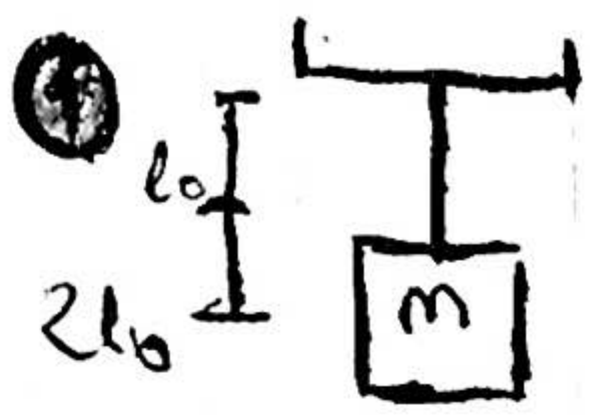
$x_T = x_H + x_p$

OSCILATORIO



Datos
$\cdot k \quad m \quad l_0$

en $t_0=0$, $2l_0$ y $v_0=0$

$$-k(x-l_0)+mg=m\ddot{x}$$

$$\ddot{x}+\frac{kx}{m}=g+\frac{kl_0}{m}$$

$$\ddot{c}+\omega_0^2 c=g+\omega_0^2 l_0$$

$$c=\frac{g}{\omega_0^2}+l_0$$

$x=x_H+x_p$

$x=A\sin(\omega_0 t+\varphi)+c$

$\rightarrow x(t)=A\cdot\sin(\omega_0 t+\varphi)+\frac{g}{\omega_0^2}+l_0$

$x_0=2l_0=A\cdot\sin(\omega_0\cdot 0+\varphi)+\frac{g}{\omega_0^2}+l_0$

$x_0=A\sin\varphi+\frac{g}{\omega_0^2}-l_0$

$\dot{x}=v_0=+A\cos(\omega_0 t+\varphi)\cdot\omega_0=0$

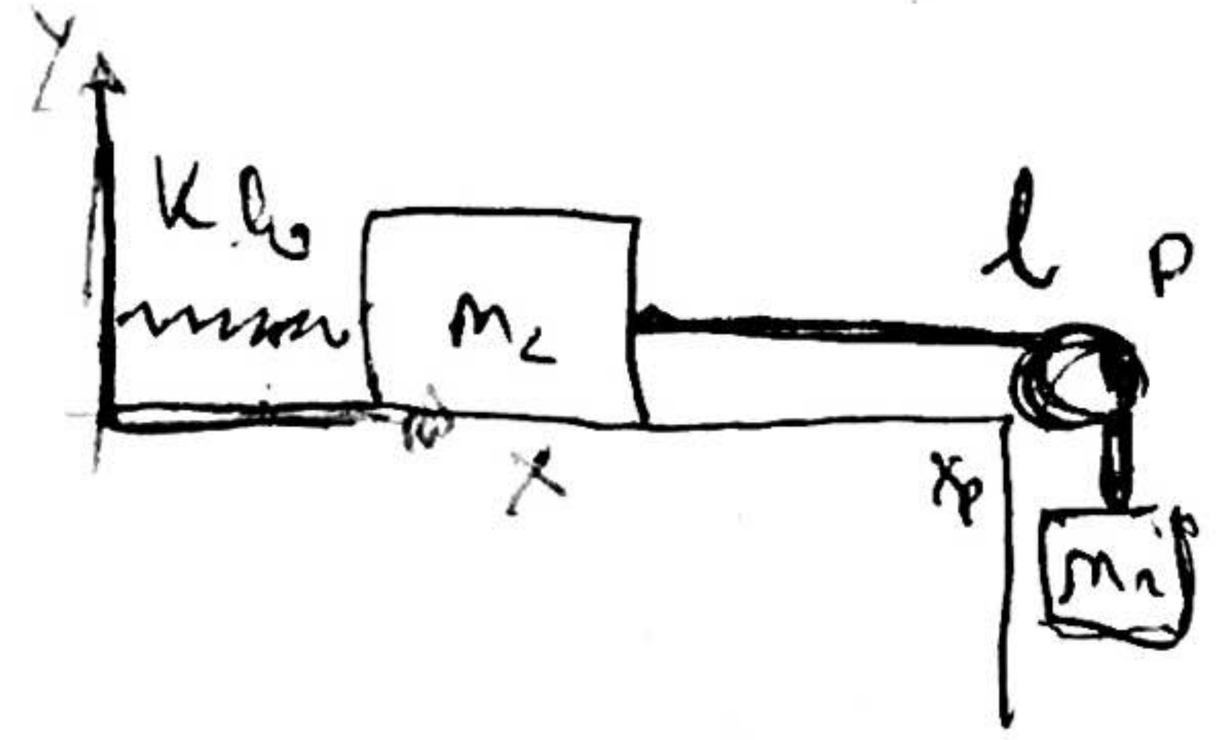
$\cos\varphi=0$

$\varphi=\frac{\pi}{2}$

$-\frac{g}{\omega_0^2}+l_0=A\sin\frac{\pi}{2}$

$A=-\frac{g}{\omega_0^2}+l_0$

$$\boxed{x(t)=\left(-\frac{g}{\omega_0^2}+l_0\right)\left(\sin\left(\omega_0 t+\frac{\pi}{2}\right)\right)+\frac{g}{\omega_0^2}+l_0}$$

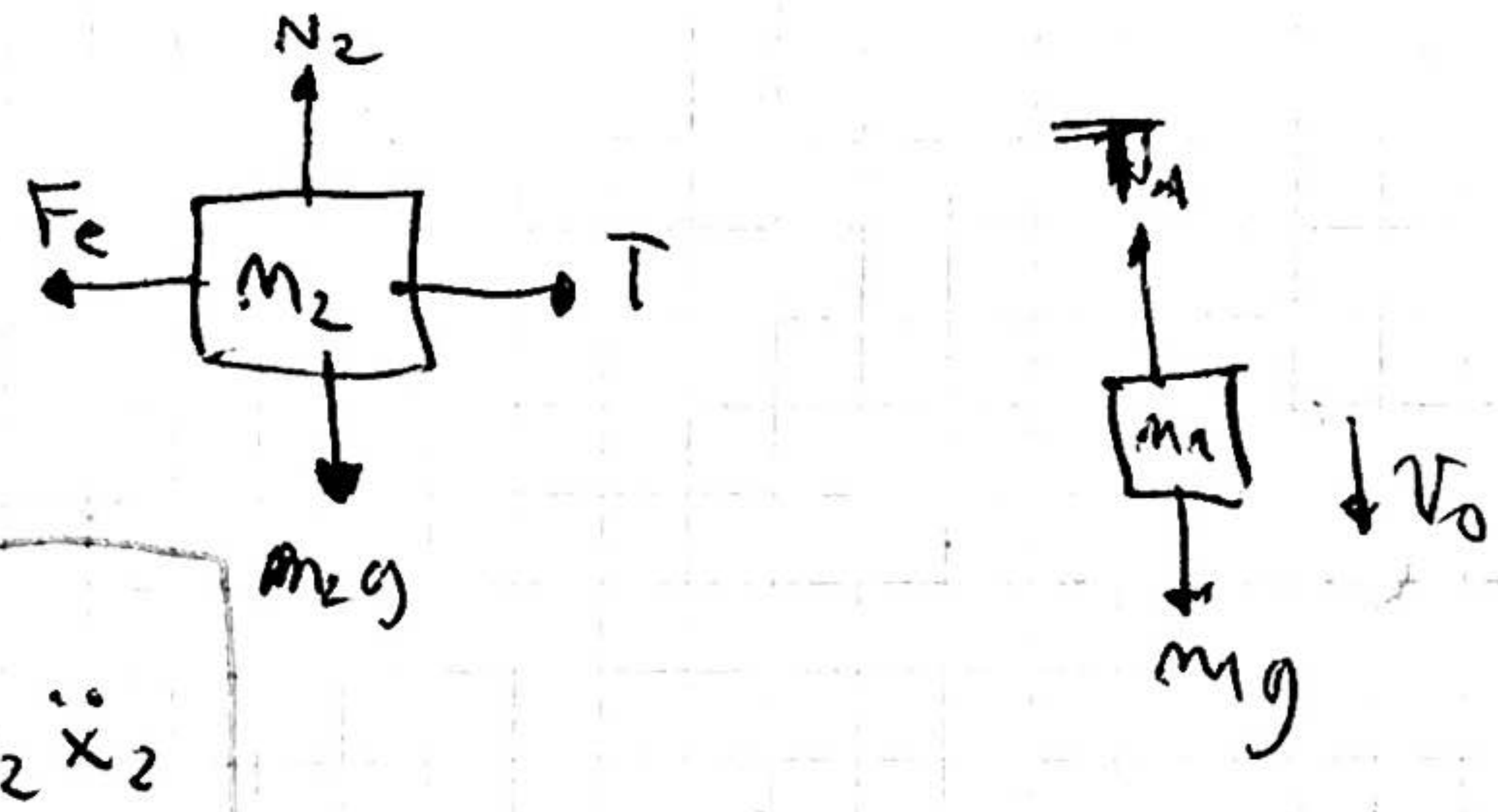


Datos

m_1, m_2, k, l_0 Eq

luego se da v_0 a m_1 hacia abajo,

sin roz



a) Ec Newton y vínculo

$m_1)$

$$(\hat{x})\ T - k(x_2 - l_0) = m_2 \ddot{x}_2$$

$$(\hat{y})\ N_2 - m_2 g = m_2 \ddot{y}_2$$

Vínculo $\ddot{y}_2 = 0$

$m_2)$

$$(\hat{y})\ T - m_1 g = m_1 \cdot \ddot{y}_1$$

$$l = (x_p - x_2) + (y_p - y_1)$$

$$0 = -\ddot{x}_2 - \ddot{y}_1$$

$$\ddot{x}_2 = -\ddot{y}_1$$

b) $x_2(t) = ?$

$$x_2(t) = x_H(t) + x_p(t)$$

$$x_2(t) = A \sin(\omega_0 t + \varphi) + x_p(t)$$

$$\cancel{T - k(x_2 - l_0)} - T + m_1 g = m_2 \ddot{x}_2 + m_1 \ddot{x}_2$$

$$-k(x_2 - l_0) + m_1 g = \ddot{x}_2 (m_1 + m_2)$$

$$\ddot{x}_2 + \frac{k x_2}{m_1 + m_2} = \frac{k l_0}{m_1 + m_2} + \frac{m_1 g}{m_1 + m_2}$$

