Cinemática

1) Un cuerpo se mueve a lo largo de una línea recta de acuerdo a la ecuación $x = -kt^3 + bt^2$, con k, b, constantes ≥ 0

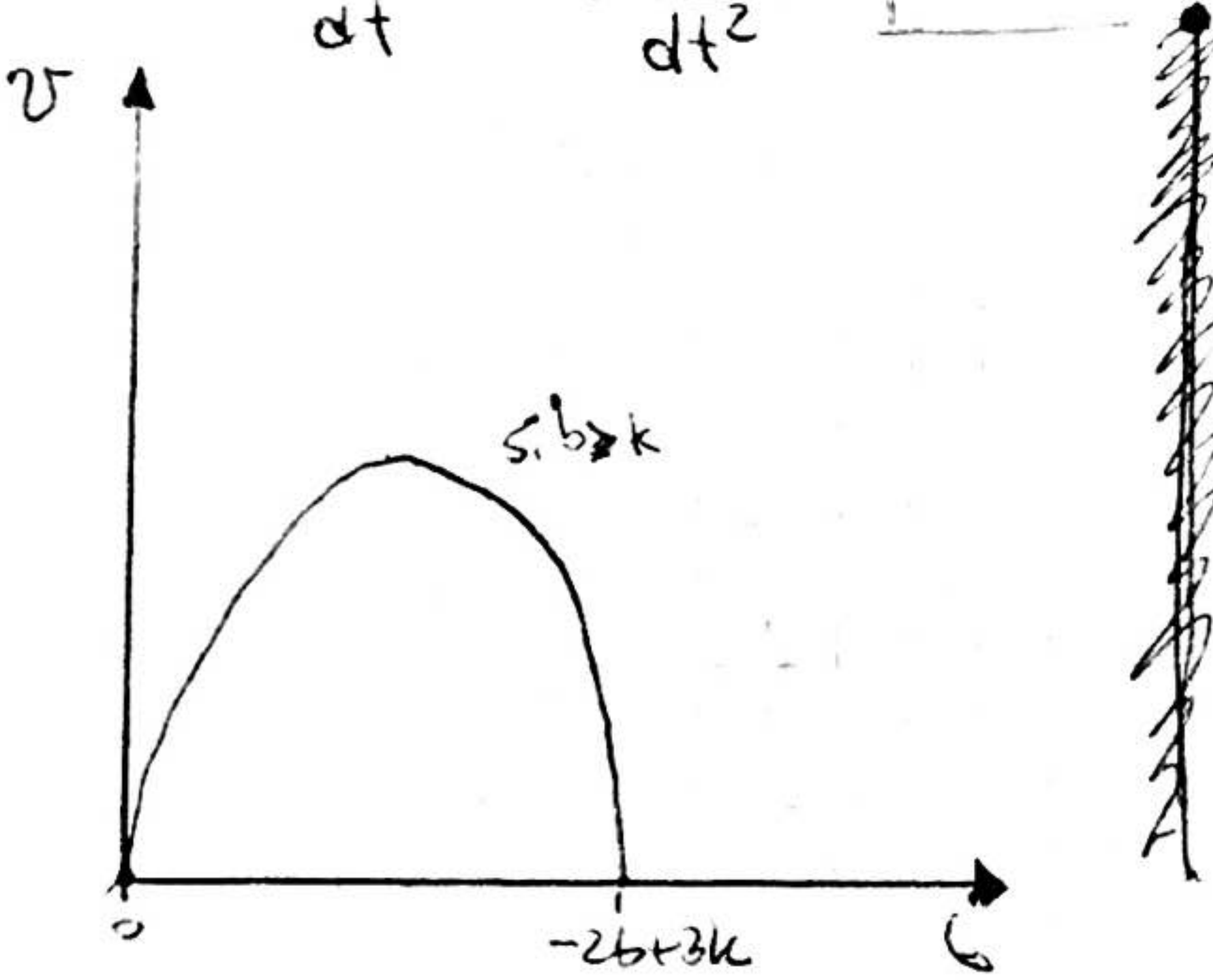
a) $v(t)$ ∧ $a(t)$. Graficar

$$v(t) = \frac{dx(t)}{dt} = x'(t) = \boxed{-3kt^2 + 2bt}$$

$t(-3kt^2 + 2b) = \ldots$

$t^* = -2b + 3k$

$$a(t) = \frac{d\,v(t)}{dt} = \frac{d^2x(t)}{dt^2} = \boxed{-6kt + 2b}$$



b) Halle t' y $x(t)$ / $v=0$

$v(t) = -3kt^2 + 2bt = 0$

$t(-3kt + 2b) = 0$

$$\boxed{t = 0 \quad \vee \quad +3kt = +2b, \quad t = \frac{2b}{3k}}$$

$$x = -k\left(\frac{2b}{3k}\right)^3 + b\left(\frac{2b}{3k}\right)^2 = \frac{-8b^3}{27k^2} + \frac{4b^3}{9k^2} = \frac{4b^3}{9k^2}\left(-\frac{2}{3} + 1\right) = \boxed{\frac{4b^3}{27k^2}}$$

c)

Una partícula se desplaza en línea recta de acuerdo a la ecuación

$x=\sqrt{x_0^2+2kt}$, con x_0, k constantes >0

a) Calcular $v(t)$ y $a(t)$.

$$v(t) = x'(t) = \frac{dx(t)}{dt} = \sqrt{x_0^2+2kt}\,' = \left(x_0^2+2kt\right)^{1/2} = \frac{\left(x_0^2+2kt\right)^{-1/2}}{2}\cdot(2k)$$

$$= \frac{\cancel{2}k}{\cancel{2}\left(x_0^2+2kt\right)^{1/2}} = \boxed{v(t) = \frac{k}{\left(x_0^2+2kt\right)^{1/2}}}$$

$$a(t) = v'(t) = \frac{dv(t)}{dt} = \frac{d^2x(t)}{dt^2} = \frac{-k\left(x_0^2+2kt\right)^{-3/2}}{\cancel{2}}\cdot\cancel{2}k = \boxed{\frac{-k^2}{\left(x_0^2+2kt\right)^{3/2}}}$$

b) Exprese las magnitudes del punto a) en función de la posición y grafíquelas partiendo desde ~~t=0~~ $x(0)$

