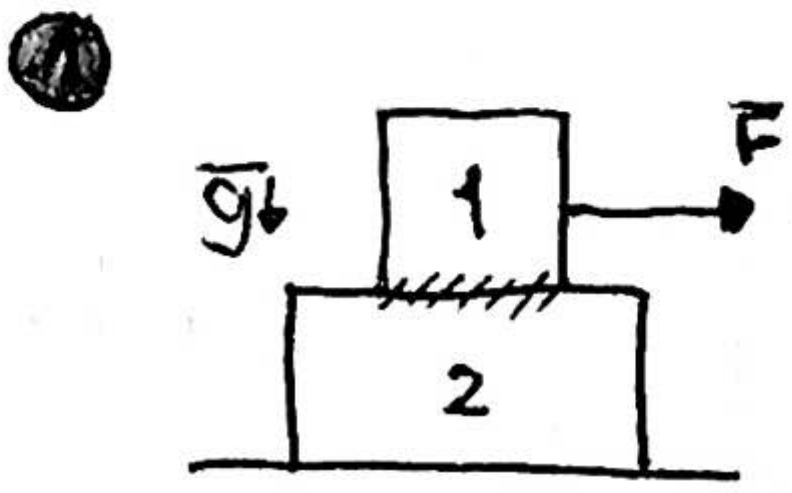
DINÁMICA I
ROZ

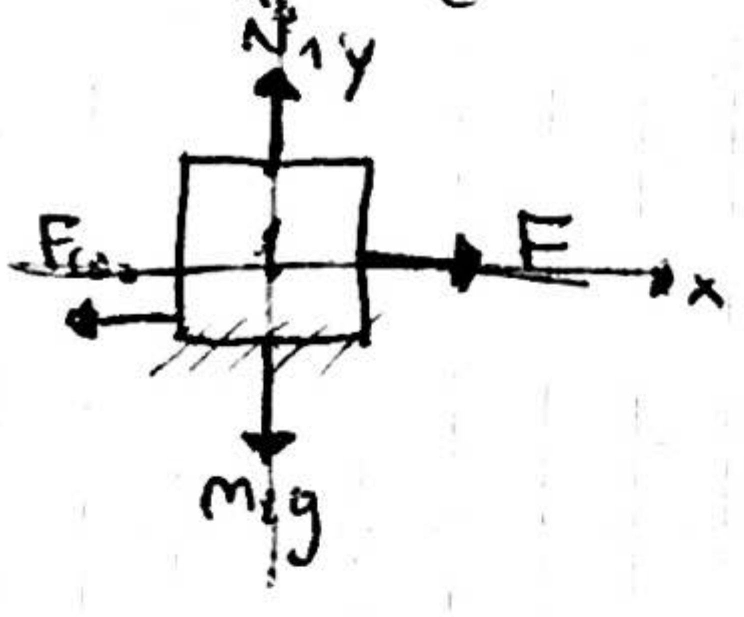
1

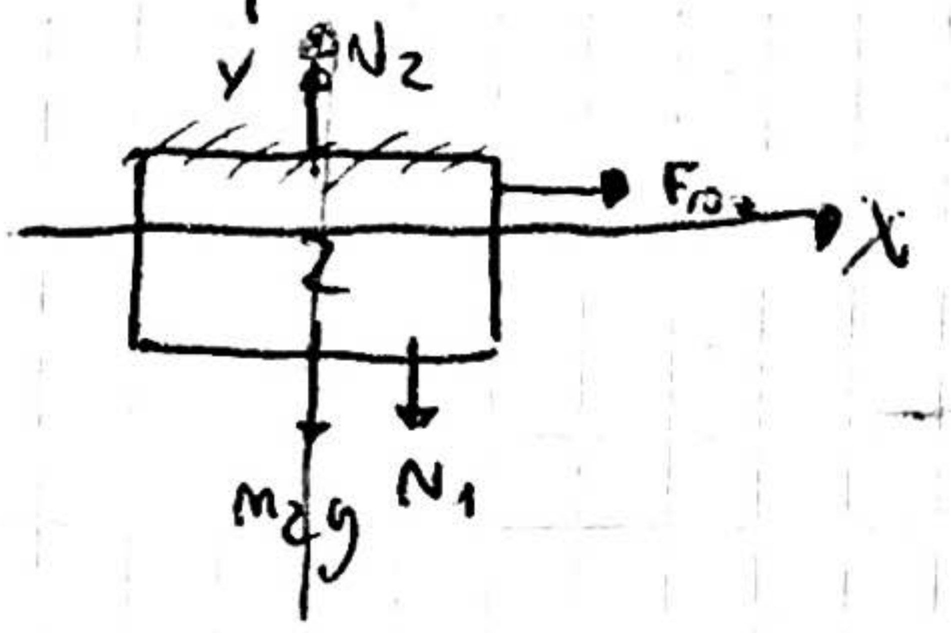


Datos

$m_1 \quad m_2 \quad \mu_E$

$F_{MÁX}$ que acelere ambos cuerpos pero no deslicen uno respecto del otro





$\Rightarrow (1) = (\hat{x}) \quad F - F_{roz} = m_1 \cdot \ddot{x}_1$

$(\hat{y}) \quad N_1 - m_1 g = m_1 \ddot{y}_1 \overset{=0}{} \Rightarrow N_1 = m_1 g$

$(2) = (\hat{x}) \quad F_{roz} = m_2 \cdot \ddot{x}_2$

$(\hat{y}) \quad N_2 - N_1 - m_2 g = m_2 \cdot \ddot{y}_2 \overset{=0}{} \Rightarrow N_2 = N_1 + m_2 g$

Situación estática

$F_{roz} = F_{RE} \qquad \ddot{x}_1 = \ddot{x}_2$

$|F_{ROZ\,MÁX}| = \mu_E \cdot N_1$

$\Rightarrow F_{roz} = F - m_1 \ddot{x}_1 \qquad\qquad F_{roz} = m_2 \cdot \ddot{x}_1$

$\frac{F_{ROZ}}{m_2} = \ddot{x}_1$

$\cancel{F_{ROZ\,MÁX} = F_{MÁX}}$

$\cancel{\mu_E \cdot N_1 = F_{MÁX}}$

$\cancel{\mu_E\, m_1\, g = F_{MÁX}}$

$\Rightarrow F_{ROZ} = F - \frac{m_1 F_{ROZ}}{m_2}$

$F_{ROZ} + \frac{F_{ROZ} m_1}{m_2} = F$

$F_{ROZ}\left(1 + \frac{m_1}{m_2}\right) = F$

$F_{ROZ} = \frac{F}{\left(1 + \frac{m_1}{m_2}\right)} \qquad |F_{RE\,MÁX}| = \mu_E \cdot N_1 = \mu_E \cdot m_1 g$

$\cancel{\mu_E m_1 g} = \frac{F_{MÁX}}{\left(1 + \frac{m_1}{m_2}\right)} = \mu_E \cdot m_1 g$

F

$$\boxed{F_{máx} = \mu_e \cdot m_1 \cdot g \left(1 + \frac{m_1}{m_2}\right)}$$

¿Cual es la aceleración del sistema?

$$\boxed{\ddot{x}_1 = \frac{F - F_{roz}}{m_1} = \frac{F_{roz}}{m_2}}$$

Para el otro

$$\boxed{\begin{array}{l} F_{máx} = \mu_e g \left(1 + \frac{m_2}{m_1}\right) m_1 = \mu_e g (m_1 + m_2) \\ \ddot{x} = \frac{F_{re}}{m_1} = \frac{-F_{re} + F_r}{m_2} \end{array}}$$

Se aplica sobre m_2 una $f = 2F_{máx}$ calculada en C) ¿cual es la aceleración de m_1 y m_2 si μ_D?

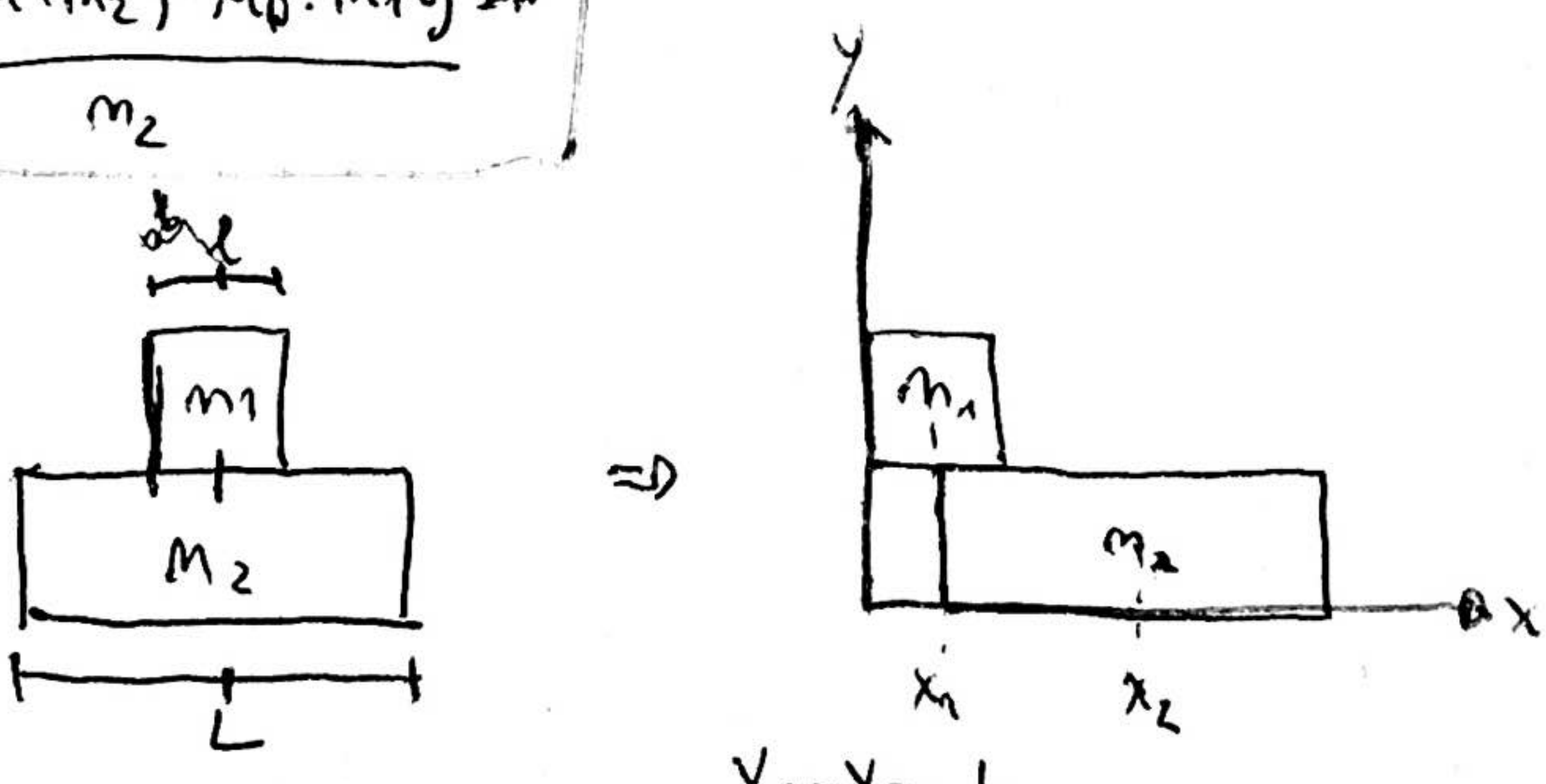
$$F = 2\mu_e \cdot g (m_1 + m_2)$$

$\Rightarrow$ Rozamiento dinámico $\quad \ddot{x}_1 \neq \ddot{x}_2 \wedge F_R = F_{RD}$