si $x_p = C$

$$\omega_0^2 = \frac{k}{m_1 + m_2}$$

$$\ddot{C}_2 + \frac{kC}{m_1 + m_2} = \frac{k l_0}{m_1 + m_2} + \frac{m_1 g}{m_1 + m_2}$$

$$\ddot{\theta} + \omega_0^2 C = \omega_0^2 l_0 + \frac{m_1 g}{m_1+m_2}$$

$$C = l_0 - \frac{m_1 g}{(m_1+m_2)\omega_0^2}$$

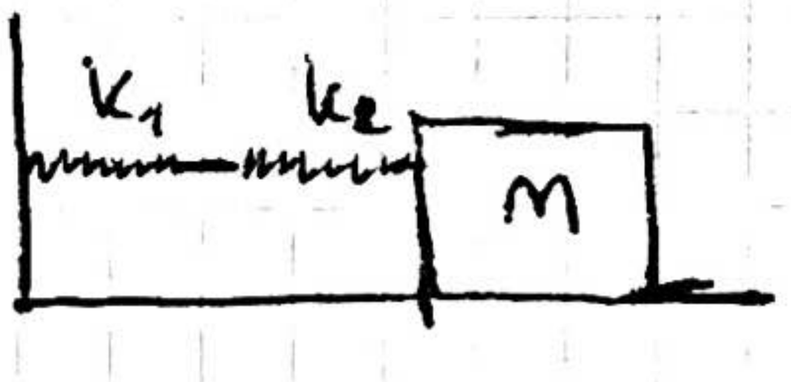
$$X_2(t) = A\sin(\omega_0 t + \varphi) + l_0 + \frac{m_1 g}{(m_1+m_2)\omega_0^2}$$

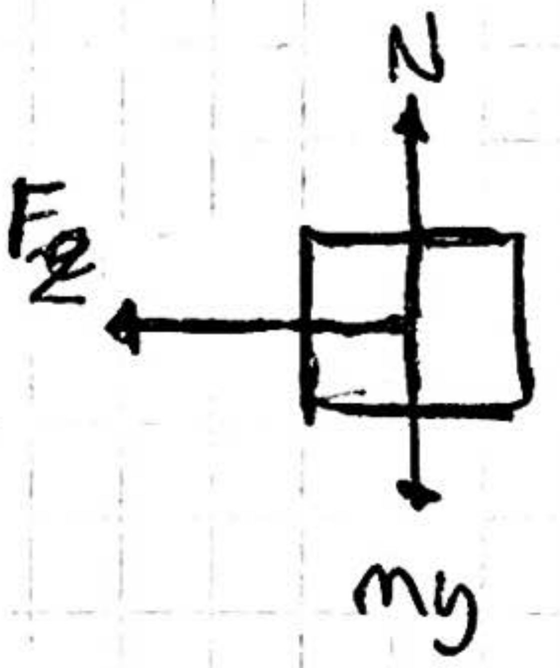
$$\boxed{X_2(t) = A\sin(\omega_0 t + \varphi) + l_0 + \frac{m_1 g}{K}}$$

Dados,

k_1, k_2, m

Demostrar que en a $f = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_1+k_2)m}}$ y en b,c $= \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$

a) 



$$T = \frac{2\pi}{\dot{\theta}}$$

$$-k_2(\Delta x_2 - l_{02}) = m\ddot{x}$$

$x_1 - l_{01} = \Delta x_1 \quad x_2 - l_{02} = \Delta x_2$

$$-k_1(x_1 - l_{01}) + k_2(x_2 - l_{02}) = 0$$

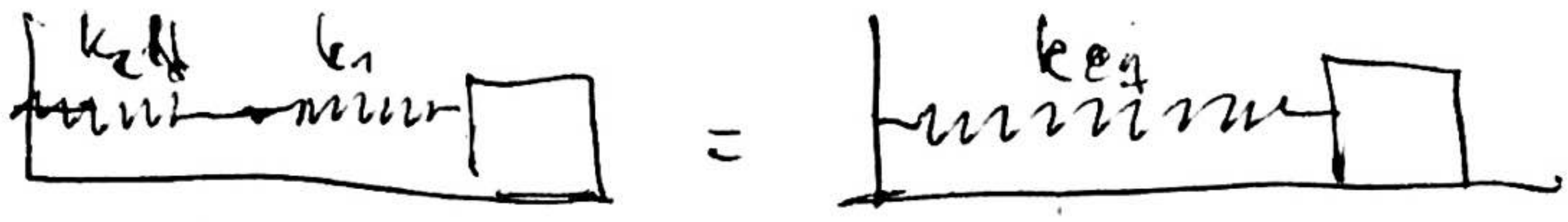
$$k_1(x_1 - l_{01}) = k_2(x_2 - l_{02})$$

$$\frac{k_1(x_1 - l_{01})}{k_2} = (x_2 - l_{02}) \Rightarrow$$

$$\Rightarrow -k_2 \Delta x_2 = m\ddot{x}$$

$$k_1 \Delta x_1 = k_2 \Delta x_2$$

Planteo que



$-k_1(\Delta x_1) = m\ddot{x}$

$-k_{eq}(\Delta x_{eq}) = m\ddot{x}$

$-k_{eq}(\Delta x_1 + \Delta x_2) = m\ddot{x}$

$k_1 \Delta x_1 = k_2 \Delta x_2$

$$-k_1(\Delta x_1) = -k_{eq}(\Delta x_1 + \Delta x_2) = m\ddot{x}$$

$\frac{k_1 \Delta x_1}{k_2} = \Delta x_2$

$$-k_1 \Delta x_1 = -k_{eq}\left(\Delta x_1 + \frac{k_1 \Delta x_1}{k_2}\right)$$

$$+k_1 \Delta x_1 = +k_{eq}\left(\frac{k_2 \Delta x_1 + k_1 \Delta x_1}{k_2}\right)$$

$$k_1 \cancel{\Delta x_1} = k_{eq} \cancel{\Delta x_1}\left(\frac{k_2 + k_1}{k_2}\right)$$

$$\frac{k_1 . k_2}{k_2 + k_1} = k_{eq}$$

$$\Rightarrow -\frac{k_1 . k_2}{k_2 + k_1}\left(\cancel{\Delta x_1 + \Delta x_2}\right) = m\ddot{x}$$

(Δx)

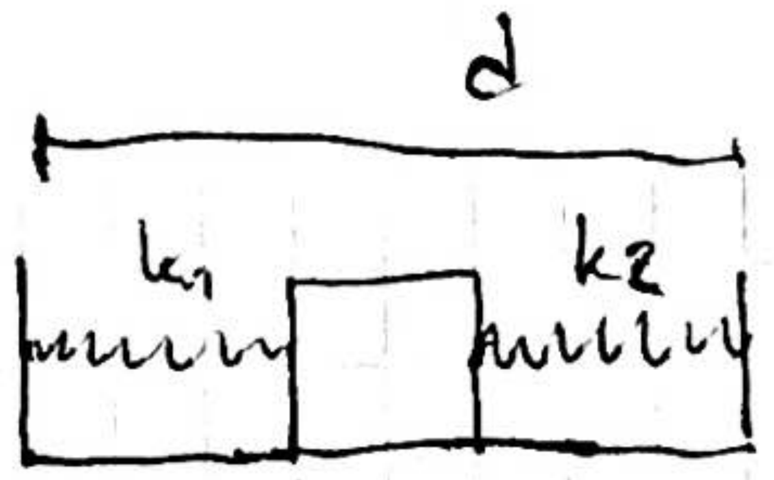
$$\ddot{x} = -\underbrace{\frac{k_1 k_2}{(k_2 + k_1)m}}_{= \omega_0^2}(\Delta x)$$

$$\omega_0^2 = \dot{\theta} = \frac{k_1 k_2}{(k_2 + k_1)m}$$

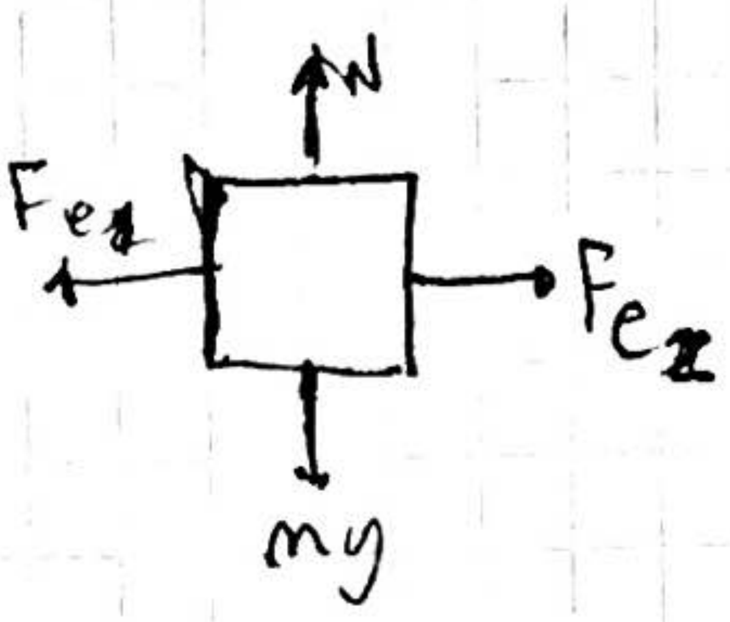
$$T = \frac{2\pi}{\omega_0} = \frac{2\pi\sqrt{(k_2 + k_1)m}}{\sqrt{k_1 k_2}}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{(k_2 + k_1)m}}}$$

b)