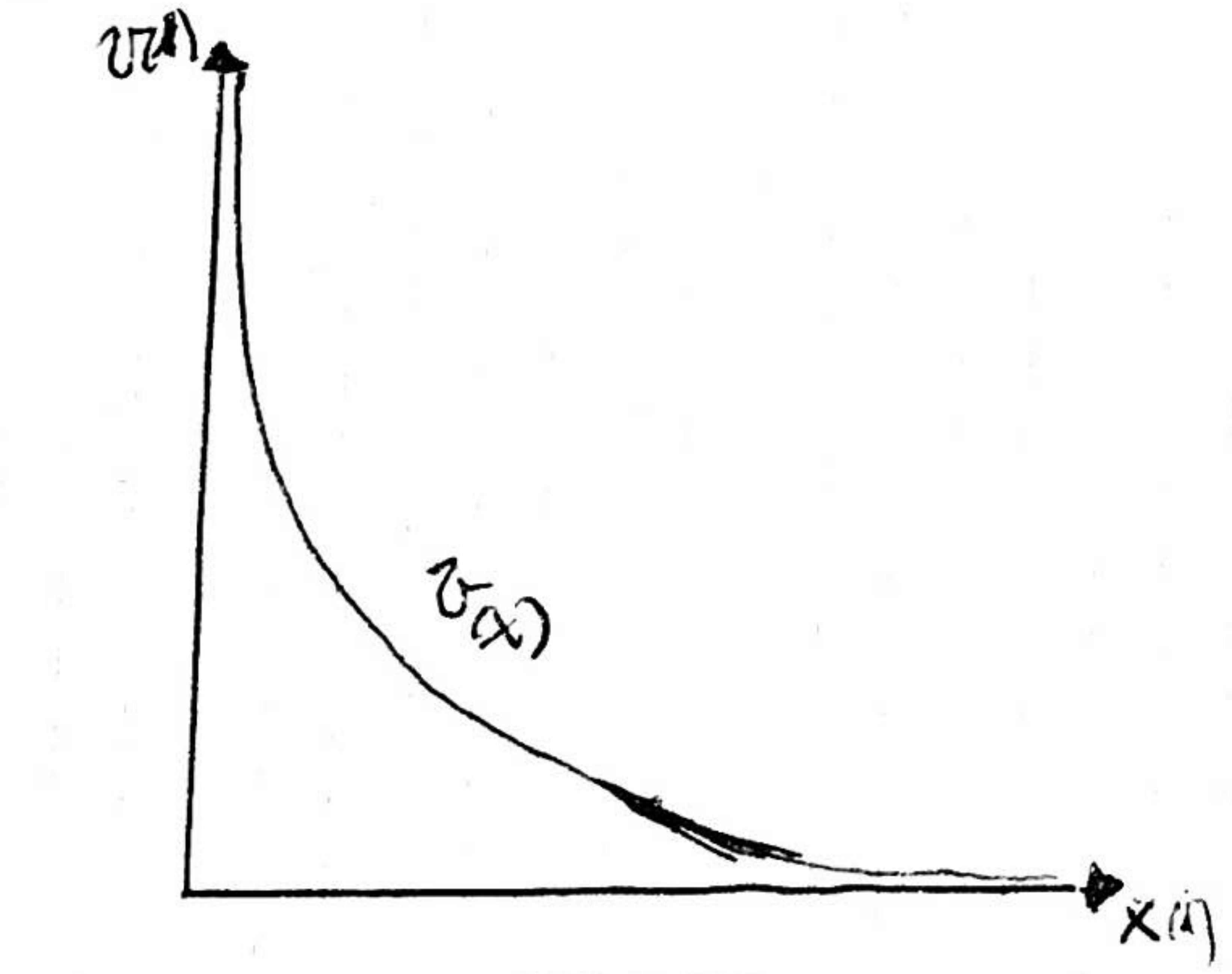
$$x^2 = x_0^2+2kt$$

$$\left(x^2-x_0^2\right) = 2kt$$

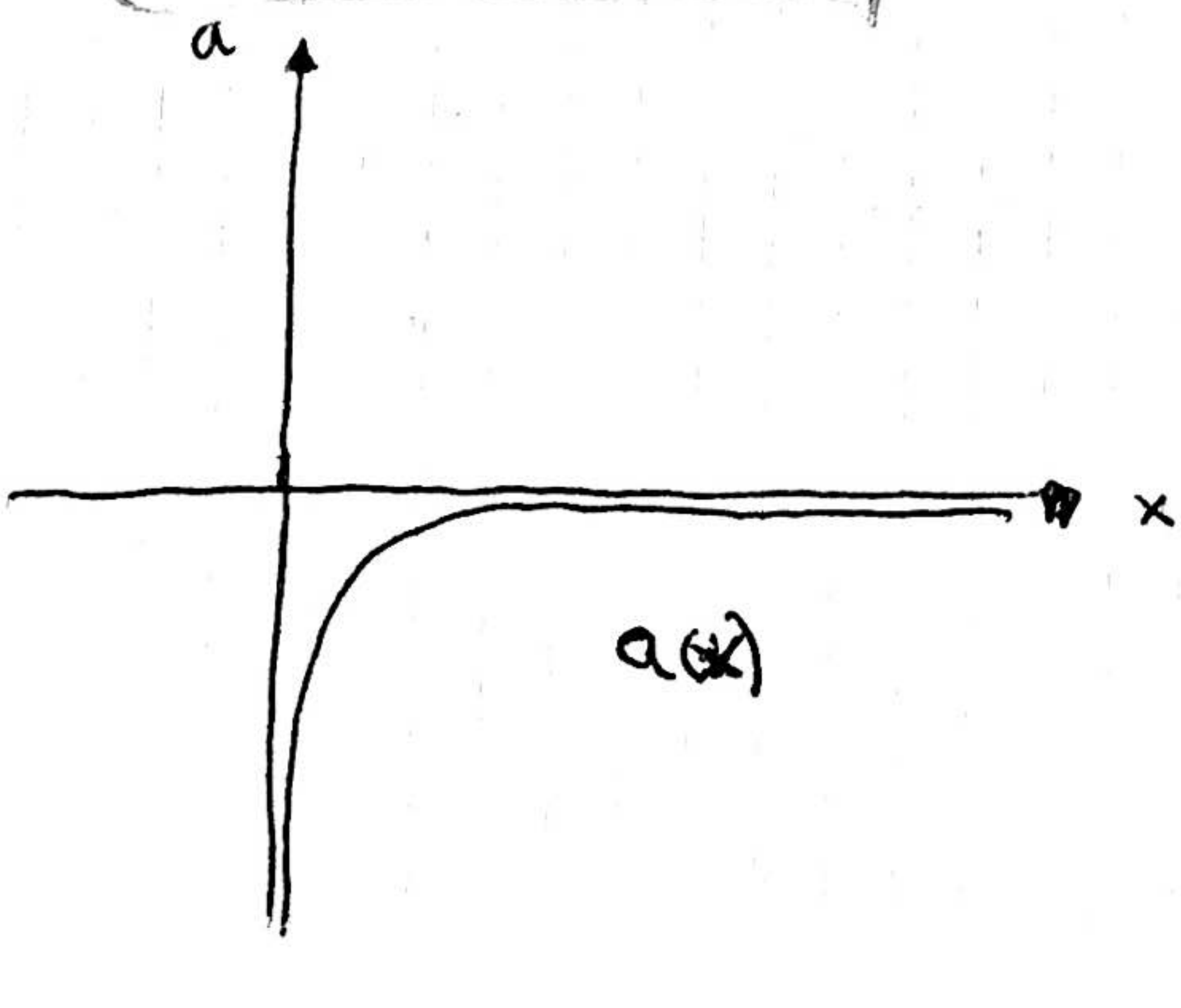
$$\frac{x^2-x_0^2}{2k} = t$$

$$v(x) = \frac{k}{\left(\cancel{x_0^2}+\cancel{2k}\cdot\frac{x^2-\cancel{x_0^2}}{\cancel{2k}}\right)^{1/2}} = \frac{k}{\left(x^2\right)^{1/2}}$$

$$= \boxed{\frac{k}{|x|} = v(x)}$$



$$\boxed{a(x) = \frac{-k^2}{\left(x^2\right)^{3/2}} = \frac{-k^2}{|x|^3}}$$



3) Un cuerpo se mueve en línea recta partiendo a $t_i=0$ de $x(0)=0$ con $v_{(0)}=V_0$

Halle $x(t)$ y $x(v)$ en los casos en que:

a) $a=kt^2$, $k>0$

$$a(t)=\frac{dv}{dt}=kt^2 \Rightarrow \int_0^t \frac{dv}{dt}\,dt=\int_0^t kt^2\,dt$$

$$\int_0^t \frac{dv}{dt}\,dt=v(t)\Big|_0^t=v(t)-v_{(0)}=$$

$$\int_0^t kt^2\,dt=\frac{kt^3}{3}$$

$$\Rightarrow v(t)-v_{(0)}=\frac{kt^3}{3}$$

$$v(t)=\frac{kt^3}{3}+v_{(0)}$$

$$v(t)=\frac{dx}{dt}=\frac{kt^3}{3}+v_{(0)}$$

$$\int_0^t \frac{dx}{dt}\,dt=\int_0^t \frac{kt^3}{3}+v_{(0)}\,dt \Rightarrow \int_0^t \frac{dx}{dt}\,dt=x(t)-x(0)$$

$$\Rightarrow \int_0^t \frac{kt^3}{3}+v_{(0)}\,dt=\frac{kt^4}{12}+v_{(0)}t\Big|_0^t$$

$$\Rightarrow x(t)-x(0)=\frac{kt^4}{12}+v_{(0)}t$$

$$\boxed{x(t)=\frac{kt^4}{12}+v_{(0)}t+\underbrace{x(0)}_{0}}$$

despejo t en función de v

$$v(t)=\frac{kt^3}{3}+v_{(0)}$$

$$\sqrt[3]{\frac{3v(t)-v_{(0)}}{k}}=t \Rightarrow \boxed{x(v)=\frac{k}{12}\left(\frac{3(v(t)-v_{(0)})}{k}\right)^{4/3}+v_{(0)}\cdot\left(\frac{3(v(t)-v_{(0)})}{k}\right)^{1/3}}$$

b) $a = -kv^2$, $k > 0$

$$a = -kv^2 = \frac{dv}{dt} =$$

$$\left(f(g(x))\right)' = f'(g(x)) \cdot g'(x)$$

$$v(x(t)) = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = \frac{dv}{dx}\cancel{v} = -k\cancel{v^2}$$

$$\frac{dv}{dx} = \frac{-kv}{x} \qquad \frac{dv}{dx\,v} = -k$$

$$\int_0^x \frac{1}{v} \frac{dv}{dx}\,dx = \int_0^x -k$$

$u = v$
$du = \frac{dv}{dx}\,dx$ $\rightarrow$ $\int_{v(x(0))}^{v(x(t))} \frac{1}{u}\,du = \int_{x(0)}^{x(t)} -k\,dx$

$$\ln u_{(x(t))} - \ln_{(0)} = -kx$$

$$\ln \frac{v}{v(0)} = -kx$$

$$\frac{v}{v(0)} = e^{-kx}$$

$$v(x) = v(0)e^{-kx}$$

$$\boxed{x(v) = \frac{\ln \frac{v}{v_0}}{-k}}$$

$$v(x) = \frac{dx}{dt} = v(0)e^{-kx}$$

$$\frac{dx}{e^{-kx}} = v(0)\,dt$$

$$\int_0^x e^{kx}\,dx = \int_0^t v(0)\,dt$$

$$\frac{e^{kx}}{k} - \frac{e^0}{k} = v(0)t - v(0) \cdot 0$$

$$e^{kx} - 1 = v(t)k$$

$$e^{kx} = v(0)tk + 1$$

$$\boxed{x = \frac{\ln v_0 tk + 1}{k}}$$

c) $a = kvx$

$$a = \frac{dv}{dt} = kvx = \frac{dv}{dx}\cdot\frac{dx}{dt} = \frac{dv}{dx}\cdot \cancel{v} = k\cancel{v}x$$

$$\Rightarrow \frac{dv}{dx} = kx$$

$$dv = kx\,dx$$

$$\int_0^x dv\,dx = \int_0^x kx\,dx \qquad \int_0^x dv\,\cancel{dx} = \int_0^x kx\,dx$$

$$v(x) - v(0) = \frac{kx^2}{2}$$

$$\boxed{\sqrt{\frac{(v(x)-v(0))2}{k}} = x(v)}$$

$$v = \frac{kx^2}{2} + v(0) = \frac{dx}{dt} \Rightarrow \frac{dx}{\frac{kx^2}{2}+v(0)} = dt \qquad \operatorname{arctg}' x = \frac{1}{1+x^2}$$

$$\int \frac{kx^2}{2} + v(0) = \int_0^t \frac{dx}{dt}\,dt$$

$$\int_0^x \left(\frac{kx^2}{2} + v(0)\right)^{-1} dx = \int_0^t dt$$

$$\int_0^t dt = \int_0^x \frac{dx}{\frac{kx^2}{2}+v(0)} = \frac{1}{v(0)}\int_0^x \frac{dx}{1+\frac{k}{2v(0)}x^2} = \frac{1}{v(0)}\int_0^x \frac{dx}{1+\left(\sqrt{\frac{k}{2v(0)}}x\right)^2}$$

$$= \frac{1}{v(0)}\int_0^{\sqrt{\frac{k}{2v(0)}}x} \frac{\sqrt{\frac{2v(0)}{k}}\,du}{1+u^2} = \sqrt{\frac{2}{v(0)k}}\int_0^{\sqrt{\frac{k}{2v(0)}}x} \frac{du}{1+u^2}$$

$$u = \sqrt{\frac{k}{2v(0)}}x$$

$$du = \sqrt{\frac{k}{2v(0)}}\,dx$$

$$t = \sqrt{\frac{2}{v(0)k}}\operatorname{arctg}\left(\sqrt{\frac{k}{2v(0)}}x\right)$$

A $t=0$ se deja caer un cuerpo sin velocidad inicial desde una altura H del piso. Además del peso actúa una fuerza en la dirección horizontal que provoca una aceleración en esa dirección que puede expresarse como $a_x = -kt^2$ con $k > 0$

a) Escriba las ecuaciones de movimiento y halle la ecuación de trayectoria

$a_x = -kt^2$

$a_y = g$

$$a_x = -kt^2 = v_{(x)}' = \frac{dv}{dt} \qquad \frac{dv}{dt} = -kt^2$$

$$\int_0^t \frac{dv}{dt}\,dt = \int_0^t -kt^2$$

$$\int_0^t \frac{dv}{dt}\,dt = v(t) - v(0) \qquad \int_0^t -kt^2 = -\frac{kt^3}{3}$$

$$\Rightarrow v_x(t) = -\frac{kt^3}{3} + \overset{=0}{v(0)} \qquad v_x(t) = -\frac{kt^3}{3}$$

$$v_x(t) = -\frac{kt^3}{3} = x_x(t) = \frac{dx}{dt}$$

$$\int_0^t \frac{dx}{dt}\,dt = \int_0^t -\frac{kt^3}{3}\,dt$$

$$x(t) - \overset{=0}{x(0)} = -\frac{kt^4}{12}$$

$$\boxed{x(t) = -\frac{kt^4}{12}}$$

$$a_y = v_y(t)' = \frac{dv_y}{dt} = g \Rightarrow \int_0^t \frac{dv_y}{dt}\,dt = \int_0^t g\,dt \Rightarrow v_y(t) - \overset{=0}{v_y(0)} = g(t)$$

$$v_y(t) = g(t)$$

$$\boxed{Y = \frac{gt^2}{2} + Y_0} \Rightarrow \quad \text{Trayectoria } Y(x)$$

$\cancel{x(t) = -\frac{k t^4}{12}}$

$\cancel{\frac{12x}{-k}}$

$t^4 = \frac{12x}{k}$

$t = \sqrt[4]{\frac{12x}{k}}$

$$\Rightarrow y = \frac{g \sqrt[2]{\frac{12x}{k}}}{2} + y_0$$

$$\boxed{y = \frac{g}{2}\sqrt[2]{\frac{12x}{k}} + H}$$

b)

