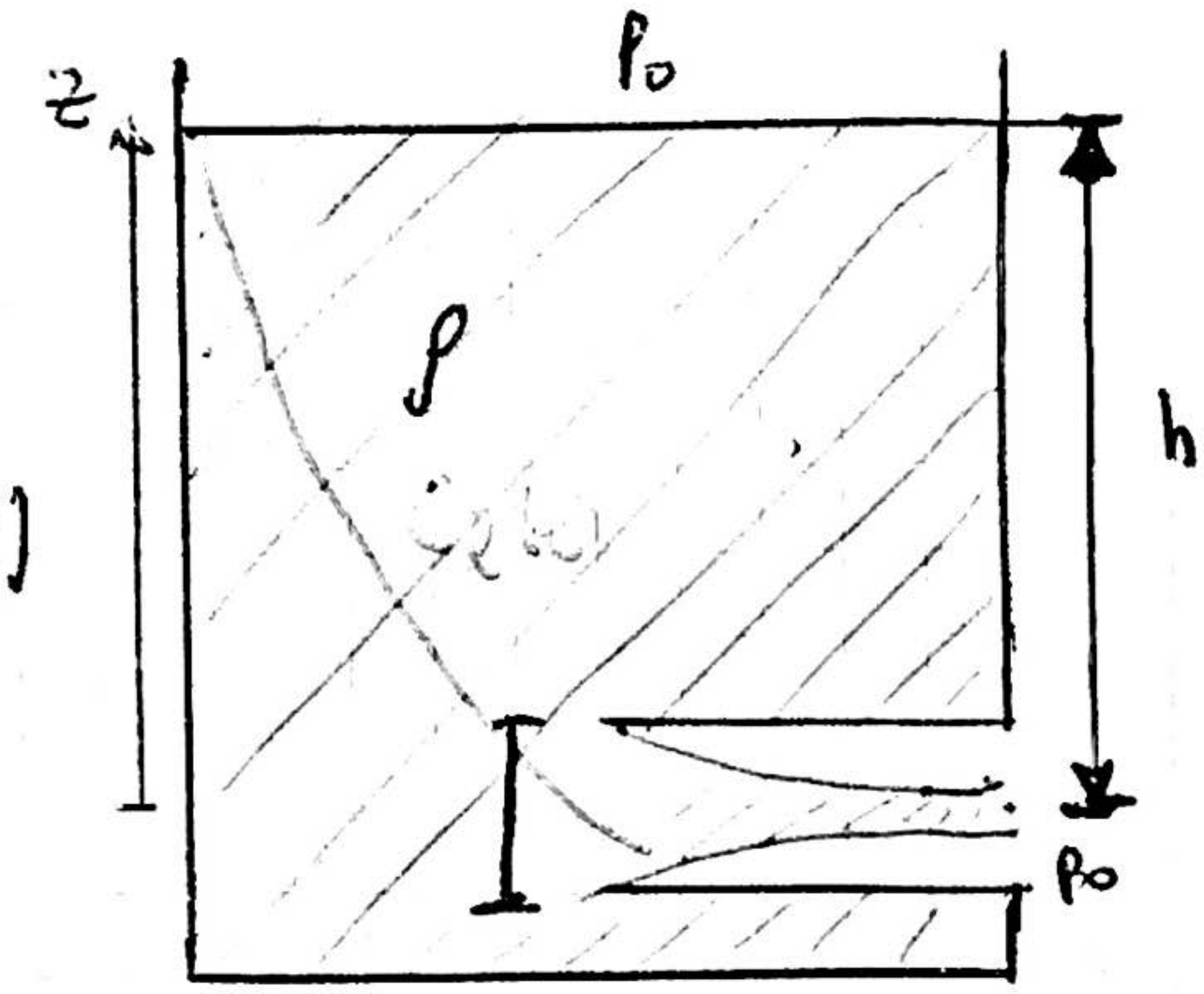
$$\Rightarrow \underline{F} = -\rho u_1^2 A_1 \hat{x} - \rho u_2^2 A_2 \hat{y}$$

$$\Rightarrow \boxed{\begin{array}{l} \underline{F} = -\rho u_1^2 A_1 \hat{x} - \rho u_2^2 A_2 \hat{y} \\ Q = u_1 A_1 \\ u_1 A_1 = u_2 A_2 \end{array}}$$

u_1, u_2, F incógnitas ✓

$$\frac{u_1^2}{2} + \frac{p_1}{\rho} = \frac{u_2^2}{2} + \frac{p_2}{\rho}$$

3.8)



