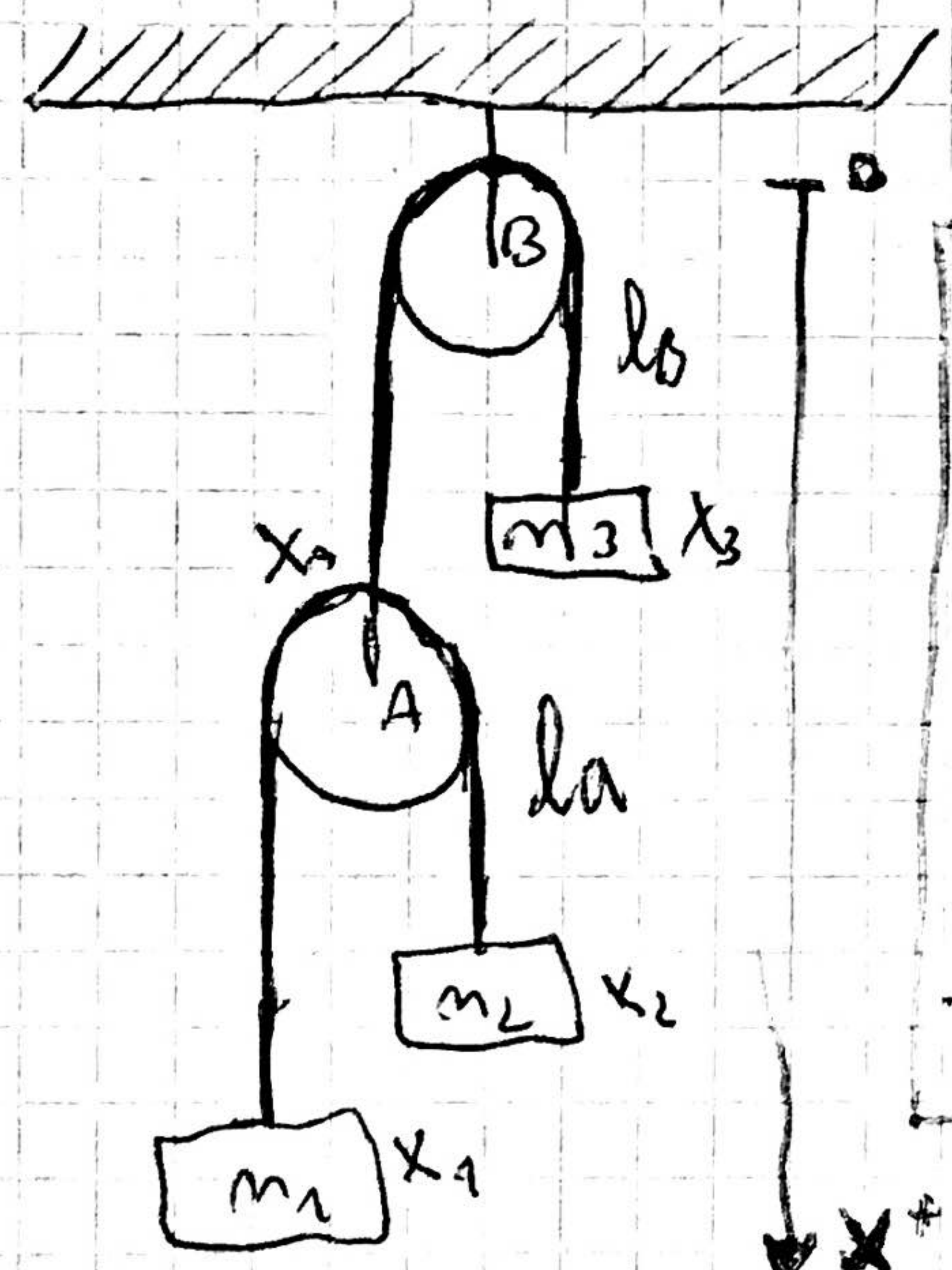
Dinámica

1) Inicialmente reposo, poleas e hilos masas despreciables y hilos inextensibles



$l_B = x_{3(t)} + x_A \Rightarrow 0 = \dot{x}_{3(t)} + \dot{x}_A = \ddot{x}_{3(t)} + \ddot{x}_A$

$\cancel{l_A = x_{2(t)} + x_{1(t)}}$

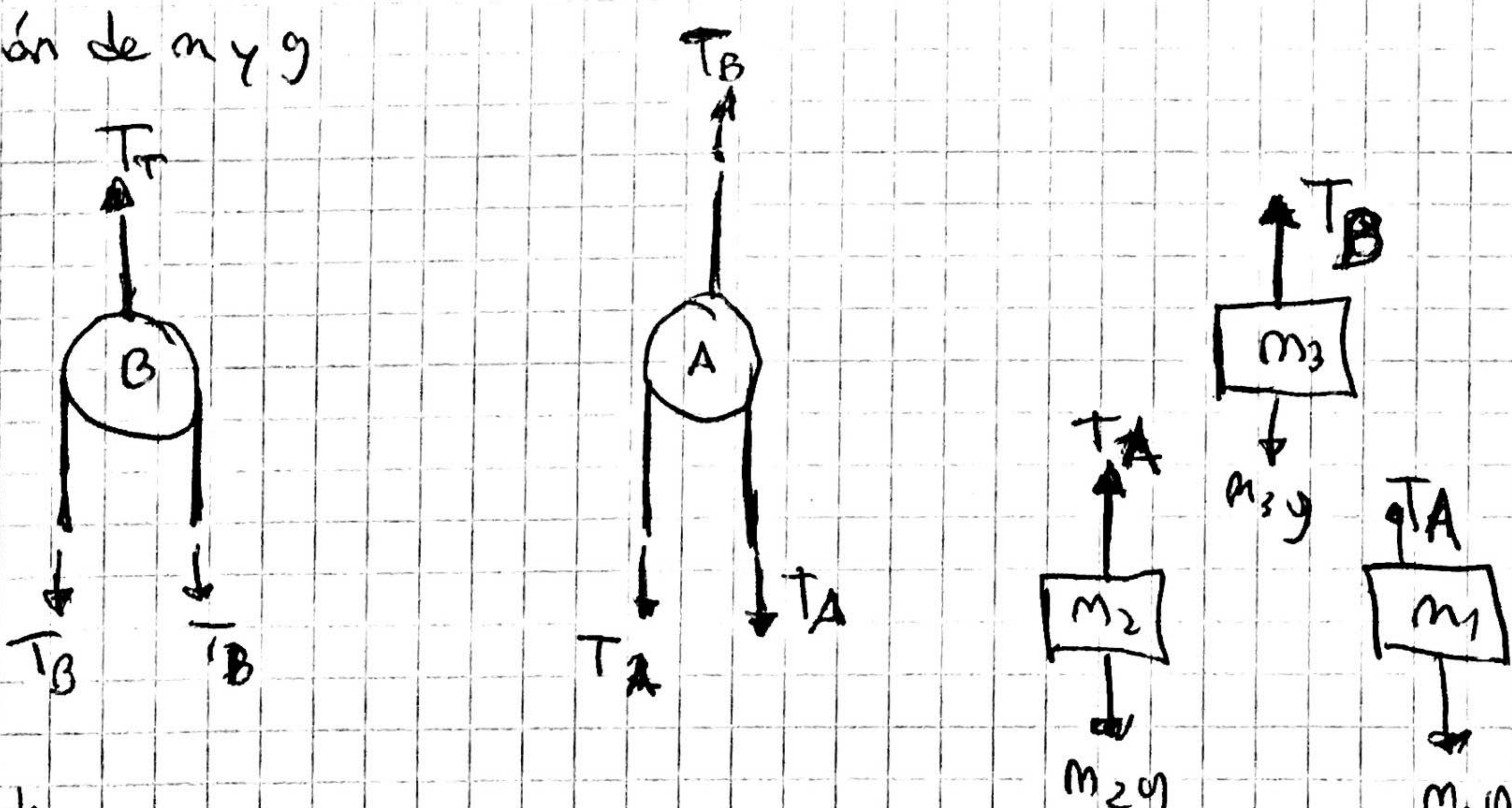
$l_A = x_2(t) - x_A + x_1(t) - x_A$

$l_A = x_2(t) + x_1(t) - 2x_A$

$\Rightarrow 0 = \ddot{x}_2(t) + \ddot{x}_1(t) - 2\ddot{x}_A$

a) Escriba las ecuaciones de Newton para las masas y la condición de vínculo que relaciona sus posiciones

b) Halle la aceleración de cada cuerpo y las tensiones en los hilos en función de m y g



l = cte

(1) $\hat{x}$ $m_1 g - T_A = m_1 \ddot{x}_1(t)$ (3) $\hat{x}$ $m_3 g - T_B = m_3 \ddot{x}_3(t)$

(2) $\hat{x}$ $m_2 g - T_A = m_2 \ddot{x}_2(t)$ $T_B = 2T_A$

b) Halle la aceleración de cada cuerpo y las tensiones en función de masas y g

$0 = \ddot{X}_{B(t)} + \ddot{X}_A$ $\quad$ $0 = \ddot{x}_2 t + \ddot{x}_1 t - 2\ddot{X}_A$

$T_B = 2T_A$

$m_1 g - T_A = m_1 \ddot{x}_1(t)$ $\quad$ $m_2 g - T_A = m_2 \ddot{x}_2(t)$ $\quad$ $m_3 g - T_B = m_3 \ddot{x}_3(t)$

$T_A = m_1 \ddot{x}_1 + m_1 g$ $\quad$ $T_A = -m_2 \ddot{x}_2(t) + m_2 g$ $\quad$ $T_B = -m_3 \ddot{x}_3(t) + m_3 g$

$\ddot{x}_3 = -\ddot{x}_A$ $\quad$ $\ddot{x}_2 = 2\ddot{x}_A - \ddot{x}_1$ $\quad$ $2T_A = -m_3 \ddot{x}_3(t) + m_3 g$

$\cancel{-m_1 \ddot{x}_1 + m_1 g = m_2 (2\ddot{x}_A - \ddot{x}_1)}$

$m_1 g - T_A =$

$$m_1 g - T_A = m_1 \ddot{x}_1$$

$$m_2 g - T_A = m_2 \ddot{x}_2$$

$$m_3 g - T_B = m_3 \ddot{x}_3$$

$$T_B = 2T_A$$

$$\ddot{x}_3 = -\ddot{x}_A \qquad \ddot{x}_2 + \ddot{x}_1 = 2\ddot{x}_A$$

$$\frac{m_2 g - T_A}{m_2} + \frac{m_1 g - T_A}{m_1} = 2\ddot{x}_A$$

$$\frac{m_3 g - T_B}{m_3} = -\ddot{x}_A$$

$$\frac{m_2 g - T_A}{m_2} + \frac{m_1 g - T_A}{m_1} = -2\frac{m_3 g - T_B}{m_3}$$

$$\frac{m_2 g - T_A}{m_2} + \frac{m_1 g - T_A}{m_1} + \frac{2m_3 g - 2T_A}{m_3} = 0$$

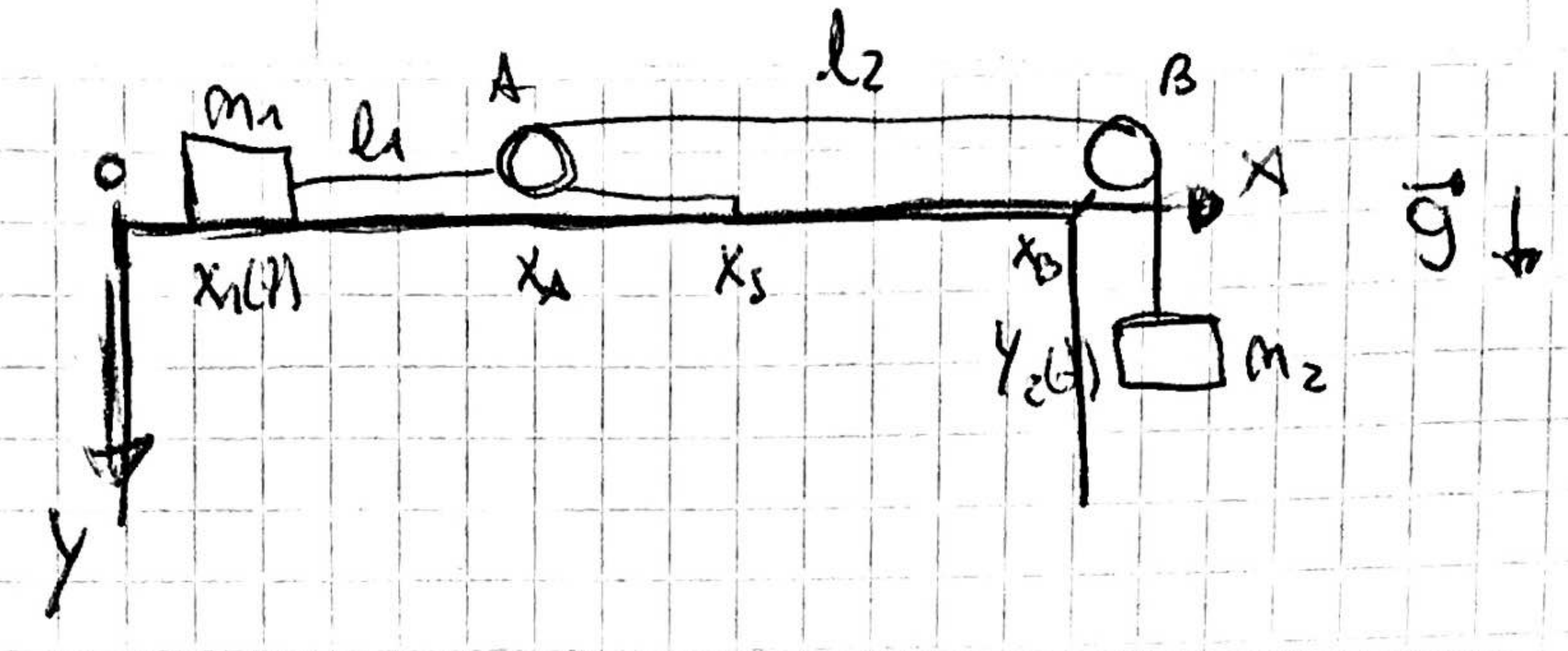
$$\cancel{T_A(m_2} \quad +\frac{T_A}{m_2} + \frac{T_A}{m_1} + \frac{2T_A}{m_3} = +\frac{m_2 g}{m_2} + \frac{m_1 g}{m_1} + \frac{2m_3 g}{m_3}$$

$$T_A\left(\frac{1}{m_2} + \frac{1}{m_1} + \frac{2}{m_3}\right) = \frac{m_2 g}{m_2} + \frac{m_1 g}{m_1} + \frac{2m_3 g}{m_3}$$

$$T_A\left(\frac{m_1 m_3 + m_2 m_3 + 2m_1 m_2}{m_2 m_1 m_3}\right) = \frac{m_2 m_1 m_3 g + m_1 m_2 m_3 g + 2m_1 m_2 m_3 g}{m_2 m_1 m_3}$$

$$T_A = \frac{4\cancel{m_1 m_2 m_3}\, g}{\cancel{m_2 m_1 m_3}} \cdot \left(\frac{m_1 m_2 m_3}{m_1 m_3 + m_2 m_3 + 2m_1 m_2}\right)$$

$$\boxed{T_A = \frac{4g\, m_1 m_2 m_3}{m_1 m_3 + m_2 m_3 + 2m_1 m_2}} \qquad \boxed{T_B = \frac{2g\, m_1 m_2 m_3}{m_1 m_3 + m_2 m_3 + 2m_1 m_2}}$$



a)