$$\Rightarrow 0 = \frac{g}{2}\sqrt[2]{\frac{12x_F}{k}} + H$$

$$-H = \frac{g}{2}\sqrt[2]{\frac{12x_F}{k}}$$

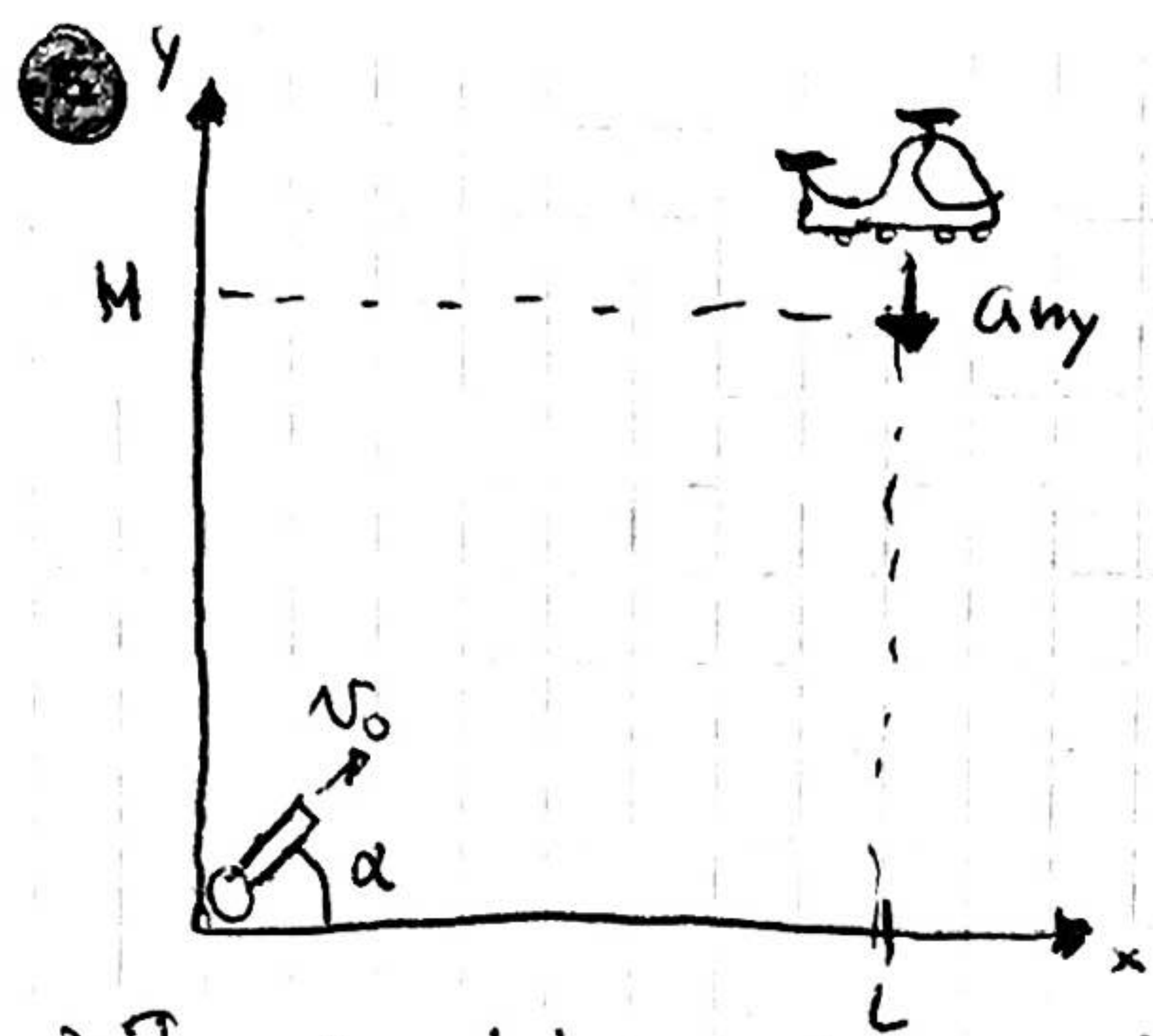
$$\left(-\frac{2H}{g}\right)^2 = \left(\sqrt[2]{\frac{12x_F}{k}}\right)^2$$

$$\frac{4H^2}{g^2} = \frac{12x_F}{k}$$

$$\boxed{\frac{H^2 k}{3g^2} = x_F}$$

Para $a_x = 0$ $\quad x_F' = 0 =$

$$\boxed{|x_F' - x_F| = \frac{H^2 k}{3g^2}}$$



$a_{Hy} = -kt$

a) Trayectoria del proyectil y graficar

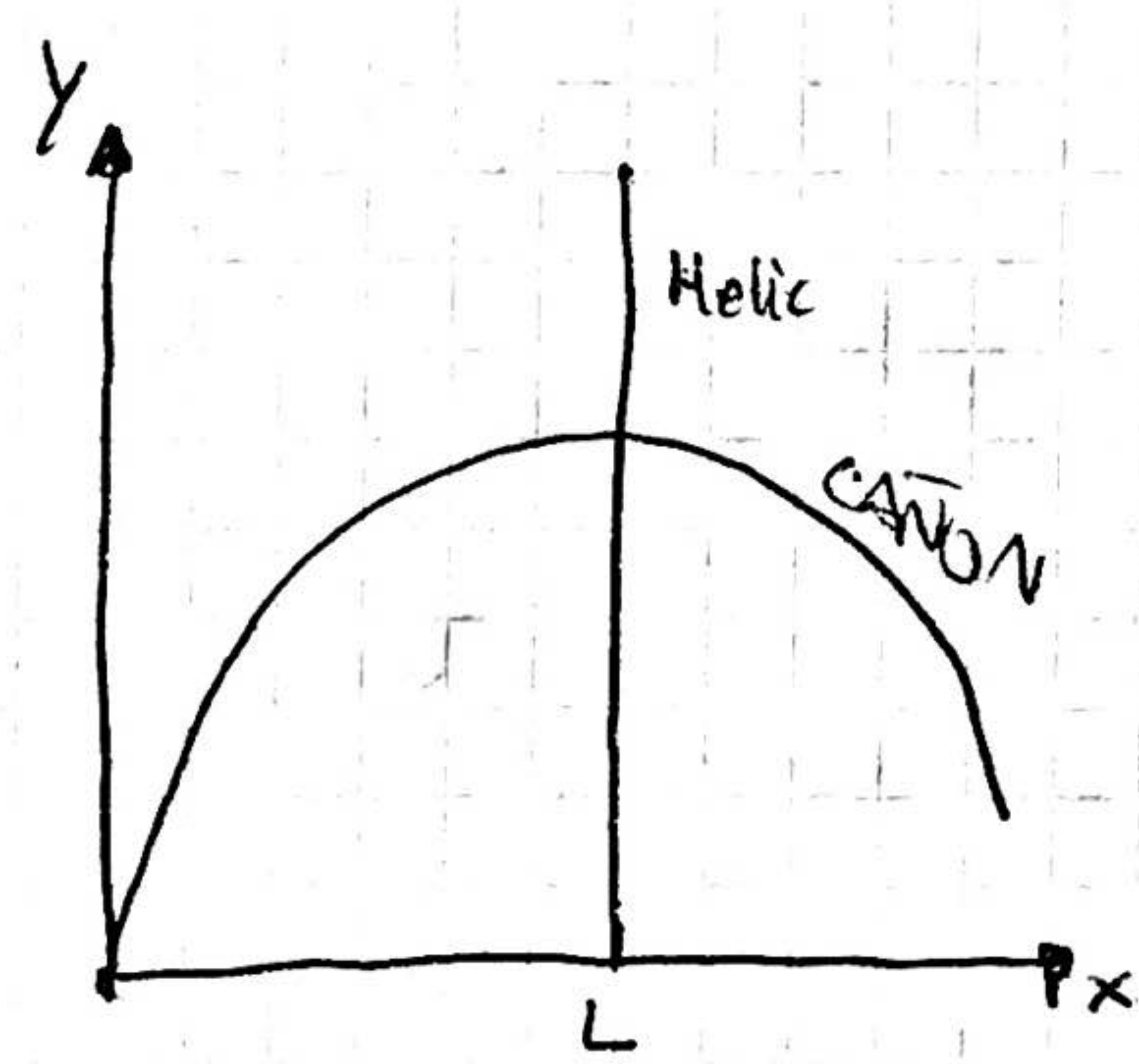
$\cancel{x_P} = v_0 \cdot \cos\alpha \, (t - t_0)$ $\qquad$ $t - t_0 = \dfrac{x}{v_0 \cos\alpha}$

$\cancel{y_P} = v_0 \cdot \operatorname{sen}\alpha \, (t - t_0) - \dfrac{g}{2}(t - t_0)^2$

$Y_P(x_P) = \dfrac{\cancel{v_0} \cdot \operatorname{sen}\alpha}{\cancel{v_0} \cos\alpha} x - \dfrac{g}{2} \dfrac{x^2}{v_0^2 \cos^2\alpha}$

$Y_P(x_P) = x \operatorname{tg}\alpha - \dfrac{x^2 g}{2 v_0^2 \cos^2\alpha}$

$$\boxed{Y_P(x_P) = x \left(\operatorname{tg}\alpha - \frac{x g}{2 v_0^2 \cos^2\alpha} \right)}$$



Helicoptero

$x_H = L$

$a_{Hy} = -kt = \dfrac{dv}{dt} = v(t)' \Rightarrow \displaystyle\int_0^t \frac{dv}{dt} \, dt = \int_0^t -kt \, dt$

$\Rightarrow v_{Hy}(t) - \overbrace{v(0)}^{=0} = -\dfrac{kt^2}{2}$

$v_{Hy}(t) = \dfrac{-kt^2}{2}$

$v_H(t) = \dfrac{dx}{dt} = x_H' = \dfrac{-kt^2}{2} \Rightarrow Y_H = \dfrac{-kt^3}{6} + Y_0$

$Y_H = \dfrac{-kt^3}{6} + H$

b) ¿Para qué valores de V_0 la trayectoria del proyectil y la del helic se intersectan?

$$X_p = L \quad 0 \leq Y_p \leq H$$

$$v_0 \cos\alpha (t - t_0) = L$$

$$0 \leq L\left(tg\alpha - \frac{Lg}{2v_0^2 \cos^2\alpha}\right) \leq H$$

$$0 \leq L\left(tg\alpha - \frac{Lg}{2v_0^2\cos^2\alpha}\right)$$

$$\frac{Lg}{2v_0^2 \cos^2\alpha} \leq tg\alpha$$

$$\frac{Lg}{2\,tg\alpha \cos^2\alpha} \leq v_0^2$$

$$v_0 \geq \sqrt{\frac{Lg}{2 \operatorname{sen}\alpha \cos\alpha}}$$

$$L\left(tg\alpha - \frac{Lg}{2v_0^2\cos^2\alpha}\right) \leq H$$

$$tg\alpha L - H \leq \frac{L^2 g}{2v_0^2\cos^2\alpha}$$

$$tg\alpha - \frac{H}{L} \leq \frac{Lg}{2v_0^2\cos^2\alpha}$$

$$v_0^2\left(tg\alpha - \frac{H}{L}\right) \leq \frac{Lg}{2\cos^2\alpha}$$

↳ 3 casos

⇒ Primer caso $\quad tg\alpha - \frac{H}{L} > 0$

$$v_0^2\left(tg\alpha - \frac{H}{L}\right) \leq \frac{Lg}{2\cos^2\alpha}$$

$$v_0 \leq \sqrt{\frac{Lg}{2\cos^2\alpha\left(tg\alpha - \frac{H}{L}\right)}} \quad v_{máx}$$

⇒ Segundo caso

$$tg\alpha - \frac{H}{L} < 0$$

$$v_0^2\left(tg\alpha - \frac{H}{L}\right) \geq \frac{Lg}{2\cos^2\alpha}$$

$$v_0^2 \geq \frac{Lg}{2\cos^2\alpha\left(tg\alpha - \frac{H}{L}\right)} \quad ✓$$