(1) $(\hat{x})$ $F_{RD} = m_1 \ddot{x}_1$ $\quad (\hat{y})$ $N_1 = m_1 g$

(2) $(\hat{x})$ $2\mu_e g (m_1 + m_2) - F_{RD} = m_2 \ddot{x}_2$ $\quad (\hat{y})$ $N_2 = m_2 g + N_1$

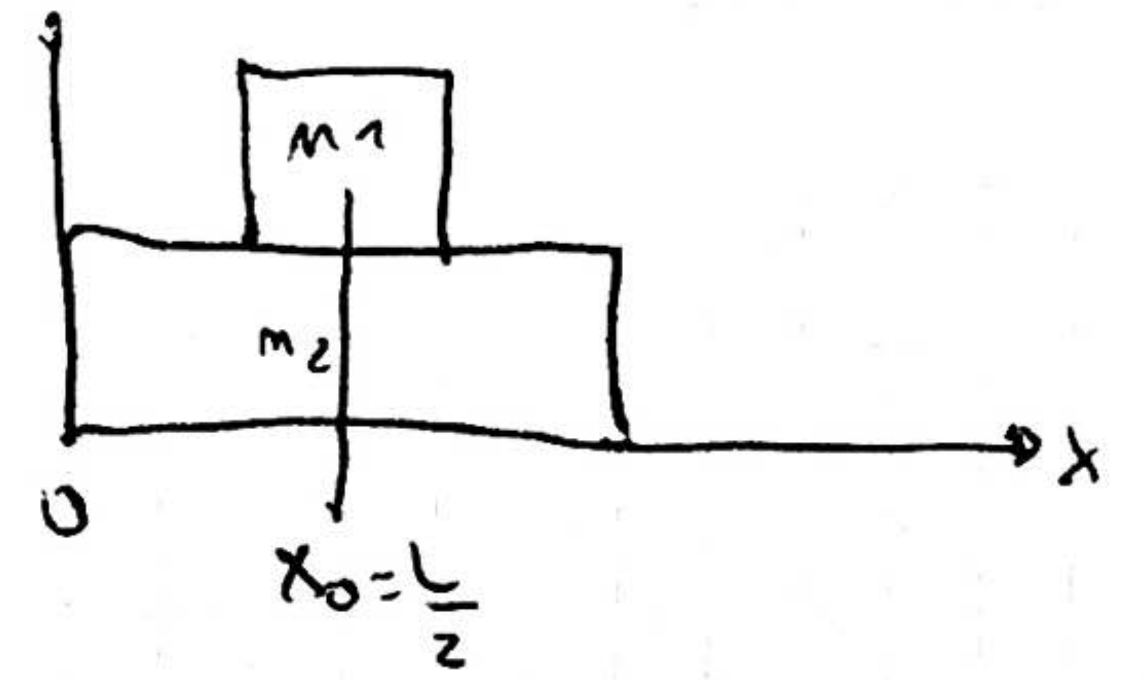
$$\boxed{\begin{array}{l} \ddot{x}_1 = \frac{\mu_D \cdot \cancel{m_1} \cdot g}{\cancel{m_1}} \quad \ddot{x}_1 = \mu_D \cdot g \\ \ddot{x}_2 = \frac{2\mu_e g (m_1 + m_2) - \mu_D \cdot m_1 g}{m_2} \end{array}}$$

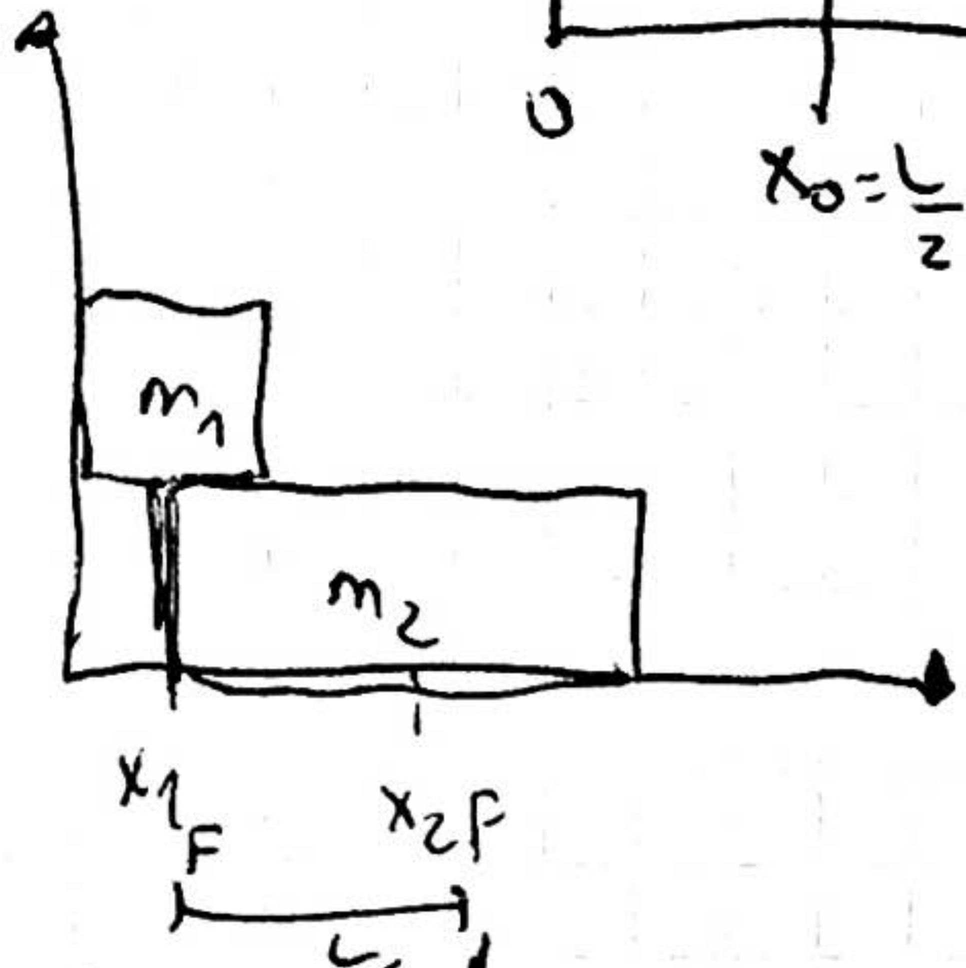


$$x_1 - x_2 = \frac{L}{2}$$

$$x_1 = \frac{\mu_D g t^2}{2}$$

$$x_2 = \frac{2\mu_e g (m_1 + m_2) - \mu_D m_1 g}{2m_2} t^2$$





$$x_{2F} - x_{1F} = \frac{L}{2}$$

$$\ddot{x}_1 = \frac{\mu_D \cdot g t^2 + L}{2}$$

$$\ddot{x}_2 = \frac{2\mu_E g (m_1 + m_2) - \mu_D m_1 g t^2}{2m_2} + \frac{L}{2}$$

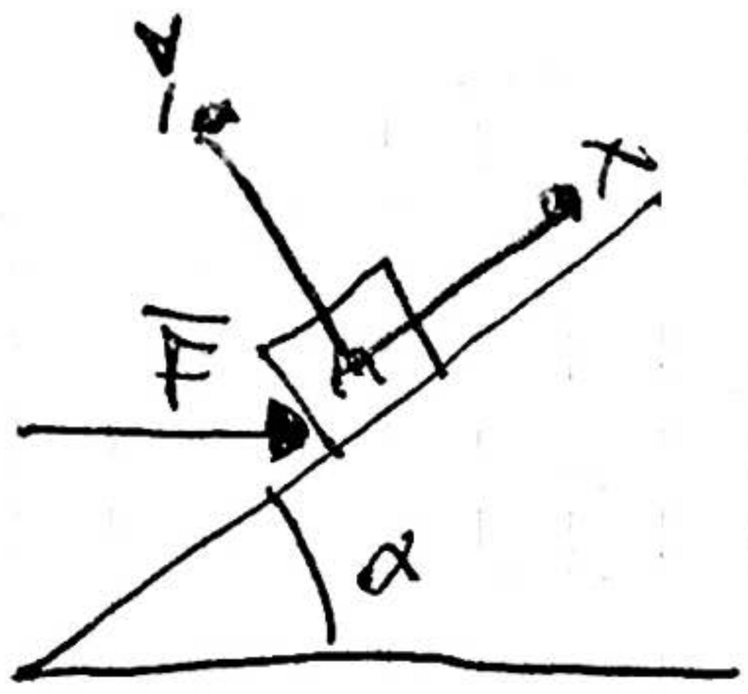
$$\frac{(2\mu_E g (m_1 + m_2) - \mu_D m_1 g) t^2}{2m_2} + \cancel{\frac{L}{2}} - \frac{\mu_D \cdot g \cdot t^2}{2} - \cancel{\frac{L}{2}} = \frac{L}{2}$$

$$t^2 \left(\frac{2\mu_E g (m_1 + m_2) - \mu_D m_1 g}{2m_2} - \frac{\mu_D \cdot g}{2} \right) = \frac{L}{2}$$

$$t = \sqrt{\frac{L}{2} \cdot \left(\frac{1}{\frac{2\mu_E g (m_1 + m_2) - \mu_D m_1 g - \mu_D g m_2}{2m_2}} \right)}$$

$$\boxed{t_F = \sqrt{\frac{L}{2} \cdot \frac{2m_2}{(2\mu_E g (m_1 + m_2) - \mu_D m_1 g - \mu_D g m_2)}}}$$