Probar que vale $f = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}$



$$-k_1 \Delta x_1 + k_2 \Delta x_2 = m\ddot{x}$$

$$\frac{-k_1(x-l_{01}) + k_2(d-x-l_{02})}{m} = \ddot{x}$$

$$\frac{-k_1 x - k_2 x}{m} + \frac{l_{01}k_1}{m} - \frac{l_{02}k_2}{m} + \frac{k_2 d}{m} = \ddot{x}$$

$$\ddot{x} + \underbrace{\frac{(k_1+k_2)}{m}}_{\omega_0^2} x = \frac{k_1 l_{01} - k_2 l_{02} + k_2 d}{m}$$

$$\omega_0 = \sqrt{\frac{k_1+k_2}{m}}$$

$$\boxed{f = \frac{\dot{\theta}}{2\pi} = \frac{\omega_0}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}}$$

c) $-k_1(x-l_{01}) - k_2(x-l_{02}) = m\ddot{x}$

$$\frac{-k_1 x}{m} - \frac{k_2 x}{m} + \frac{k_1 l_{01} + k_2 l_{02}}{m} = \ddot{x}$$

$$\ddot{x} + \underbrace{\frac{(k_1+k_2)}{m}}_{\omega_0^2} x = \frac{k_1 l_{01} + k_2 l_{02}}{m}$$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{k_1+k_2}{m}}}$$

iii) Para eq $\Rightarrow$ $\ddot{x}(x_{eq}) = 0$

Para a) $\frac{-k_1 x + k_1 l_{01}}{m} = \ddot{x}$

$$\ddot{x} + \frac{k_1 x}{m} = \frac{k_1 l_{01}}{m}$$

$$\frac{k_1 x_{eq}}{\cancel{m}} = \frac{k_1 l_{01}}{\cancel{m}}$$

$$\boxed{x_{eq} = l_{01}}$$

Para b)

$$\ddot{x}_1 + \frac{(k_1+k_2)x}{m} = \frac{k_1 l_{01} - k_2 l_{02} + k_2 d}{m}$$

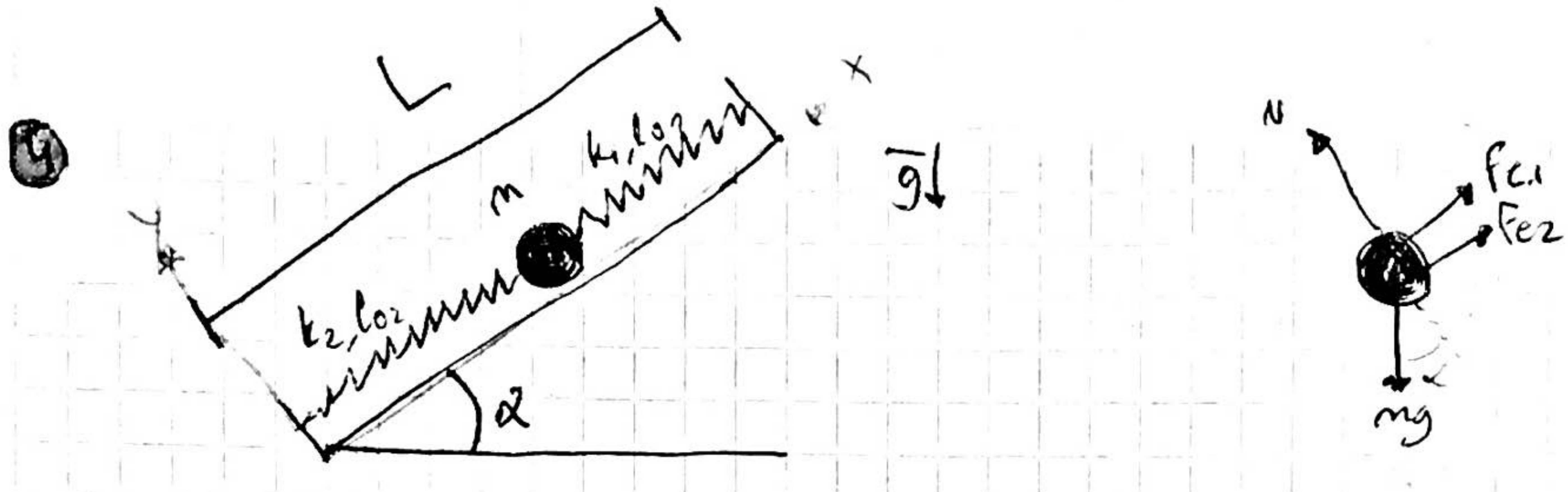
$$0 + \frac{(k_1+k_2)x_{eq}}{\cancel{m}} = \frac{k_1 l_{01} - k_2 l_{02} + k_2 d}{\cancel{m}}$$

$$\boxed{x_{eq} = \frac{k_1 l_{01} - k_2 l_{02} + k_2 d}{k_1+k_2}}$$

Para c)

$$\boxed{x_{eq} = \frac{k_1 l_{01} + k_2 l_{02}}{k_1+k_2}}$$

4



a) Ec Newton y mov

$$(\hat{x})\quad -mg\cdot\operatorname{sen}\alpha - k_2(x-l_{o2}) + k_1(L-x-l_{o1}) = m\ddot{x}$$

$$(\hat{y})\quad N = mg\cdot\cos\alpha$$

$$\ddot{x} = \frac{-mg\operatorname{sen}\alpha - k_2x + k_2l_{o2} + k_1L - k_1x - k_1l_{o1}}{m}$$

$$\ddot{x} + \frac{k_2x}{m} + \frac{k_1x}{m} = -g\operatorname{sen}\alpha + \frac{k_2l_{o2} + k_1L - k_1l_{o1}}{m}$$

$$\boxed{\ddot{x} + \frac{(k_1+k_2)x}{m} = -g\operatorname{sen}\alpha + \frac{k_2l_{o2} + k_1L - k_1l_{o1}}{m}}$$

b) Halle x_{eq} y det si estable o inestable

$$\ddot{x}(x_{eq}) = 0$$

$$\Rightarrow 0 + \frac{(k_1+k_2)}{m}x_{eq} = -g\operatorname{sen}\alpha + \frac{k_2l_{o2} + k_1L - k_1l_{o1}}{m}$$

$$x_{eq} = \frac{-g\operatorname{sen}\alpha\, m}{k_1+k_2} + \frac{k_2l_{o2} + k_1L - k_1l_{o1}}{k_1+k_2}$$

$$\boxed{x_{eq} = \frac{-mg\operatorname{sen}\alpha + k_2l_{o2} + k_1L - k_1l_{o1}}{k_1+k_2}}$$

Para equilibrio estable $\frac{dF}{dx} < 0$

$$\Rightarrow \left.\frac{d\ddot{x}}{dx}\right|_{x_{eq}} = -\frac{(k_1+k_2)}{m}$$ signo negativo $\therefore$ estable

c) $x(t) = A \cdot \sin(\omega_0 t + \varphi) + x_p$

$x(0) = A \cdot \sin(\omega_0 \cdot 0 + \varphi) + \cancel{x_p} = x_{eq}$ $\qquad A\sin(\omega_0 t + \varphi) = 0$

$\dot{x}(0) = A\cos(\omega_0 \cdot 0 + \varphi)\,\omega_0 = v_0$ $\qquad \sin\varphi = 0$