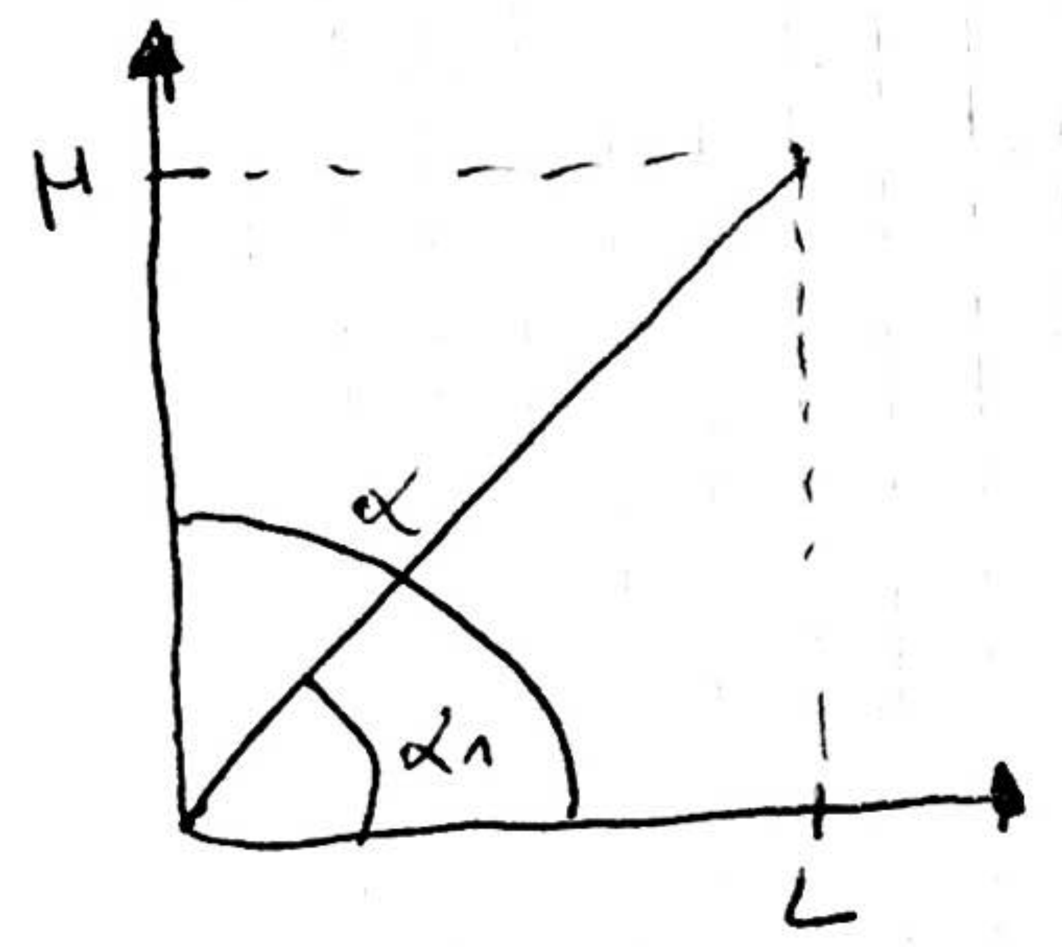
$\forall\, V_0 \geq v_{mín}$

⇒ Tercer caso $\quad tg\alpha - \frac{H}{L} = 0$

$$0 \leq \frac{Lg}{2\cos^2\alpha} \quad ✓$$

Solución $\sqrt{\dfrac{Lg}{2\operatorname{sen}\alpha\cos\alpha}} \leq v_0 \leq \sqrt{\dfrac{Lg}{2\cos^2\alpha\left(tg\alpha - \frac{H}{L}\right)}}$

c) Si V_0 es alguno de los valores hallados en b) diga en qué instante debe efectuarse el disparo para que el proyectil haga impacto en el helicóptero



$x_p(t_e) = x_H(t_e) \Rightarrow v_0.\cos\alpha.(t_e - t_0) = L$

$y_p(t_e) = y_H(t_e) \Rightarrow v_0.\operatorname{sen}\alpha\,(t_e - t_0) - \frac{g}{2}(t_e - t_0)^2 = \frac{-k t_e^3}{6} + H$

$t_e = \frac{L}{v_0.\cos\alpha} + t_0$

$\Rightarrow v_0.\operatorname{sen}\alpha\left(\frac{L}{v_0.\cos\alpha} + t_0 - t_0\right) - \frac{g}{2}\left(\frac{L}{v_0.\cos\alpha} + t_0 - t_0\right)^2 = \frac{-k\left(\frac{L}{v_0.\cos\alpha}\right)^3}{6} + H$

$L\,tg\alpha - \frac{gL^2}{2v_0^2.\cos^2\alpha} = \frac{-kL^3}{6v_0^3.\cos^3\alpha} + H$

$(t_e - t_0) = \frac{L}{v_0.\cos\alpha}$

$\Rightarrow \cancel{v_0}.\operatorname{sen}\alpha.\frac{L}{\cancel{v_0}.\cos\alpha} - \frac{g}{2}\frac{L^2}{v_0^2.\cos^2\alpha} = \frac{-k t_e^3}{6} + H$

$-L\,tg\alpha + \frac{g}{2}\frac{L^2}{v_0^2.\cos^2\alpha} = \frac{+k t_e^3}{6} + H \Rightarrow -L\,tg\alpha + \frac{gL^2}{2v_0^2.\cos^2\alpha} + H = \frac{k t_e^3}{6}$

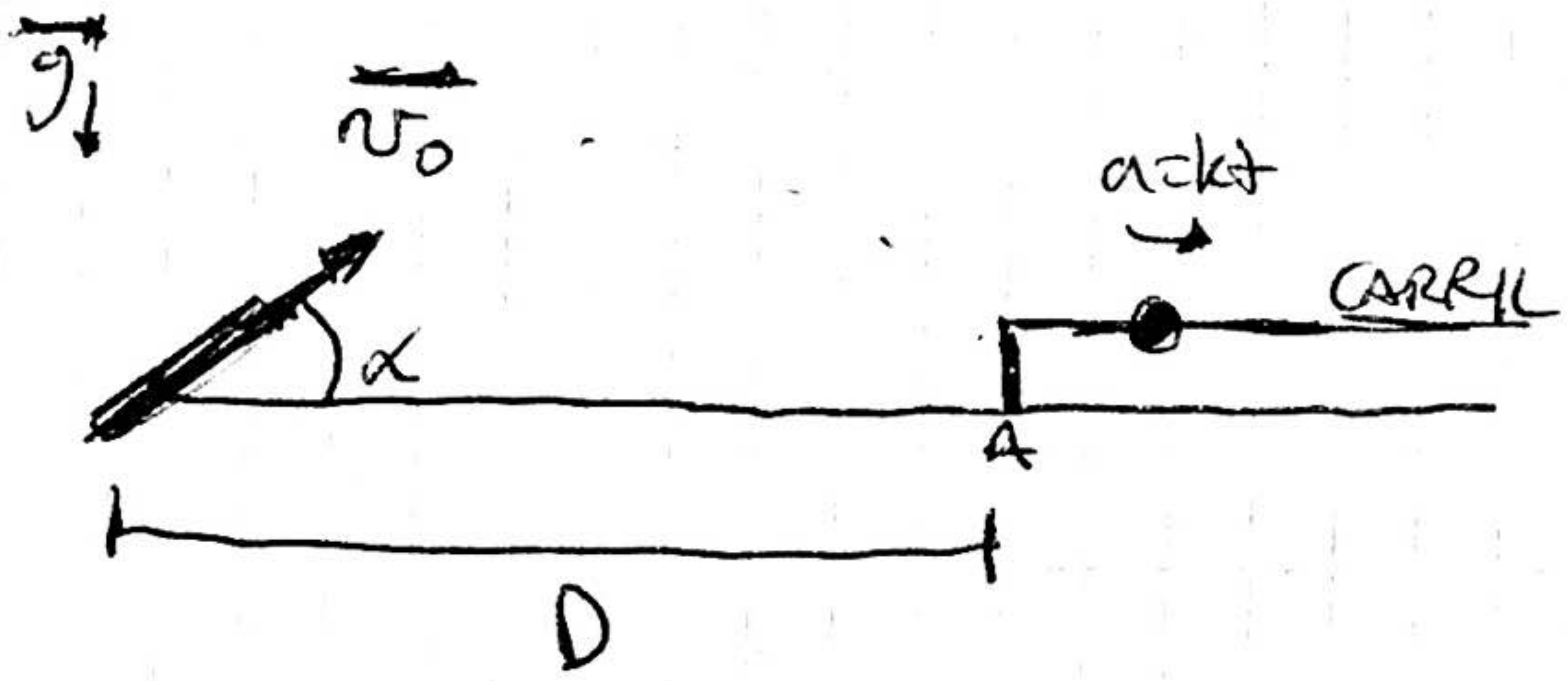
$\frac{6L}{k}\left(-tg\alpha + \frac{gL}{2v_0^2.\cos^2\alpha} + \frac{H}{L}\right) = t_e^3$

$t_e^3 = \frac{6L}{k}\left(\frac{v_{max}}{v_0^2\left(tg\alpha - \frac{H}{L}\right)} - tg\alpha + \frac{H}{L}\right) \Rightarrow t_e^3 = \frac{6L}{k}\left(tg\alpha - \frac{H}{L}\right)\left(\frac{v_{max}}{v_0^2} - 1\right)$

~~$t_0 = t_e t_0$~~ $t_0 = -\frac{L}{v_0 \cos\alpha} + t_e$

$$\boxed{\Rightarrow t_0^* = -\frac{L}{v_0 \cos\alpha} + \sqrt[3]{\frac{6L}{k}\left(tg\alpha - \frac{H}{L}\right)\left(\frac{v_{max}}{v_0^2} - \frac{1}{L}\right)}}$$

6 Un juego de un parque de diversiones consiste en una pelotita que se mueve por un carril rectilíneo con $a = kt$ hacia derecha. k cte > 0. A $t = 0$, pelotita en reposo en punto A. Rifle a distancia D de A, disparando bala con v_0 variable pero α fijo.



a) v_0 / sea posible acertar a la pelotita. Es decir, para que v_0 las trayectorias de la bala se intersectan

Pelotita

$a = kt$ $\quad v_0 = 0$ REPOSO

$v = \frac{kt^2}{2}$

$x = \frac{kt^3}{6} + D$

Rifle

$x_R = v_0 \cdot \cos\alpha \, (t - t_0)$

$y_R = v_0 \cdot \operatorname{sen}\alpha \, (t - t_0) - \frac{g}{2}(t - t_0)^2$

$y(x)_R = tg\alpha \, x_R - \frac{g x_R^2}{2 v_0^2 \cos^2\alpha}$

Para que intersecten trayectorias

$y_R = 0 \quad x_R \geq D$

$\Rightarrow 0 = tg\alpha \, D - \frac{g D^2}{2 v_0^2 \cos^2\alpha} \qquad \frac{g D}{2 v_0^2 \cos^2\alpha} = tg\alpha$

$\sqrt{\frac{g D}{2 \operatorname{sen}\alpha \cdot \cos\alpha}} = v_0$

t_e

$Y_e = 0 \quad X_R \geq D$

$\Rightarrow D \leq v_0 . \cos\alpha (t - t_0)$

$0 = v_0 . \text{sen}\alpha (t - t_0) - \frac{g}{2}(t - t_0)^2$

$v_0 . \text{sen}\alpha (t - t_0) = \frac{g}{2}(t - \cancel{t_0})^{\cancel{2}}$

$v_0 . \text{sen}\alpha = \frac{g}{2}(t - t_0)$

$\frac{2 v_0 \text{sen}\alpha}{g} = (t - t_0)$

$\Rightarrow A \leq v_0 . \cos\alpha \frac{2v_0 . \text{sen}\alpha}{g}$

$$\boxed{v_0 \geq \sqrt{\frac{g D}{2\cos\alpha \, \text{sen}\alpha}}}$$

b) Si V_0 es alguna de v_0 hallados en a).

