Práctica 1

$\underline{v} = \frac{dx}{dt} = \frac{\partial x}{\partial t} + \underline{u}\cdot\nabla$

Ejercicio 1.1

Campo de velocidades en notación Euleriana

$v_1 = v_2 = 0 \quad ; v_3 = f(x_3) \;, \quad t \geq 0 \;, \quad x_3 \geq 0$

Encuentre la expresión lagrangeana. Luego aplíquelo a la cascada cayendo.

$\underline{v} = (0, 0, f(x_3)) \leftarrow$ campo de velocidades $\quad \underline{v}(x_3) = f(x_3)\hat{z}$

$$\frac{dx_3}{dt} = v_3 = f(x_3) \rightarrow \frac{1}{f(x_3)}\, dx_3 = \int_0^t dt \; .$$

euler indep (x, t)

$$\underline{v} = \frac{\partial x}{\partial t} + \underline{u}\cdot\underbrace{\nabla x}_{\mathrm{I}}$$

Si es la caída del agua $\quad \underline{v} = (0, 0, -gt)$

$$\frac{dv_3}{\partial t} = -g \Rightarrow v_3 = -gt$$

$$\frac{d^2 x_3}{dt^2} = \frac{dv_3}{dt} = \frac{dv_3}{dx_3}\cdot\frac{dx_3}{dt} = \frac{dv_3}{dx_3}\, v_3 = -g$$

$$\Rightarrow \int_{v_{03}}^{v_3} v_3\, dv_3 = -g\int_{x_{03}}^{x_3} dx_3 \quad \Leftrightarrow \quad \frac{v_3^2}{2} - \frac{v_{03}^2}{2} = -g(x_3 - x_{03})$$

$$\frac{dx_3}{dt} = -gt \Rightarrow x_3 - x_{03} = -\frac{gt^2}{2}$$

$$\Rightarrow \boxed{x_3 = x_{03} - \frac{gt^2}{2}} \qquad \dot{x}_3 = -gt$$

$$\boxed{x_3(x_{03}, t)}$$

$$\Rightarrow \frac{dT}{dt} = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial x}\cdot\frac{\partial x}{\partial t} = \frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x} \qquad \frac{\partial T}{\partial t} = -\alpha e^{-x/L}\left(\frac{2\pi}{\tau}\right)\cos(2\pi t/\tau)$$

$$\frac{\partial T}{\partial x} = \frac{\alpha}{L}\, e^{-x/L}\sin(2\pi t/\tau)$$

$$\Rightarrow \boxed{\frac{dT}{dt} = e^{-x/L}\left[-\frac{2\pi\alpha}{\tau}\cos(2\pi t/\tau) + \frac{U\alpha}{L}\sin(2\pi t/\tau)\right]}$$

$$\underline{v} = \frac{dx}{dt} = \frac{\partial x}{\partial t} + \underline{u}.\underline{\nabla}$$

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Ejercicio 1.1

Campo de velocidades en notación Euleriana

$v_1 = v_2 = 0 \quad ; v_3 = f(x_3) \;,\; t \geq 0 \;,\; x_3 \geq 0$

Encuentre la expresión Lagrangeana. Luego aplíquelo a la cascada cayendo.

$\underline{v} = (0, 0, f(x_3))$ ← campo de velocidades $\quad \underline{v}(x_3) = f(x_3)\hat{z}$

$$\frac{dx_3}{dt} = v_3 = f(x_3) \rightarrow \frac{1}{f(x_3)} dx_3 = \int_0^t dt \;.$$

$$\underline{v} = \frac{\partial \overset{\text{euler indep}(x,t)}{x}}{\partial t} + \underline{u}.\underbrace{\underline{\nabla}\underline{x}}_{\mathbb{I}}$$

Si es la caída del agua $\underline{v} = (0, 0, -gt)$

$$\frac{dv_3}{\partial t} = -g \Rightarrow v_3 = -\cancel{t}$$

$$\frac{d^2 x_3}{dt^2} = \frac{dv_3}{dt} = \frac{dv_3}{dx}.\frac{dx_3}{dt} = \frac{dv_3}{dx}.v_3 = -g$$

Ejercicio 1.2 $T = T_0 - \alpha e^{-x/L} \sin(2\pi t/\tau)$; T_0, α, L, τ datos, constantes