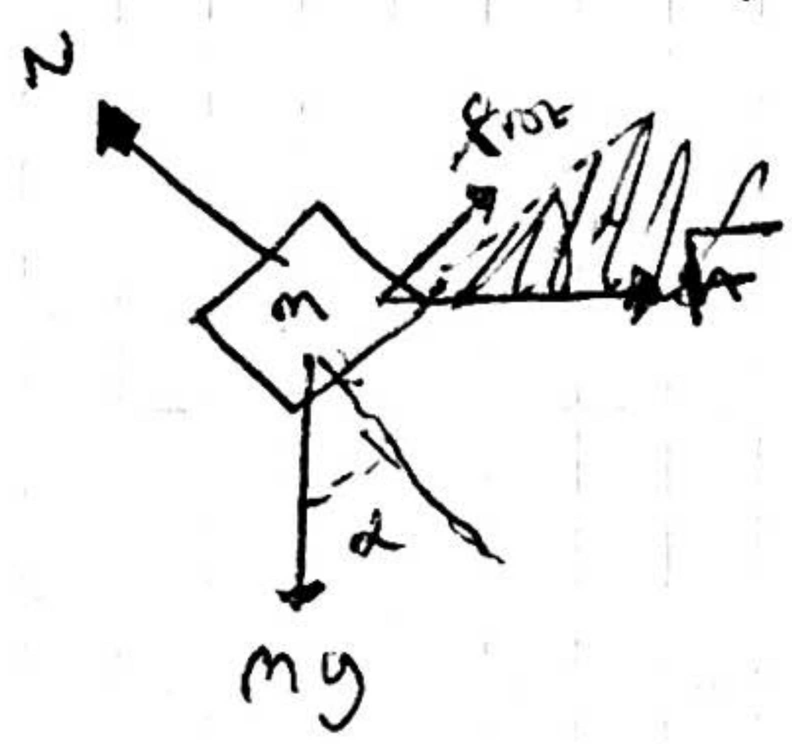
2



DATOS
m μ_E

$\bar{g}\downarrow$

a) se conoce m y μ_E y $\bar{F}=0$ ¿para que α estará el bloque en reposo?



si $F=0$

$(\hat{x})\ -mg.\operatorname{sen}\alpha \pm F_{r_E}=0 \quad \Rightarrow F_{R_E}=mg\operatorname{sen}\alpha$

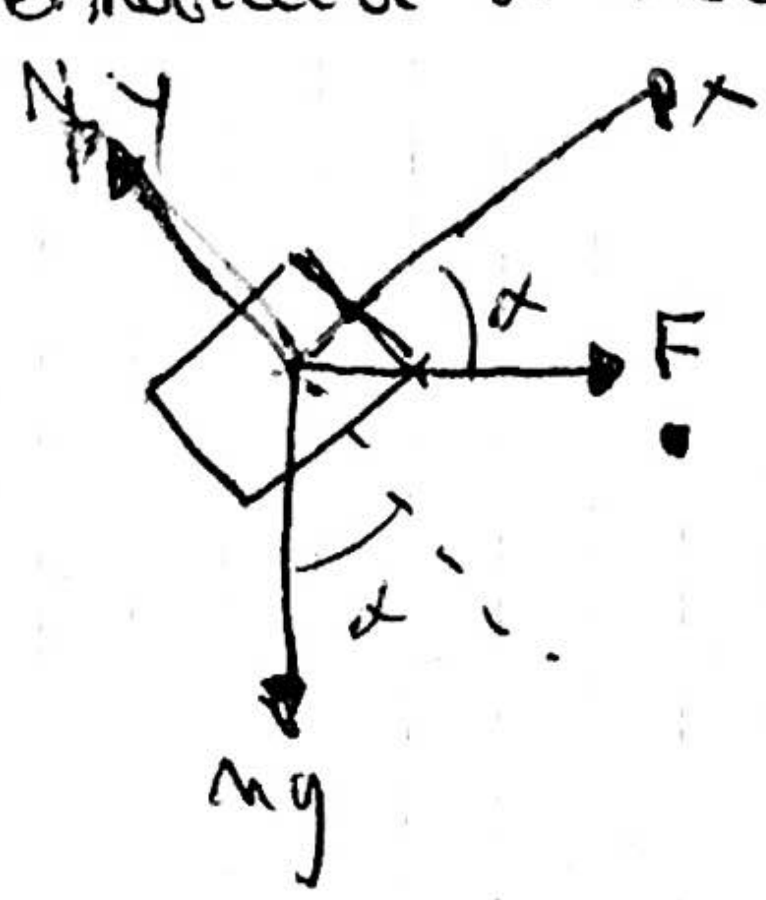
$(\hat{y})\ N-mg\cos\alpha=0 \qquad N=mg\cos\alpha$

$\Rightarrow\ F_{r_{E_{máx}}}=\mu_E.N \geq |F_{r_E}|$ como $\alpha\in[0,\frac{\pi}{2}]$ no importa el módulo

$\Rightarrow\ \mu_E.\cancel{mg}\cos\alpha \geq \cancel{mg}\operatorname{sen}\alpha$

$\boxed{\mu_E \geq tg\ \alpha}$

b) si α es alguno de los valores hallados en a) ¿Para que valores de $\bar{F}$ permanecerá el bloque en reposo?



$(\hat{x})\ +F_{r_E}+F\cos\alpha-mg.\operatorname{sen}\alpha=0$

$(\hat{y})\ N-mg\cos\alpha-F\operatorname{sen}\alpha=0$

$N=mg\cos\alpha+F\operatorname{sen}\alpha$

$|F_{r_E}|\leq\mu_E N=F_{r_{E_{máx}}}$

$$\overbrace{-\mu_e(mg\cos\alpha+F\,\text{sen}\,\alpha)\le -F\cos\alpha+mg\,\text{sen}\,\alpha}^{1}\le \mu_e\left(mg\cos\alpha+F\,\text{sen}\,\alpha\right)$$

2

(1) $F\cos\alpha-\mu_e F\,\text{sen}\,\alpha \le mg\,\text{sen}\,\alpha+\mu_e mg\cos\alpha$

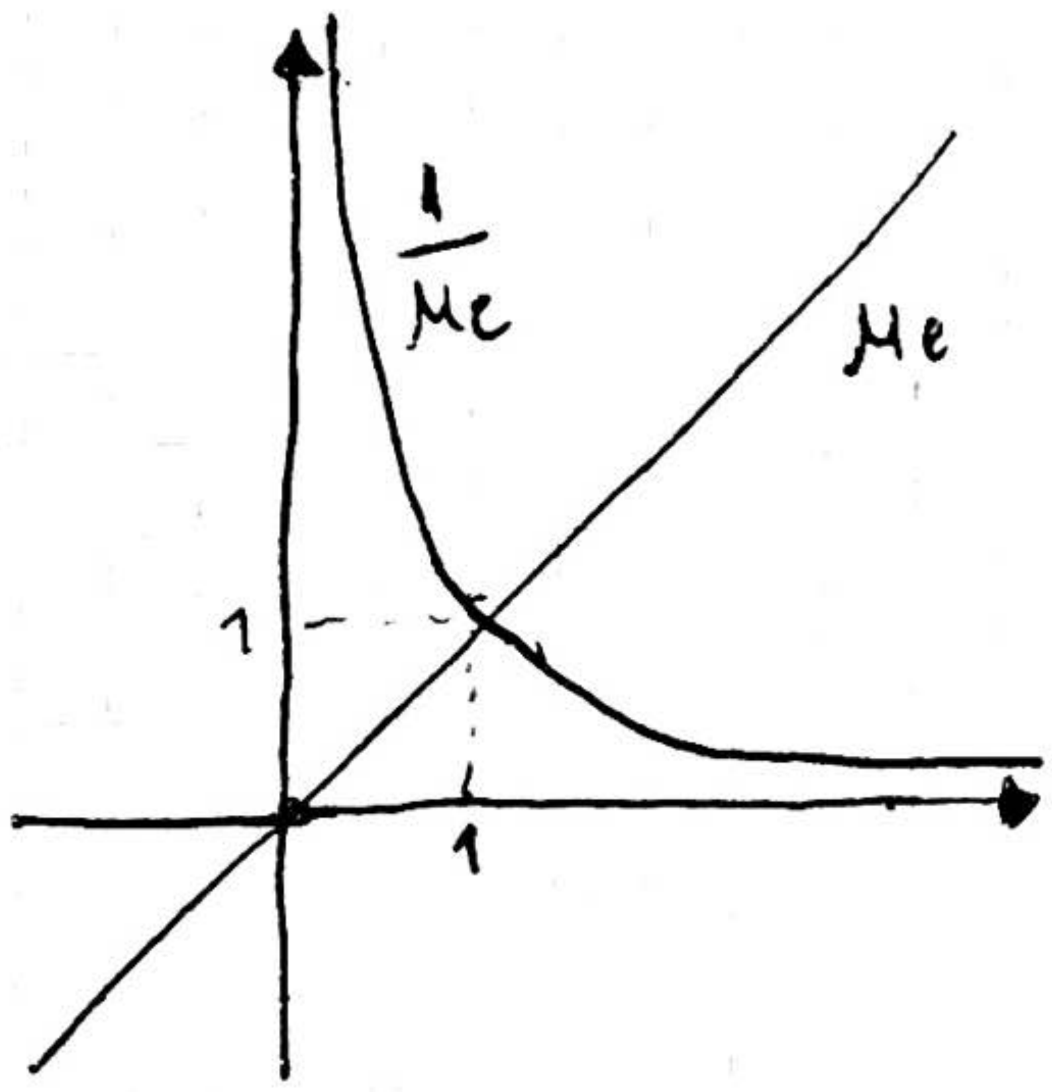
$F(\cos\alpha-\mu_e\,\text{sen}\,\alpha)\le mg(\text{sen}\,\alpha+\mu_e\cos\alpha)$

Supongo que $\cos\alpha-\mu_e\,\text{sen}\,\alpha>0$

$\Rightarrow \cos\alpha>\mu_e\,\text{sen}\,\alpha$

$\frac{1}{\mu_e}>\text{tg}\,\alpha$ y yo se que $\mu_e\geq\text{tg}\,\alpha$

Gráfico



$\Rightarrow$ Hasta 1

$\text{tg}\,\alpha\le\mu_e<\frac{1}{\mu_e}$

$\Rightarrow \cos\alpha-\mu_e\,\text{sen}\,\alpha>0$ ✓

$\therefore\ F\le\dfrac{mg(\text{sen}\,\alpha+\mu_e\cos\alpha)}{\cos\alpha-\mu_e\,\text{sen}\,\alpha}$

(2) $mg\,\text{sen}\,\alpha-\mu_e mg\cos\alpha\le\mu_e F\,\text{sen}\,\alpha+F\cos\alpha$

$mg(\text{sen}\,\alpha-\mu_e\cos\alpha)\le F(\mu_e\,\text{sen}\,\alpha+\cos\alpha)$