$$l_2 = y_2 + x_B - x_A + x_s - x_A \Rightarrow 0 = \ddot{y}_2 + \overset{=0}{\ddot{x}_B} + \overset{=0}{\ddot{x}_s} - 2\ddot{x}_A$$

$$l_1 = x_A - x_1(t) \Rightarrow 0 = \ddot{x}_A - \ddot{x}_1(t)$$

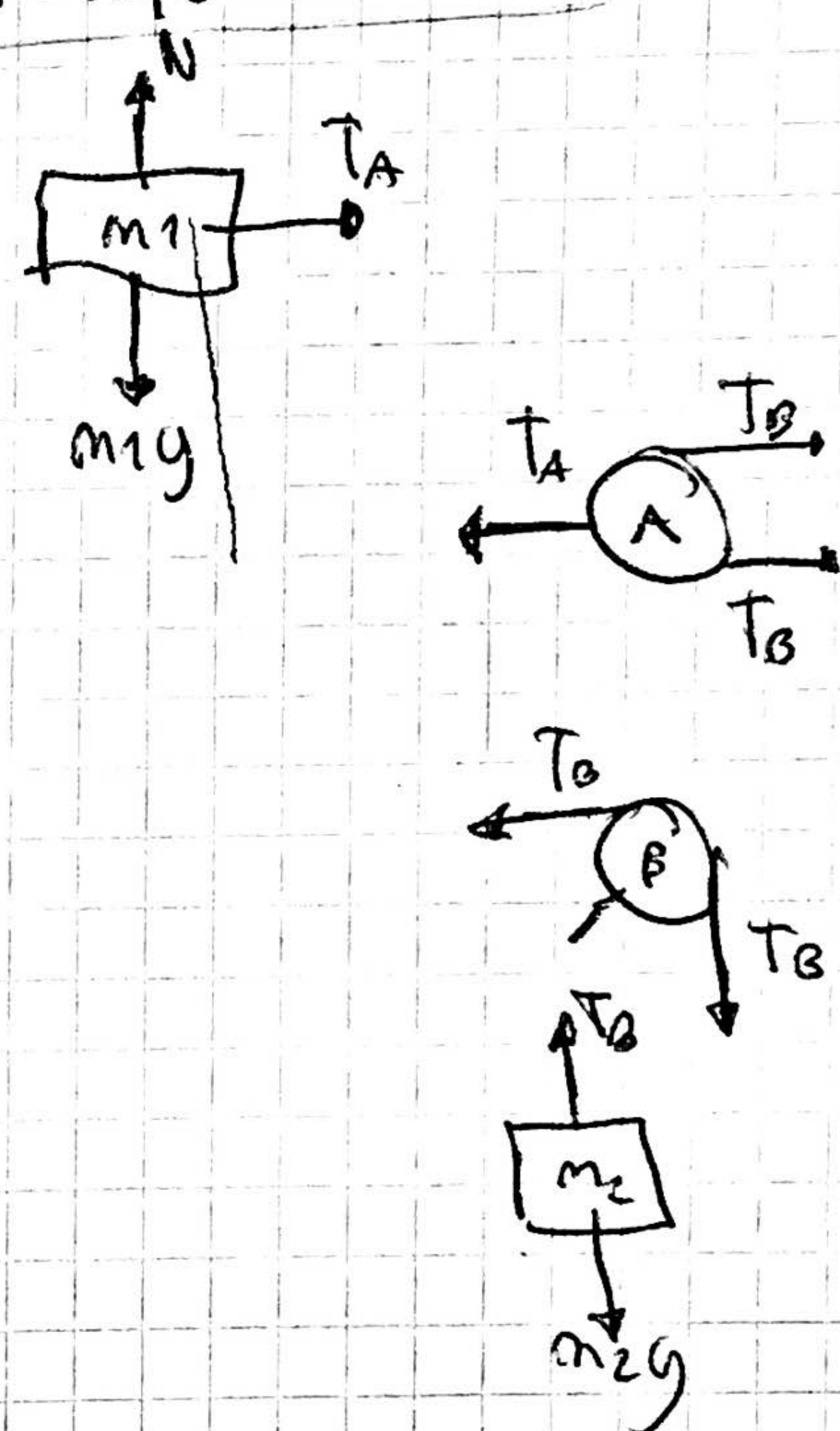
(1) $\hat{x}$ $T_A = m_1 . \ddot{x}_1$

$\hat{y}$ $m_1 g - N = m_1 \overset{=0}{\ddot{y}_1}$

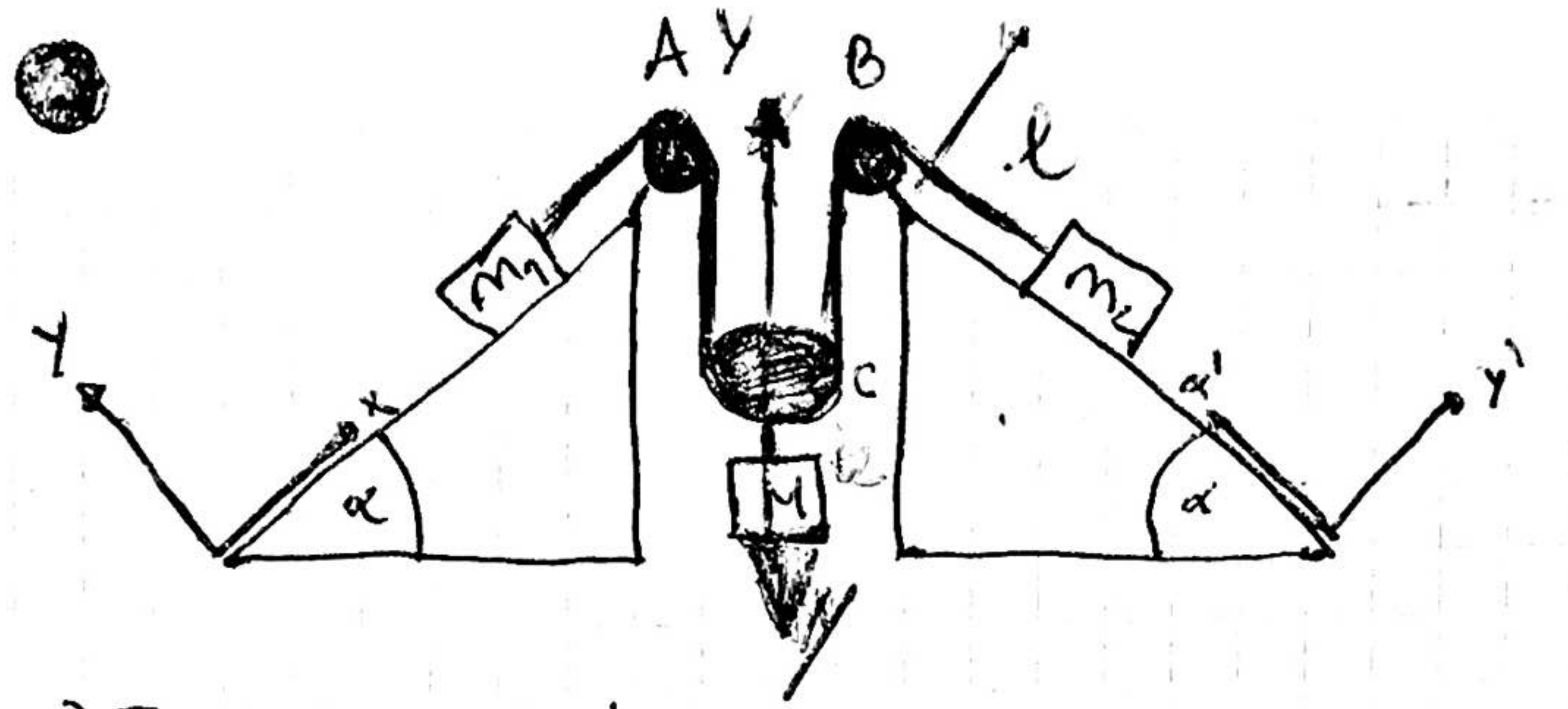
(2) $\hat{x}$ $0=0$

$\hat{y}$ $m_2 . g - T_B = m_2 . \ddot{x}_2$

Vinculos → $T_A = 2T_B$

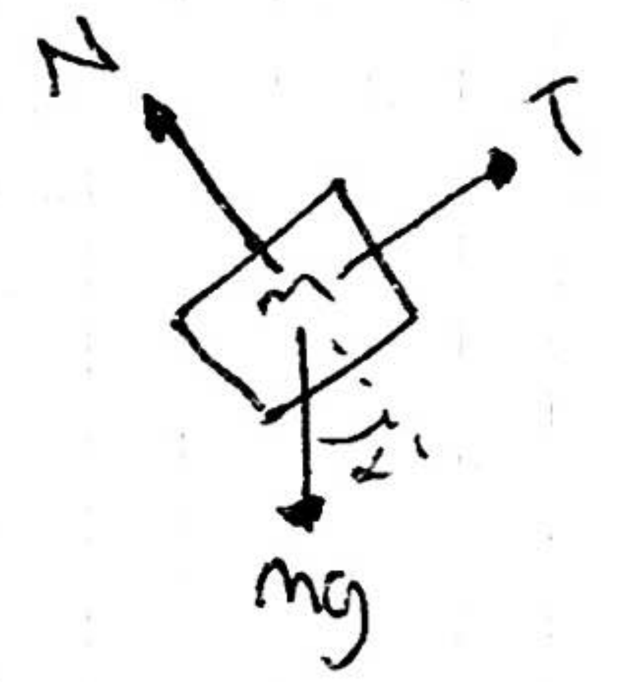


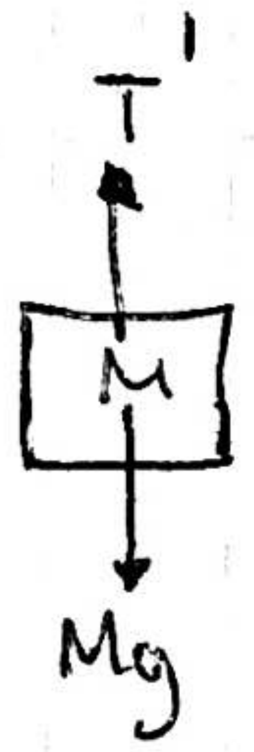
b)

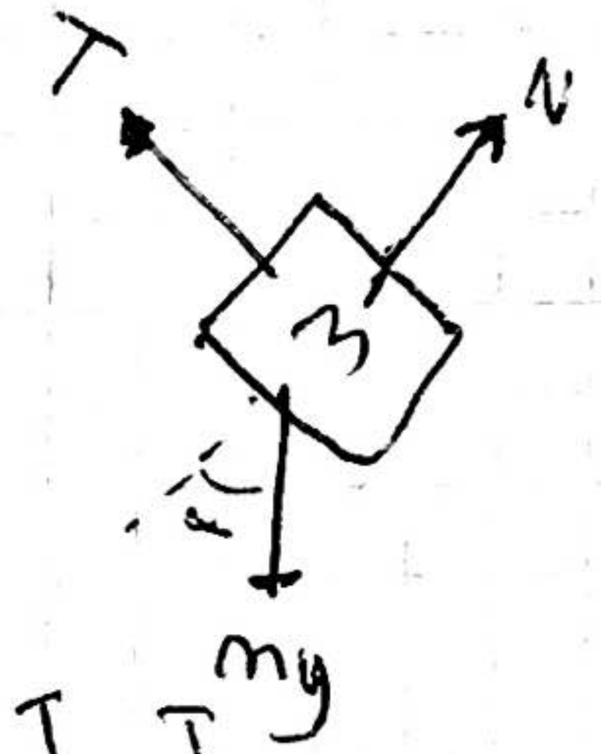


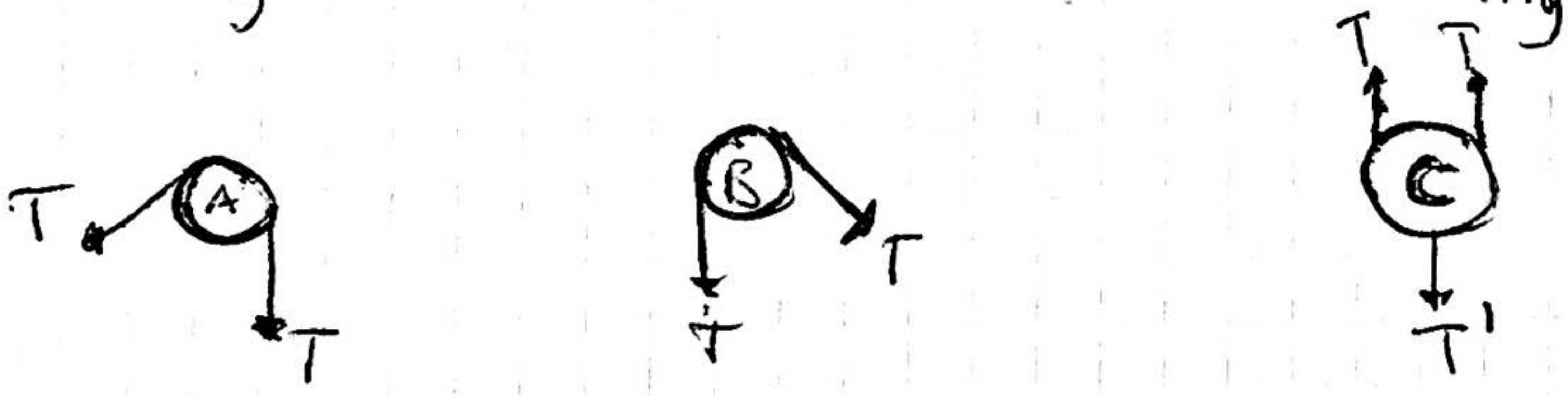
$m_1 = m_2 = m$

a) Ec Newton y vínculo









$m_1)$ $-(x)$ $T - m.g.\operatorname{sen}\alpha = m\ddot{x}_1$

(y) $N = mg.\cos\alpha$

$m_2)$ $-T + mg\operatorname{sen}\alpha = m\ddot{x}_2$

$N = mg\cos\alpha$

$M)$ $-T' + Mg = M\ddot{y}_M$ $-$

$\cancel{x_A - x_B}$

$x_A - x_1 = x_B - x_2$

$\ell = (x_A - x_1) + (x_B - x_2) + y_C + y_C$

$\ell = 2(x_A - x_1 + y_C) \rightarrow 0 = 2\ddot{x}_A - 2\ddot{x}_1 + 2\ddot{y}_C \rightarrow \ddot{x}_1 = \ddot{y}_C$