Una partícula se mueve con U cte

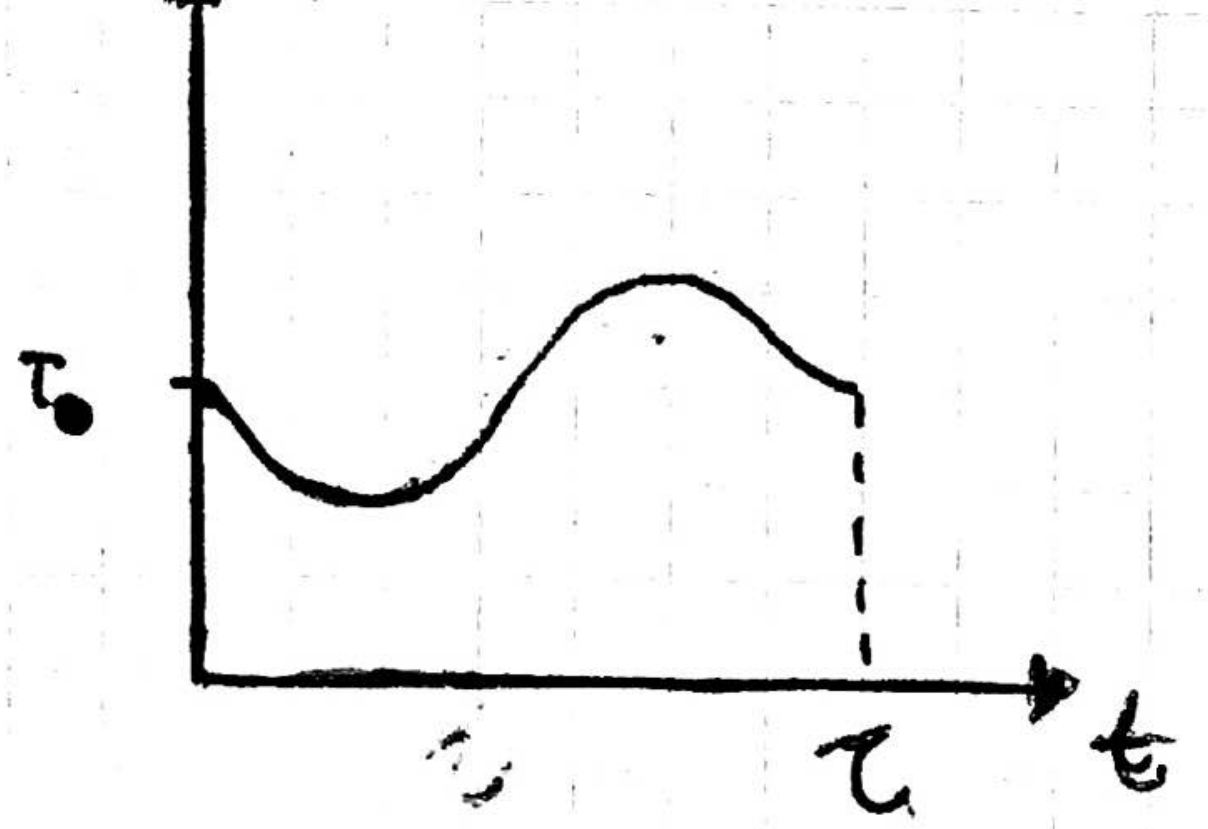
$x(x,t) = x(x(t), t)$

a) $\frac{dT}{dt}$ en descripción Euleriana.