$F\geq\dfrac{mg\overbrace{(\text{sen}\,\alpha-\mu_e\cos\alpha)}^{\le 0}}{\mu_e\,\text{sen}\,\alpha+\cos\alpha}$ ya que $\mu_E\geq\text{tg}\,\alpha \Leftrightarrow \mu_E\cdot\cos\alpha\geq\text{sen}\,\alpha$

c) Si $m = 2kg$ $\mu_e = tg\alpha = 0,3$ Hallar $\bar{F}$ máx / bloque quieto

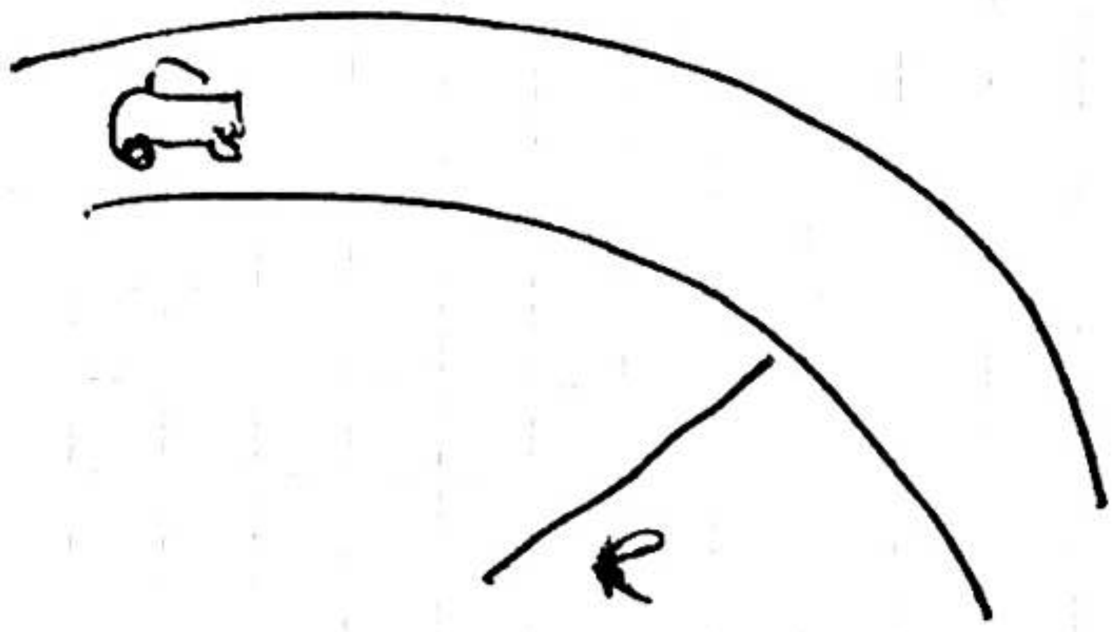
$$F_{máx} = \frac{m \cdot g\,(\operatorname{sen}\alpha + \mu_e \cos\alpha)}{\cos\alpha - \mu_e \operatorname{sen}\alpha}$$

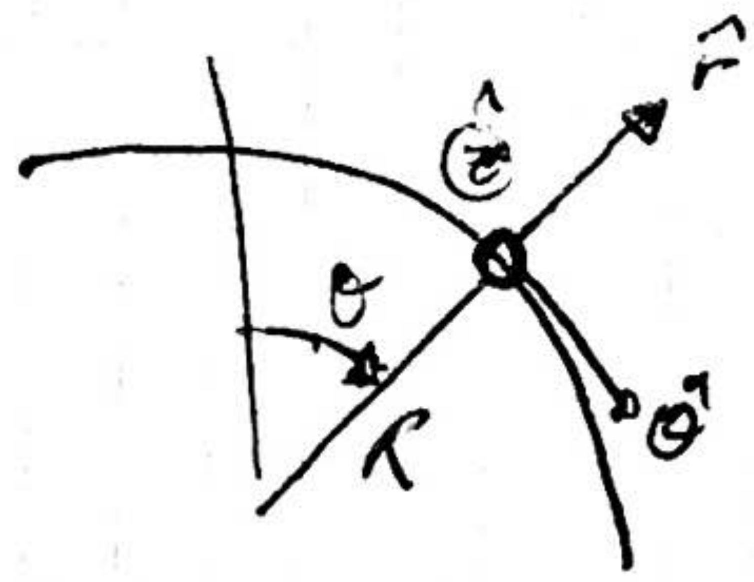
$tg\,\alpha = 0,3 \Rightarrow \alpha = 16,7°$

$\cancel{\frac{\operatorname{sen}\alpha}{\cos\alpha} = 0,3}$

$$\boxed{\Rightarrow F_{máx} = \frac{2kg \cdot 9,8\,m/s^2 \cdot (0,2873 + 0,3 \cdot 0,95782)}{0,95782 - 0,3 \cdot (0,2873)} \approx 12,92}$$

3 Versión polar





$(\hat{\theta})= 0=0$

$(\hat{r}) = +F_R = \cancel{+}R\dot{\theta}^2 . m$

$(\hat{z}) = N - mg = m.\ddot{z}^{=0} \quad N = mg$

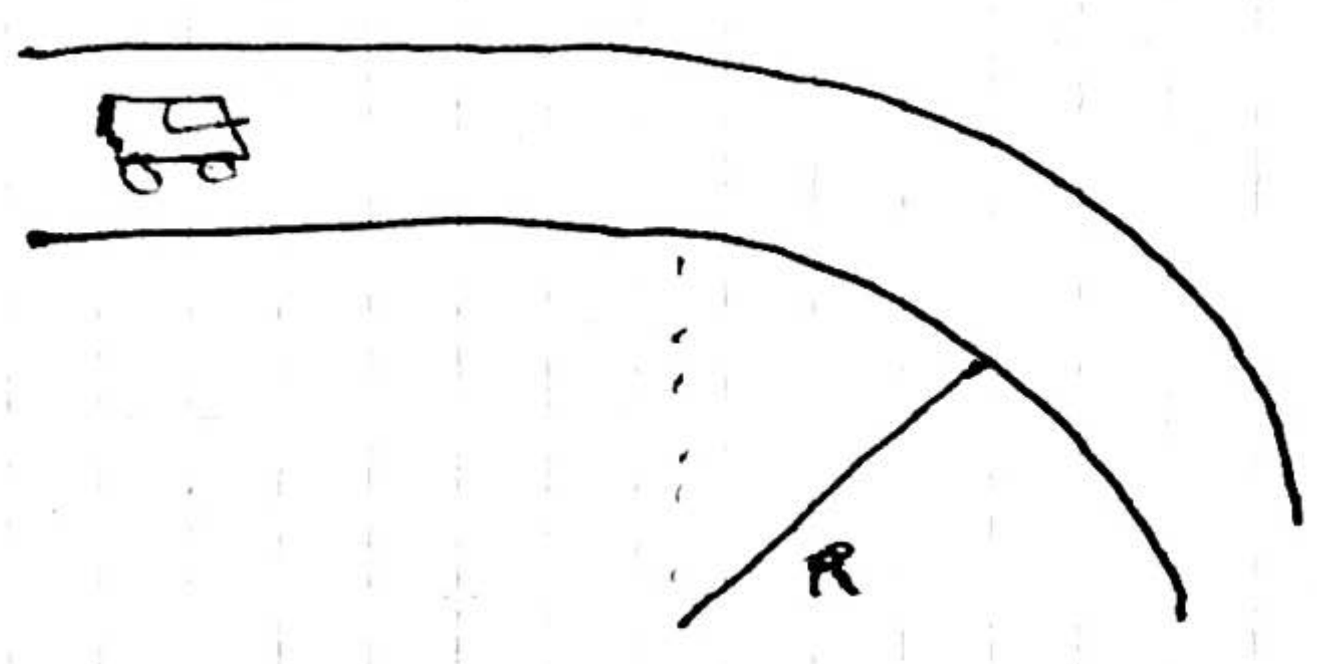
$|F_{RE}| \leq \mu_E . mg$ Para que μ_E sea mín se debe obtener $F_{máx}$ con la vcte

$\Rightarrow \cancel{F_{RE}} = R\dot{\theta}^2_{\cancel{M}} = \mu_{Emin} . mg$

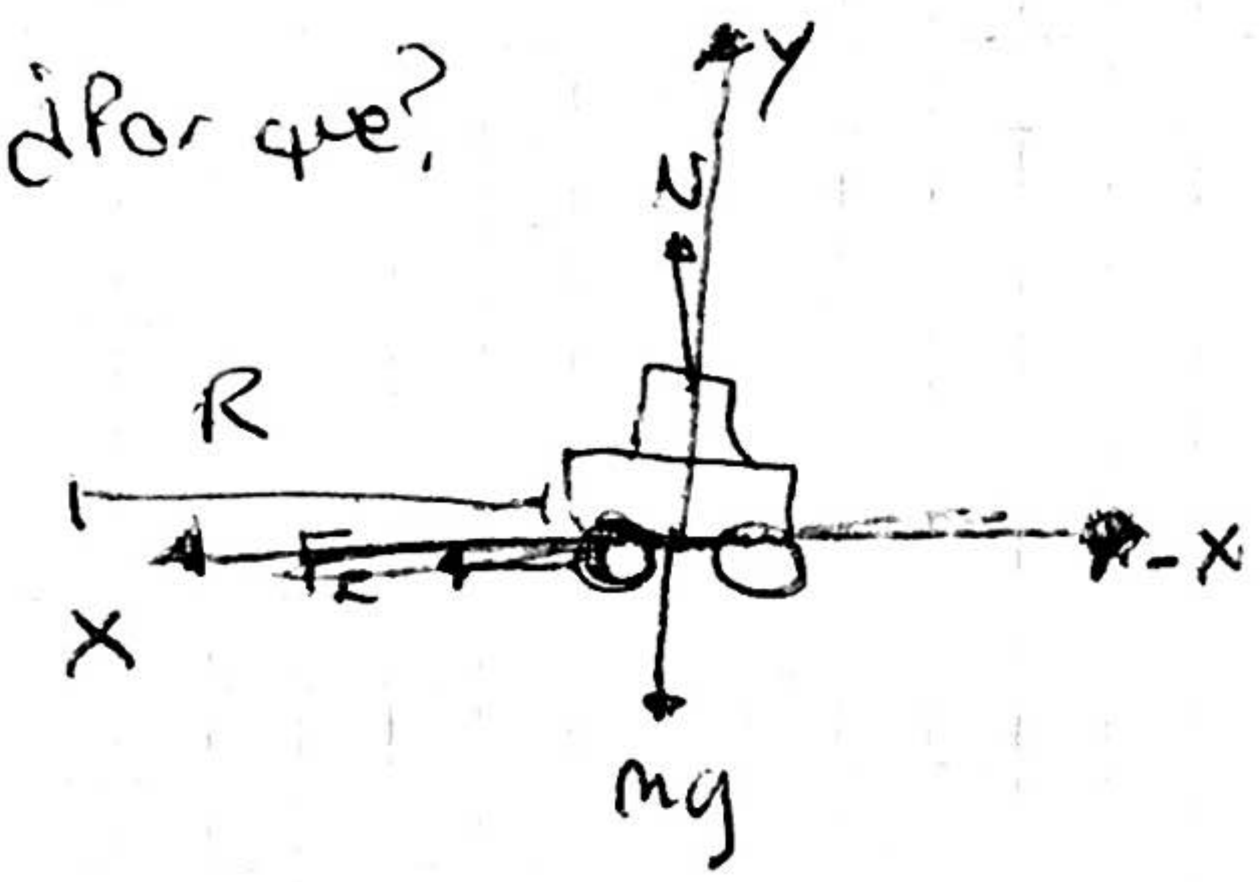
$\dfrac{R\dot{\theta}^2}{g} = \mu_{EMIN}$ $\quad v = \dot{\theta}R \Rightarrow v^2 = \dot{\theta}^2 . R^2 \Rightarrow \dfrac{v^2}{R^2} = \dot{\theta}^2$

$\therefore \boxed{\dfrac{\cancel{R} v^2}{g R^2} = \mu_{Emin}}$

v cte $= v$



a) Cual debe ser el mínimo coef de roz para que no deslice? ¿Estático o dinámico? ¿Por que?