¿En que instante debe disparar el jugador para acertar?

$\Rightarrow X_R(t_e) = X_P(t_e) \quad Y_R(t_e) = 0$

$\frac{k t_e^3}{6} + A = v_0 . \cos\alpha (t_e - t_0)$

$0 = v_0 . \text{sen}\alpha (t_e - t_0) - \frac{g}{2}(t_e - t_0)^2$

$v_0 \text{sen}\alpha = \frac{g}{2}(t_e - t_0)$

$\frac{2 v_0 \text{sen}\alpha}{g} = (t_e - t_0)$

$\Rightarrow \frac{k t_e^3}{6} + A = \frac{2 v_0^2 . \cos\alpha . \text{sen}\alpha}{g}$

$\frac{k t_e^3}{6} = \frac{2 v_0^2 . \cos\alpha . \text{sen}\alpha}{g} - A$

$t_e^3 = \frac{6}{k}\left(\frac{2 v_0^2 . \cos\alpha . \text{sen}\alpha}{g} - A\right)$

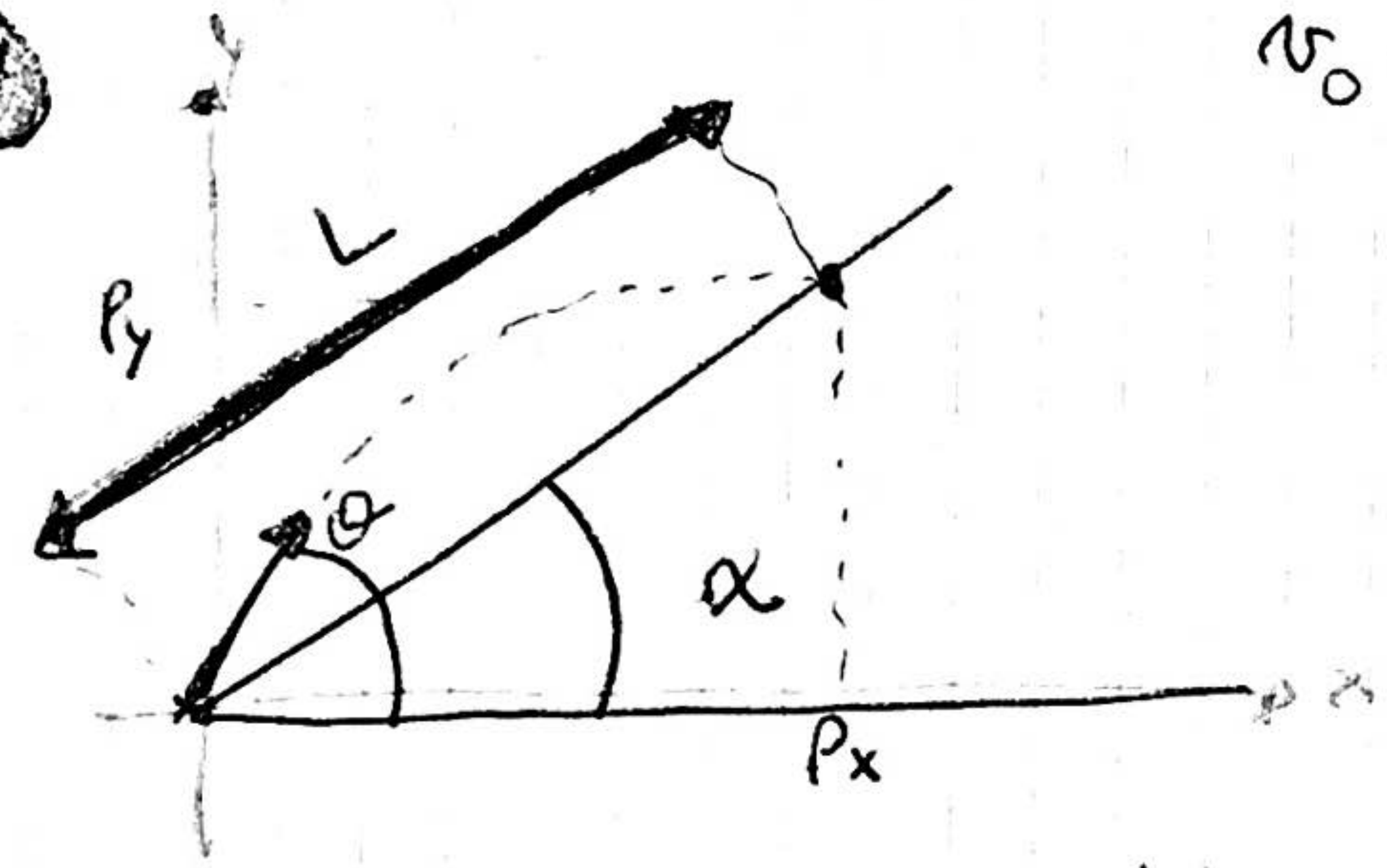
$$\frac{2v_0\, sen\alpha}{g} = (t_e - t_0)$$

$$-\frac{2v_0\, sen\alpha}{g} + t_e = t_0$$

$$\boxed{t_0 = \frac{-2v_0\, sen\alpha}{g} + \sqrt[3]{\frac{6}{k}\left(\frac{2v_0^2 \cos\alpha\, sen\alpha}{g} - A\right)}}$$

7

$v_0 = v_0 \quad \theta = \theta$



a) Muestre que alcance L en función del ángulo θ es $L = \frac{2v_0^2}{g\cos^2\alpha}\, sen(\theta - \alpha)\cos\theta$

~~$X_p = v_0$~~.

$P_x = L \cdot \cos\alpha$

$P_y = L \cdot sen\alpha$

$Y(t) = v_0 \cdot sen\theta \cdot (t) - \frac{g}{2}t^2$

$X(t) = v_0 \cdot \cos\theta\, (t)$

$$Y(x) = tg\,\theta \cdot x - \frac{g x^2}{2v_0^2 \cos^2\theta} \Rightarrow \cancel{L} \cdot sen\alpha = tg\,\theta \cdot \cancel{L}\cos\alpha - \frac{g \cdot L^{\cancel{2}}\cos^2\alpha}{2v_0^2 \cdot \cos^2\theta}$$

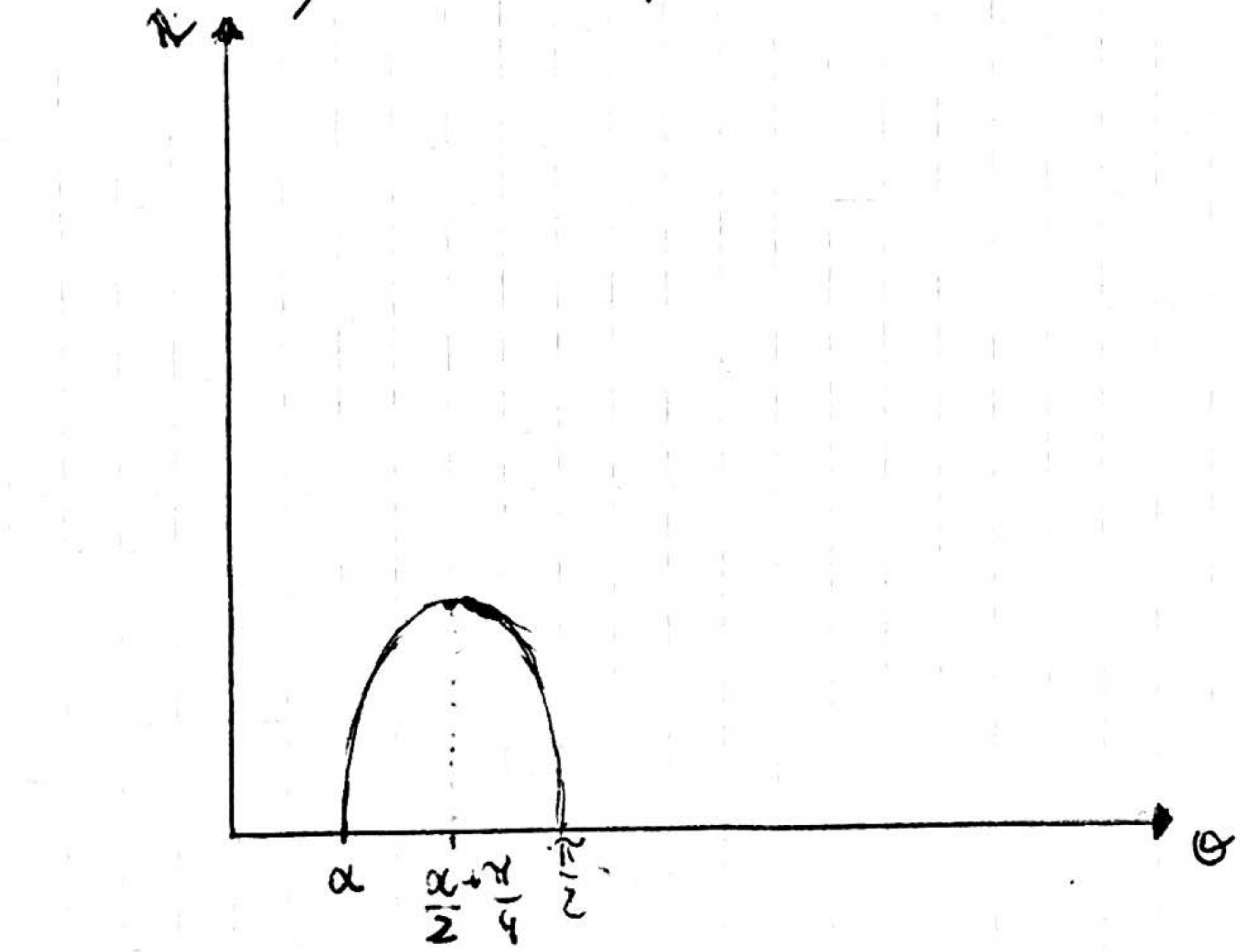
$$\frac{gL\cos^2\alpha}{2v_0^2 \cos^2\theta} = tg\theta \cdot \cos\alpha - sen\alpha$$

$$L = (tg\theta \cdot \cos\alpha - sen\alpha)\frac{2v_0^2 \cdot \cos^2\theta}{g \cdot \cos^2\alpha} = L = \left(\frac{sen\theta}{\cos\theta} \cdot \cos\alpha \cos^2\theta - sen\alpha \cdot \cos^2\theta\right)\frac{2v_0^2}{g\cos^2\alpha}$$

$$L = (sen\theta \cdot \cos\alpha - sen\alpha \cdot \cos\theta)\frac{\cos\theta\, 2v_0^2}{g\cos^2\alpha}$$

$$\boxed{L = \frac{2v_0^2}{g\cos^2\alpha}\left(sen(\theta - \alpha)\right) \cdot \cos\theta}$$

b) Gráfique el alcance L en función de θ y demuestre que para cada valor de L hay dos valores posibles de θ (tiro rasante y de elevación)



$$L=\left(\operatorname{sen}\theta.\cos\alpha\cos\theta-\operatorname{sen}\alpha.\cos^2\theta\right)\frac{2v_0^2}{g\cos^2\alpha}$$