$\rightarrow T' = 2T$

$\ell_2 = -y_M + y_C$

$\ddot{y}_M = \ddot{y}_C$ $-$

b) $\cancel{2T} - 2mg\operatorname{sen}\alpha = 2m\ddot{y}_1$

$+$ $\cancel{-2T} + Mg = M\ddot{y}_M$

$\Rightarrow$ $-\cancel{2mg\operatorname{sen}\alpha - 2T}$

$-2mg\operatorname{sen}\alpha + Mg = 2m\ddot{y}_M + M\ddot{y}_M$

$-2mg\operatorname{sen}\alpha + Mg = \ddot{y}_M(2m+M)$

$$\boxed{\ddot{y}_M = \frac{-2mg\operatorname{sen}\alpha + Mg}{2m+M} = \ddot{x}_1 = \ddot{x}_2}$$

c) mínimo m para levantar el cuerpo a una altura H en un tiempo T

$$Y_M = \frac{-2mg\sin\alpha + Mg}{2m+M}\,\frac{t^2}{2} + Y_{M0}$$

$$H = \frac{-2mg\sin\alpha + Mg}{2m+M}\cdot\frac{T^2}{2} + Y_{M0}$$

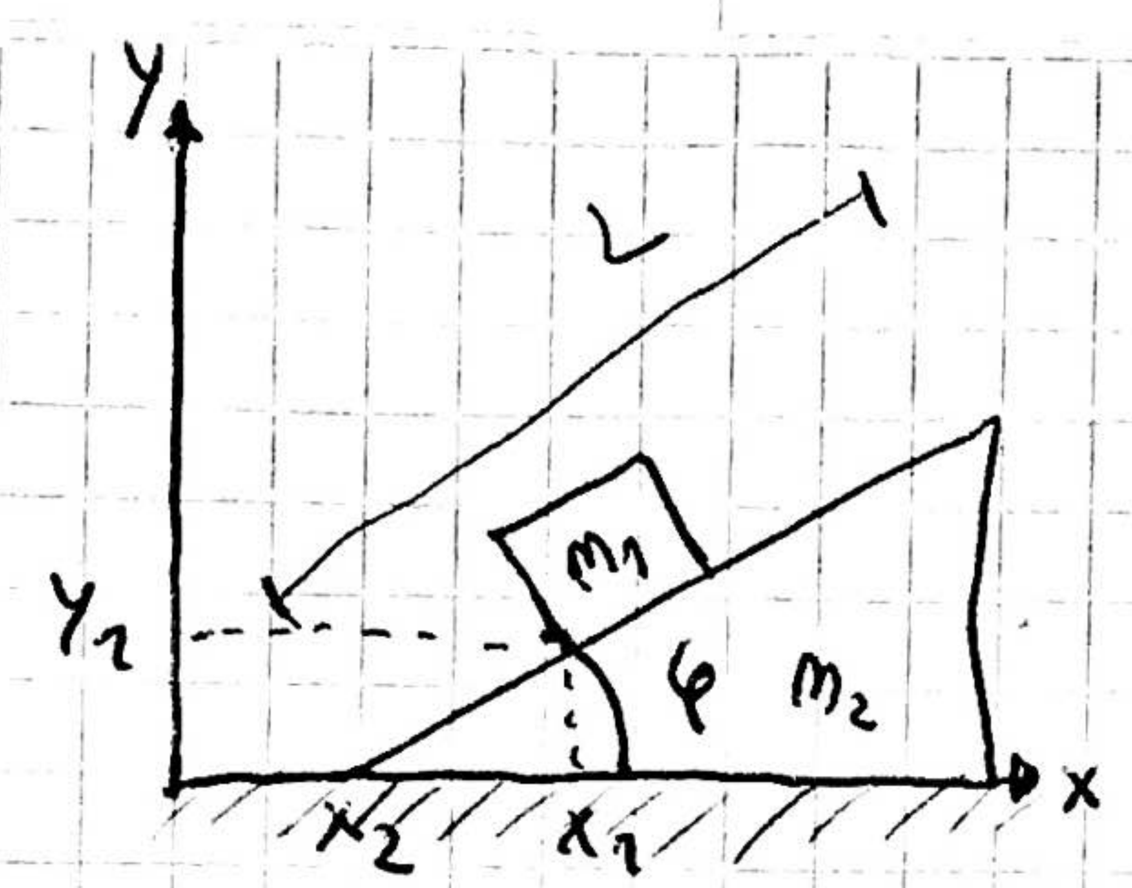
$$\frac{(H - Y_{M0})}{T^2} = \frac{-2mg\sin\alpha + Mg}{2m+M}$$

$$(2m+M)\left(\frac{2H - 2Y_{M0}}{T^2}\right) = -2mg\sin\alpha + Mg$$

$$2m\left(\frac{2H-2Y_{M0}}{T^2} + g\sin\alpha\right) = Mg - M\left(\frac{2H-2Y_{M0}}{T^2}\right)$$

$$\boxed{m_{min} = \frac{Mg - M\left(\frac{2H-2Y_{M0}}{T^2}\right)}{2\left(\frac{2H-2Y_{M0}}{T^2} + g\sin\alpha\right)}}$$

M mín

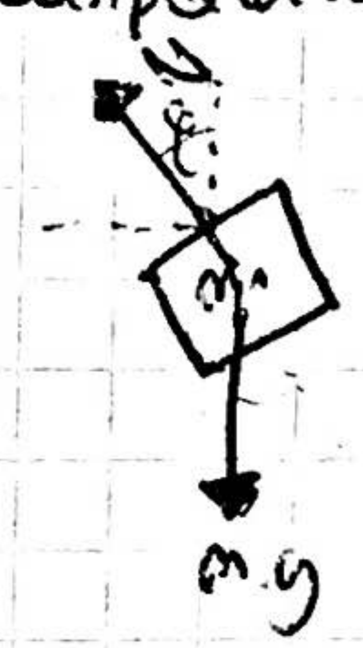


~~$x_1 \varphi$~~

$\cancel{(x_1 - x_2)^2 + y^2}$

c) Componentes x e y

$\Rightarrow$



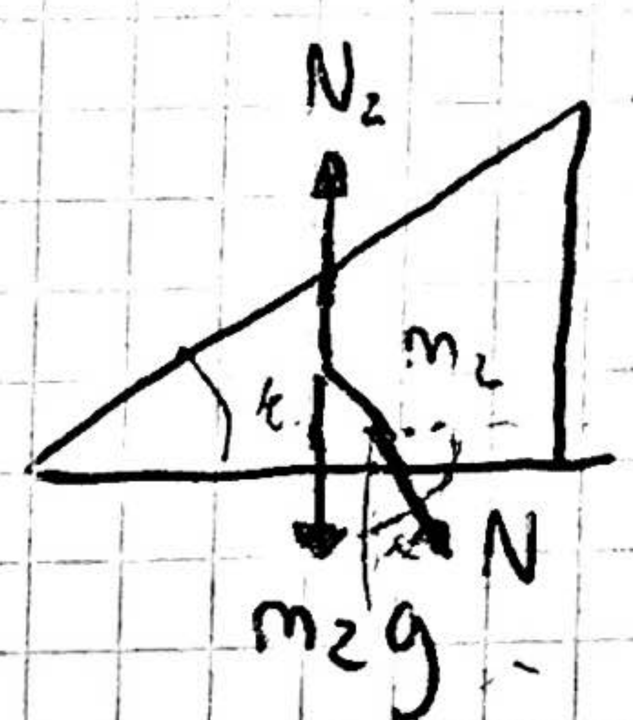
$\Rightarrow$ Ec. de Newton

$(\hat{x})\ -N \operatorname{sen} \varphi = m_1 \cdot \ddot{X}_1$

$(\hat{y})\ -m_1 g + N\cos\varphi = m_1 \cdot \ddot{Y}_1$

ii) El plano se mueve

a) Pruebe que $\ddot{x}$ del bloque es $\Rightarrow a_{1x} = \dfrac{-m_2 g \operatorname{tg}\varphi}{(m_2 \sec^2\varphi + m_1 \operatorname{tg}^2\varphi)}$



Ec Newton

$(\hat{x})\ N \operatorname{sen}\varphi = m_2 \cdot \ddot{x}_2$

$(\hat{y})\ -m_2 g - N\cos\varphi + N_2 = m_2 \ddot{y}_2 = 0 \Rightarrow N_2 = m_2 g + N\cos\varphi$

Vínculo

$\operatorname{tg}\varphi = \dfrac{Y_1}{x_1 - x_2}$ $\qquad \operatorname{tg}\varphi (x_1 - x_2) = Y_1$

$\operatorname{tg}\varphi (\ddot{x}_1 - \ddot{x}_2) = \ddot{y}_1$

$-m_1 g + N\cos\varphi = m_1 \operatorname{tg}\varphi (\ddot{x}_1 - \ddot{x}_2)$

$$tg\,\ddot{x}_1 - tg\,\ddot{x}_2 = \ddot{y}_1$$

$$\ddot{x}_1 = \frac{-N\,sen\,\varphi}{m_1}$$

$$\ddot{x}_2 = \frac{N\,sen\,\varphi}{m_2}$$

$\Rightarrow \cancel{tg}$

$$\ddot{y}_1 = -\frac{\cancel{m_1}g}{\cancel{m_1}} + \frac{N\cos\varphi}{m_1}$$

$$\Rightarrow +\frac{tg\,N\,sen\,\varphi}{m_1} + \frac{tg\,N\,sen\,\varphi}{m_2} + \frac{N\cos\varphi}{m_1} = +g$$

$$\frac{tg\,N\,sen\,\varphi + N\cos\varphi}{m_1} + \frac{tg\,N\,sen\,\varphi}{m_2} = g$$

$$N\left(\frac{tg\varphi\,sen\,\varphi + \cos\varphi}{m_1} + \frac{tg\,sen\,\varphi}{m_2}\right) = g$$

$$N\left(\frac{m_2(tg\,\varphi\,sen\,\varphi + \cos\varphi) + m_1\,tg\,sen\,\varphi}{m_1 m_2}\right) = g$$

$$N = \frac{m_1 m_2 g}{m_2 tg\varphi\left(sen\,\varphi + \frac{\cos\varphi}{tg\,\varphi}\right) + m_1\,tg\,sen\,\varphi}$$

$$\ddot{x}_1 = \frac{-\cancel{m_1} m_2 g\,sen\,\varphi}{\left(m_2\,tg\,\varphi\left(sen\,\varphi + \frac{\cos\varphi}{tg\,\varphi}\right) + m_1\,tg\,sen\,\varphi\right)\cancel{m_1}}$$

$$\ddot{x}_1 = \frac{-m_2 g\,sen\,\varphi}{\cos\varphi\left(m_2\,tg\,\varphi\left(\frac{sen\,\varphi}{\cos\varphi} + \frac{1}{tg\,\varphi}\right) + m_1\,tg\,\varphi\,\frac{sen\,\varphi}{\cos\varphi}\right)}$$

$$\boxed{\ddot{x}_1 = \frac{-m_2\,g\,tg\,\varphi}{m_2\underbrace{(tg^2\varphi + 1)}_{= sec^2\varphi} + m_1\,tg^2\varphi}}$$

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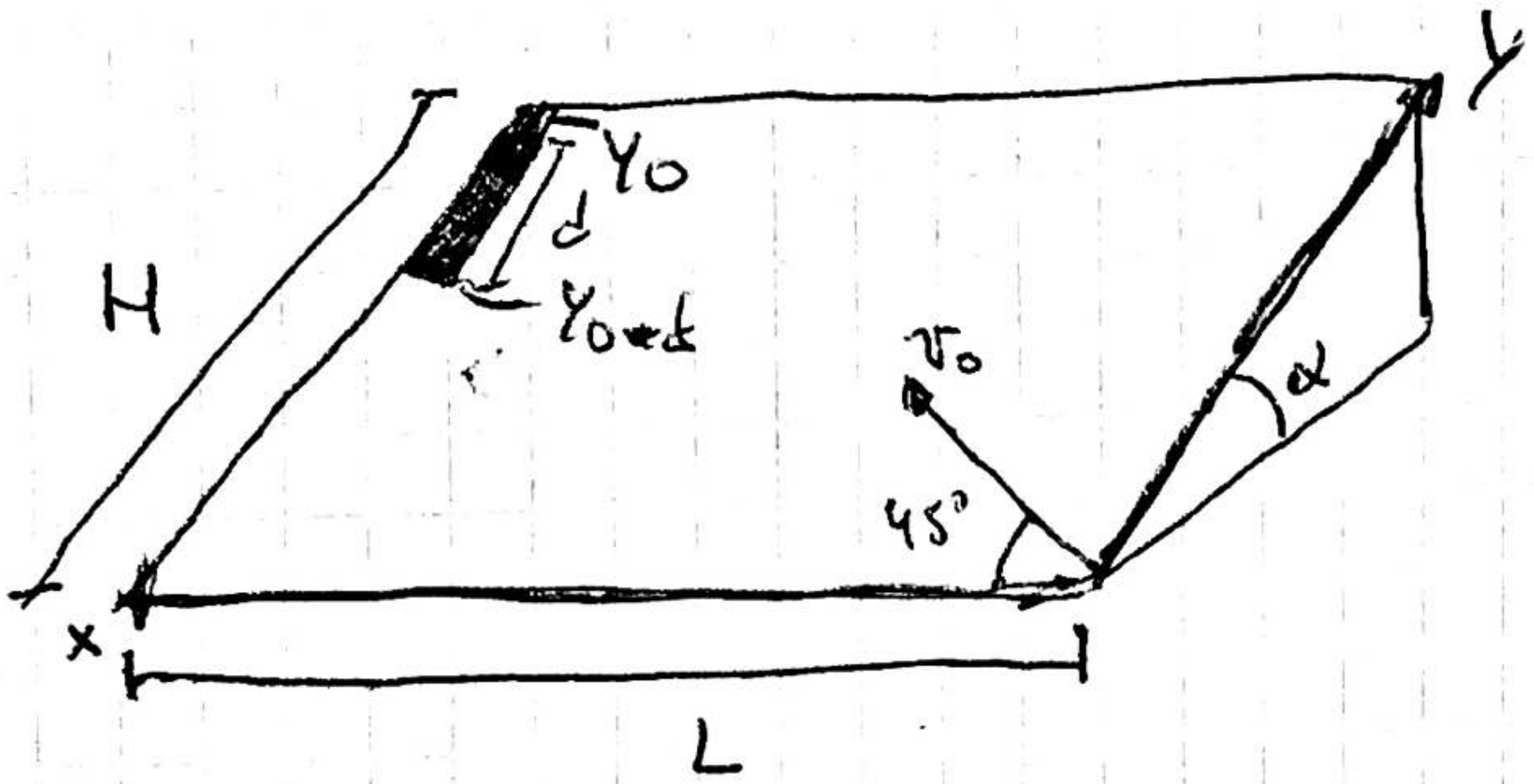


$g_y = g \cdot \operatorname{sen}\alpha$

a)

Varilla

$(\hat{y})\ -\cancel{m} g \operatorname{sen}\alpha = \cancel{m} \cdot \ddot{y}$

$(\hat{x})\ \ 0 = 0$

Proyectil

$(\hat{y})\ -\cancel{m_p} g \operatorname{sen}\alpha = \cancel{m_p} \cdot \ddot{y}_p$

$(\hat{x}) = \ 0 = 0$

b) $\ddot{y} = -g \operatorname{sen}\alpha \qquad \ddot{y}_p = -g \operatorname{sen}\alpha$

Cinemática

Varilla

$y = -\frac{g \operatorname{sen}\alpha}{2} t^2 + H$

$x = d$

Proyectil

$y_p = v_0 . \operatorname{sen}45° (t - 1s) - \frac{g \operatorname{sen}\alpha}{2} (t - 1s)^2$