$\varphi = 0$

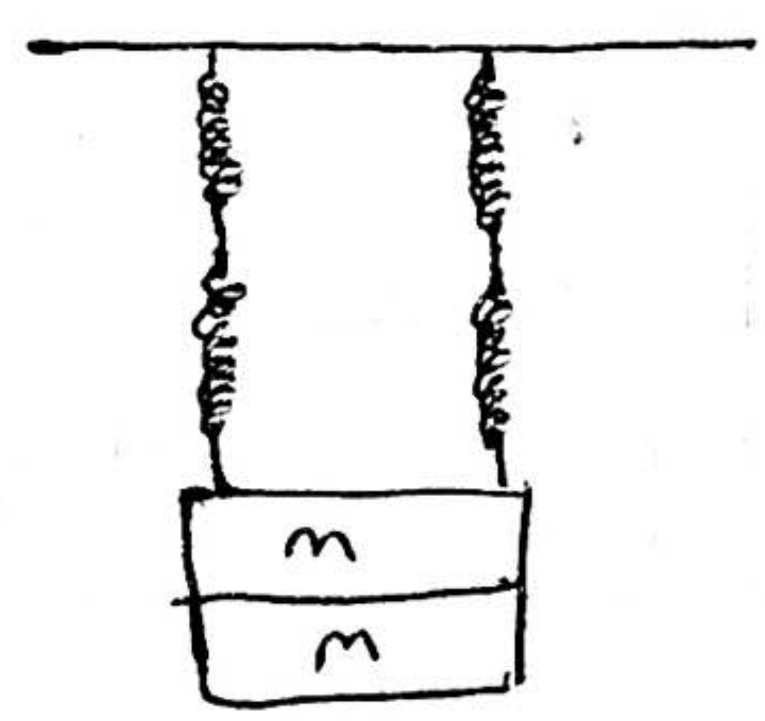
$\Rightarrow A\cos 0\; \omega_0 = v_0$

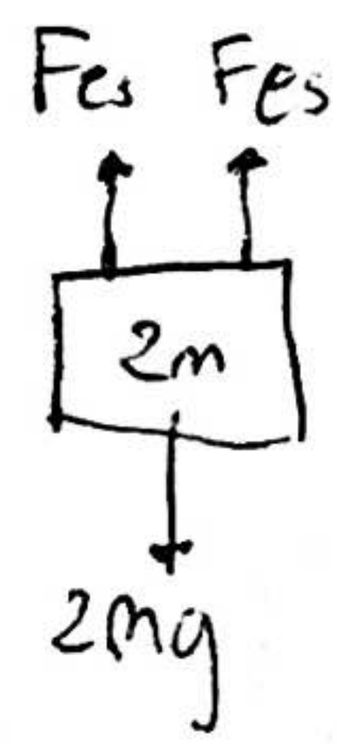
$A = \frac{v_0}{\omega_0}$

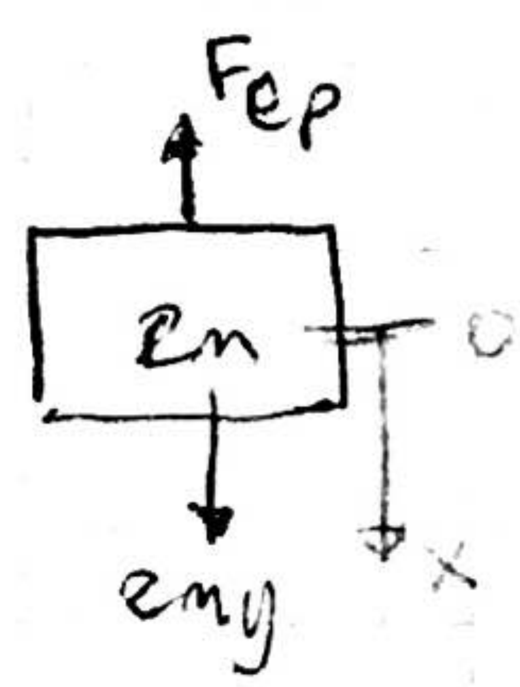
$$\Rightarrow \boxed{x(t) = \frac{v_0}{\omega_0}\cdot \sin(\omega_0 t) + x_{eq}}$$

5) Datos l_0, m

$d = x_{eq}$







En serie $l_0 = 2l_0$ $\qquad$ En paralelo $l_0 = 2l_0$

$\frac{k^2}{2k} = \frac{k}{2}$ $\qquad \frac{k}{2} + \frac{k}{2} = k$

Cada resorte $\qquad$ En equilibrio

$(\hat{x}) \quad -k(d - 2l_0) + 2mg = 0$

$k(d - 2l_0) = 2mg$

$$\boxed{k = \frac{2mg}{d - 2l_0}}$$

b) Ec mov. T oscilación y nuevo eq si ahora mg

$$x_0 = d$$

$$-k(x - 2l_0) + mg = m\ddot{x}$$

$$\ddot{x} + \frac{kx}{m} = \frac{k2l_0}{m} + g \qquad \frac{k}{m} = \omega_0^2$$

$$\boxed{\ddot{x} + \omega_0^2 x = \omega_0^2 2l_0 + g}$$

$$\boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

Nuevo eq

$$\ddot{x}(x_{eq}') = 0$$

$$\Rightarrow 0 + \omega_0^2 x_{eq}' = \omega_0^2 2l_0 + g$$

$$x_{eq}' = 2l_0 + \frac{g}{\omega_0^2}$$

$$\boxed{x_{eq}' = 2l_0 + \frac{gm}{k}}$$

c) $x(t) = ?$

$$x(t) = A\sin(\omega_0 t + \varphi) + C$$

$$x(t) = A\sin(\omega_0 t + \varphi) + 2l_0 + \frac{mg}{k}$$

$$x(0) = d = A\sin\varphi + 2l_0 + \frac{mg}{k}$$

$$\dot{x}(0) = 0 = A\cos\varphi\,\omega_0$$

$$\cos\varphi = 0$$

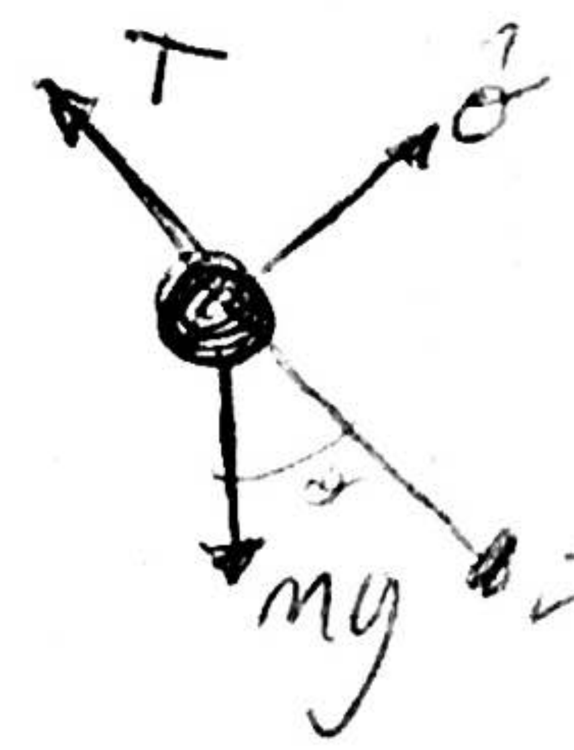
$$\varphi = \frac{\pi}{2}$$

$$\Rightarrow d = A\sin\frac{\pi}{2} + 2l_0 + \frac{mg}{k}$$

$$d - 2l_0 - \frac{mg}{k} = A$$

$$\boxed{x(t) = \left(d - 2l_0 - \frac{mg}{k}\right)\sin\left(\omega_0 t + \frac{\pi}{2}\right) + 2l_0 + \frac{mg}{k}}$$

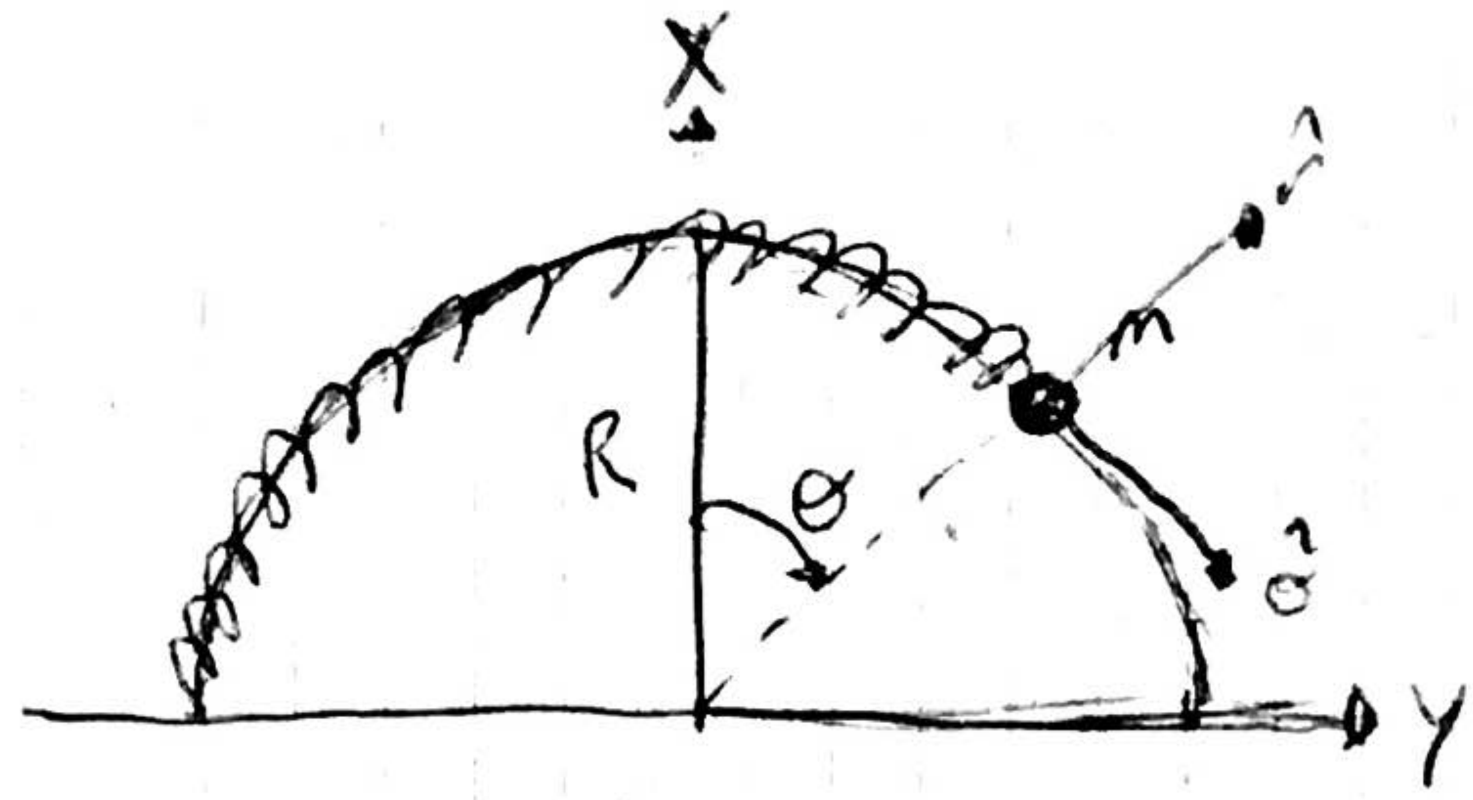
6



a) $-T + mg\cos\theta = m(-R\dot{\theta}^2)$ $(\hat{r})$

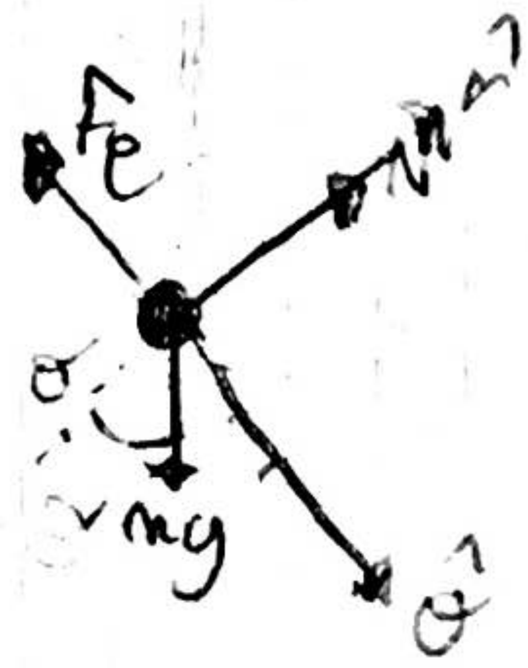
$-mg\sin\theta = mR\ddot{\theta}$ $\qquad r = R$

b) Bajo que aproximación el movimiento es armónico? ¿Que periodo tiene?