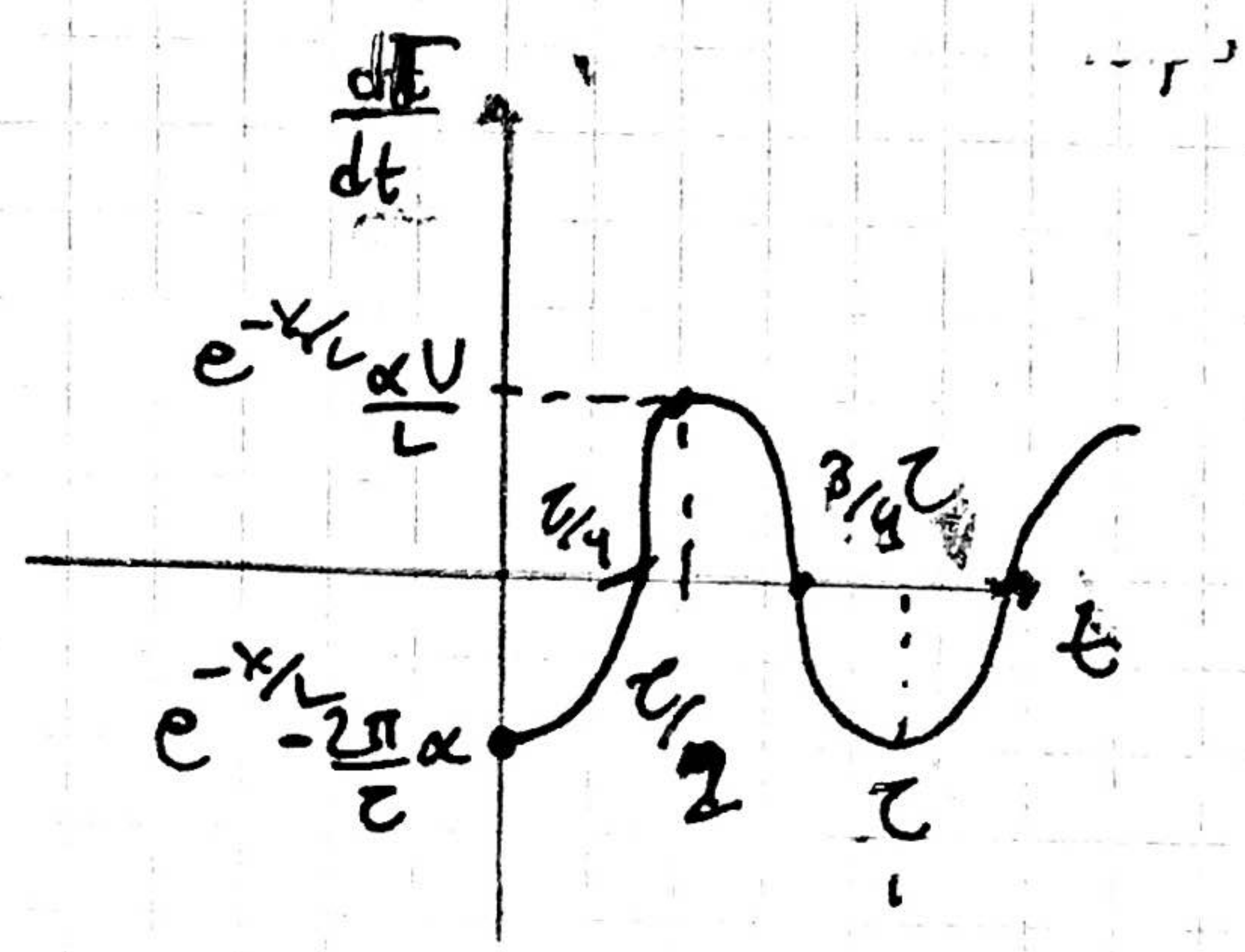
$$\Rightarrow \frac{dT}{dt} = \frac{\partial T}{\partial t} + \frac{\partial T}{\partial x}.\frac{\partial x}{\partial t} = \frac{\partial T}{\partial t} + U\frac{\partial T}{\partial x} \qquad \frac{\partial T}{\partial t} = -\alpha e^{-x/L}\left(\frac{2\pi}{\tau}\right)\cos(2\pi t/\tau)$$

$$\frac{\partial T}{\partial x} = \frac{\alpha}{L} e^{-x/L} \sin(2\pi t/\tau)$$

$$\Rightarrow \boxed{\frac{dT}{dt} = e^{-x/L}\left[-\frac{2\pi\alpha}{\tau}\cos(2\pi t/\tau) + \frac{U\alpha}{L}\sin(2\pi t/\tau)\right]}$$



$$\frac{dT}{dt} = e^{-x/L}\alpha\left[-\frac{2\pi}{\tau}\cos(2\pi t/\tau) + \frac{U}{L}\sin(2\pi t/\tau)\right]$$



$\Rightarrow$

b) Descripción Lagrangeana

$\Rightarrow X(t, x_0)$ $\qquad X = X_0 + Ut$

$$\Rightarrow T = T_0 - \alpha e^{\frac{-(x_0+Ut)}{L}}\sin(2\pi t/\tau)$$

$$\frac{dT}{dt} = -\alpha e^{\frac{-(x_0+Ut)}{L}}\left[-\frac{U}{L}\sin(2\pi t/\tau) + \frac{2\pi}{\tau}\cos(2\pi t/\tau)\right]$$

Que si comparamos, es igual a la obtenida en la descripción Euleriana

Ejercicio 1.3

a) Corriente uniforme

$\underline{u} = u\hat{x}$ $\quad \Rightarrow \frac{dx}{dt} = u \quad \frac{dy}{dt} = 0 \quad \Rightarrow \underline{x}(t, \underline{x_0}) = \underline{x_0} + ut\hat{x}$