$x_p = v_0 . \cos 45° (t - 1s)$

Para encuentro

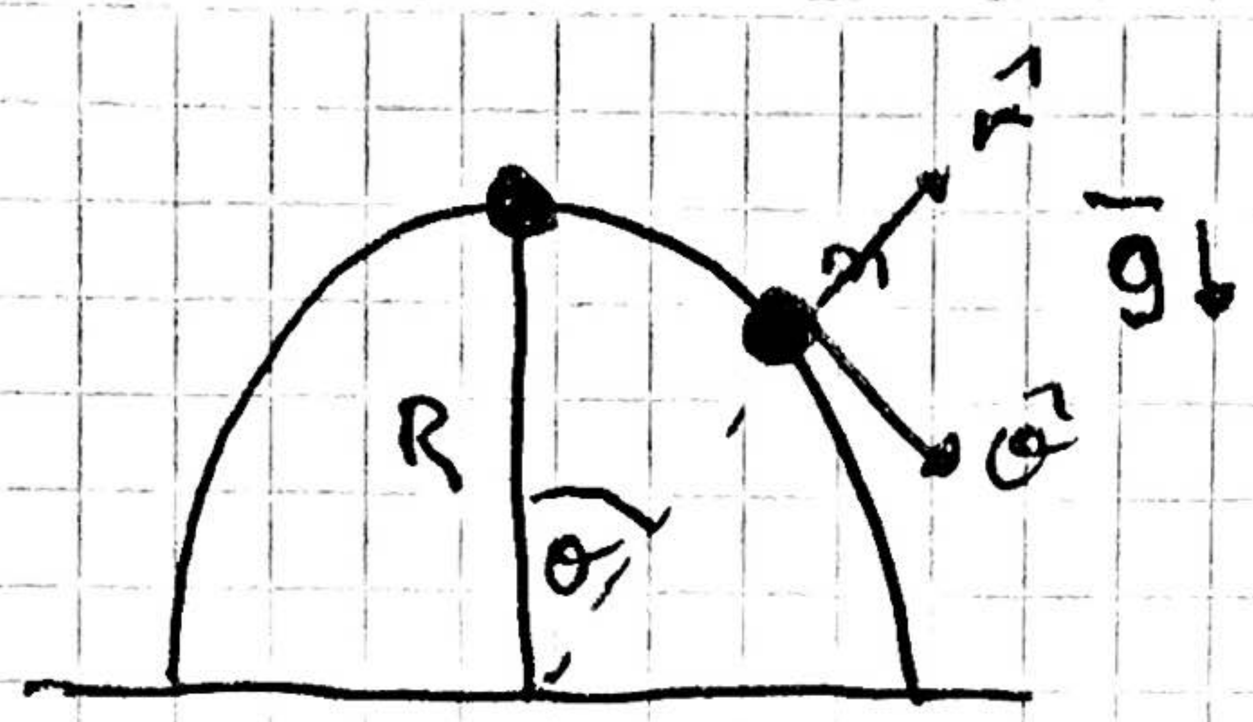
$x_p = L$

$L = v_0 \cos 45° (t_e - 1s) \qquad \frac{L}{v_0 \cos 45°} + 1s = t_e$

$y_v - d \leq y_p \leq y_v$

$$\boxed{-\frac{g \sin\alpha\, t_e^2}{2} + H - d \leq v_0 . \operatorname{sen}45° (t_e - 1_s) - \frac{g \sin\alpha}{2} (t_e - 1s)^2 \leq -\frac{g \sin\alpha\, t_e^2}{2} + H}$$

6



θ / separa de sup esférica

$v_0 = 0$

Para que θ(de) $N = 0$

$(\hat{r})$ $N - m.g.\cos\theta = m.(\ddot{r} - r\dot{\theta}\dot{\theta})$ $\quad \ddot{r}=0$

$(\hat{\theta})$ $\cancel{m}.g.\text{sen}\,\theta = \cancel{m}(r\ddot{\theta} + 2\dot{r}\dot{\theta})$ $\quad \dot{r}=0$

$N = m(-R\dot{\theta}\dot{\theta} + g\cos\theta)$

si $N = 0 \Rightarrow R\dot{\theta}^2 = g\cos\theta$

$\dot{\theta}^2 \frac{R}{g} = \cos\theta$

$\frac{g\,\text{sen}\,\theta}{R} = \ddot{\theta}$

$\ddot{\theta} = \frac{d\dot{\theta}}{d\theta} \cdot \frac{d\theta}{dt} = \frac{g\,\text{sen}\,\theta}{R}$ $\quad \frac{d\theta}{dt} = \dot{\theta}$

$\int_0^{\dot{\theta}(\theta)} \dot{\theta}\,d\dot{\theta} = \int_0^{\theta} \frac{g\,\text{sen}\,\theta}{R}\,d\theta$

$\frac{\dot{\theta}^2}{2} = \frac{-g\cos\theta}{R} + \frac{g\cos 0}{R}$

$\dot{\theta} = \sqrt{\frac{2g}{R}(-\cos\theta + 1)}$

$\frac{2g\cancel{R}}{\cancel{R}\cancel{g}}(-\cos\theta + 1) = \cos\theta$

$-2\cos\theta + 2 = \cos\theta$

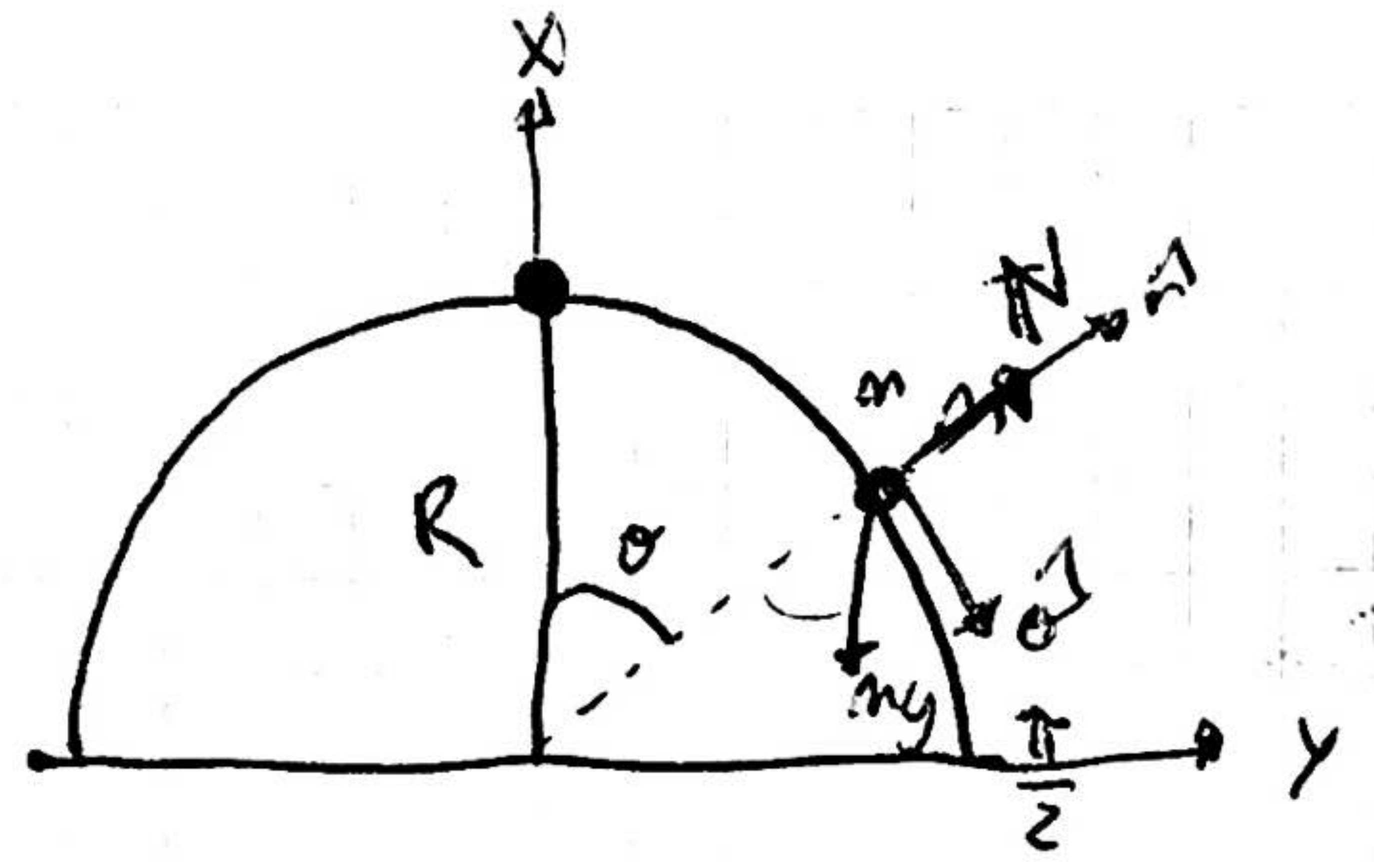
$2 = 3\cos\theta$

$\frac{2}{3} = \cos\theta$

$\boxed{\arccos\frac{2}{3} = \theta}$

~~arc cos~~

b)



$\hat{r}$) $N - mg \cdot \cos\theta = -m . R\dot{\theta}^2$

$\hat{\theta}$) $\cancel{m}g \,\text{sen}\, \theta = \cancel{m} R \ddot{\theta} \Rightarrow \frac{g \,\text{sen}\, \theta}{R} = \ddot{\theta}$

$\bar{v}_p = ?$ $\quad \dot{\theta} = \sqrt{\frac{2g}{R}(-\cos\theta + 1)}$ $\quad$ No hay V_t en θ_f

$a_t = ?$

$\dot{\theta} =$

$\bar{v} = \underbrace{\dot{r}\hat{r}}_{=0} + \dot{\theta}\hat{\theta} r$

$\ddot{\theta} = \frac{1 \cdot \frac{\cancel{2}g}{R} \,\text{sen}\,\theta \dot{\theta}}{\cancel{2}\sqrt{\frac{2g}{R}(-\cos\theta + 1)}}$

$\bar{v} = R\sqrt{\frac{2g}{R}\left(-\cos\frac{\pi}{2} + 1\right)}$

$\boxed{\bar{v} = R\sqrt{\frac{2g}{R}}}$

$\ddot{\theta} = \frac{g}{R} \,\text{sen}\,\theta \frac{\sqrt{\frac{2g}{R}(-\cos\theta + 1)}}{\sqrt{\frac{2g}{R}(-\cos\theta + 1)}}$

$\bar{a} = (\overset{=0}{\ddot{r}} - r\dot{\theta}^2)\hat{r} + (\overset{=0}{2\dot{r}\dot{\theta}} + r\ddot{\theta})\hat{\theta}$

$\bar{a} = -R\sqrt{\frac{2g}{R}} + \frac{\cancel{R} g \,\text{sen}\,\theta}{\cancel{R}}$

$\bar{a} = -R\sqrt{\frac{2g}{R}} + g \cdot \text{sen}\,\frac{\pi}{2}$

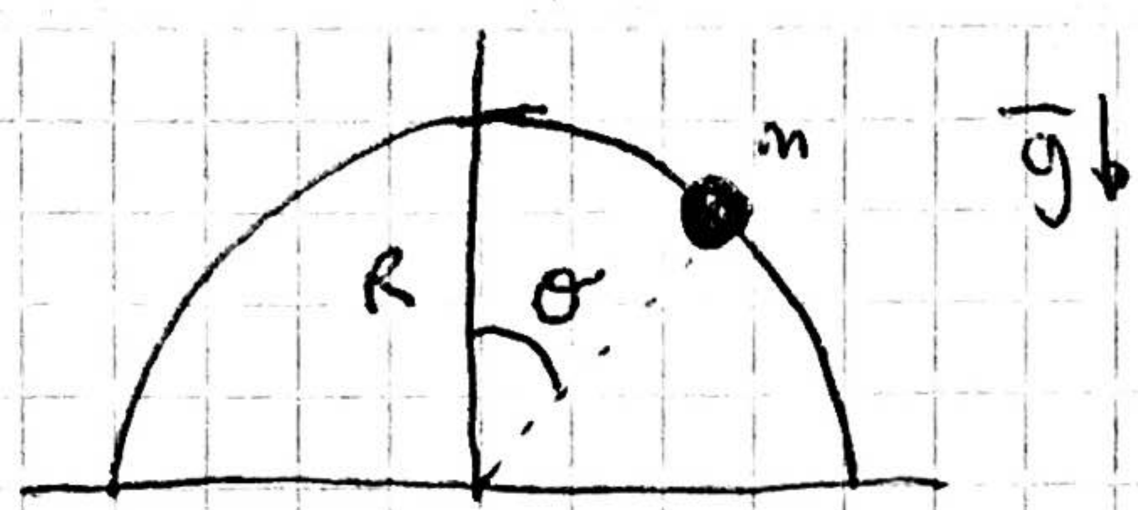
$\boxed{\bar{a} = -R\sqrt{\frac{2g}{R}} + g}$

c) $\dot{\theta} = \frac{d\theta}{dt}$ $\quad \int_0^t \dot{\theta}\, dt = \int_0^{\theta(t)} d\theta$

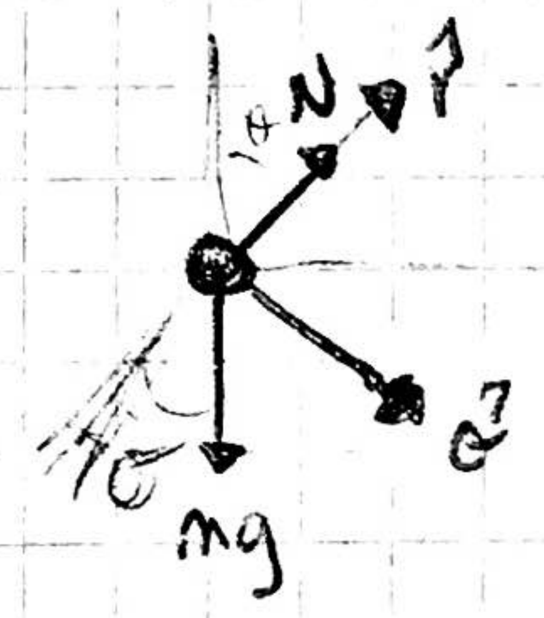
$\rightarrow \dot{\theta} t = \theta(t)$

$t\sqrt{\frac{2g}{R}(-\cos\theta + 1)} = \theta(t)$

6



a) Calcular el ángulo θ para que la bola se separe si $\theta_{(0)}=0$ y $v_0=0$



$(\hat{\theta})$ $\cancel{m}g \cdot \sin\theta = \cancel{m}R\ddot{\theta}$ $\quad g\sin\theta = R\ddot{\theta}$

$(\hat{r})$ $N - mg\cos\theta = -mR\dot{\theta}^2$

$$\ddot{\theta} = \frac{g\sin\theta}{R} = \frac{d\dot{\theta}}{d\theta} \cdot \frac{d\theta}{dt} = \frac{d\dot{\theta}}{d\theta}\dot{\theta}$$

$$\Rightarrow \int_0^{\theta_{(0)}} \dot{\theta}\, d\dot{\theta} = \int_0^{\theta} \frac{g\sin\theta}{R}\, d\theta$$

$$\frac{\dot{\theta}^2}{2} = -\frac{g\cos\theta}{R} + \frac{g}{R}$$

$$\dot{\theta}^2 = \frac{2g}{R}(\cos\theta + 1)$$

$\Rightarrow$ Momento en que se separa $N=0$

$$\Rightarrow \cancel{m}\cancel{g}\cos\theta = \cancel{m}\cancel{R}\frac{2\cancel{g}}{\cancel{R}}(-\cos\theta + 1)$$

$$\cos\theta = -2\cos\theta + 2$$

$$3\cos\theta = 2$$

$$\boxed{\theta = \arccos\frac{2}{3}}$$

b) Si m se empieza en cied semicírculo hallar v_F ¿Que aceleración tangencial tiene m a ese instante?