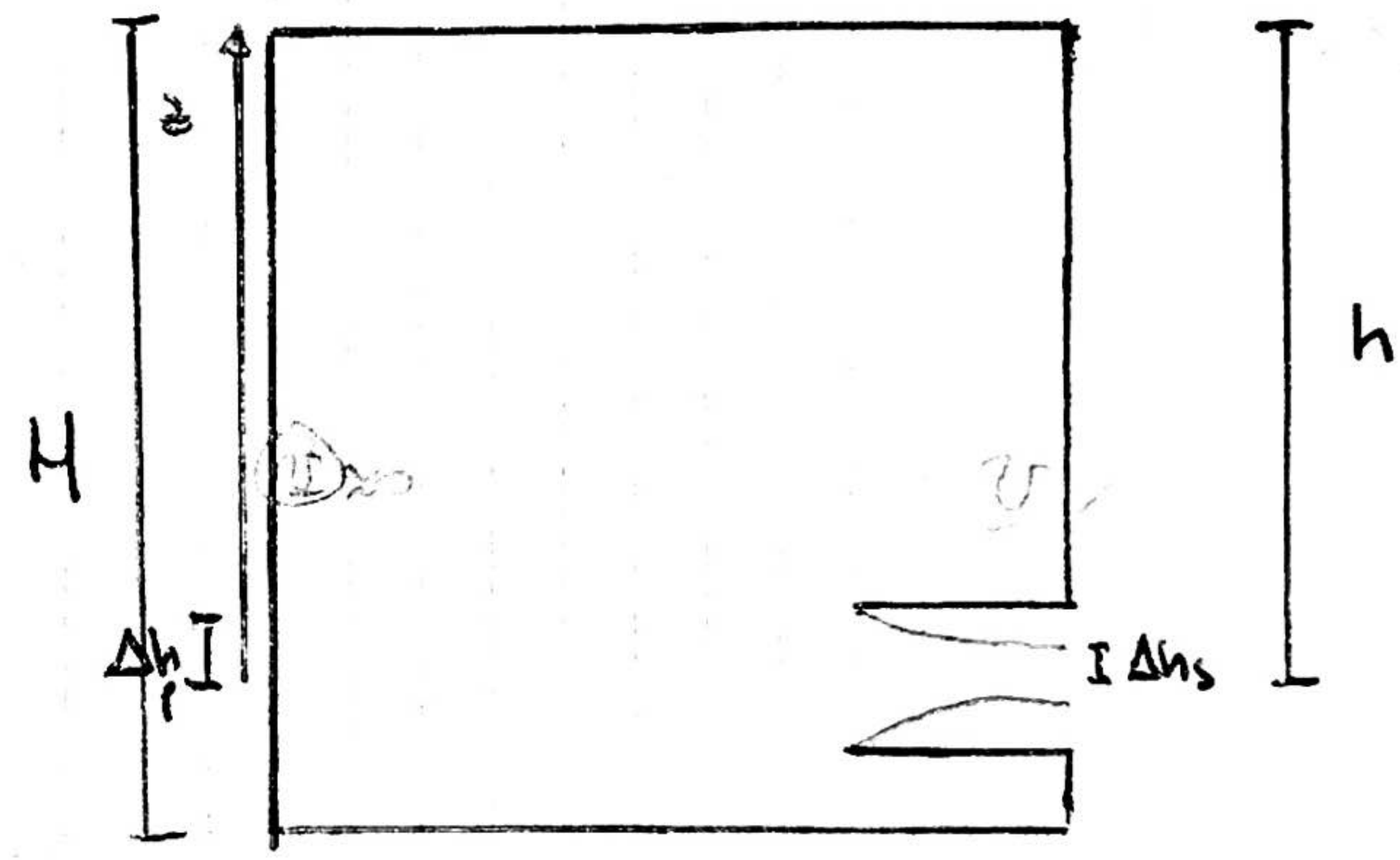
Hipótesis chart

- líquido incompresible $\Rightarrow \frac{d\rho}{dt} = 0$
- flujo estacionario en períodos de t considerables
- tanque grande / $\underline{v}$ superficie ≈ 0

a) ~~Cllt~~ $\frac{|\underline{u}_{sup}|^2}{2} + \frac{p_0}{\rho} \pm mgh = \frac{|\underline{u}_{sal}|^2}{2} + \frac{p_0}{\rho}$, con $\frac{|\underline{u}_{sup}|^2}{2} \approx 0$

Bernoulli estacionario

$\therefore \boxed{|u_{sal}| \approx \sqrt{2mgh}}$



b) En estacionario

$$\frac{\partial P}{dt} = 0 = \underline{f}_v + \underline{\nabla} \cdot \underline{\underline{T}}$$

$$\Rightarrow 0 = \int_V \underline{f}_v \, dV + \oint_{S(V)} \underline{\underline{T}} \cdot d\underline{S}$$

$$\Rightarrow \int_V -\rho g \hat{z} \, dV + \left(-\oint_{S(V)} \rho \underline{u}(\underline{u} \cdot \hat{n}) dS - \oint_S p \, \hat{n} \, dS \right) = 0$$