Datos

$m, k, R, l_0 = \frac{\pi}{2}R$



a) Halle la ecuación de mov

$(\hat{r})\quad N - mg\cos\theta = -mR\dot{\theta}^2$

$(\hat{\theta})\quad +F_e + mg\sin\theta = mR\ddot{\theta}$

$N - mg\cos\theta = -mR\dot{\theta}^2$

$-k\left(\frac{\pi}{2}R + \theta R - \frac{\pi}{2}R\right) = F_e$

$-k\theta R + mg\sin\theta = mR\ddot{\theta}$

$$\boxed{\ddot{\theta} = -\frac{k\theta}{m} + \frac{g\sin\theta}{R}}$$

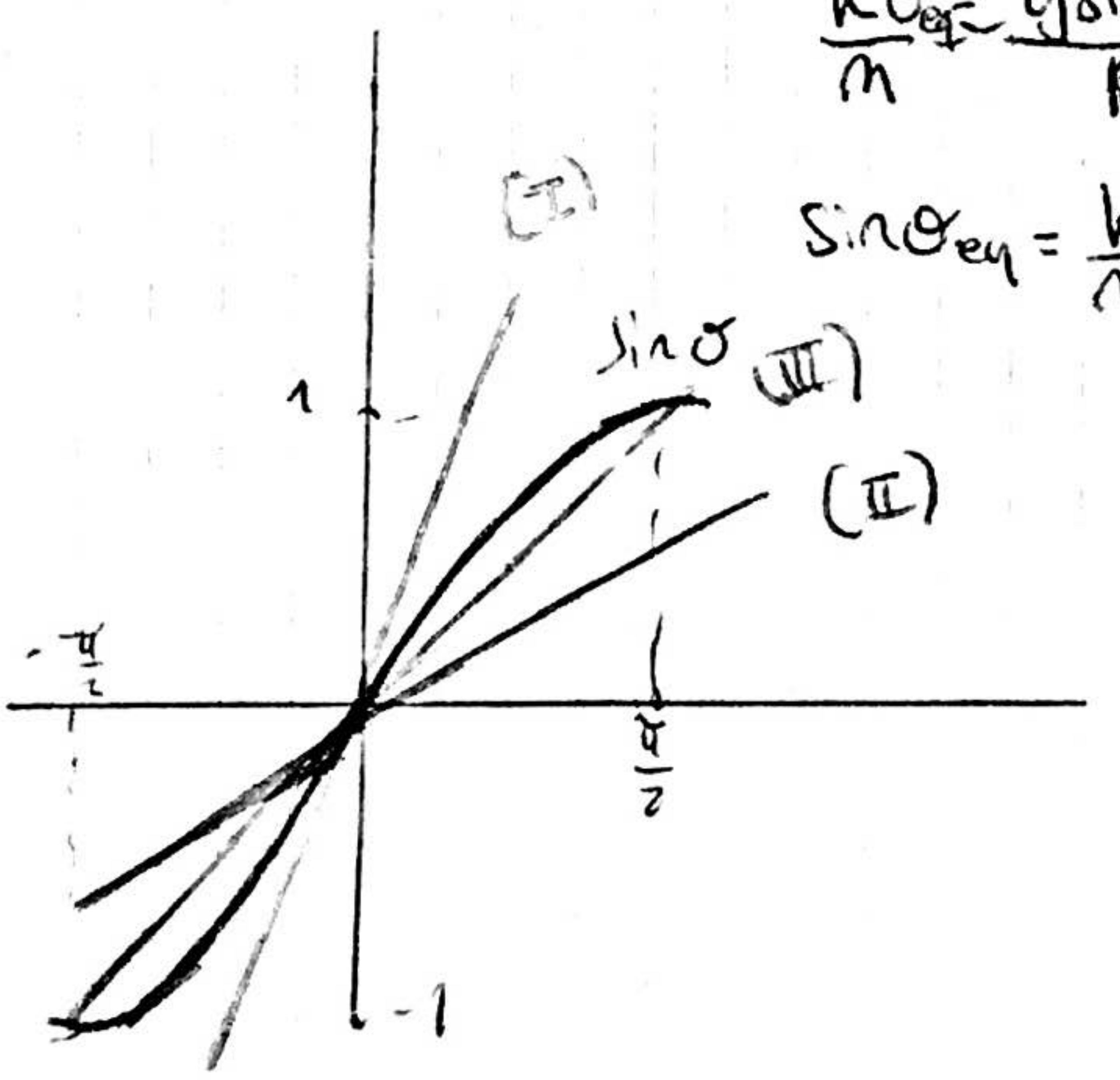
c)

b) Posiciones de equilibrio y (diga cuando es estable)

$\ddot{\theta}(\theta_{eq}) = 0 \Rightarrow 0 = \frac{k}{m}\theta_{eq} + \frac{g\sin\theta_{eq}}{R}$