~~¿θ para que L máx?~~

$$\cos^2\theta=\frac{1+\cos 2\theta}{2}$$

$$2\operatorname{sen}2\theta=2\operatorname{sen}\theta.\cos\theta$$

$$L=\frac{2v_0^2}{g\cos^2\alpha}\left(\operatorname{sen}(\theta-\alpha)\right)\cos\theta \Rightarrow L'=0 \text{ para max}$$

$$L'=\frac{2v_0^2}{g\cos^2\alpha}\left[\left(\operatorname{sen}(\theta-\alpha)\right)'.\cos\theta+\left(\operatorname{sen}(\theta-\alpha)\right).(\cos\theta)'\right]$$

$$L'=\frac{2v_0^2}{g\cos^2\alpha}\cos(\theta-\alpha).1.\cos\theta+\operatorname{sen}(\theta-\alpha).(-\operatorname{sen}\theta)$$

$$L'=\frac{v_0^2}{g\cos^2\alpha}2\left(\cos(\theta-\alpha)\cos\theta-\operatorname{sen}(\theta-\alpha)\operatorname{sen}\theta\right)=\frac{v_0^2}{g\cos^2\alpha}2\cos(2\theta-\alpha)$$

$$2\cos(2\theta-\alpha)=0$$

$$\cos(2\theta-\alpha)=0$$

$$2\theta-\alpha=\frac{\pi}{2} \Rightarrow 2\theta=\frac{\pi}{2}+\alpha$$

$$\theta=\frac{\pi}{4}+\frac{\alpha}{2}$$

$$L = \frac{2V_0^2}{g\cos^2\alpha}\left(\operatorname{sen}(\theta-\alpha)\right)\cos\theta = 0$$

$\operatorname{sen}(\theta-\alpha)\cos\theta = 0$ $\qquad\qquad$ $\theta = \frac{\pi}{2}$

$\operatorname{sen}(\theta-\alpha) = 0$

$\theta - \alpha = \pi$

$\theta = \pi + \alpha$

8. Un cuerpo inicialmente en reposo $(\theta(t=0)=0,\ \omega(t=0)=0)$ es acelerado en una trayectoría circular de 1,3m de radio, de acuerdo a $\gamma = 120s^{-4}t^2 - 48s^{-3}t + 16s^{-2}$ donde γ es la aceleración angular medida en seg^{-2}

Halle

a) $\theta = \theta(t)$

b) $\omega = \omega(t)$

c) $\vec{\gamma}$ (descomposición polar)

d) $\vec{v}$ en $t=2$ seg?

$$\gamma = 120\frac{t^2}{s^4} - \frac{48t}{s^3} + \frac{16}{s^2} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

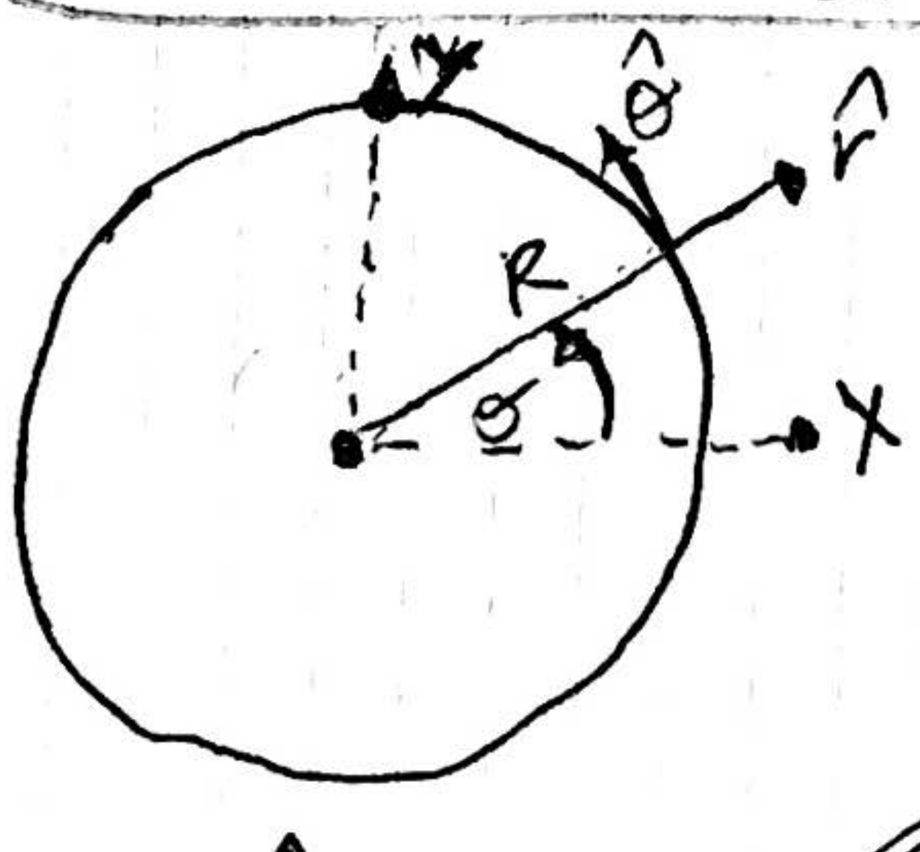
$$\int_0^{\omega(t)} d\omega = \int_0^t 120\frac{t^2}{s^4} - \frac{48t}{s^3} + \frac{16}{s^2}\,dt$$

$$\boxed{\omega(t) - \underbrace{\boxed{\omega(0)}}_{=0} = \frac{40t^3}{s^4} - \frac{24t^2}{s^3} + \frac{16t}{s^2}}$$

$$\omega(t) = \frac{d\theta}{dt} = \frac{40t^3}{s^4} - \frac{24t^2}{s^3} + \frac{16t}{s^2}$$

$$\Rightarrow \int_0^{\theta(t)} d\theta = \int_0^t \frac{40t^3}{s^4} - \frac{24t^2}{s^3} + \frac{16t}{s^2}\,dt$$

$$\boxed{\theta(t) - \underbrace{\boxed{\theta(0)}}_{=0} = \frac{10t^4}{s^4} - \frac{8t^3}{s^3} + \frac{8t^2}{s^2}}$$



$$\cancel{\vec{\gamma} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}}$$

$$\cancel{\vec{\gamma} = -r\dot{\theta}^2\hat{r} + r\ddot{\theta}\hat{\theta}}$$

$$\cancel{\vec{\gamma} = [1,3m\,(120\frac{t^2}{s^4} - \frac{48t}{s^3} + \frac{16}{s^2})]\hat{r} + (1,3m}$$

$$\overline{\gamma} = -R\omega^2\,\hat{r} + R\gamma\,\hat{\theta}$$

$$\boxed{\overline{\gamma} = (-1{,}3)\left(\frac{40t^3}{s^4} - \frac{24t^2}{s^3} + \frac{16t}{s^2}\right)^2 + (1{,}3)\left(\frac{10t^4}{s^4} - \frac{8t^3}{s^3} + \frac{8t^2}{s^2}\right)}$$

$$\overline{v} = \omega R\,\hat{\theta}$$

$$\overline{v}_{(2s)} = 1{,}3m\left(\frac{40(2s)^3}{s^4} - \frac{24(2s)^2}{s^3} + \frac{16(2s)}{s^2}\right)$$

$$\boxed{\overline{v} = 332{,}8\,m/s}$$

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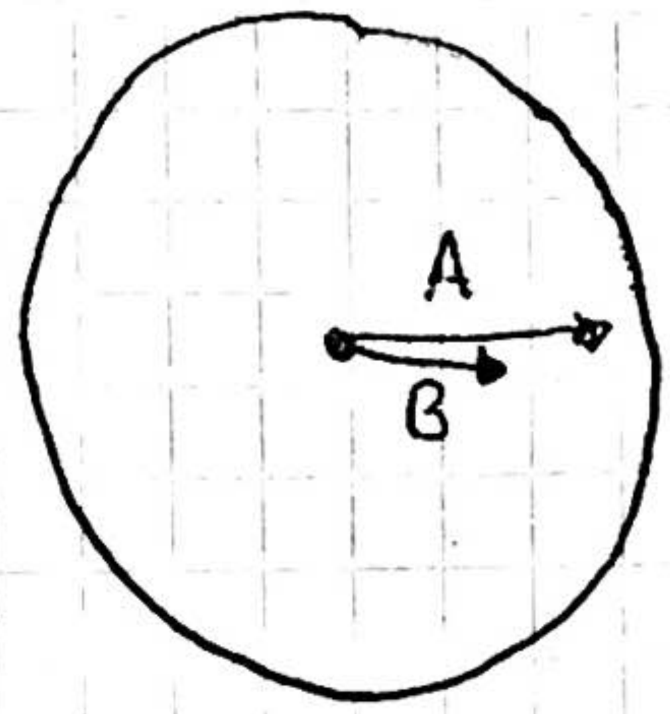


$\omega_{A0} = cte = \omega_0$, $\varphi_A(t=0) = 0$

γ_B cte $\omega_{B0}(t=0) = 2\omega_0$ de la posicion $\varphi_B(t=0) = 0$

a) Calcule en que instante ambas agujas coinciden

$$\gamma_B = \frac{d\omega_B}{dt}$$

$$\int_0^t \gamma_B \, dt = \int_{2\omega_0}^{\omega} d\omega_B$$

$$\gamma_B t = \omega - 2\omega_0$$

$$\gamma_B = \frac{\omega - 2\omega_0}{t}$$

$$\gamma_B t + 2\omega_0 = \omega_B$$

$$\omega_B = \frac{d\varphi_B}{dt} = \gamma_B t + 2\omega_0$$

$$\int_0^{\varphi_B(t)} d\varphi_B = \int_0^t \gamma_B t + 2\omega_0 \, dt$$

$$\varphi_B(t) = \frac{\gamma_B t^2}{2} + 2\omega_0 t$$

$\cancel{\varphi_A = \omega}$

$\varphi_A = \omega_0 \cdot t$

$\varphi_B = 2\omega_0 \cdot t +$

$$\Rightarrow \varphi_B - \varphi_A = 2k\pi$$

$$\frac{\gamma_B t^2}{2} + 2\omega_0 t - \omega_0 \cdot t = 2k\pi \Rightarrow \frac{\gamma_B}{2} t^2 + \omega_0 \cdot t - 2k\pi = 0$$