$(\hat{x})$ $+F_R = m\,\ddot{x}$

$(\hat{y})$ $N - mg = m\ddot{y}^{=0} \Rightarrow N = mg$

$$a_{rad} = \gamma = \frac{v^2}{R}$$

$$|F_r| \leq \mu_E \cdot N = F_{roz\,máx}$$

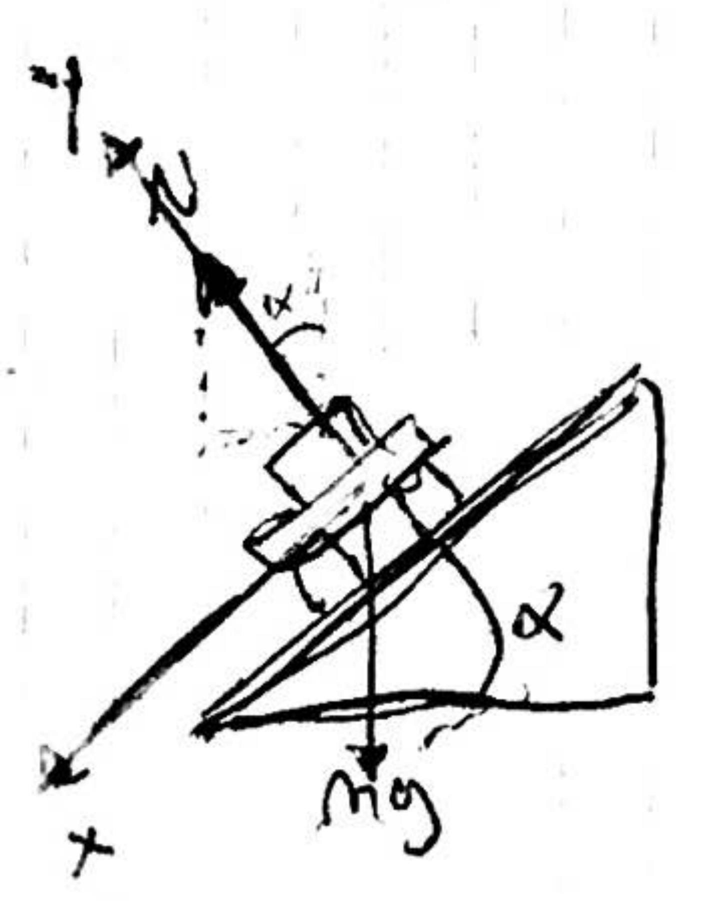
El menor μ_E se da cuando con la v cte dada la Froz e máx.

$$F_R = m.\frac{v^2}{R} \qquad F_{roz\,máx} = \mu_{E\,min} \cdot N = m.\frac{v^2}{R}$$

$$\mu_{E\,min} . \cancel{m}g = \frac{\cancel{m}v^2}{R}$$

$$\boxed{\mu_{E\,min} = \frac{v^2}{gR}}$$

b) No $\exists$ roz



$(\hat{x})$ $\cancel{m}g\,sen\alpha = \cancel{m}\ddot{x}$

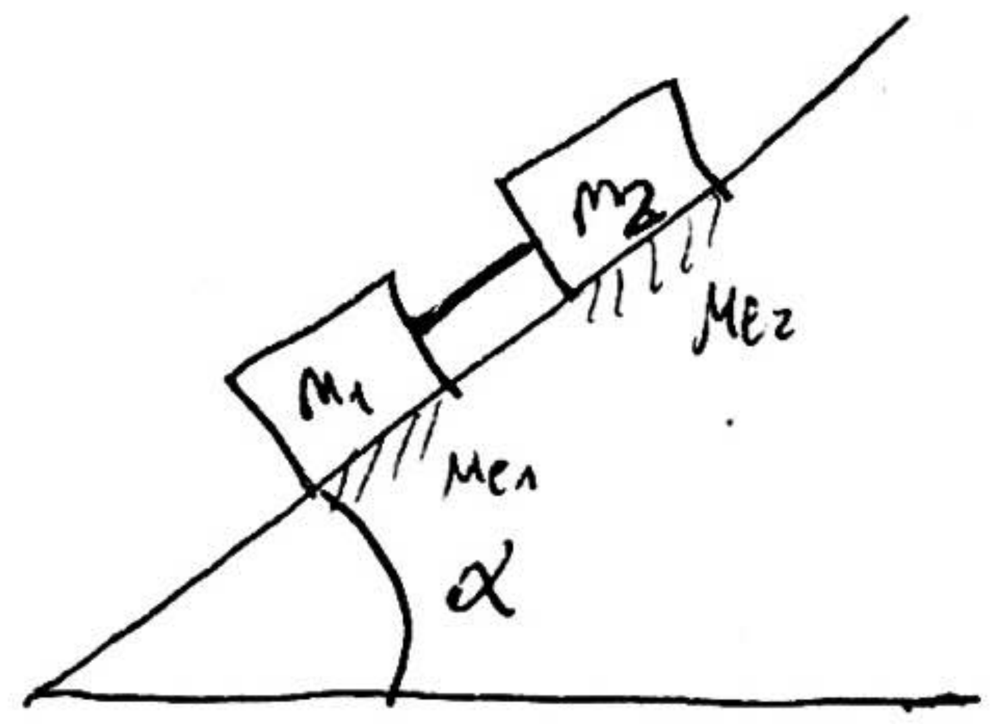
$(\hat{y})$ $N - mg\cos\alpha = m\ddot{y}^{=0} \Rightarrow N = mg\cos\alpha$

$$g\,sen\,\alpha = \frac{v^2}{R}$$

$$\boxed{\alpha = arcsen\,\frac{v^2}{gR}}$$

● A partir de las ecuaciones de Newton

③



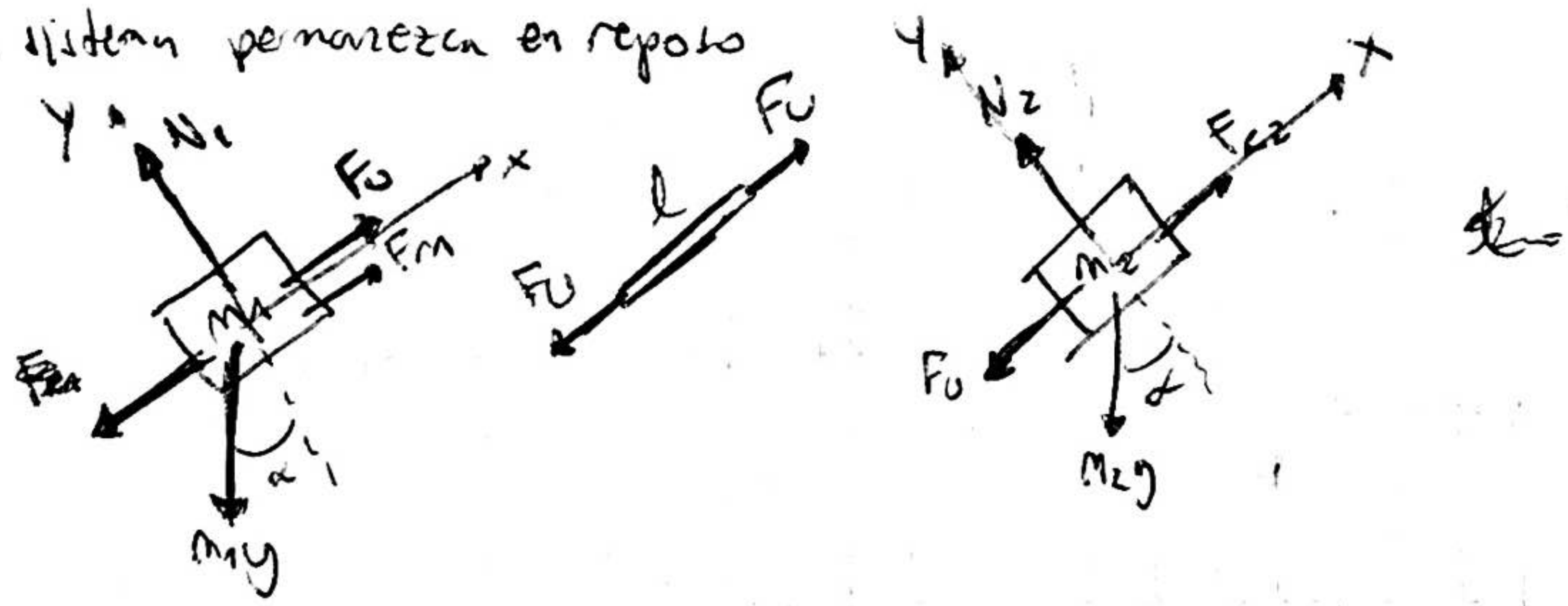
Suponga que están en reposo y encuentre una relación entre f_{r1}, f_{r2}, m_1, m_2 y α (f_r = fuerza rozamiento). Grafique la relación f_{r2} vs f_{r1}

Si $\mu_{e2}=0{,}6$, $\mu_{e1}=0{,}9$, $m_1=5kg$, $m_2=10kg$, $\alpha=30°$. Dibuje en el gráfico anterior la zona donde el rozamiento es estático

Diga si se puede con estos datos el estado de reposo que hemos supuesto

¿Puede determinar los valores de f_{r1} y f_{r2}? Diga qué valores puede tomar α para que el sistema permanezca en reposo

Supongo reposo



m_1 $(\hat{x})$ $F_U + F_{R1} - m_1 g \operatorname{sen}\alpha = 0$

$(\hat{y})$ $N_1 - m_1 g \cos\alpha = 0$

m_2 $(\hat{x})$ $F_{R2} - F_U - m_2 g \operatorname{sen}\alpha = 0$

$(\hat{y})$ $N_2 - m_2 g \cos\alpha = 0$

$\cancel{F_U + F_{R1}}$ $F_{R2} - m_2 g \operatorname{sen}\alpha + F_{R1} - m_1 g \operatorname{sen}\alpha = 0$