$\frac{k\theta_{eq}}{m} = \frac{g\sin\theta_{eq}}{R}$

$\sin\theta_{eq} = \frac{kR}{mg}\theta_{eq}$



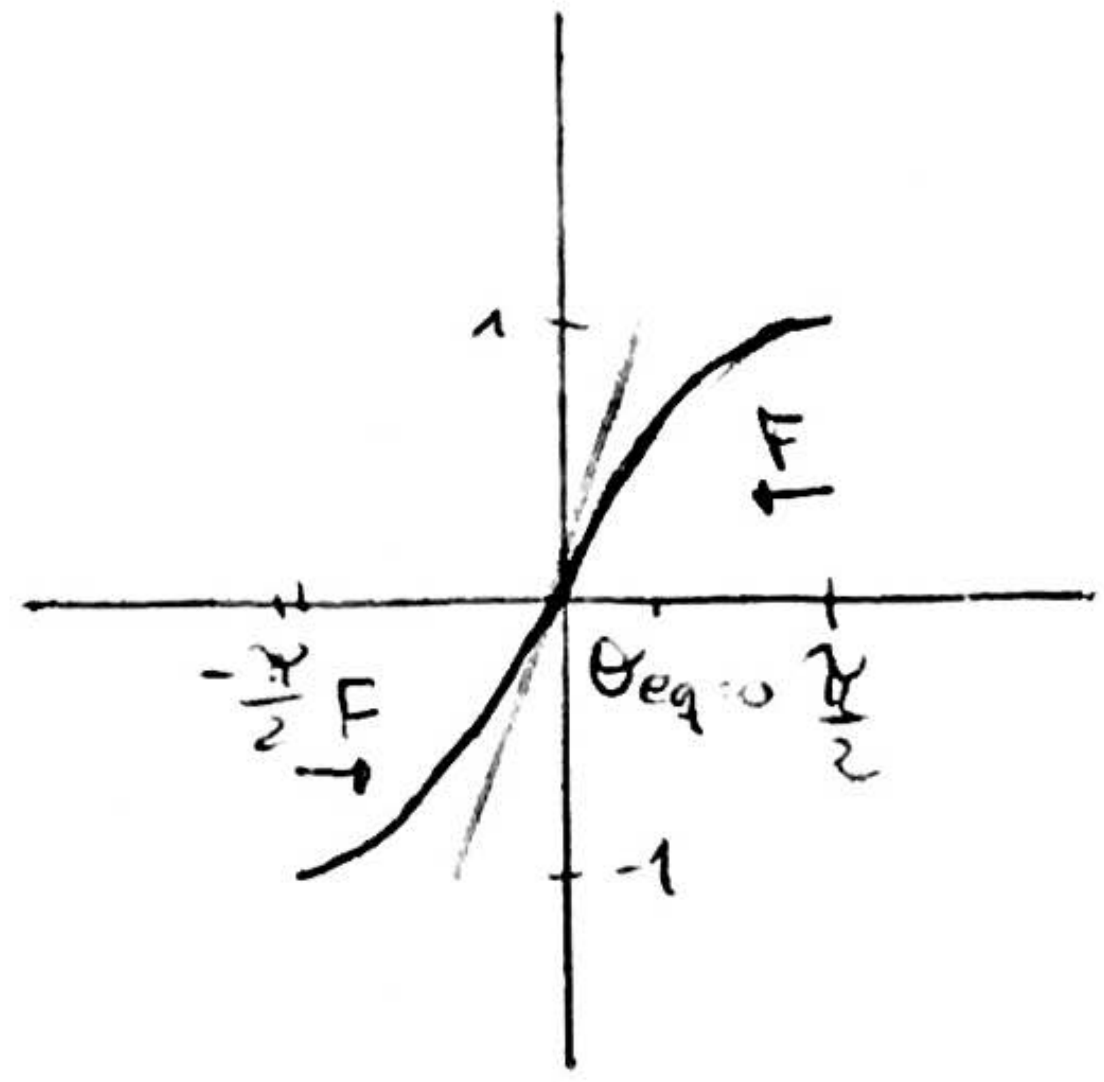
(I) si $\frac{k_g}{R_m} \geq 1$

$\theta_{eq} = 0$

$\sin\theta_{>0} - \frac{k_g}{R_m}\theta_{>0} = \ominus$

$\sin\theta_{<0} - \frac{k_g}{R_m}\theta_{<0} = \oplus$

$\therefore$ $\theta_{eq} = 0$ estable



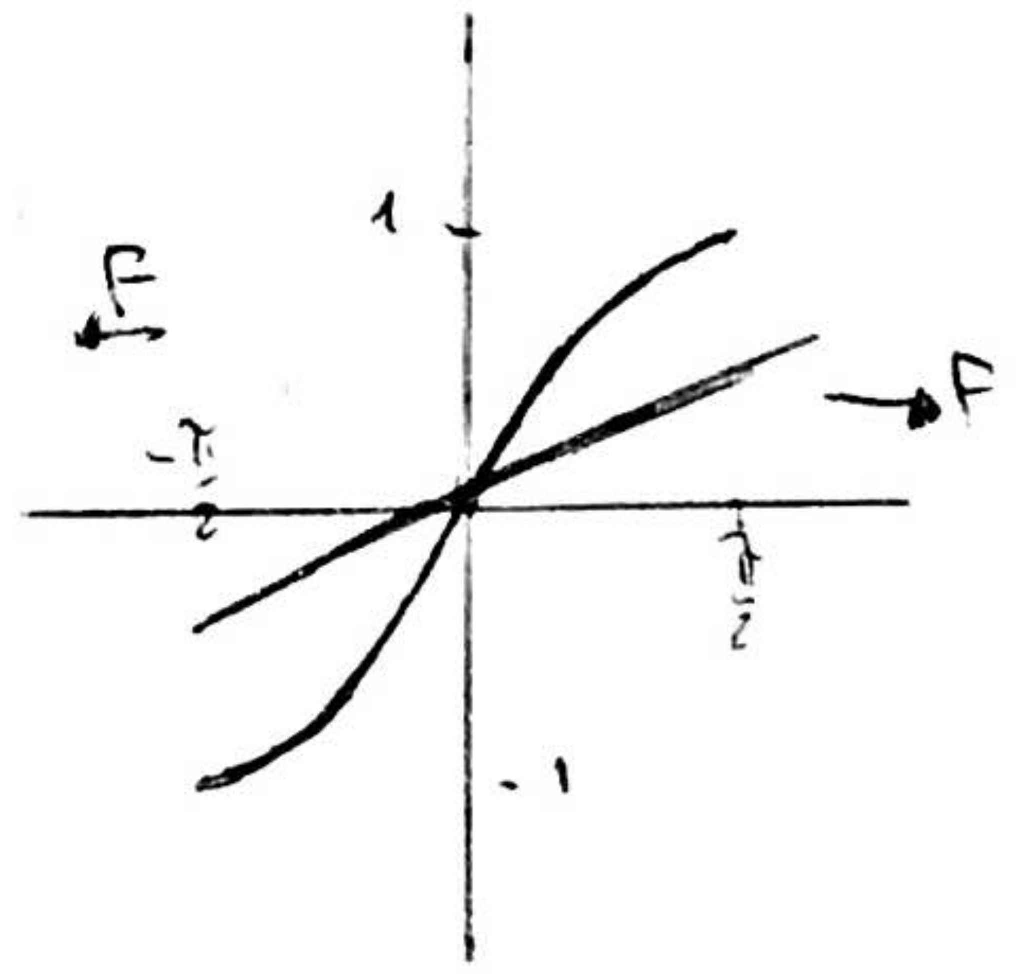
(II)

$\frac{k_g}{R_m} < \frac{2}{\pi}$

$\theta_{eq} = 0$ e inestable

$1 = m\frac{\pi}{2}$

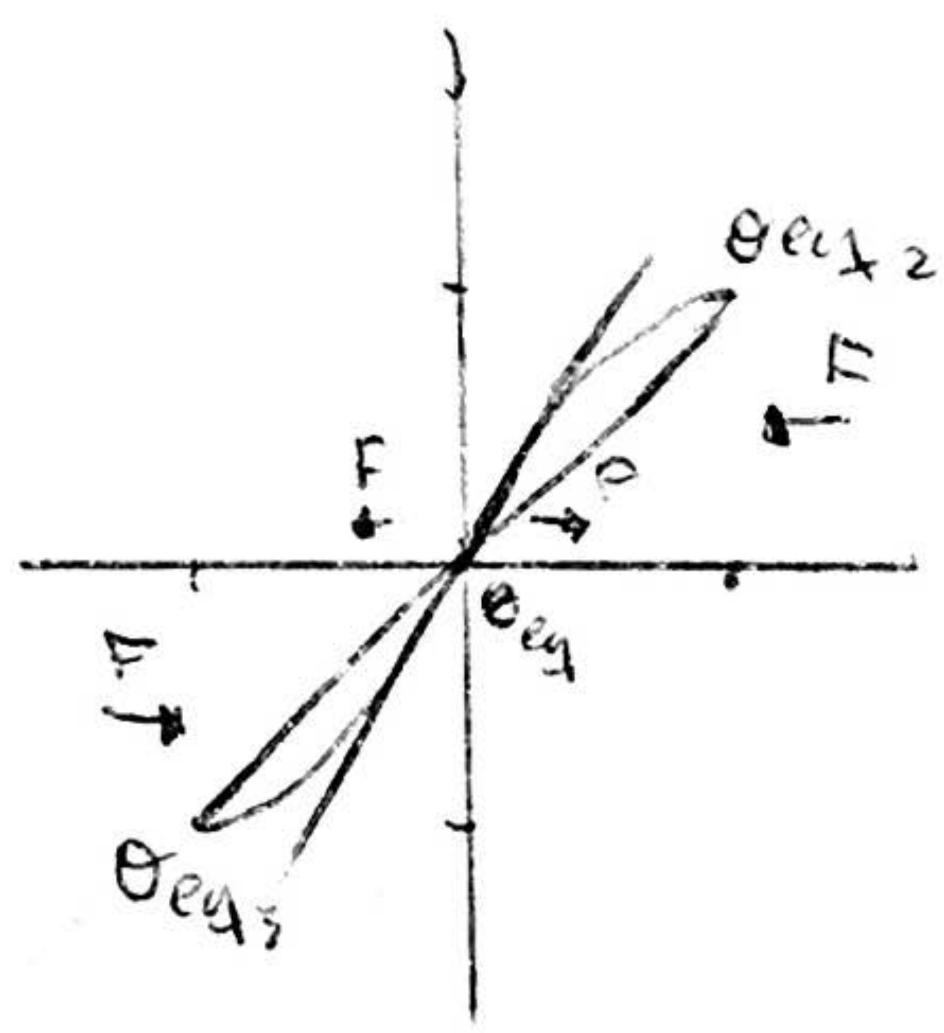
$m = \frac{2}{\pi}$



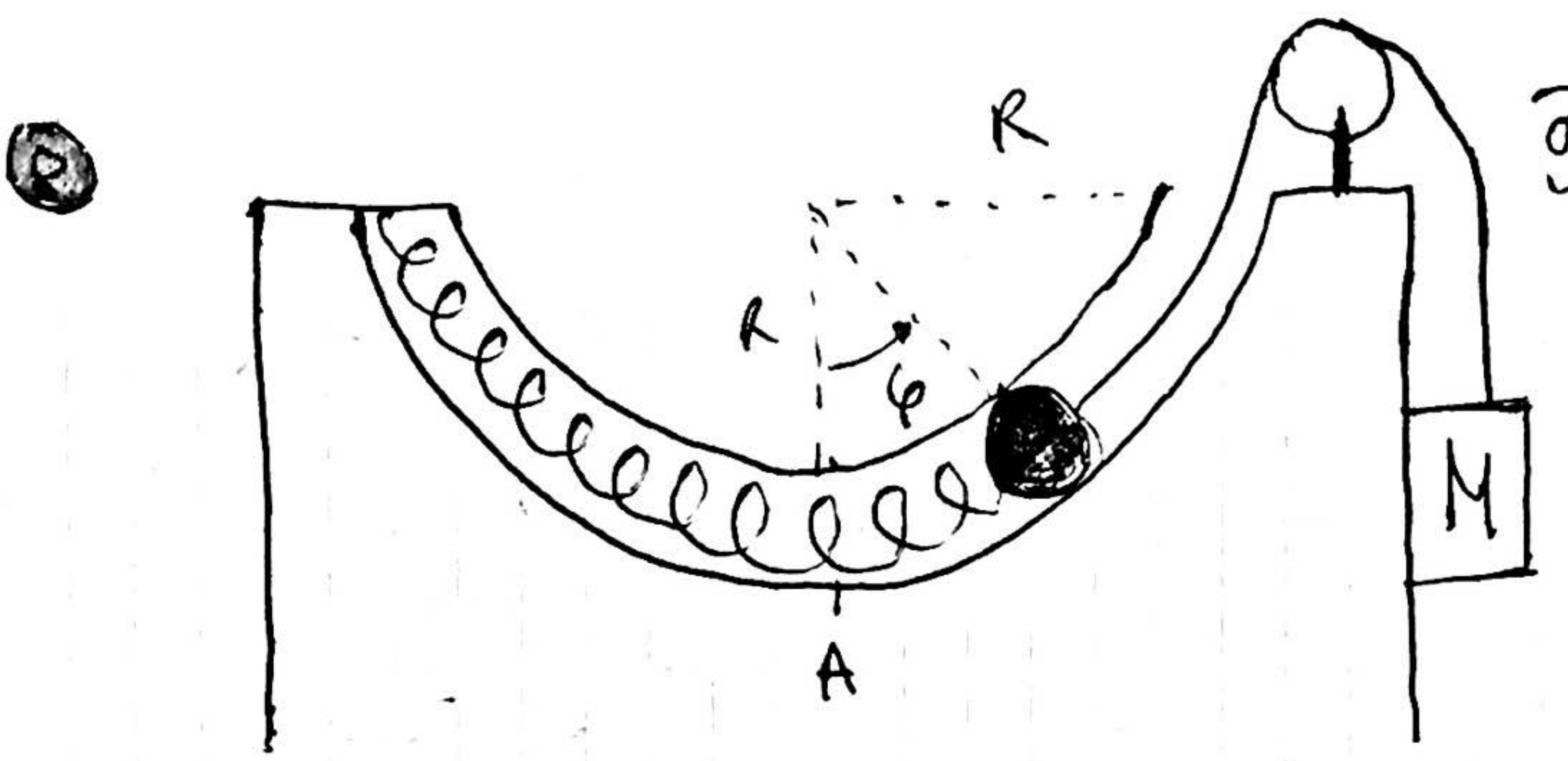
(III) $\frac{2}{\pi} \leq \frac{k_g}{R_m} < 1$