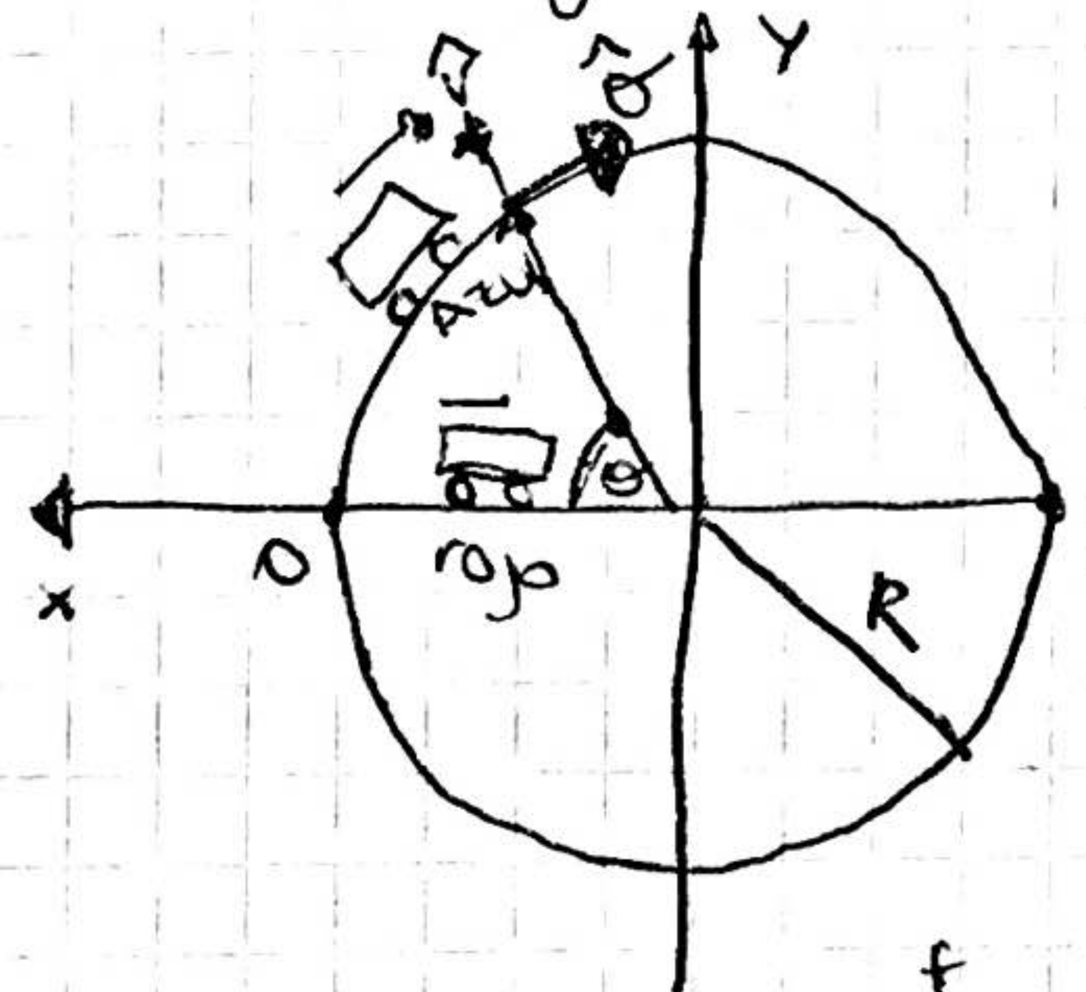
$$\frac{-\omega_0 \pm \sqrt{\omega_0^2 - 4(1)(-2k\pi)\left(\frac{\gamma_B}{2}\right)}}{\gamma_B}$$

10 $R = 90m$ $A_{zul} \Rightarrow \gamma_\theta = kt$ $\left(k = \frac{\pi}{6}\ s^{-3}\right)$

$t = 3s$ parte auto rojo $a_r = -a_0\hat{x}$

a) ¿Cuanto tarda azul en llegar a P?

b) ¿a_0 para que auto rojo alcance a azul en P?



a)

$\gamma_a = kt \Rightarrow = \frac{d\omega}{dt} \Rightarrow \int_0^t \frac{d\omega}{dt} dt = \int_0^t kt\, dt$

$= \int_0^{\omega(t)} d\omega = \int_0^t kt\, dt \Rightarrow \omega(t) - \omega(0) = \frac{kt^2}{2}$

$\omega(t) = \frac{kt^2}{2}$

$\omega(t) = \frac{d\theta}{dt} = \int_{\theta(0)}^{\theta(t)} d\theta = \int_0^t \frac{kt^2}{2} dt$

$\theta(t) = \frac{kt^3}{6}$

~~$R = 2R$~~ $\Rightarrow$ $P = -R$

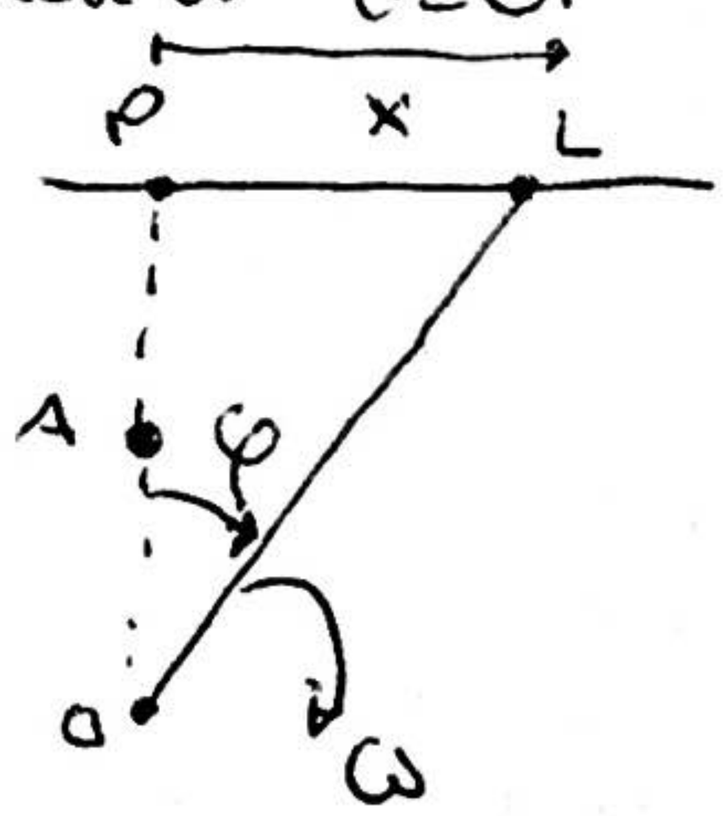
$\pi = \frac{kt^3}{6}$ $\boxed{\Rightarrow \sqrt[3]{\frac{6\pi}{k}} = t_f}$

b) $X_R = R + \frac{a_0}{2}(t_f - t_0)^2 \hat{x}$

$-R = R - \frac{a_0}{2}\left(\sqrt[3]{\frac{6\pi}{k}} - 3s\right)^2$ $\qquad +2R = +\frac{a_0}{2}\left(\sqrt[3]{\frac{6\pi}{k}} - 3s\right)^2$

$\boxed{\frac{4R}{\left(\sqrt[3]{\frac{6\pi}{k}} - 3s\right)^2} = a_0}$

11) Un faro que gira con ω cte $=\omega$, proyecta su luz sobre una pantalla ubicada a $d = OP$



a) Halle V_{lineal} de punto luminoso sobre la pantalla en función de datos y de x

b) Calcule en función de datos y de x la velocidad angular del punto luminoso para un observador situado a una distancia $D = AP$ de la pantalla

c) ¿Cómo debería ser la velocidad angular del faro para que el punto luminoso se mueva a $V =$ cte?

a) $\frac{x}{d} = tg\varphi \Rightarrow x = tg\varphi \cdot d$ $\quad V = \dot{x} = \frac{1}{\cos^2\varphi} \cdot \frac{d\varphi}{dt}$ $\quad \dot{\varphi} = \omega$

$\Rightarrow V = \frac{d\omega}{\cos^2\varphi}$

$\cos^2\varphi + sen^2\varphi = 1$

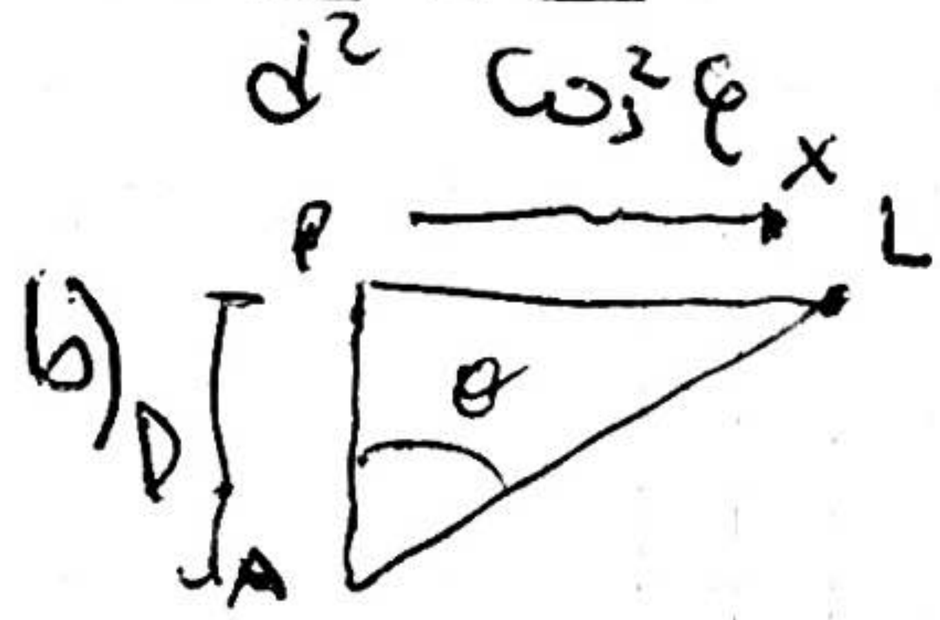
$1 + tg^2\varphi = \frac{1}{\cos^2\varphi}$

$1 + \frac{x^2}{d^2} = \frac{1}{\cos^2\varphi}$

$V_{(x)} = d\omega\left(1 + \frac{x^2}{d^2}\right)$

$x = tg\theta D$

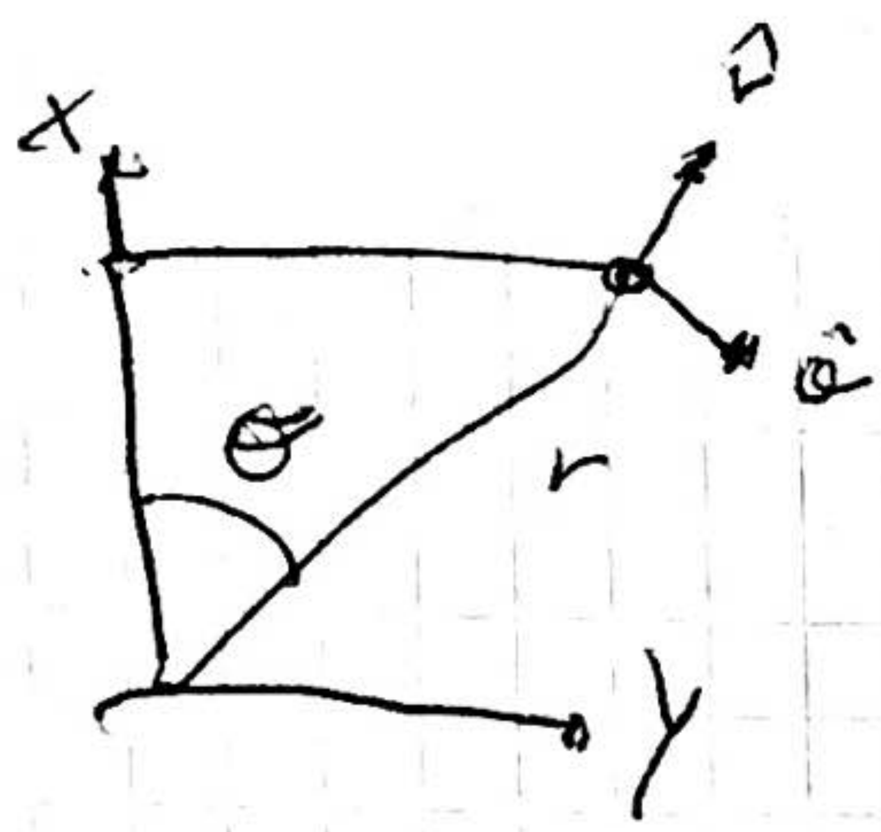
b)