$F_{R1} + F_{R2} = g \operatorname{sen}\alpha (m_1 + m_2)$

$|F_1| \leq \mu_{e1} m_1 g \cos\alpha$

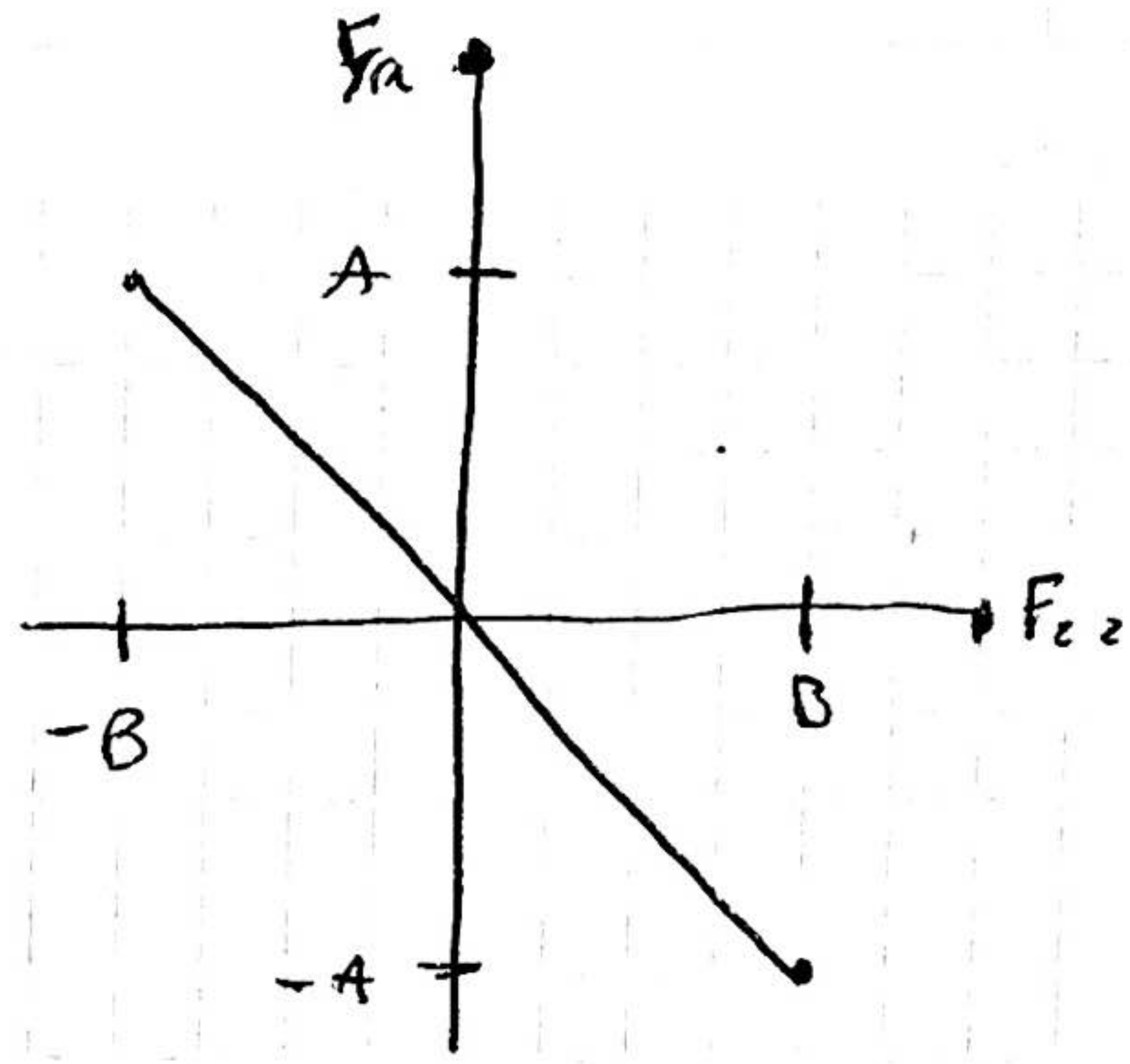
$|F_2| \leq \mu_{e2} m_2 g \cos\alpha$

$|F_{R1} + F_{R2}| \leq |F_{r1}| + |F_{r2}| \leq \mu_{e1} m_1 g \cos\alpha + \mu_{e2} m_2 g \cos\alpha$

$F_{R1} = g \operatorname{sen}\alpha (m_1 + m_2) - F_{R2}$

$|F_{R1}| \leq \mu_e m_1 g \cos\alpha$ A

$|F_{R2}| \leq \mu_e m_2 g \cos\alpha$
B



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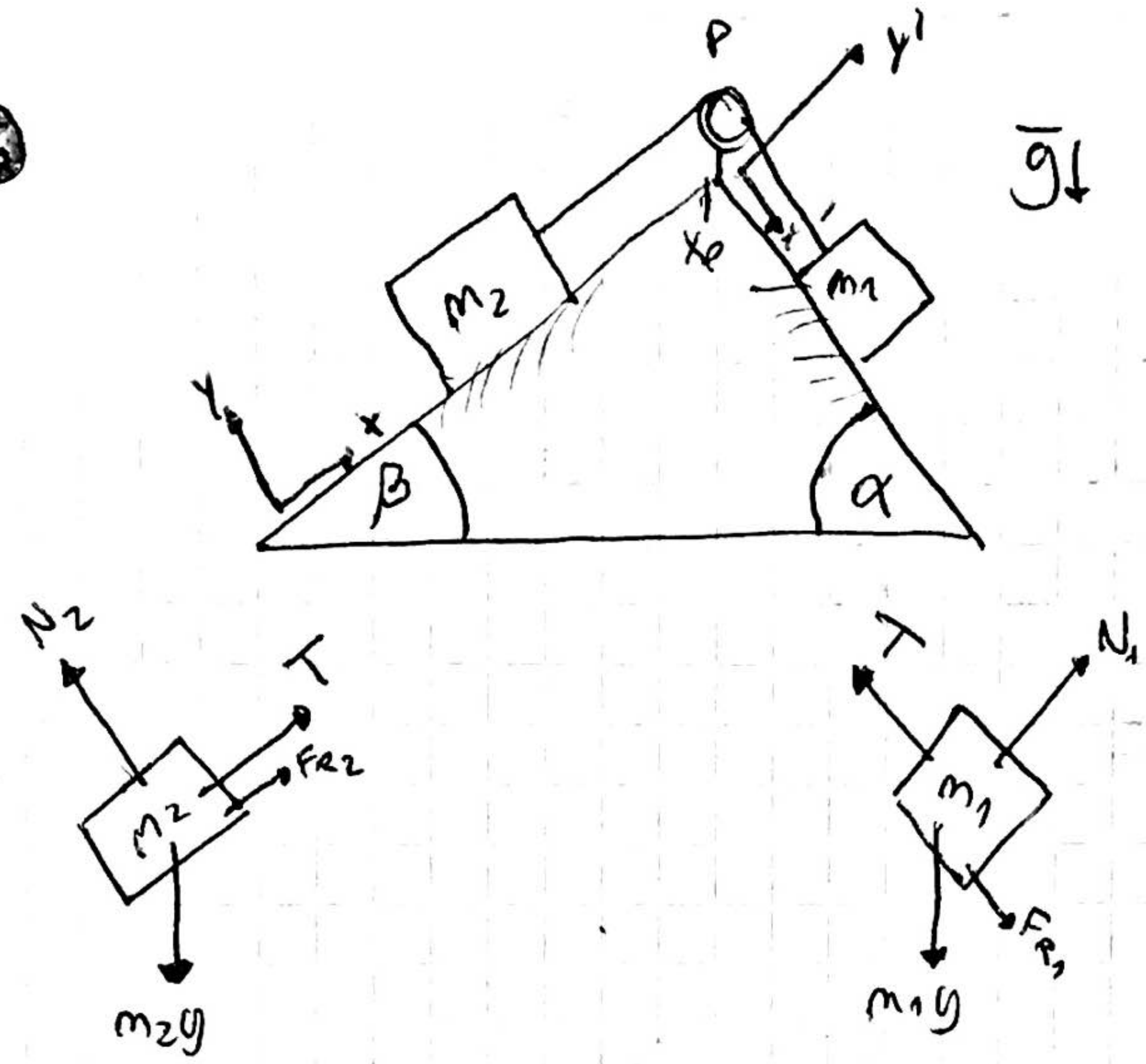


$\bar{g}\downarrow$ $\mu_D = 0{,}25$

$\mu_E = 0{,}3$

m_2: $(\hat{x})$ $T + F_{e_2} - m_2 g \cdot \operatorname{sen}\beta = m_2 \ddot{x}_2$

$(\hat{y})$ $N_2 - m_2 g \cos\beta = m_2 \ddot{y}_2 = 0$

m_1: $(\hat{x})$ $F_{e_1} - T + m_1 g \cdot \operatorname{sen}\alpha = m_1 \ddot{x}_1$