$\theta_{eq_1} = 0$ inestable

$\theta_{eq_2} = -\theta_{eq_3}$ estables



9b

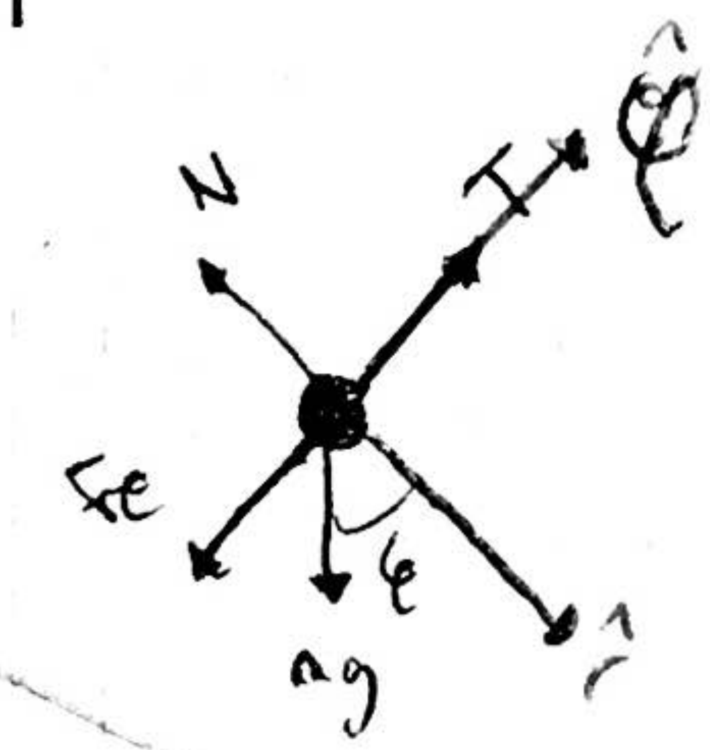


<u>datos</u>

$m,\ R,\ k,\ l_0 = \frac{\pi}{2}R,\ M$

$A(\varphi = 0) \to v_0 = v_0$

a) Ecuaciones de Newton y de mov



Bola

$(\hat{\varphi})\ T - k\left(\left(\frac{\pi}{2}+\varphi\right)R - \frac{\pi}{2}R\right) - mg\,\text{sen}\,\varphi = mR\ddot{\varphi}$

$(\hat{r})\ -N + mg\cos\varphi = -mR\dot{\varphi}^2$

Masa

$(\hat{x})\ -T + Mg = M\ddot{x}_M$

$l = \varphi_p - \varphi = x_p + x_M$

$\ddot{\varphi} = \ddot{x}_M$

$r = R$

$-k\varphi R - mg\,\text{sen}\,\varphi + Mg = \ddot{\varphi}\,(mR + M)$

$\ddot{\varphi} = \dfrac{-k\varphi R - mg\,\text{sen}\,\varphi + Mg}{mR + M}$ Ec mov

b) Halle gráficamente la o las posiciones de Eq. Ver si son estables o inestables

Para eq $\ddot{\varphi}(\varphi_{eq}) = 0$

$\Rightarrow mg\sin\varphi = -k\varphi R + Mg$

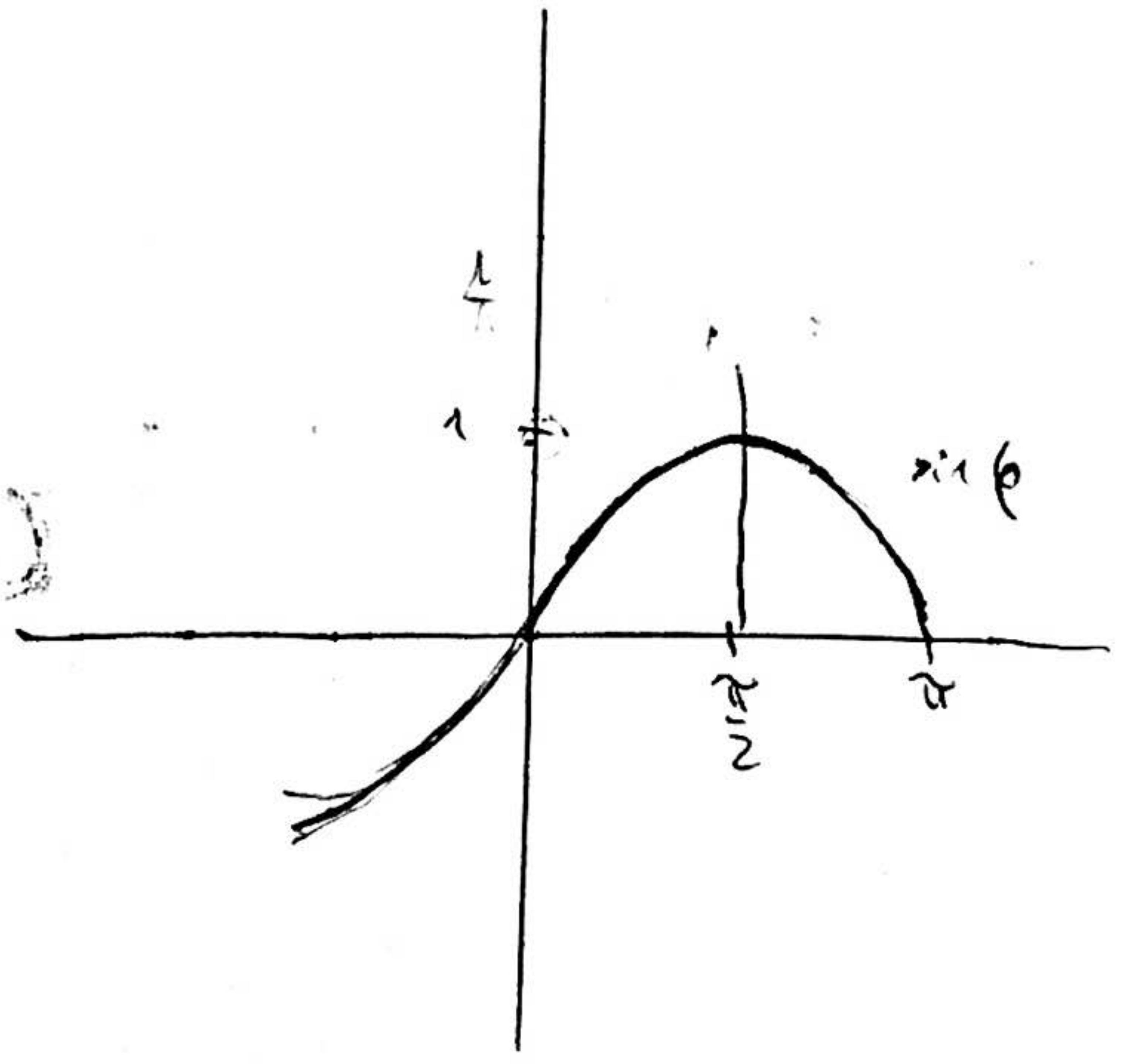
$\sin\varphi = -\underbrace{\frac{k\varphi R}{mg}}_{\rho} + \frac{M}{m}$

$\sin\varphi = -\rho\varphi + \frac{M}{m}$

$\sin \varphi = -\beta \varphi + \frac{M}{m}$

$-\beta \varphi + \frac{M}{m}$

Recta decreciente con o.o $\frac{M}{m}$



c) $N - mg\cos\varphi = mR\dot{\varphi}^2$

$$\ddot{\varphi} = \frac{d\dot{\varphi}}{dt} = \frac{d\dot{\varphi}}{d\varphi}\frac{d\varphi}{dt} = \frac{d\dot{\varphi}}{d\varphi}\cdot\dot{\varphi} = \frac{-k\varphi R - mg\sin\varphi + Mg}{mR + M}$$

$$\int_{v_0/R}^{\dot{\varphi}(\varphi)} \dot{\varphi}\, d\dot{\varphi} = \int_0^{\varphi} \frac{-k\varphi R - mg\sin\varphi + Mg}{mR+M}\, d\varphi$$

$$\int_{v_0/R}^{\dot{\varphi}(\varphi)} \dot{\varphi}\, d\dot{\varphi} = \frac{1}{mR+M}\left(\int_0^{\varphi} -k\varphi R - mg\sin\varphi + Mg\, d\varphi\right)$$

$$\left.\frac{\dot{\varphi}^2}{2}\right|_{v_0}^{\dot{\varphi}(\varphi)} = \frac{1}{mR+M}\left(\left.-\frac{kR\varphi^2}{2} + mg\cos\varphi + Mg\varphi\right|_0^{\varphi}\right)$$