$tg\theta = \frac{x}{D}$

$V = \frac{1}{\cos^2\theta} \cdot \omega . D$ $\quad V = \frac{D\omega}{\cos^2\theta}$ $\quad V_{(x)} = D\omega\left(1 + \frac{x^2}{D}\right)$

$\omega = \frac{V\cos^2\theta}{D}$ $\quad \omega = \frac{V}{\left(1 + \frac{x^2}{D}\right) . D}$



$\rightarrow \bar{v} = \dot{x}\hat{x}$

$\bar{r} = r\hat{r}$

$\Rightarrow \bar{v} = \dot{\bar{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$

$|\bar{v}| = v^2 = \dot{x}^2 = \dot{r}^2 + (r\dot{\theta})^2$

$r = \sqrt{x^2+d^2}$

$\dot{r} = \dfrac{\cancel{2}x\dot{x}}{\cancel{2}\sqrt{x^2+d^2}}$

$\dot{r}^2 = \dfrac{\cancel{4}x^2\dot{x}^2}{\cancel{4}x^2+d^2} + \omega^2 \cdot (x^2+d^2)$

$\dot{x} = (x^2+d^2)$

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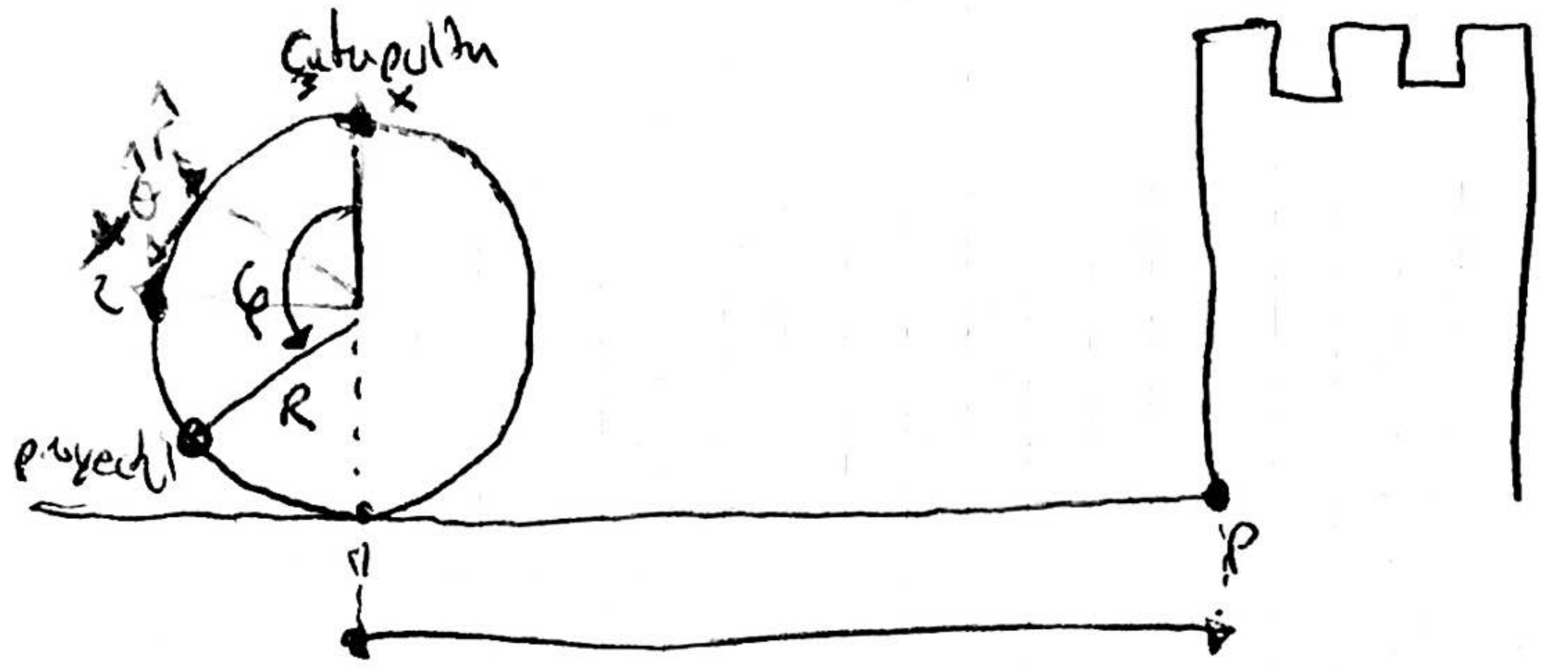


$v_0 = 0$ $\qquad \ddot{\varphi} = -\dfrac{(n+1)k}{\pi^{n+1}}\varphi^n$ $\qquad$ K y n constantes, n=4

a) v_r en función de K, R y φ. Calcule $v_{(2)}$

$$\bar{v}_t = r \cdot \dot{\theta}\,\hat{\theta}$$
$$\bar{v}_t = R\dot{\varphi}\,\hat{\theta}$$

$$\ddot{\varphi} = -\frac{(n+1)k}{\pi^{n+1}}\varphi^n = \frac{d\dot{\varphi}}{dt} = \frac{d\dot{\varphi}}{d\varphi}\frac{d\varphi}{dt}$$

$$-\frac{(n+1)k}{\pi^{n+1}} \cdot \varphi^n = \frac{d\dot{\varphi}}{d\varphi} \cdot \dot{\varphi}$$

$$\int_0^{\dot{\varphi}} \dot{\varphi}\, d\dot{\varphi} = \int_{\pi}^{\varphi} -\frac{(n+1)k}{\pi^{n+1}}\varphi^n \cdot d\varphi$$

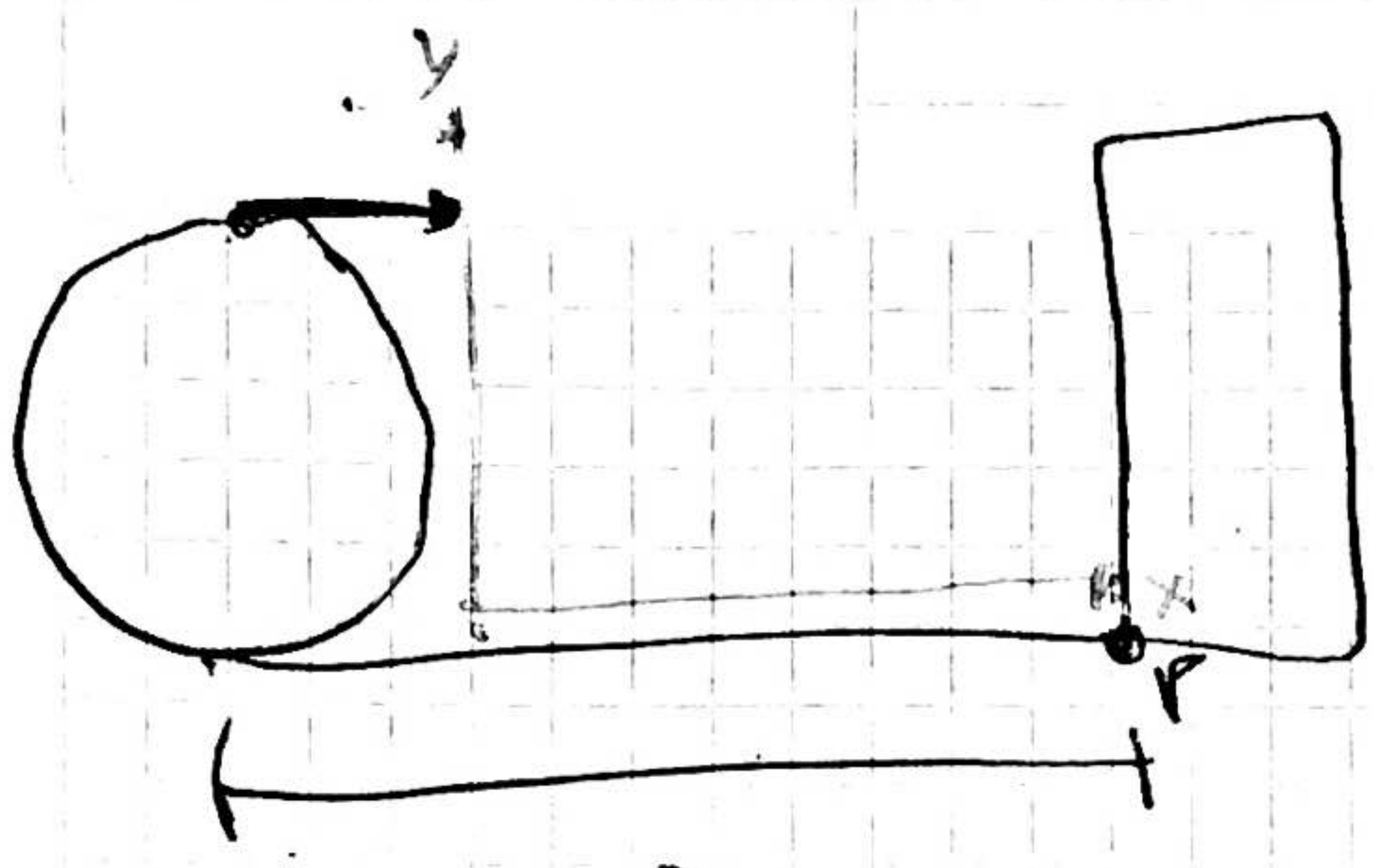
$$\frac{\dot{\varphi}^2}{2} = -\frac{\cancel{(n+1)}k}{\pi^{n+1}}\left(\frac{\varphi^{n+1}}{\cancel{n+1}} - \frac{\pi^{n+1}}{\cancel{n+1}}\right)$$

$$\frac{\dot{\varphi}^2}{2} = \frac{-k}{\pi^{n+1}}\left(\varphi^{n+1} - \pi^{n+1}\right)$$

$$\dot{\varphi} = \sqrt{\frac{-2k}{\pi^{n+1}}\left(\varphi^{n+1} - \pi^{n+1}\right)}$$ cumple siempre.

$$\boxed{\bar{v}_t = R\sqrt{\frac{-2k}{\pi^{n+1}}\left(\varphi^{n+1} - \pi^{n+1}\right)}} \qquad \boxed{v_{(2)} = R\sqrt{\frac{-2k}{\pi^{n+1}}\left(\frac{\pi}{2}^{n+1} - \pi^{n+1}\right)}}$$

b) Calcule D que hay que desplazar la catapulta para que los proyectiles lanzados peguen en P



$$y(t) = 2R - \frac{g}{2}(t-t_0)^2$$

$$x(t) = R\sqrt{2k\cancel{\ldots}}\,(t-t_0)$$

$$\Rightarrow x(t) = R\sqrt{2k}\,(t-t_0)$$

$$\Rightarrow \begin{array}{l} x = D \\ y = 0 \end{array}$$

$$0 = 2R - \frac{g}{2}(t-t_0)^2 \qquad D = R\sqrt{2k}\,(t-t_0)$$

$$\frac{D}{R\sqrt{2k}} = (t-t_0)$$

$$\frac{g}{2}\,\frac{D^2}{R^2 2k} = 2R$$

$$D^2 = \frac{8R^3k}{g} \qquad \boxed{D = \sqrt{\frac{8R^3k}{g}}}$$

⑬ Un nadador puede nadar 0,7 m/s respecto al agua. Quiere cruzar un río de 80 m de ancho. Corriente a 0,5 m/s

a) Si quiere ir al punto opuesto ¿En qué dirección debe nadar? ¿Cuánto tarda?