$(\hat{\theta})$

$(\hat{r})$

$$t = \frac{\theta(t)}{\sqrt{\frac{2g}{R}(-\cos\theta+1)}} \qquad t = \frac{\theta(t)}{\sqrt{\frac{2g}{R}}} \qquad t_F = \frac{\pi}{2\sqrt{\frac{2g}{R}}}$$

si $R = 1cm$

$$t_F = \frac{\pi}{2\sqrt{\frac{2g}{1cm}}}$$

si $R = 10cm$

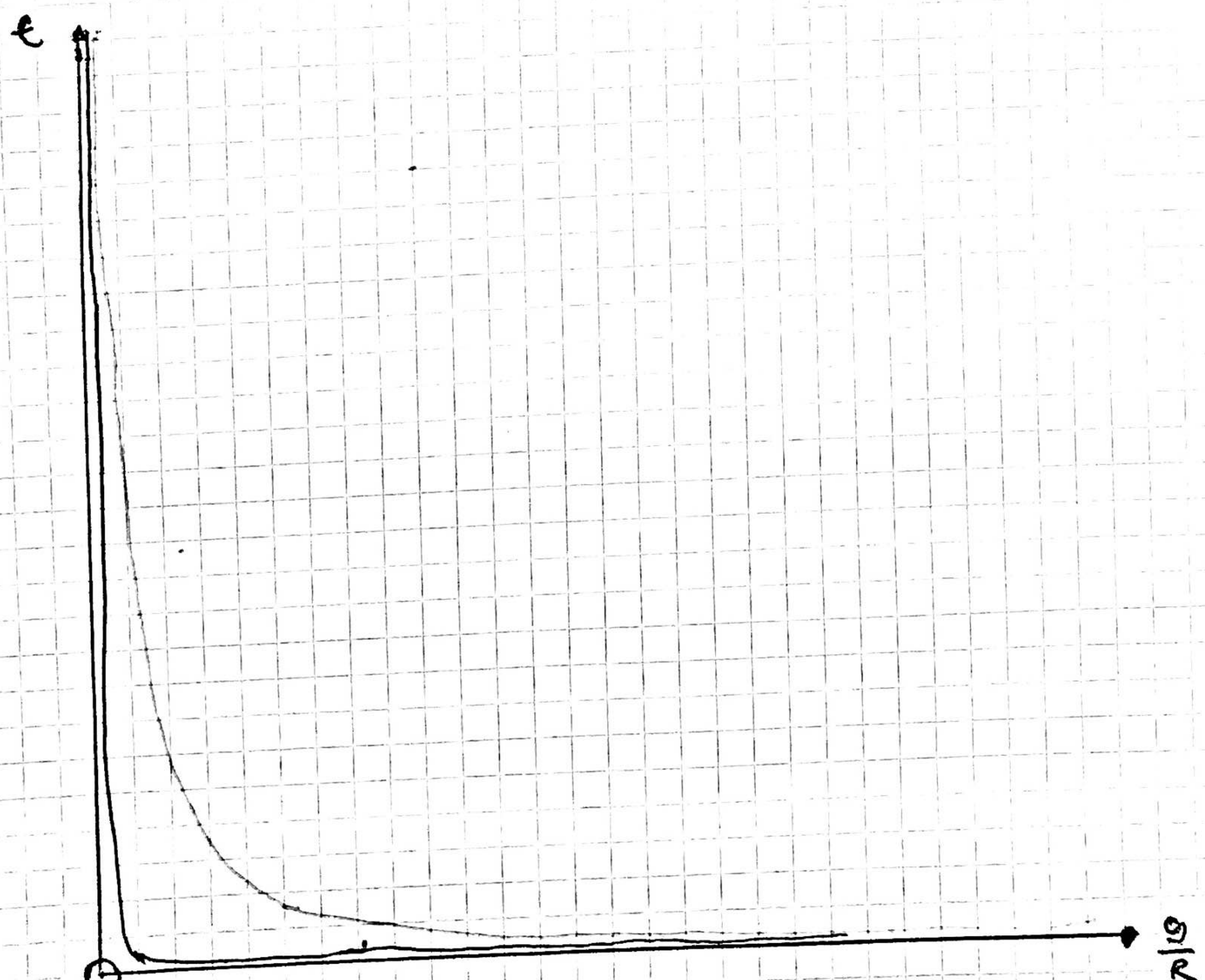
$$t_F = \frac{\pi}{2\sqrt{\frac{2g}{10cm}}} = \frac{\pi}{2\sqrt{\frac{g}{5cm}}}$$

si $R = 50cm$

$$\Rightarrow t_F = \frac{\pi}{2\sqrt{\frac{g}{25cm}}} = \frac{5\pi}{2\sqrt{g}} cm$$

si $R = 100cm$

$$\frac{\pi}{2\sqrt{\frac{g}{50cm}}}$$



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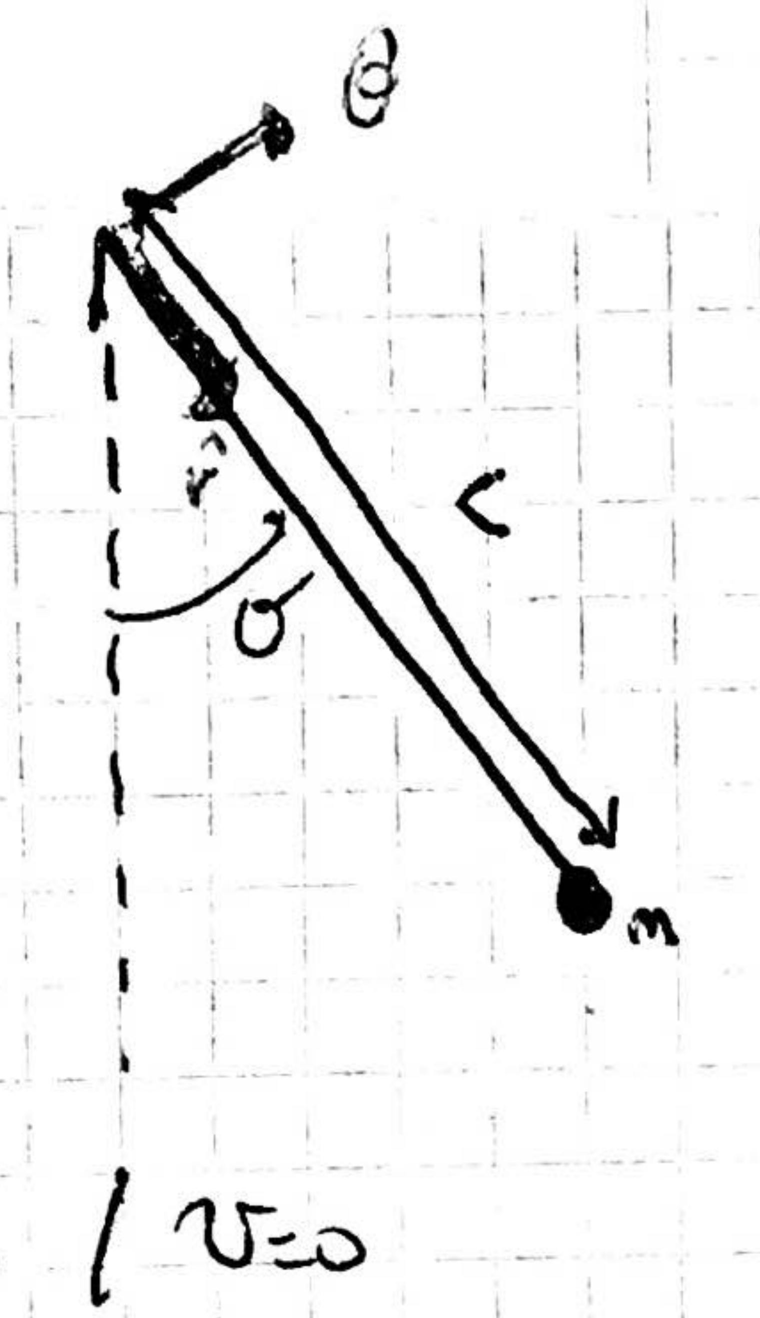


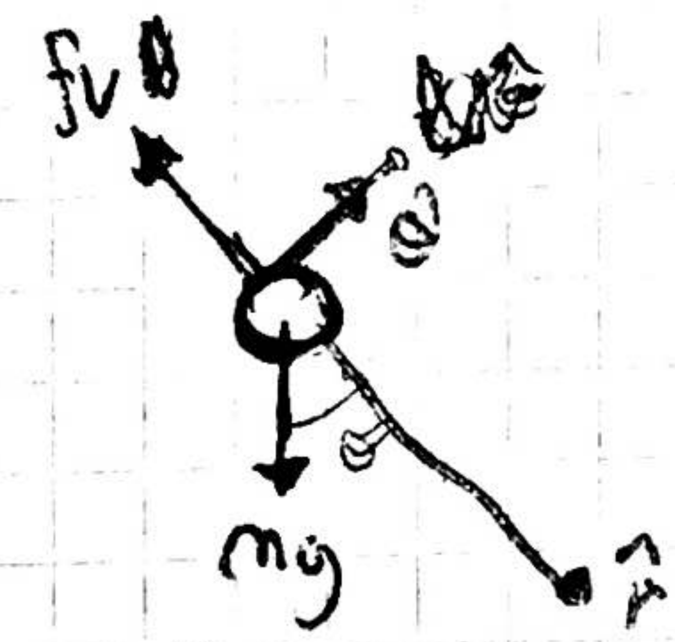
$\vec g \downarrow$

$\bar r(t)$ = vínculo $r = cte$ $\quad r = L$

$\bar f_v$ = fuerza de vínculo barra sobre masa

$\bar f_v = f_v \hat r$

a) θ_v / $v=0$



$$\Rightarrow \quad (\hat\theta) \quad -m.g.\operatorname{sen}\theta + \underbrace{f_v\,\hat\theta}_{=0} = m.a\,\hat\theta$$

$$(\hat r) \quad -f_v\,\hat r + m.g.\cos\theta = m.a\,\hat r$$

$$\ddot{\vec r}(t) = (\underbrace{\ddot r}_{=0} - r\dot\theta\dot\theta)\hat r + (r\ddot\theta + 2\dot\theta\underbrace{\dot r}_{=0})\hat\theta$$

ya que R=cte

$$\Rightarrow (\hat\theta) \quad -m.g.\operatorname{sen}\theta + \underbrace{f_v\,\hat\theta}_{=0} = m r\ddot\theta \qquad \Rightarrow -\cancel{m} g \operatorname{sen}\theta = \cancel{m} r\ddot\theta$$

$$(\hat r) \quad -f_v\hat r + mg\cos\theta = m r\dot\theta^2 \qquad g\operatorname{sen}\theta = r\ddot\theta$$

$$(\hat\theta)\ -g\operatorname{sen}\theta = L\ddot\theta \qquad (\hat r)\ -f_v + mg\cos\theta = mL\dot\theta^2$$

a) θ_v / $\bar v = 0$

$$\bar v = \underbrace{\dot r}_{=0}\hat r + \dot\theta\hat\theta \qquad \bar v = 0 \Leftrightarrow \dot\theta = 0$$

$$\bar\gamma = \frac{-g\operatorname{sen}\theta}{L} = \frac{d\dot\theta}{dt} = \frac{d\dot\theta}{d\theta}\frac{d\theta}{dt} \Rightarrow \int_{\dot\theta(\theta(0))}^{\dot\theta(\theta(t))} d\dot\theta\,\dot\theta = \int_{\theta(0)}^{\theta(t)} \frac{-g\operatorname{sen}\theta}{L}\,d\theta$$

$$= \frac{\dot\theta^2}{2}\Bigg|_{\dot\theta(\theta(0))}^{\dot\theta(\theta(t))} = \frac{+g\cos\theta}{L}\Bigg|_{\theta(0)}^{\theta(t)}$$

$\cancel{\dot{\theta}^2 = \frac{2g}{L}(\cos\theta - 1) + \frac{v_0^2}{L^2}}$

$$\frac{\dot{\theta}^2}{2} = \frac{g}{L}(\cos\theta - 1) + \frac{v_0^2}{2L} \qquad \dot{\theta} = \sqrt{\frac{2g}{L}(\cos\theta - 1) + \frac{v_0^2}{L^2}}$$

$$v_T = \omega . R$$

$$v_0 = \dot{\theta}_{(0)} L$$

$$\frac{v_0}{L} = \dot{\theta}_{(0)}$$

$\cancel{\dot{\theta} = \sqrt{\frac{1}{L}\left(2g(\cos\theta - 1) + v_0^2\right)}}$

$\cancel{0 = \frac{1}{L}\left(2g\cos\theta - 2g + v_0^2\right)}$

$\cancel{\frac{2g}{L} - \frac{v_0^2}{L} = 2g\cos\theta}$

$$0 = \frac{2g}{L}(\cos\theta - 1) + \frac{v_0^2}{L^2}$$

$$-\frac{v_0^2}{2gL} + 1 = \cos\theta$$

$$\theta = \arccos\left(-\frac{v_0^2}{2gL} + 1\right)$$

b) θ_f para la cual la fuerza que hace la barra sobre la partícula se anula

Observe que θ_f puede no existir

$$F\hat{v} = mg\cos\theta - mL\dot{\theta}^2$$

$$F\hat{v} = mg\cos\theta + \left(\frac{2g}{L}(\cos\theta - 1) + \frac{v_0^2}{L^2}\right) mL$$