Además de hidrostática

$$-\rho g + \underline{\nabla} p = 0$$

$$\Rightarrow p = -\rho g z + C$$

con $p(h) = p_0 \Rightarrow C = +\rho g h + p_0$

$$\Rightarrow p = \begin{cases} p_0 & z > h \\ -\rho g(z-h) + p_0 & z < h \end{cases}$$

$$\Rightarrow -\rho g V_\ell \hat{z} - \oint_{S(V)} \rho \underline{u}(\underline{u} \cdot \hat{n}) dS - \oint_{S(V)} (p - p_0) \hat{n} \, dS =$$

suponiendo $\underline{u} \perp \hat{n}$ salvo en las salidas

$V_\ell \cong H \cdot A_\ell$ A_ℓ es el área de todo el pote sin tener abertura

~~$V_\ell = A_\ell H$~~

A_p es el área de la ranura / $A_\ell + A_p$ completan el recipiente

$\Rightarrow V_p \approx A_p \Delta h_p 2$

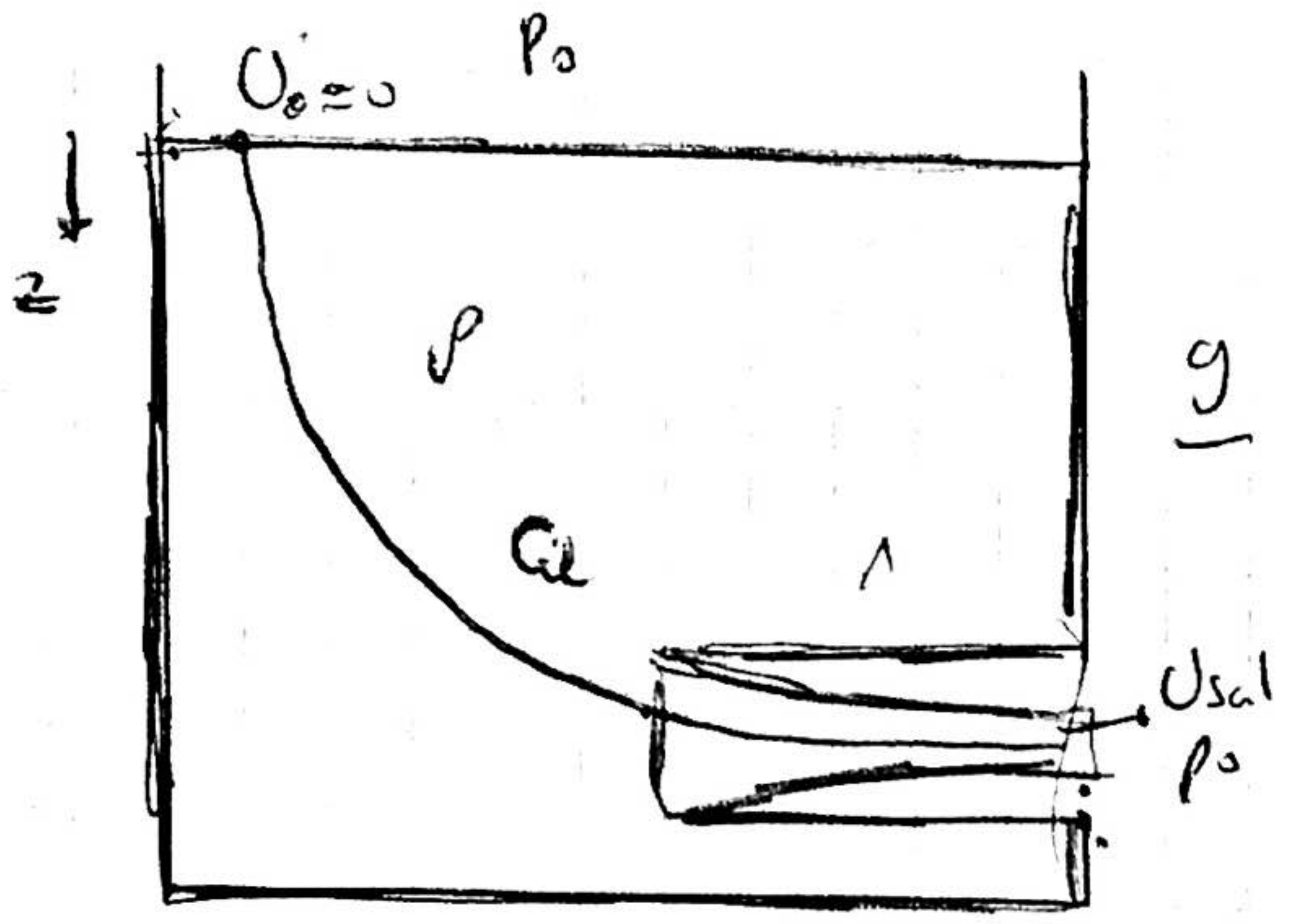
Y $V_s = A_0 \Delta h_s 2$ deducible

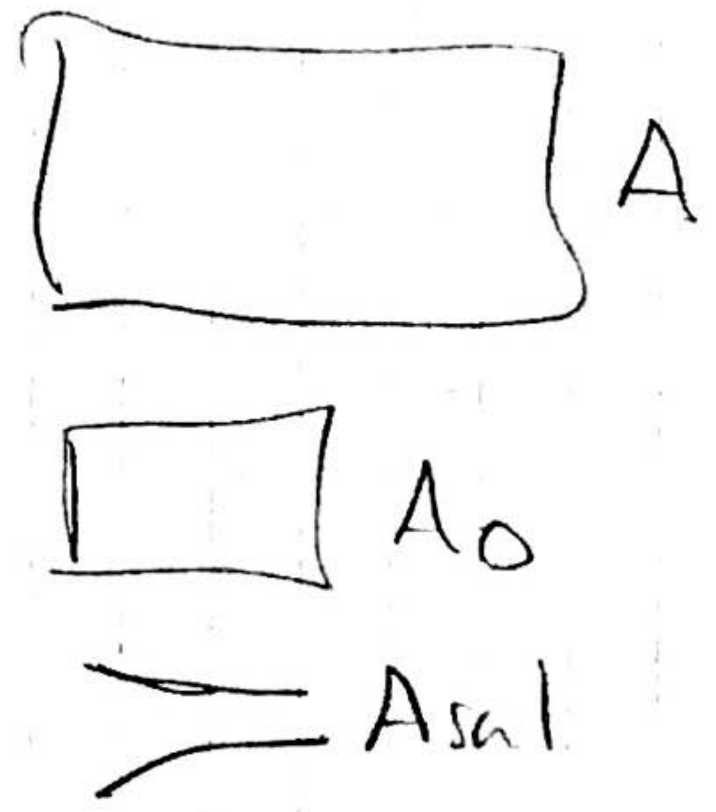
$$A_\ell >> A_p > A_s$$

$-\rho g H A \hat{z} - A_0 \rho u_{sal}^2 \hat{x} - \ldots$ cosas recordemos solo importa lo en x

$\Rightarrow \quad -A_0 \rho u_{sal}^2 \hat{x} + \int_0^h -\rho g (z-h)\, dz\, \hat{x} \cdot A_0 =$

3.8)





a) Como el recipiente es grande, puedo suponer el flúido quieto sobre periodos de t

$\Rightarrow$ además U_{sal} la supongo uniforme y unidireccional