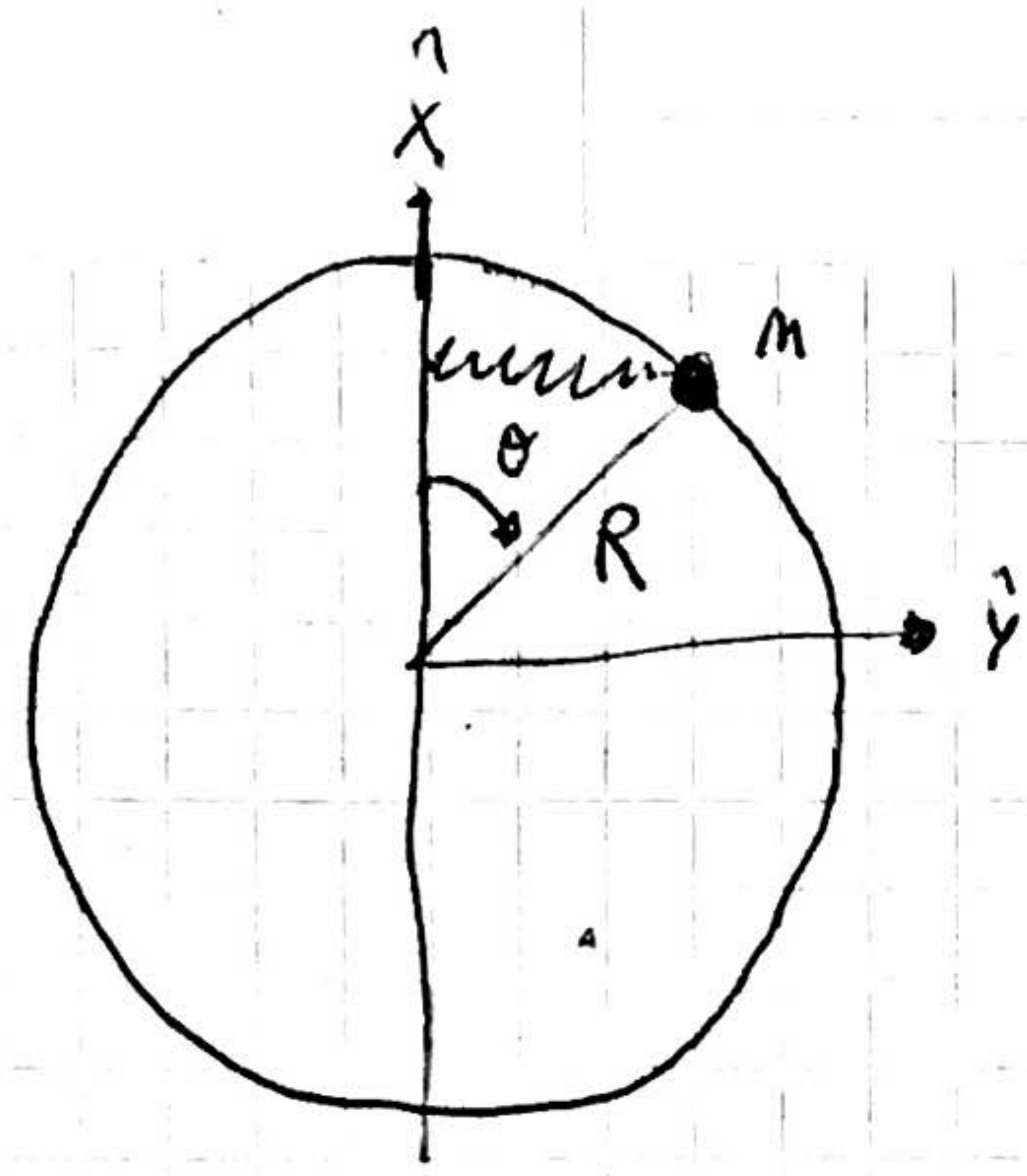
$$\frac{\dot{\varphi}^2}{2} - \frac{v_0^2}{2R^2} = \frac{1}{mR+M}\left(-\frac{kR\varphi^2}{2} + mg\cos\varphi + Mg\varphi - mg\cos\varphi\right)$$

$$\frac{\dot{\varphi}^2}{2} = \frac{-kR\varphi^2}{2(mR+M)} + \frac{Mg\varphi}{mR+M} + \frac{v_0^2}{2R^2} + \frac{mg\cos\varphi - mg}{mR+M}$$

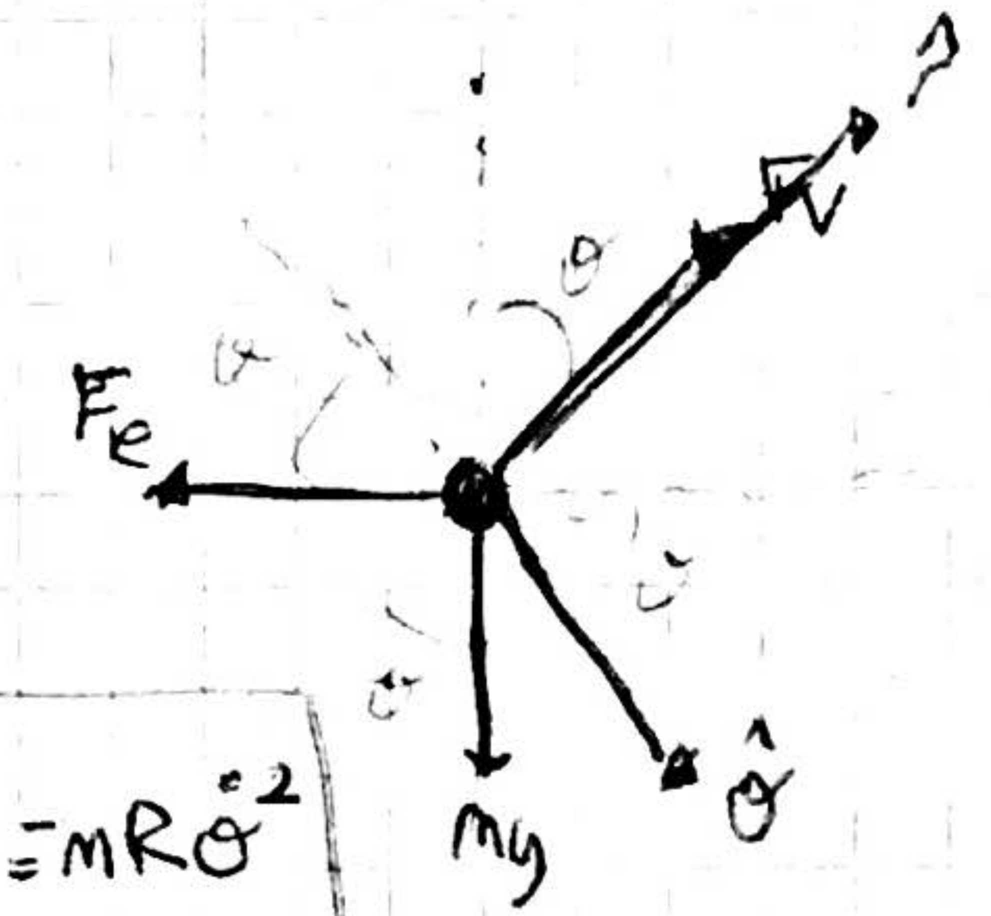
$$\dot{\varphi}^2 = \frac{-kR\varphi^2 + 2Mg\varphi}{mR+M} + \frac{v_0^2}{R^2} + \frac{2mg\cos\varphi - 2mg}{mR+M}$$

$$N = mR\dot{\varphi}^2 + mg\cos\varphi$$

$$\boxed{N = mR\left(\frac{-kR\varphi^2 + 2Mg\varphi}{mR+M} + \frac{v_0^2}{R^2} + \frac{2mg\cos\varphi - 2mg}{mR+M}\right) + mg\cos\varphi}$$



Datos
R k, $l_0 = 0$



a) Ec de Newton para m

$$\boxed{\begin{aligned}(\hat{r})\ & F_V - mg\cdot\cos\theta - k(R\cdot\sin\theta)\sin\theta = mR\dot{\theta}^2 \\ (\hat{\theta})\ & mg\sin\theta - k(R\sin\theta)\cdot\cos\theta = mR\ddot{\theta}\end{aligned}}$$

b) Si inicialmente $\theta_{(0)} = \frac{\pi}{2}$ $v_{(0)} = 0$, Halle $F_V(\theta)$

$$\ddot{\theta} = \frac{mg\sin\theta - kR\sin\theta\cos\theta}{mR} = \frac{d\dot{\theta}}{dt} = \frac{d\dot{\theta}}{d\theta}\cdot\frac{d\theta}{dt} = \frac{d\dot{\theta}}{d\theta}\cdot\dot{\theta}$$

$$\int_0^{\dot{\theta}(\theta)} d\dot{\theta}\,\dot{\theta} = \int_{\frac{\pi}{2}}^{\theta} \frac{mg\sin\theta - kR\sin\theta\cos\theta}{mR}\,d\theta$$

$$\frac{\dot{\theta}^2}{2}\Bigg|_0^{\dot{\theta}(\theta)} = \frac{-mg\cos\theta}{mR}\Bigg|_{\frac{\pi}{2}}^{\theta} + \int_{\frac{\pi}{2}}^{\theta}\frac{-kR\sin\theta\cos\theta\,d\theta}{mR}$$

CA: $\displaystyle\int_{\frac{\pi}{2}}^{\theta}\frac{-kR\sin\theta\cos\theta\,d\theta}{mR}$

$z = \sin\theta$
$dz = \cos\theta\,d\theta$

$$\int^{\theta}\frac{-kRz\,dz}{mR} = \frac{-kRz^2}{2mR}\Bigg|_1^{\sin\theta}$$

$$\frac{\dot{\theta}^2}{2} - \frac{\dot{\theta}_0^2}{2} = -\frac{mg\cos\theta}{mR} + \frac{mg\cos\theta_0}{mR} - \frac{kR\sin^2\theta}{2mR} + \frac{kr}{2mR}$$

$=0$

$$\frac{\dot{\theta}^2}{\cancel{2}} = \frac{-2mg\cos\theta - kR\sin^2\theta + kR}{\cancel{2}mR}$$

$$\dot{\theta}^2 = -2mg\cos\theta - kR\sin^2\theta + kR$$

$$\boxed{F_v = -mR\left(-2mg\cos\theta - kR\sin^2\theta + kR\right) + mg\cos\theta + kR\sin^2\theta}$$

c) Halle posiciones de eq y analice si estables o inestables

Para eq $\ddot{\theta}_{(\theta_{eq})} = 0$

$$\Rightarrow \ddot{\theta} = \frac{mg\sin\theta - kR\sin\theta\cos\theta}{mR} = 0$$

$\cancel{mg\sin\theta}$ $\sin\theta\,(mg - kR\cos\theta) = 0$

$\sin\theta = 0$

$\theta = \frac{\pi}{2}$

$mg = kR\cos\theta$

$\frac{mg}{kR} = \cos\theta$ $\qquad \theta = \arccos\frac{mg}{kR}$

~~$\cos\theta = \arccos$~~