$(\hat{y})$ $N_1 - m_1 g \cos\alpha = m_1 \ddot{y}_1 = 0$

$\ddot{x}_2 = \ddot{x}_1$

$l = x_p - x_2 + x_1 - x_p$ $\qquad l = -x_2 + x_1$

En estática

$\ddot{x}_2 = \ddot{x}_1 = 0$

$F_{R1} = F_{RE1}$

$F_{R2} = F_{RE2}$

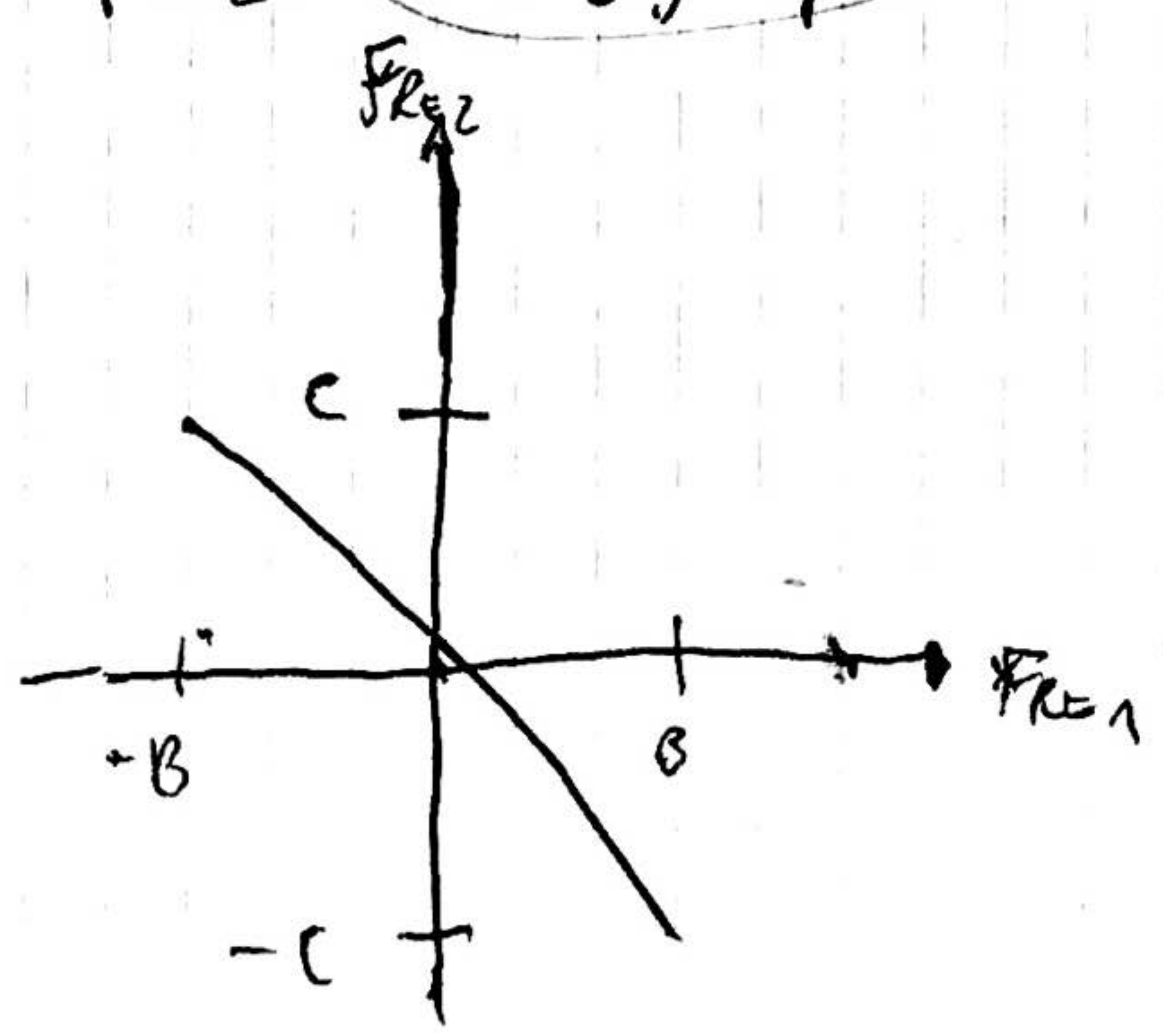
$F_{RE2} - m_2 g \operatorname{sen}\beta + F_{RE1} + m_1 g \operatorname{sen}\alpha = 0$

$F_{RE2} + F_{RE1} = m_2 g \operatorname{sen}\beta - m_1 g \operatorname{sen}\alpha$ A

$|F_{RE1}| \leq \mu_E \cdot m_1 g \cos\alpha$ B

$|F_{RE2}| \leq \mu_E \cdot m_2 g \cos\beta$ C

$F_{RE2} = A - F_{RE1}$



$$||F_{RE_1}+F_{RE_2}| \leq |F_{RE_1}| + |F_{RE_2}| \leq \mu_E \cdot (m_1 g \cos\alpha + m_2 g \cos\beta)$$

$$|m_2 \cancel{g} \operatorname{sen}\beta - m_1 \cancel{g} \operatorname{sen}\alpha| \leq \mu_E \cancel{g} (m_1 \cos\alpha + m_2 g \cos\beta)$$

Si $m_1 = 1kg$, $m_2 = 2kg$, $\alpha = 60^\circ$, $\beta = 30^\circ$ ¿Se podrán mover el sistema?

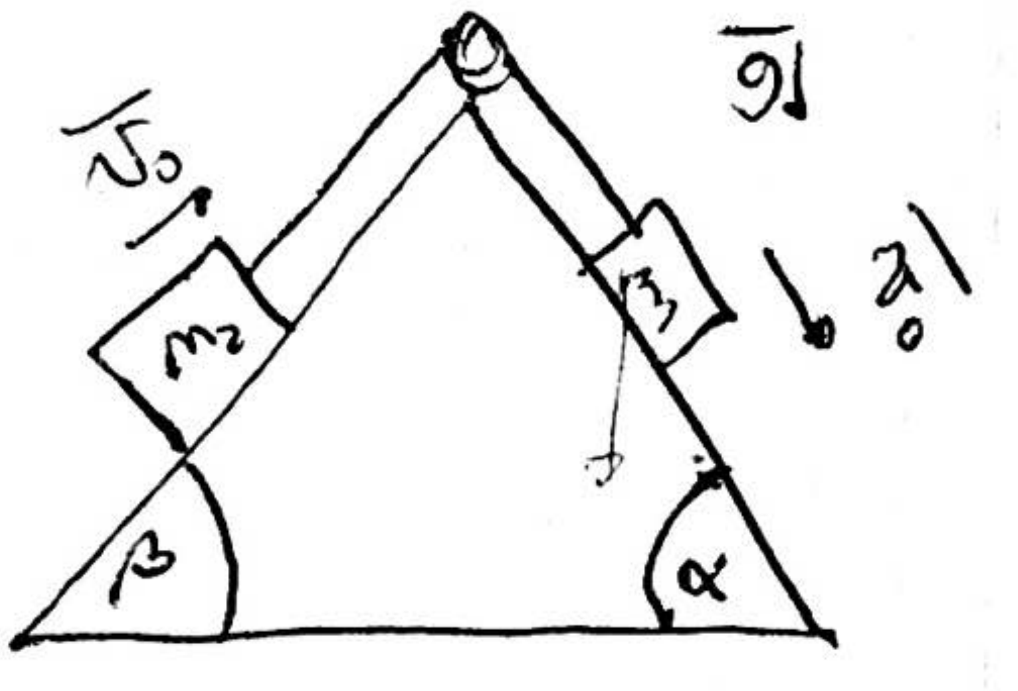
$$|2kg \cdot \operatorname{sen} 30^\circ - 1kg \cdot \operatorname{sen} 60^\circ| \leq 0{,}3 \cdot (1kg \operatorname{sen} 60^\circ + 2kg \operatorname{sen} 30^\circ)$$

$$\left|1kg - \frac{\sqrt{3}kg}{2}\right| \leq 0{,}3\left(\frac{\sqrt{3}}{2}kg + 1kg\right)$$

$$0{,}13 \leq 0{,}55$$

Sí.

c)



m_1: $(\hat{x})$ $-F_{ROZD} - T + m_1 \cdot g \operatorname{sen}\alpha = m_1 \cdot \ddot{x}_1$

$(\hat{y})$ $N_1 = m_1 g \cos\alpha$

m_2: $(\hat{x})$ $-F_{ROZD} + T - m_2 g \operatorname{sen}\beta = m_2 \ddot{x}_2$

$(\hat{y}) = N_2 = m_2 g \cos\beta$

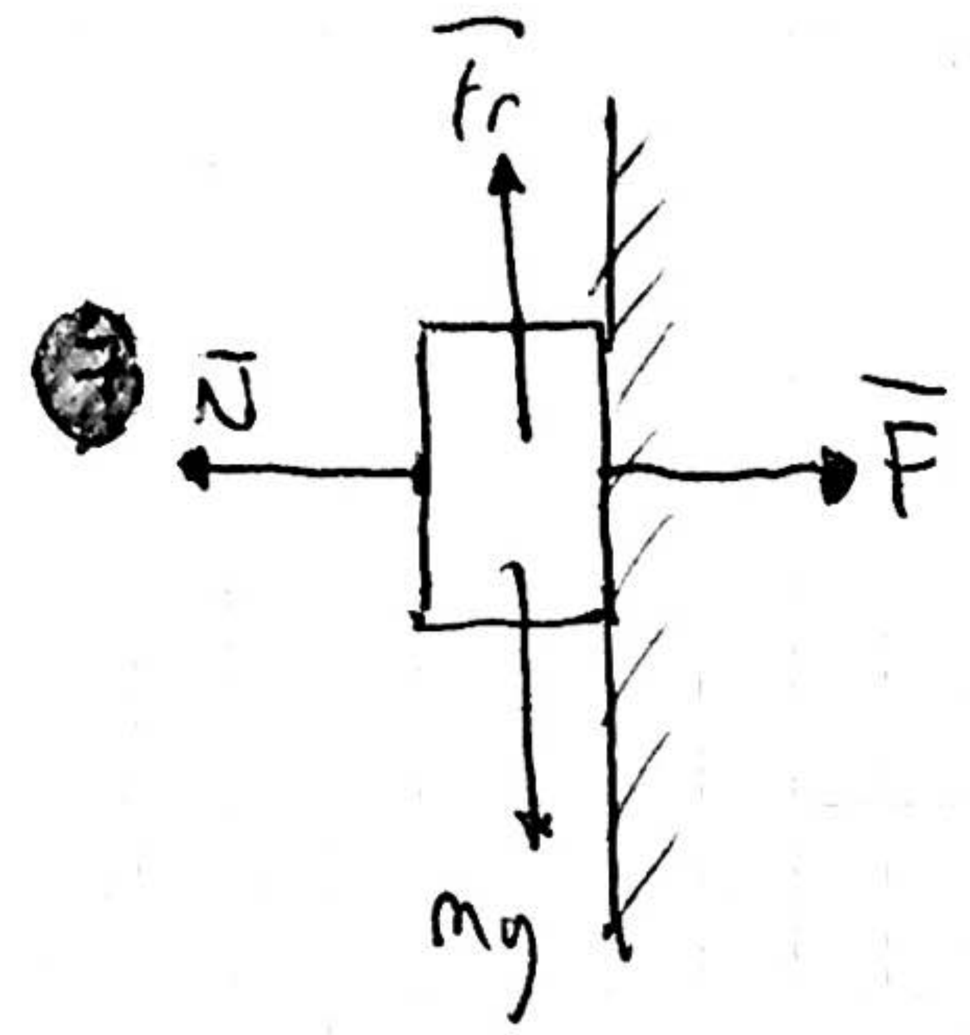
$\ddot{x}_1 = \ddot{x}_2$

$$\Rightarrow -2F_{ROZD} + m_1 g \operatorname{sen}\alpha - m_2 g \operatorname{sen}\beta = m_1 \ddot{x}_1 + m_2 \ddot{x}_1$$