i) incompresible $\Rightarrow \frac{D\rho}{Dt} = 0 \therefore \rho = cte$

ii) $\underline{F_m} = \underline{g}$ = conservativa

iii) desprecio viscosidad

iv) flujo estacionario por dtn

$\Rightarrow$ Bernoulli $\quad \frac{1}{2}|U|^2 + \frac{p}{\rho} + \varphi_m = C_{\underline{\text{línea}}}$

$\Rightarrow \frac{U_0^2}{2} + \frac{p_0}{\rho} + \rho gh = \frac{U_{sal}^2}{2} + \frac{p_0}{\rho} \quad \Leftrightarrow \boxed{U_{sal} = \sqrt{2gh}}$ $2gh$

b) $\oint_S \rho \underline{u}(\underline{u}\cdot\hat{n})ds + \oint_S (p-p_0)\cdot\hat{n}ds = 0$ → solo importan las componentes en x

p con hidrostática $\quad 0 = +\rho g\hat{z} - \underbrace{\nabla p}_{presión} \quad \Rightarrow \quad p = \rho g z + C$

si $z=0 \quad p = p_0$

$$\Rightarrow p = \begin{cases} p_0 & z>0 \\ \rho g z + p_0 & z<0 \end{cases}$$

$\Rightarrow \oint (p-p_0)\cdot\hat{n}ds$ Se anula en $z=0$ y

Solo importa $p(h) \Rightarrow -\rho g h A_0 \hat{x}$

de la U solo importa la que sale $\Rightarrow \rho u_s^2 A_s \hat{x} - \rho g h A_0 \hat{x} = 0$

$\Rightarrow 2gh\rho = \rho g h \frac{A_0}{A_s}$ $\quad \Leftrightarrow \boxed{\frac{A_s}{A_0} = \frac{1}{2}}$