$$F\hat{v} = mg\cos\theta + 2mg\cos\theta - 2mg + \frac{v_0^2 m}{L}$$

$$F\hat{v} = 3mg\cos\theta - 2mg + \frac{v_0^2 m}{L}$$

$$F\hat{v} = gm\left(3\cos\theta - 2 + \frac{v_0^2}{gL}\right)$$

$$0 = 3\cos\theta - 2 + \frac{v_0^2}{gL}$$

$$3\cos\theta = 2 - \frac{v_0^2}{gL}$$

$$\boxed{\cos\theta = \frac{2}{3} - \frac{v_0^2}{3gL}}$$

$$\theta_f = \arccos\left(\frac{2}{3} - \frac{v_0^2}{3gL}\right)$$

$$-1 \leq \frac{2}{3} - \frac{v_0^2}{3gL} \leq 1$$

$$-1 \leq \frac{2}{3}\left(1 - \frac{v_0^2}{2gL}\right) \leq 1$$

$$-\frac{3}{2} \leq 1 - \frac{v_0^2}{2gL} \leq \frac{3}{2}$$

$$-\frac{5}{2} \leq -\frac{v_0^2}{2gL} \leq \frac{1}{2}$$

$$-5 \leq -v_0^2 \leq 1$$

$$-gL \leq v_0^2 \leq 5gL$$

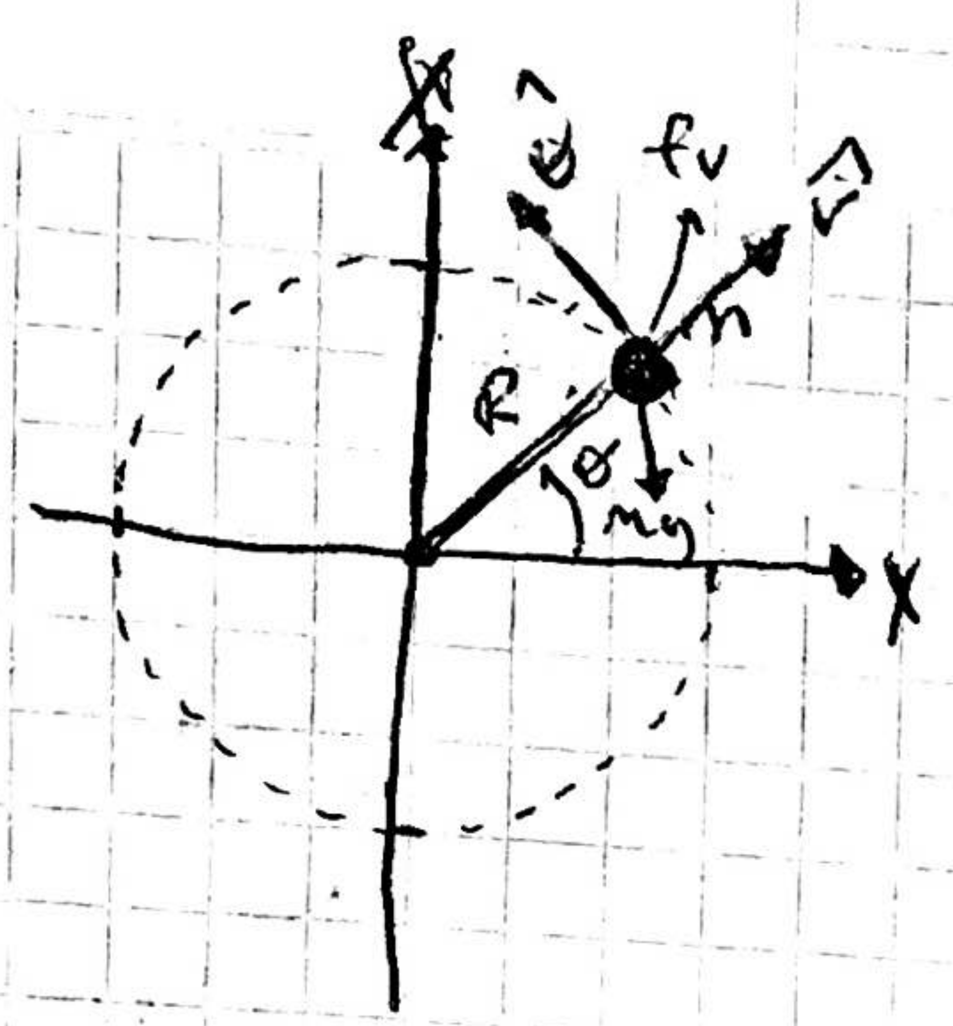
$$\boxed{v_0 \leq \sqrt{5gL}}$$

Para θ_v $\quad -1 \leq 1 - \frac{v_0^2}{2gL} \leq 1$

$$-2 \leq -\frac{v_0^2}{2gL} \leq 2$$

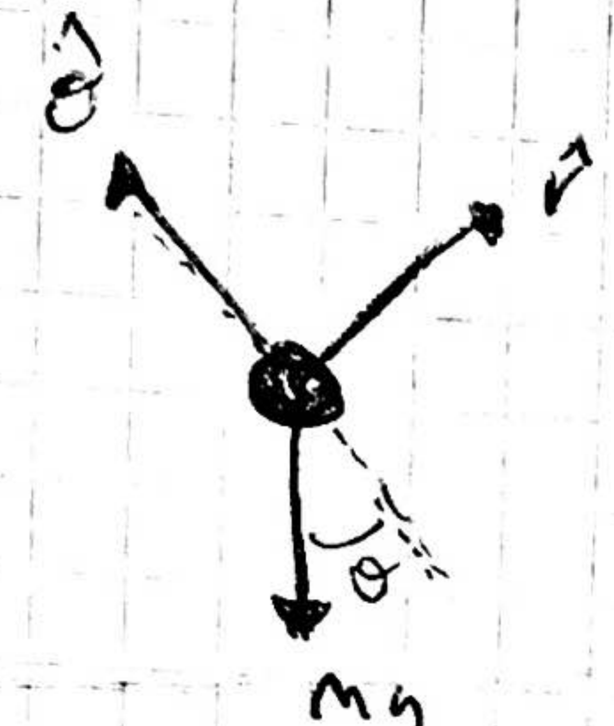
$$4gL \geq v_0^2 \geq -4gL$$

$$v_0 \leq \sqrt{4gL}$$



varilla rígida

$\bar{\omega} = \omega$ + módulo



Ec vínculo

$\bar{r} = R \qquad \omega = cte$

$\bar{f_v} = f_v\hat{r} + f_v\hat{\theta}$

a)

$$(\hat{r}) : f_v\hat{r} - mg \cdot sen\theta = m\,(\overset{=0}{\ddot{r}} - r\overset{\omega=cte}{\dot{\theta}^2})$$

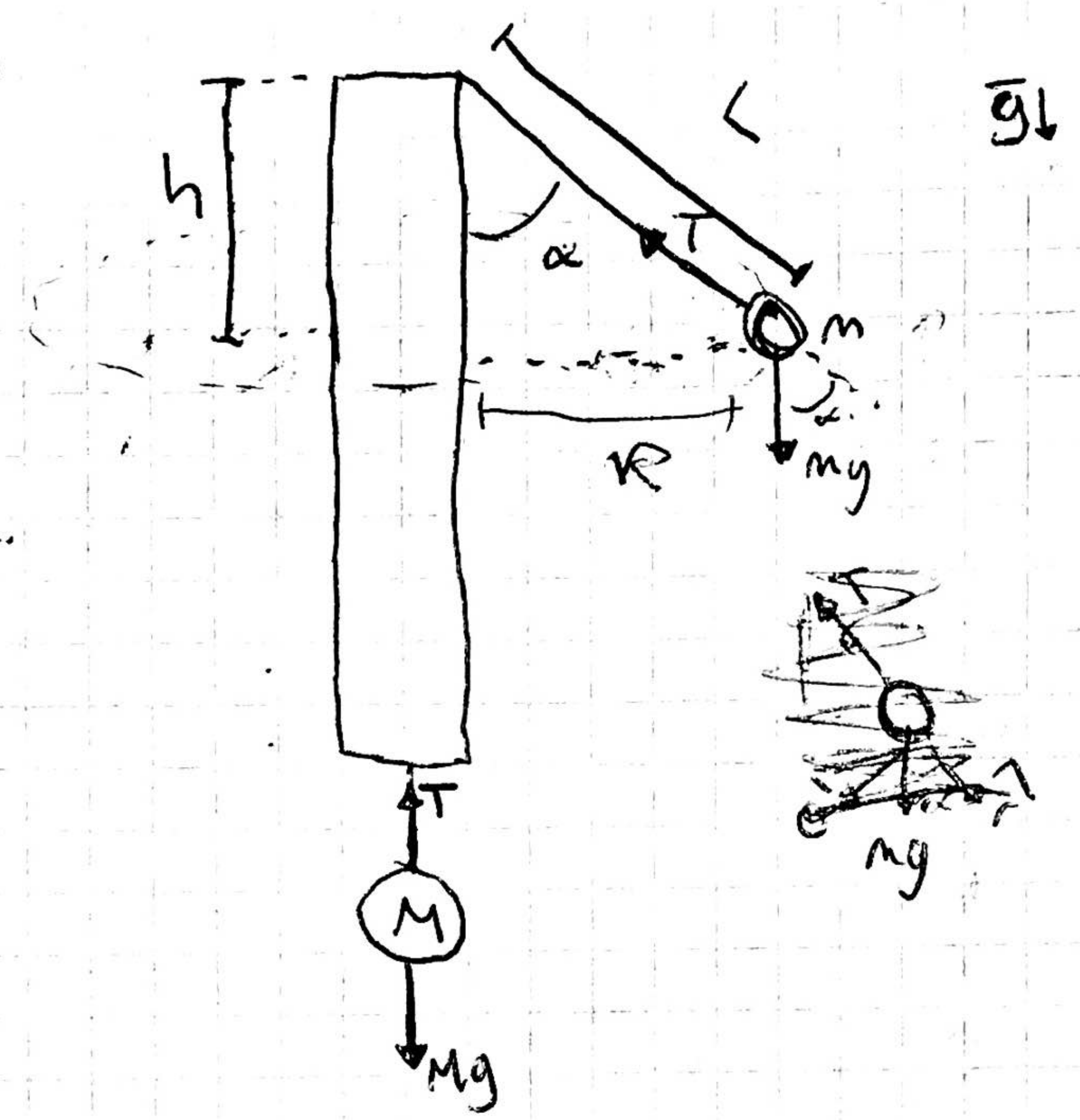
$$f_v\hat{r} = mg\,sen\theta + mR\omega^2$$

$$(\hat{\theta}) : f_v\hat{\theta} - mg\cos\theta = m\,(r\overset{=0}{\ddot{\theta}} + 2\overset{=0}{\dot{r}\dot{\theta}})$$

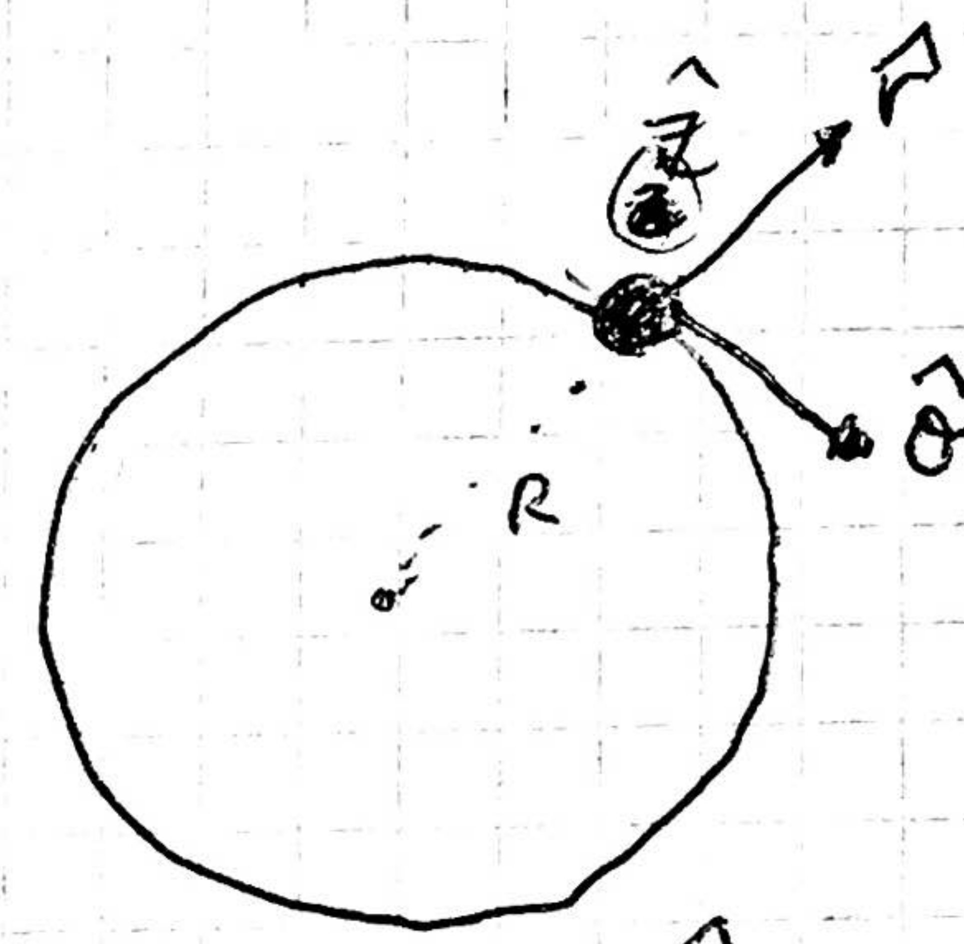
$$f_v(\hat{\theta}) = mg\cos\theta$$

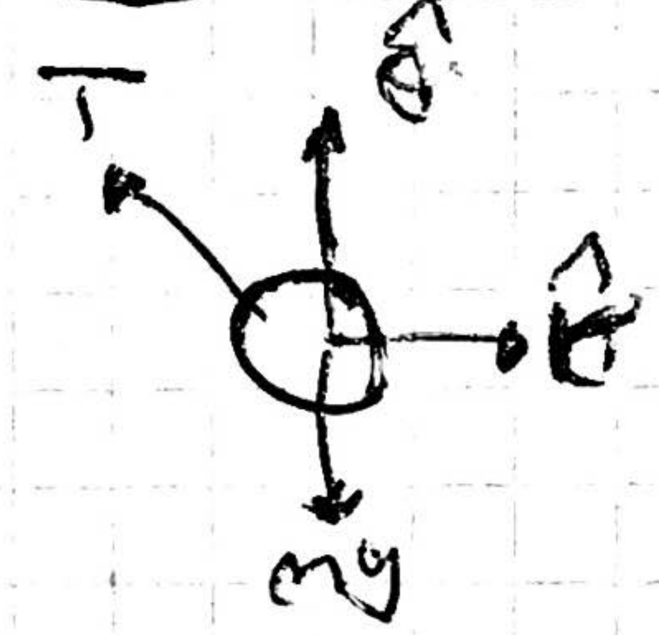
b) $\bar{f_v}(\theta) = \sqrt{m^2g^2\cos^2\theta + (mg\,sen\theta + mR\omega^2)^2} = mg\cos\theta\,\hat{\theta} + mg\,sen\theta + mR\omega^2\,\hat{r}$

9 $M > m$



$T = T(m) \cdot g$





$M)(\hat{z})\ T = Mg$

$(\hat{r})\ -T \operatorname{sen}\alpha = m\cdot(\underset{=0}{\ddot{r}} - r\dot{\theta}^2) \Rightarrow -T\operatorname{sen}\alpha = -mR\dot{\theta}^2$

$(\hat{\theta})\ T\cos\alpha - mg = m\cdot(2\underbrace{\dot{r}\dot{\theta}}_{=0} + r\ddot{\theta})$

$T\cos\alpha - mg = mR\ddot{\theta}$

$+T\operatorname{sen}\alpha = +mR\dot{\theta}^2$

$\cancel{\dfrac{Mg\operatorname{sen}\alpha}{mR}} = \dot{\theta}^2$

T en función de g y h

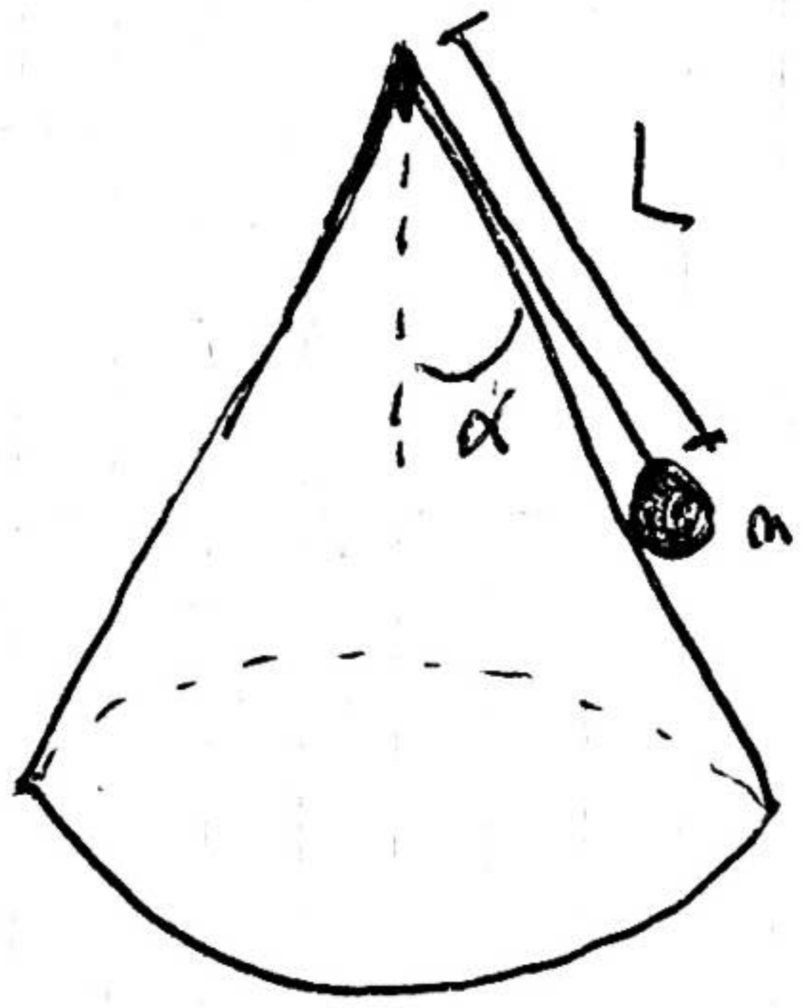
$$L = \frac{M g T^2}{m 4\pi^2} \qquad \cos\alpha = \frac{m}{M}$$