$\therefore y = y_0$ Trayectorias serán $y(x) = y_0$? Un plano en y

$dx = \alpha u_x \quad dy = \alpha u_y \Rightarrow dx = \alpha u \;, dy = 0$

$y(t) = y(x) = y_0$

Ejercicio 1.4

$u_x(x,y,t) = \dfrac{\alpha x}{1+\beta t}$; $u_y(x,y,t) = c$

$$\Rightarrow \frac{dx}{dt} = \frac{\alpha x}{1+\beta t} \Leftrightarrow \int_{x_0}^{x} \frac{dx'}{x'} = \frac{\alpha}{\beta}\int_{t_0=0}^{t} \frac{dt'}{1+\beta t'} \Leftrightarrow \ln\left(\frac{x}{x_0}\right) = \frac{\alpha}{\beta}\ln\left(\frac{1+\beta t}{1+\beta \underbrace{t_0}_{=0}}\right)$$

$$\Rightarrow e^{\ln\left(\frac{x}{x_0}\right)} = e^{\frac{\alpha}{\beta}\ln(1+\beta t)} \Leftrightarrow e^{\ln(x)-\ln(x_0)} = (1+\beta t)^{\frac{\alpha}{\beta}}$$

$$\Leftrightarrow x = x_0(1+\beta t)^{\alpha/\beta} \qquad \frac{dy}{dt} = c \Rightarrow y - y_0 = ct$$

∴ TRAYECTORIA $\boxed{x(y) = x_0\left[1+\beta\left(\frac{y-y_0}{c}\right)\right]^{\alpha/\beta}}$

lineas de corriente $\quad dx = \gamma u_x \quad dy = \gamma u_y$

$$\Leftrightarrow \frac{dx}{dy} = \frac{u_x}{u_y} = \frac{\alpha x}{c(1+\beta t)}$$

$$\Rightarrow \int_{x_0}^{x} \frac{dx'}{x'} = \int_{y_0}^{y} \frac{\alpha\, dy}{c(1+\beta t)} \Leftrightarrow \ln\left(\frac{x}{x_0}\right) = \frac{\alpha(y-y_0)}{c(1+\beta t)}$$

$$\Rightarrow \boxed{x = x_0\, e^{\frac{\alpha(y-y_0)}{c(1+\beta t)}}}$$

lineas de traza $\quad (x_R, y_R)$

$$y = ct + y_0 \qquad y_R = c\tilde{t} + y_0 \qquad \Rightarrow y_0 = y_R - c\tilde{t}$$

$$x = x_0(1+\beta t)^{\alpha/\beta} \qquad x_R = x_0(1+\beta\tilde{t})^{\alpha/\beta} \qquad x_0 = x_R(1+\beta\tilde{t})^{-\alpha/\beta}$$

$$\Rightarrow x = x_R\left[\frac{(1+\beta t)^{\alpha/\beta}}{(1+\beta\tilde{t})^{\alpha/\beta}}\right] \qquad y = ct + y_R - c\tilde{t} \qquad \tilde{t} = \left(\frac{y_R - y}{c}\right) + t$$

Hasta acá ya estoy, pero si además puedo obtener la trayectoria de t

$$x = x_R\left[\frac{(1+\beta t)^{\alpha/\beta}}{\left(1+\beta\frac{(y_R-y)}{c}+\beta t\right)^{\alpha/\beta}}\right]$$

Casos particulares $\alpha = \beta$

$$\Rightarrow \begin{array}{l} x = x_0(1+\beta t) \\ y = y_0 + ct \end{array} \qquad x(y) = x_0\left[1+\frac{\beta}{c}(y-y_0)\right]$$

$$\Rightarrow x(y) = x_0 + \frac{\beta}{c}x_0 y - \frac{x_0\beta y_0}{c}$$

$$\Rightarrow x(y) = \gamma y + \tau_0$$

lineal

y las líneas de tr:

$$x = x_R\left(\frac{1+\beta t}{1+ct+\frac{\beta}{c}(y-y_R)}\right)$$

$\alpha = 2\beta$

$$x(y) = x_0\left(1+\frac{\beta}{c}(y-y_0)\right)^2 = x_0 + \frac{2\beta x_0}{c}(y-y_0) + \frac{x_0\beta^2}{c^2}(y-y_0)^2$$

$$x(y_L, x_R, t) = \frac{x_R\ (1+\beta t)^2}{\left(1+ct+\frac{\beta}{c}(y-y_R)\right)^2}$$