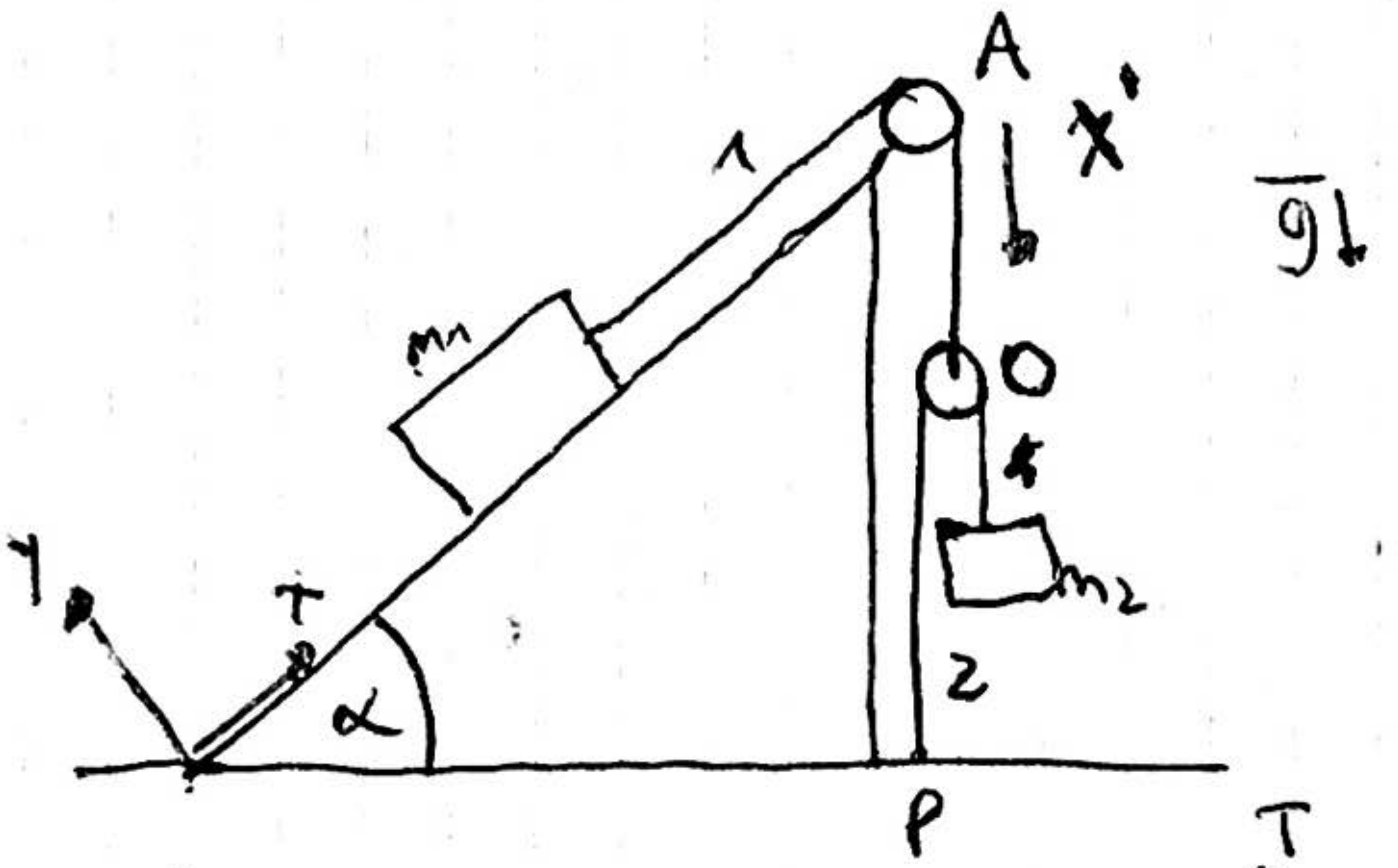
$$\Rightarrow \frac{-\mu_D \cdot m_1 g \cos\alpha - \mu_D m_2 g \cos\beta + m_1 g \operatorname{sen}\alpha - m_2 g \operatorname{sen}\beta}{m_1 + m_2} = \ddot{x}_1$$

$$\frac{-1{,}225 kg \cdot m/s^2 - 4{,}243524479 + 4{,}9 - 16{,}97409791}{3kg} = \boxed{-2{,}86 m/s^2 = \ddot{x}_1}$$

$\boxed{\text{Para el otro lado igual } \ddot{x}_1}$

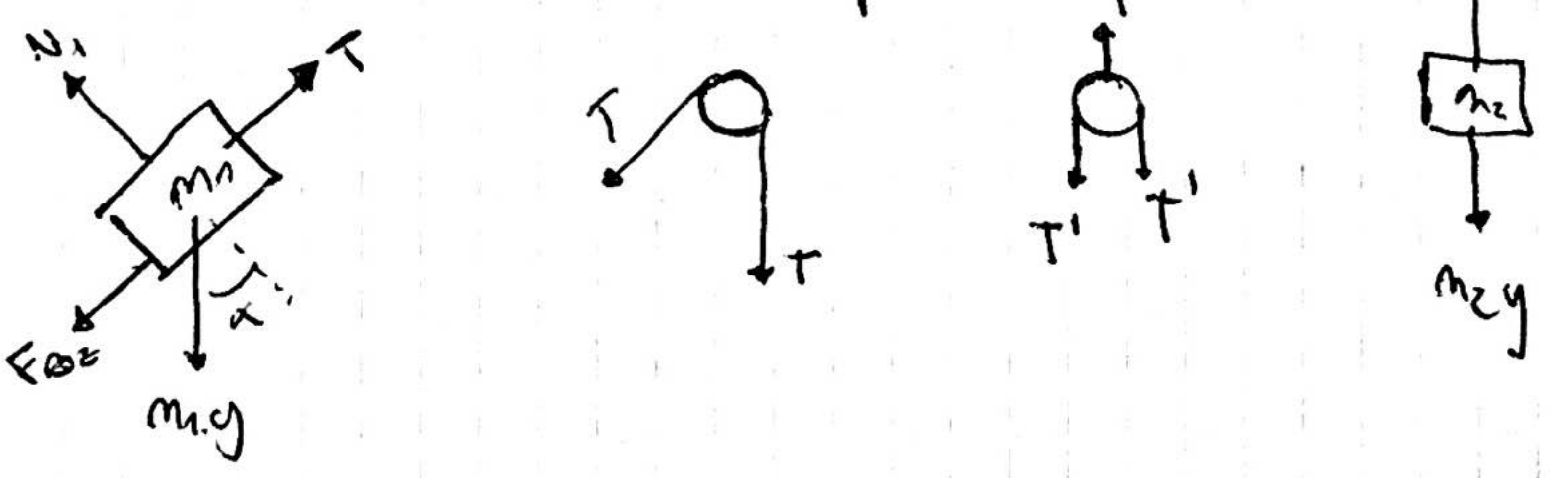


Rta: El cuerpo está en reposo porque su peso es equilibrado por la Froz, Como fr proporcional a N, podemos lograr que se mueva al aumentar la F aplicada



Datos

m_1, m_2, μ_E, μ_D



m_1: $(\hat{x})$ $T - F_{roz} - m_1 . g \operatorname{sen}\alpha = m_1 . \ddot{x}_1$

$(\hat{y})$ $N_1 - m_1 . g \cos\alpha = m_1 \ddot{y}_1 = 0 \Rightarrow N_1 = m_1 . g \cos\alpha$

m_2: $(\hat{x}')$ $m_2 g - T' = m_2 . \ddot{x}'_2$

Vínculo $T = 2T'$

$l_1 = x_A - x_1 + x'_0 \Rightarrow 0 = -\ddot{x}_1 + \ddot{x}'_0 \Rightarrow \ddot{x}_1 = \ddot{x}'_0$

$l_2 = x'_P - x'_0 + x'_2 - x'_0 \quad 0 = \ddot{x}'_2 - 2\ddot{x}'_0 \Rightarrow \ddot{x}'_2 = 2\ddot{x}'_0$

$\ddot{x}_1(\ddot{x}'_2) = ?$

$2\cancel{T - 2F_{roz} - 2m_1 g \operatorname{sen}\alpha}$

$$\cancel{2T'} - F_{ROZ} - m_1 g \operatorname{sen}\alpha + 2m_2 g - \cancel{2T'} = m_1 \ddot{x}_1 + 2m_2 \ddot{x}'_2$$

$\ddot{x}_1 = \frac{\ddot{x}'_2}{2}$. Sí, influye totalmente

Estática. ¿Rango de valores m_2 para que el sistema esté en reposo?

$\ddot{x}_1 = \ddot{x}'_2 = 0$

$F_{ROZ} = F_{ROZ_E}$

$$F_{ROZ_E} = \cancel{T} - m_1 g \operatorname{sen}\alpha + 2m_2 g$$

$$|F_{ROZ_E}| \leq F_{ROZ\,MÁX} = \mu_E . N_1 = \mu_E . m_1 . g \cos\alpha$$

$$-\mu_E m_1 g \cos\alpha \leq -m_1 g \operatorname{sen}\alpha + 2m_2 g \leq \mu_E m_1 g \cos\alpha$$

$$-\mu_E m_1 \cancel{g} \cos\alpha + m_1 \cancel{g} \operatorname{sen}\alpha \leq 2m_2 \cancel{g} \leq \mu_E m_1 \cancel{g} \cos\alpha + m_1 \cancel{g} \operatorname{sen}\alpha$$

$$\boxed{\frac{-m_1(\mu_E \cos\alpha - \operatorname{sen}\alpha)}{2} \leq m_2 \leq \frac{m_1(\mu_E \cos\alpha + \operatorname{sen}\alpha)}{2}}$$

Si m_2 desciende con aceleración constante A:

c) Calcule m_2. Diga justificando su rta. si la aceleración A puede ser tal que $A > g$

DINÁMICA

$\ddot{x}_1 = \frac{\ddot{x}'_2}{2}$ $\qquad F_{ROZ} = F_{ROZ_D}$

$$-F_{ROZ_D} - m_1 g \operatorname{sen}\alpha + 2m_2 g = m_1 \frac{A}{2} + 2m_2 A$$

$$-\mu_D . m_1 . g \cos\alpha - m_1 g \operatorname{sen}\alpha - m_1 \frac{A}{2} = 2m_2 A - 2m_2 g = m_2(2A - 2g)$$

$$\boxed{\frac{-\mu_D . m_1 g \cos\alpha - m_1 g \operatorname{sen}\alpha - \frac{m_1 A}{2}}{2A - 2g} = m_2}$$

si $A > g \Rightarrow 2A > 2g \quad 2A - 2g > 0$. Lo cual no pue ser ya que

$$\frac{-\mu_0 m_1 \cos\alpha - m_1 g \operatorname{sen}\alpha - \frac{m_1 A}{2}}{2a - 2g} < 0 \Rightarrow$$ la masa sería negativa y no puede ser así

ii) $x'_0 = h + \underbrace{v_0}_{=0} t + \frac{a t^2}{2} \Rightarrow x'_0 = h + \frac{\ddot{x}'_0 t^2}{2}$

CA

$-\mu_0 \cdot m_1 g \cos\alpha - m_1 g \operatorname{sen}\alpha + 2m_2 g = m_1 \ddot{x}'_0 + 2m_2 \, 2\ddot{x}'_0 = \ddot{x}'_0 (m_1 + 4m_2)$

$$\frac{-\mu_0 m_1 g \cos\alpha - m_1 g \operatorname{sen}\alpha + 2m_2 g}{m_1 + 4m_2} = \ddot{x}'_0$$

$$\boxed{x'_0 = h + \left(\frac{-\mu_0 m_1 g \cos\alpha - m_1 g \operatorname{sen}\alpha + 2m_2 g}{2(m_1 + 4m_2)}\right) t^2}$$