$$L \frac{m}{M} = \frac{g T^2}{4\pi^2}$$

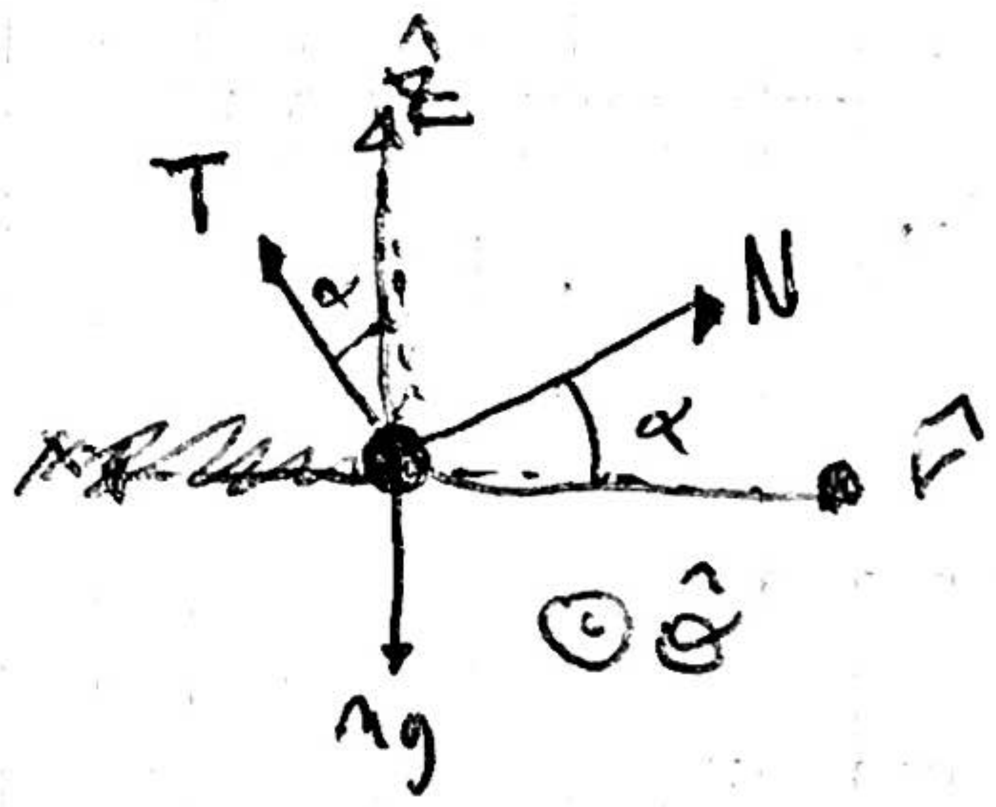
$$L\cos\alpha = h = \frac{g T^2}{4\pi^2}$$

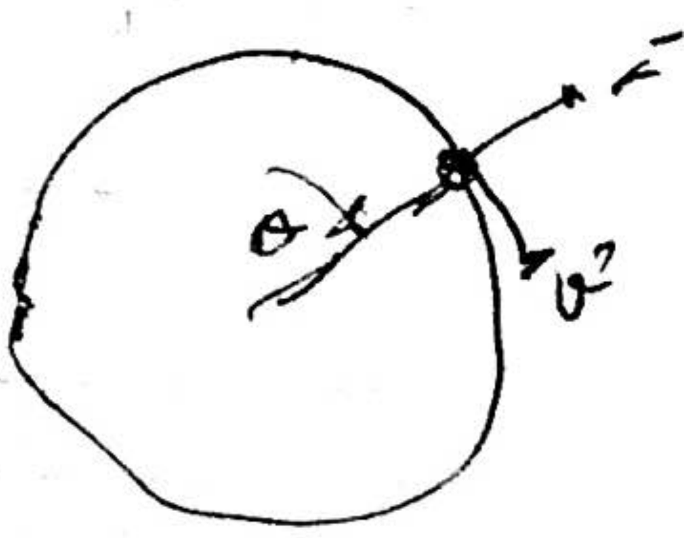
$$\sqrt{\frac{h 4\pi^2}{g}} = T \qquad \boxed{T = \sqrt{\frac{h}{g}} \cdot 2\pi}$$

6



gb Datos m. L. $\omega_0 = \omega_0$.





$(\hat{r})$ $-T\sin\alpha + N\cos\alpha = m\ddot{r} = -mR\dot{\theta}^2$

$(\hat{z})$ $T\cos\alpha + N\sin\alpha - m g = m\ddot{z} = 0$

$(\hat{\theta})$ $0 = m\ddot{\theta} = 0$

$r = R = L\sin\alpha$

vínculo $\ddot{z} = 0$

$\ddot{\theta} = 0 \Rightarrow \dot{\theta} = cte = \omega_0$

$$a = (\underbrace{\ddot{r}}_{=0} - r\dot{\theta}^2)\hat{r} + (\underbrace{2\dot{r}\dot{\theta}}_{=0} + \underbrace{r\ddot{\theta}}_{=0})\hat{\theta} + \underbrace{\ddot{z}\hat{z}}_{=0}$$

$$a = -R\dot{\theta}^2$$

$$\boxed{\bar{a} = L\sin\alpha \cdot \omega_0^2 \hat{r}}$$

c) T y N = ? Para qué valor de ω_0 N = 0?

$$T\cos\alpha + N\operatorname{sen}\alpha = mg$$

$$-T\operatorname{sen}\alpha + N\cos\alpha = -mL\operatorname{sen}\alpha \cdot \omega_0^2$$

$$T\cos\alpha = mg - N\operatorname{sen}\alpha$$

$$T = \frac{mg - N\operatorname{sen}\alpha}{\cos\alpha}$$

$$\Rightarrow -\frac{mg + N\operatorname{sen}\alpha}{\cos\alpha} \cdot \operatorname{sen}\alpha + N\cos\alpha = -mL\operatorname{sen}\alpha \cdot \omega_0^2$$

$$\cancel{N(-mg\operatorname{sen}\alpha\,tg\alpha + \cos\alpha)} \quad N\,tg\alpha \cdot \operatorname{sen}\alpha + N\cos\alpha = -mL\operatorname{sen}\alpha\,\omega_0^2 + mg\,tg\alpha$$

$$N(tg\alpha\operatorname{sen}\alpha + \cos\alpha) = -mL\operatorname{sen}\alpha\,\omega_0^2 + mg\,tg\alpha$$

$$\boxed{N = \frac{-mL\operatorname{sen}\alpha\,\omega_0^2 + mg\,tg\alpha}{tg\alpha\operatorname{sen}\alpha + \cos\alpha}}$$

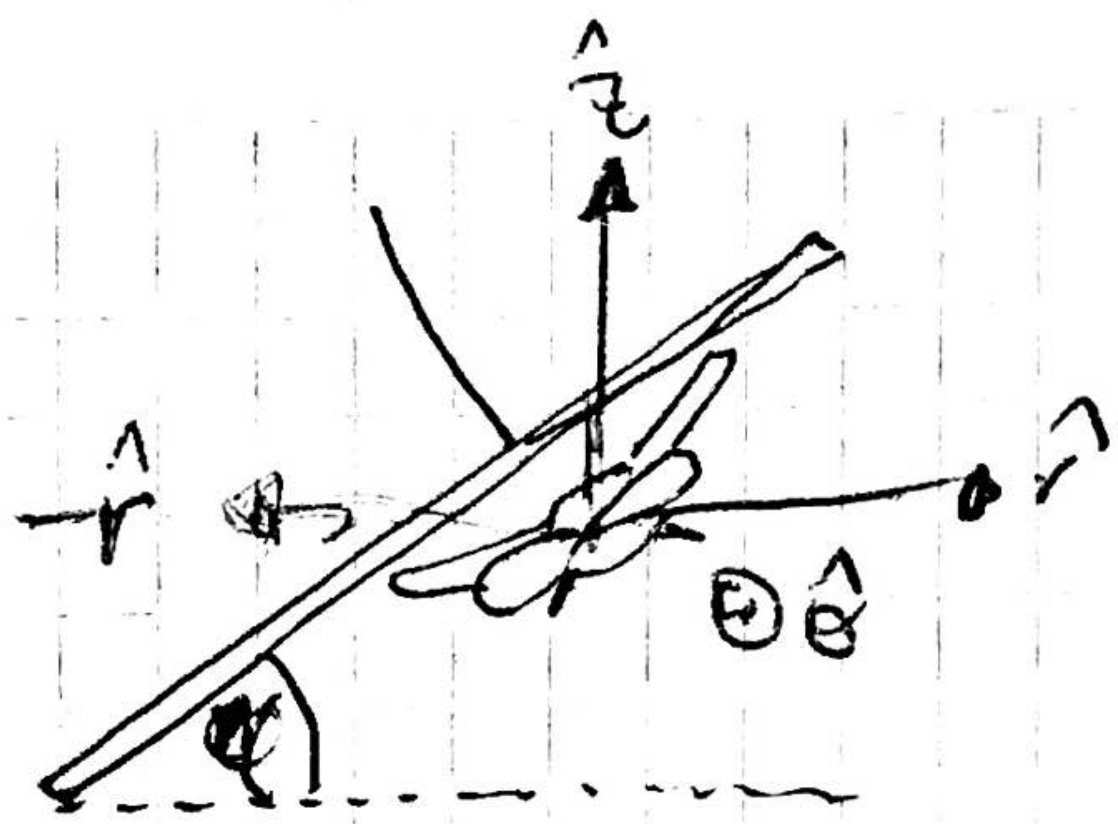
$$\boxed{T = \frac{mg}{\cos\alpha} - \left(\frac{-mL\operatorname{sen}\alpha\,\omega_0^2 + mg\,tg\alpha}{tg\alpha\operatorname{sen}\alpha + \cos\alpha}\right) \cdot \frac{1}{\cos\alpha}}$$

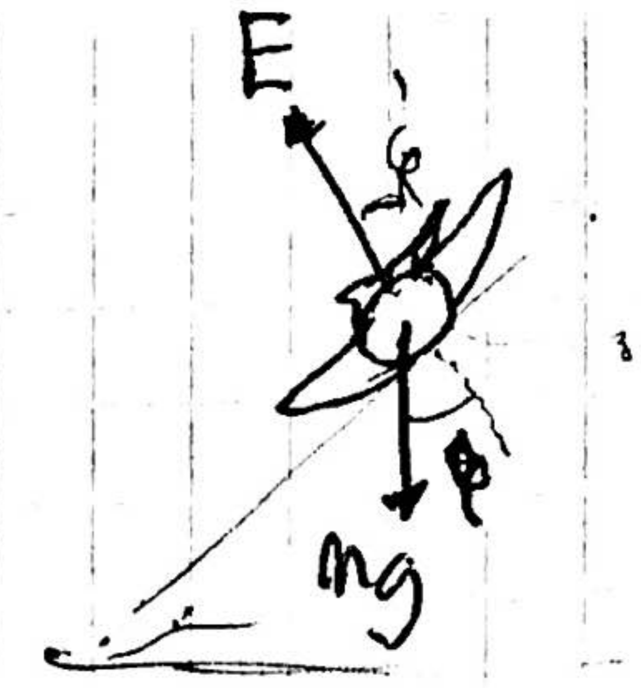
Si $N = 0$

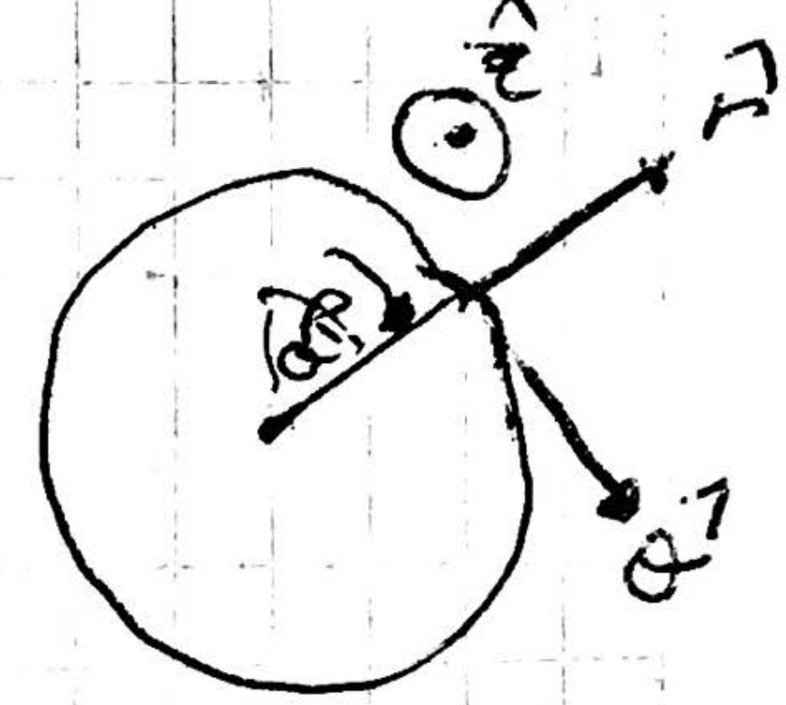
$$+T\cancel{\operatorname{sen}\alpha} = +mL\cancel{\operatorname{sen}\alpha}\,\omega_0^2$$

$$\boxed{\sqrt{\frac{T_{máx}}{mL}} = |\omega_0|}$$

(1)







$|\bar{V}| = cte \quad r = R$

a) φ en términos de $|\bar{V}|$, R y g

$(\hat{z})\quad E.\cos\varphi - m.g = m.\ddot{z} = 0$

$(\hat{r})\quad -E.\text{sen}\,\varphi = m.\ddot{r} = m(\ddot{r} - r\dot{\theta}^2)$ ($\ddot{r} = 0$)

$\Rightarrow +E\,\text{sen}\,\varphi = +mR\dot{\theta}^2$

$(\hat{\theta})\quad 0 = m.\ddot{\theta} = 0$

$E = \dfrac{mg}{\cos\varphi}$

$\Rightarrow mg\,tg\,\varphi = mR.\dot{\theta}^2$

$\cancel{m}g\,tg\,\varphi = \cancel{m}R.\dfrac{|\bar{V}|^2}{R^2}$

$$\boxed{\varphi = \text{arctg}\,\frac{\cancel{m}|V|^2}{Rg}}$$

φ para $|V| = 60\,m/s$ y $R = 1km$?

$\varphi = \text{arctg}\,\dfrac{(60\,m/s)^2}{1000m . 9{,}8\,m/s^2}$ $\quad \boxed{\varphi = 20{,}16°}$

$r = R$

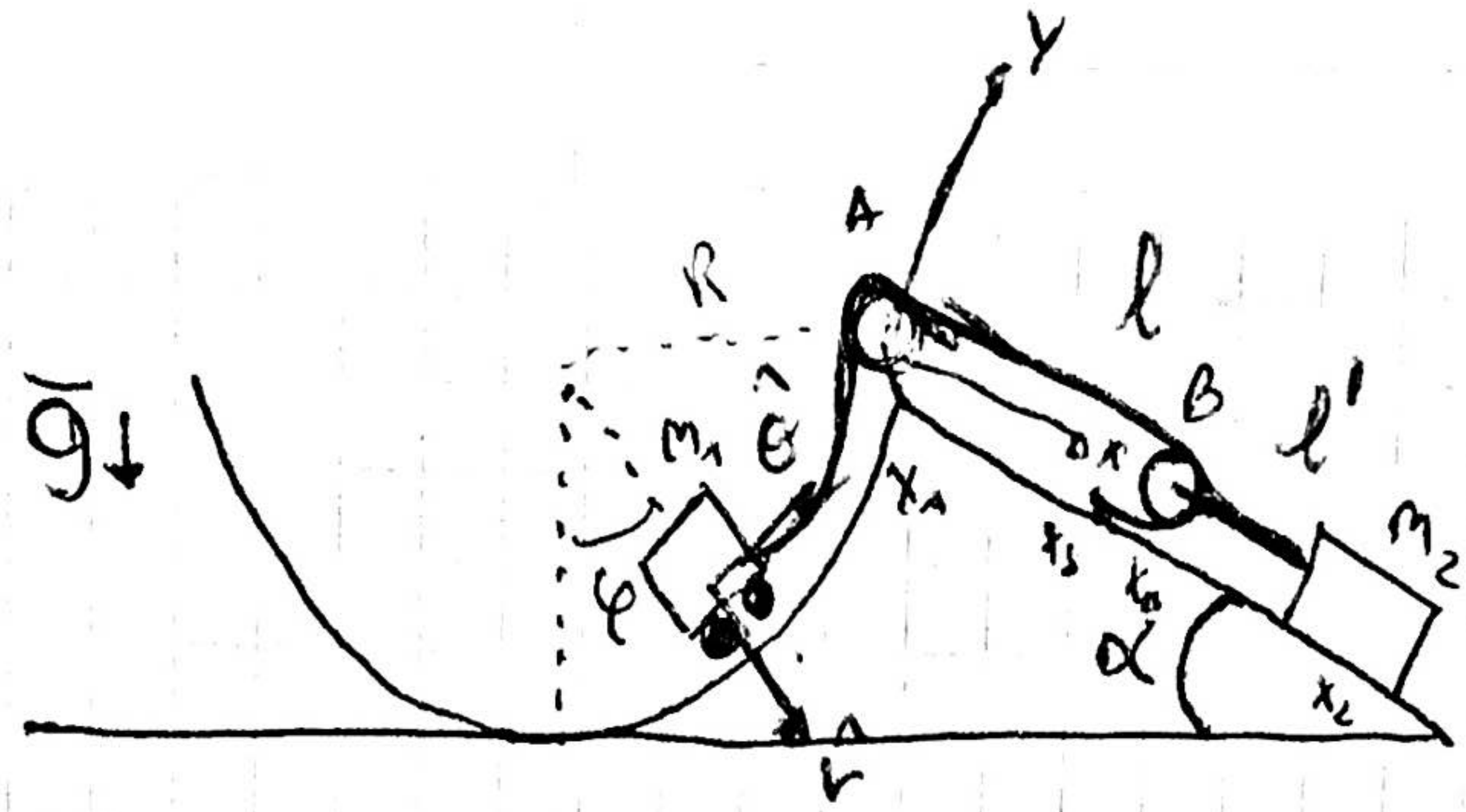
$|\bar{V}| = cte$

$|\bar{V}| = |\dot{r}\hat{r} + \dot{\theta}\hat{\theta}.r + \dot{z}\hat{z}|$ ($\dot{r} = 0$, $\dot{z} = 0$)

$|\bar{V}| = |r\dot{\theta}|$

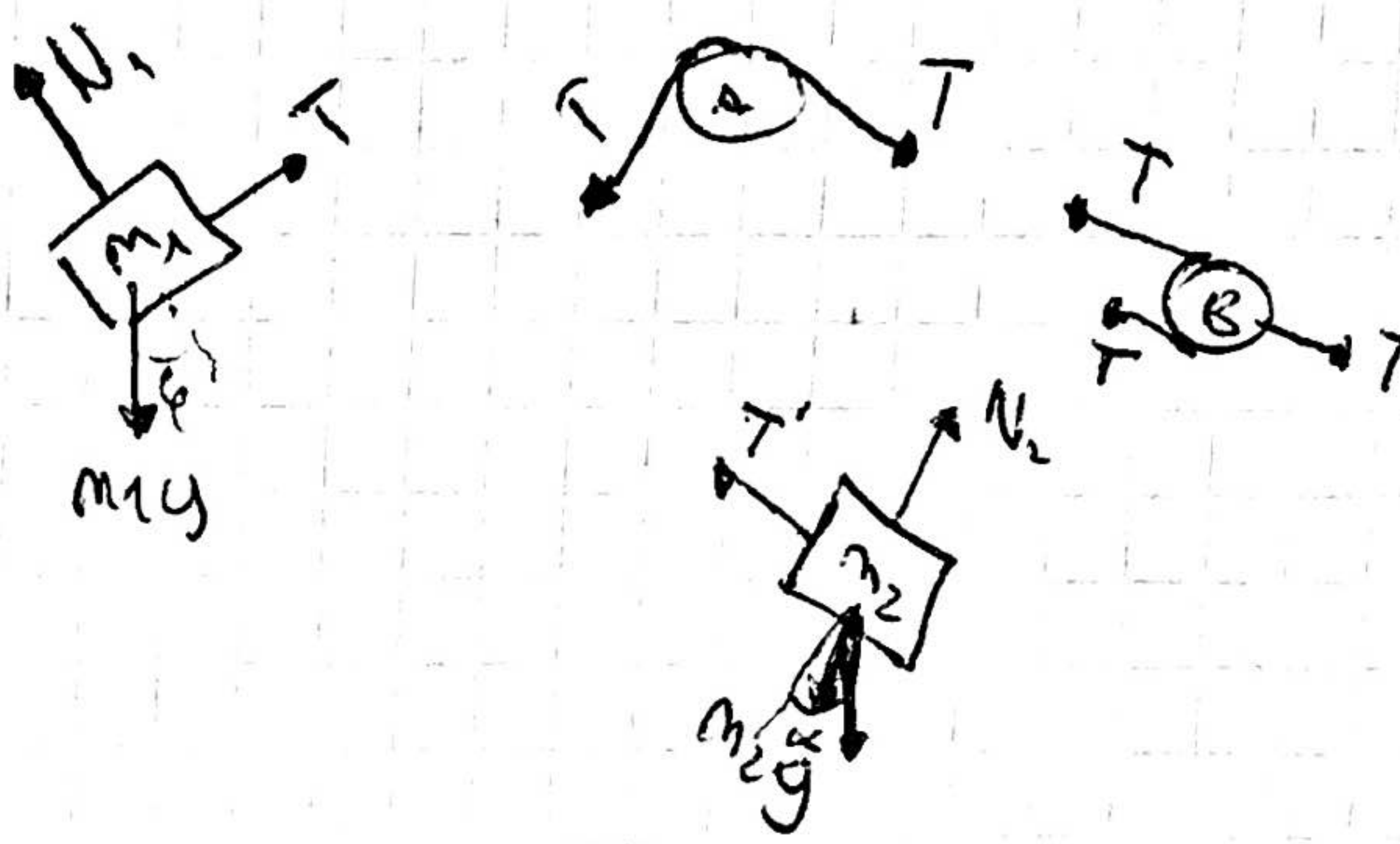
$\dfrac{|\bar{V}|}{R} = \dot{\theta}$

12



$r = R$

a) Ec de Newton



$\bar{r} = R\hat{r}$

$\dot{\bar{r}} = \underbrace{\dot{r}\hat{r}}_{=0} + R\dot{\varphi}\hat{\theta}$

$\ddot{\bar{r}} = -R\dot{\varphi}^2\hat{r} + R\ddot{\varphi}\hat{\theta}$

$m_1 \to (\hat{r})\ -N_1 + m_1 g \cdot \cos\varphi = m_1 \cdot (-R\dot{\varphi}^2)$

$(\hat{\theta})\ T - m_1 \cdot g\, \text{sen}\,\varphi = m_1 R\ddot{\varphi}$

$m_2 \to (\hat{x})\ -T' + m_2 \cdot g \cdot \text{sen}\,\alpha = m_2 \cdot \ddot{x}_2$

$(\hat{y})\ N_2 - m_2 \cdot g \cdot \cos\alpha = m_2 \overset{=0}{\ddot{y}_2} \Rightarrow N_2 = m_2 g \cos\alpha$

$T' = 2T$

$l = -x_A + x_B + x_s + x_B + R\left(\frac{\pi}{2} - \varphi\right)$ $\qquad l' = x_B - x_2$

$0 = \overset{=0}{\dot{x}_A} + 2\dot{x}_B + \overset{=0}{\dot{x}_s} + \overset{=0}{R\frac{\dot{\pi}}{2}} - R\dot{\varphi}$ $\qquad 0 = \ddot{x}_B - \ddot{x}_2$

$0 = -R\dot{\varphi} + 2\dot{x}_B \Rightarrow 0 = -R\ddot{\varphi} + 2\ddot{x}_B$

b) ¿Para que φ el sistema estara en reposo?

Para reposo $\ddot{\varphi}_{(\varphi R)} = 0 \ \wedge\ \dot{\varphi}_{(\varphi R)} = 0$ (ya es cero porque yo le doy $V_0 = 0$)

Para $\ddot{\varphi}_{(\varphi R)} = 0 \Rightarrow$

$2T - 2m_1 g\, \text{sen}\,\varphi = 2m_1 R\ddot{\varphi}$

$+ \ -2T + m_2 g\, \text{sen}\,\alpha = m_2 \ddot{x}_2$

$\Rightarrow -2m_1 g\, \text{sen}\,\varphi + m_2 g\, \text{sen}\,\alpha = 2m_1 R\ddot{\varphi} + m_2 \ddot{x}_2$

$\ddot{x}_2 = \ddot{x}_B \qquad \ddot{x}_B = +\frac{R\ddot{\varphi}}{2}$

$\Rightarrow -2m_1 g \operatorname{sen}\varphi + m_2 g \operatorname{sen}\alpha = 2m_1 R\ddot{\varphi} + \frac{m_2 R\ddot{\varphi}}{2}$

$g(-2m_1 \operatorname{sen}\varphi + m_2 \operatorname{sen}\alpha) = \ddot{\varphi}\left(2m_1 R + \frac{m_2 R}{2}\right)$

si $\ddot{\varphi} = 0 \Rightarrow -2m_1 \operatorname{sen}\varphi + m_2 \operatorname{sen}\alpha = 0$

$2m_1 \operatorname{sen}\varphi = m_2 \operatorname{sen}\alpha$

$\operatorname{sen}\varphi = \frac{m_2 \operatorname{sen}\alpha}{2m_1}$

$\boxed{\varphi = \operatorname{arcsen}\left(\frac{m_2 \operatorname{sen}\alpha}{2m_1}\right)}$

Volvamos

$\cancel{-2m_1 \operatorname{sen}\varphi + m_2 \operatorname{sen}\alpha = \frac{R}{g}\ddot{\varphi}\left(1 + \frac{m_2}{2m_1}\right)}$

$-2m_1 \operatorname{sen}\varphi + m_2 \operatorname{sen}\alpha = 0$

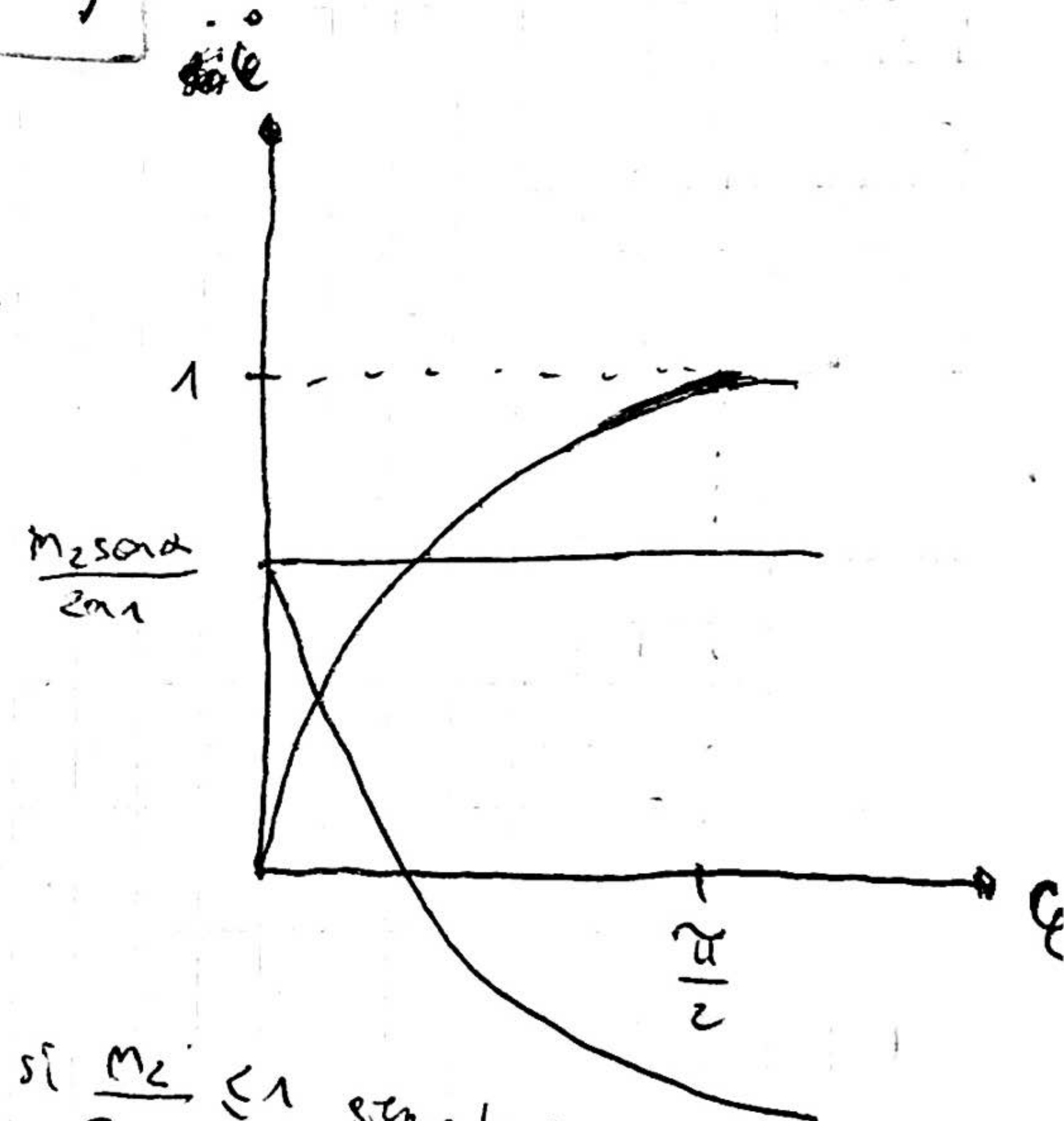
$-\operatorname{sen}\varphi + \frac{m_2 \operatorname{sen}\alpha}{2m_1} = 0$

$\cancel{\operatorname{sen}\varphi \leq \frac{m_2 \operatorname{sen}\alpha}{2m_1} \geq 0}$

Pendiente derivada
Decreciente o estable

Si $\frac{m_2 \operatorname{sen}\alpha}{m_1} \leq 1$ existe φ Rep

Si $\frac{m_2 \operatorname{sen}\alpha}{2m_1} > 1$ no existe φ Rep



entonces si $\frac{m_2}{2m_1} \leq 1$ siempre hay reposo

c) $\bar{v}_{(\varphi)} = ?$ $\quad v = R\dot{\varphi}\hat{\theta}$

$v = R$

$\dot{\varphi} = \frac{g(-2m_1 \sin\varphi + m_2 \sin\alpha)}{2m_1 R + \frac{m_2 R}{2}}$

$\int \dot{\varphi}\, d\dot{\varphi} = \int \frac{g(-2m_1 \sin\varphi + m_2 \sin\alpha)}{2m_1 R + \frac{m_2 R}{2}}\, d\varphi$

$\frac{\dot{\varphi}^2}{2} - \frac{\dot{\varphi}_0^2}{2} = \frac{g(2m_1 \cos\varphi + m_2 \sin\alpha\, \varphi)}{2m_1 R + \frac{m_2 R}{2}}$

$$\boxed{\dot{\varphi} = \sqrt{2\left(\frac{g(2m_1 \cos\varphi + m_2 \sin\alpha\, \varphi)}{2m_1 R + \frac{m_2 R}{2}} + \frac{\dot{\varphi}_0^2}{2}\right)} \qquad v = R \cdot \dot{\varphi}}$$