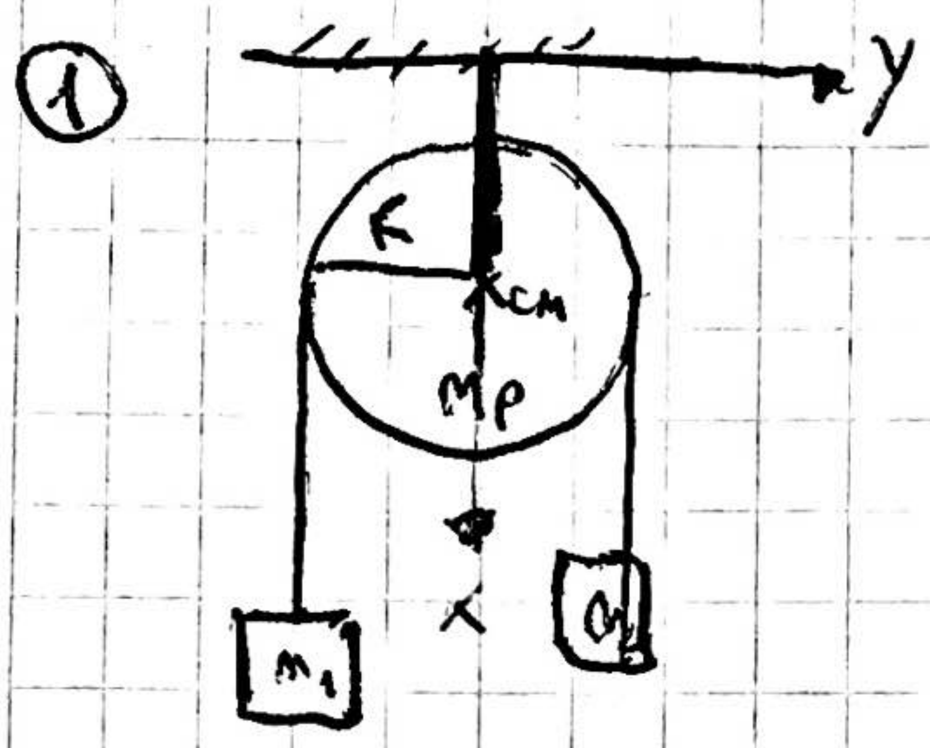
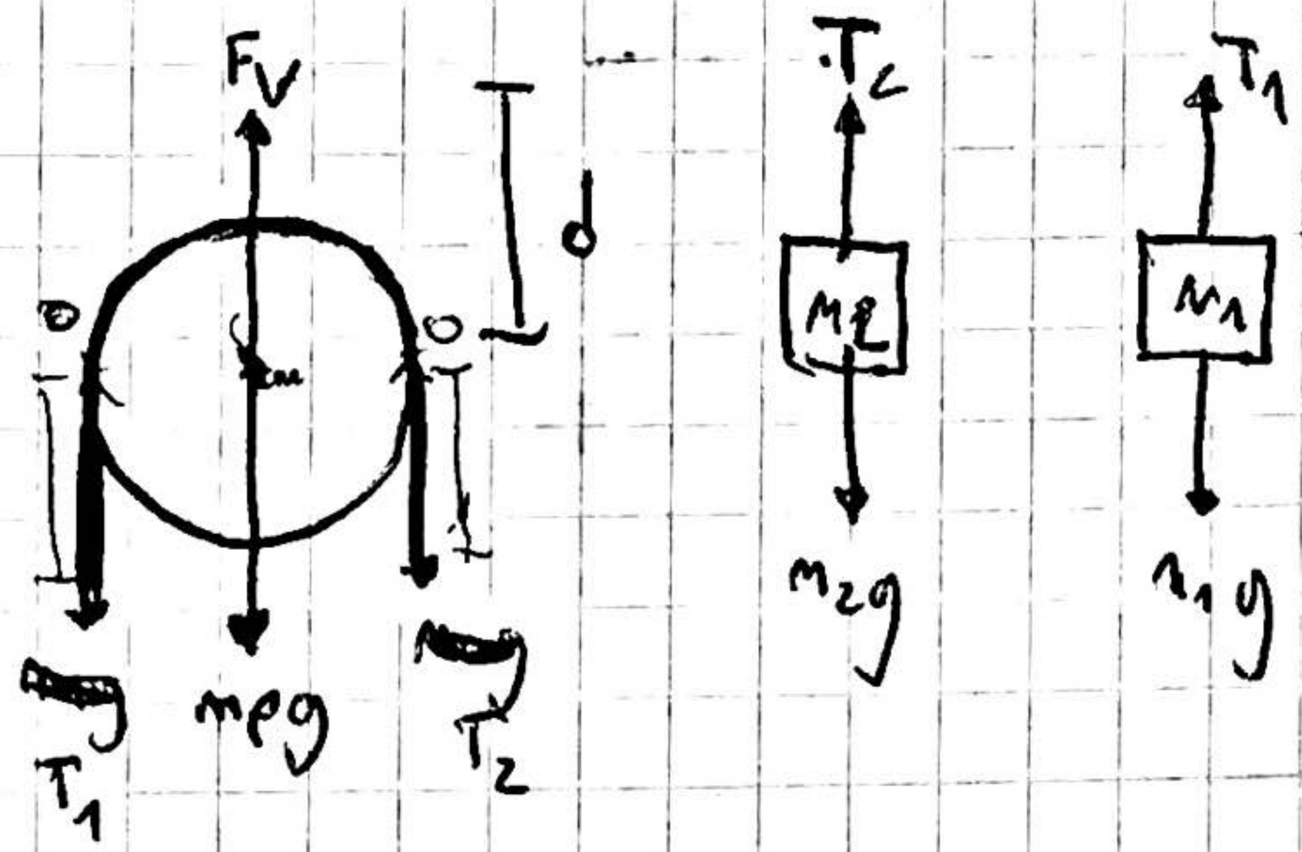
Dinámica rígido

1



aceleración de las masas . Observe que el res no depende del radio



$$I_{cm} = \frac{1R^2}{2} m_p$$

Poleas

$$m_p \ddot{x}_p = -F_V + m_p g + T_1 + T_2$$

m1) $m_1 \ddot{x}_1 = -T_1 + m_1 g$

m2) $m_2 \ddot{x}_2 = -T_2 + m_2 g$

Momentos de fuerza

$O \equiv CM$ Fijo

$$\Rightarrow \frac{d\bar{L}_o}{dt} = \bar{N} = R\hat{y} \times T_2 \hat{x} - R\hat{y} \times T_1 \hat{x} = -RT_2 \hat{z} + RT_1 \hat{z} = R(T_1 - T_2)\hat{z} = I_{cm}\bar{\gamma}$$

$$R(T_1 - T_2)\cancel{\hat{z}} = I_{cm}\gamma\cancel{\hat{z}} \qquad \frac{R(T_1 - T_2)}{I_{cm}} = \boxed{\gamma = \frac{2(T_1 - T_2)}{R m_p}}$$

$$l = X_2 - d + X_1 - d + \pi R$$

$$\dot{x}_2 = -\dot{x}_1$$

$$\bar{v}_{cm} = \bar{v}_0 + \bar{\Omega} \times (\bar{r}_{cm} - \bar{r}_0)$$

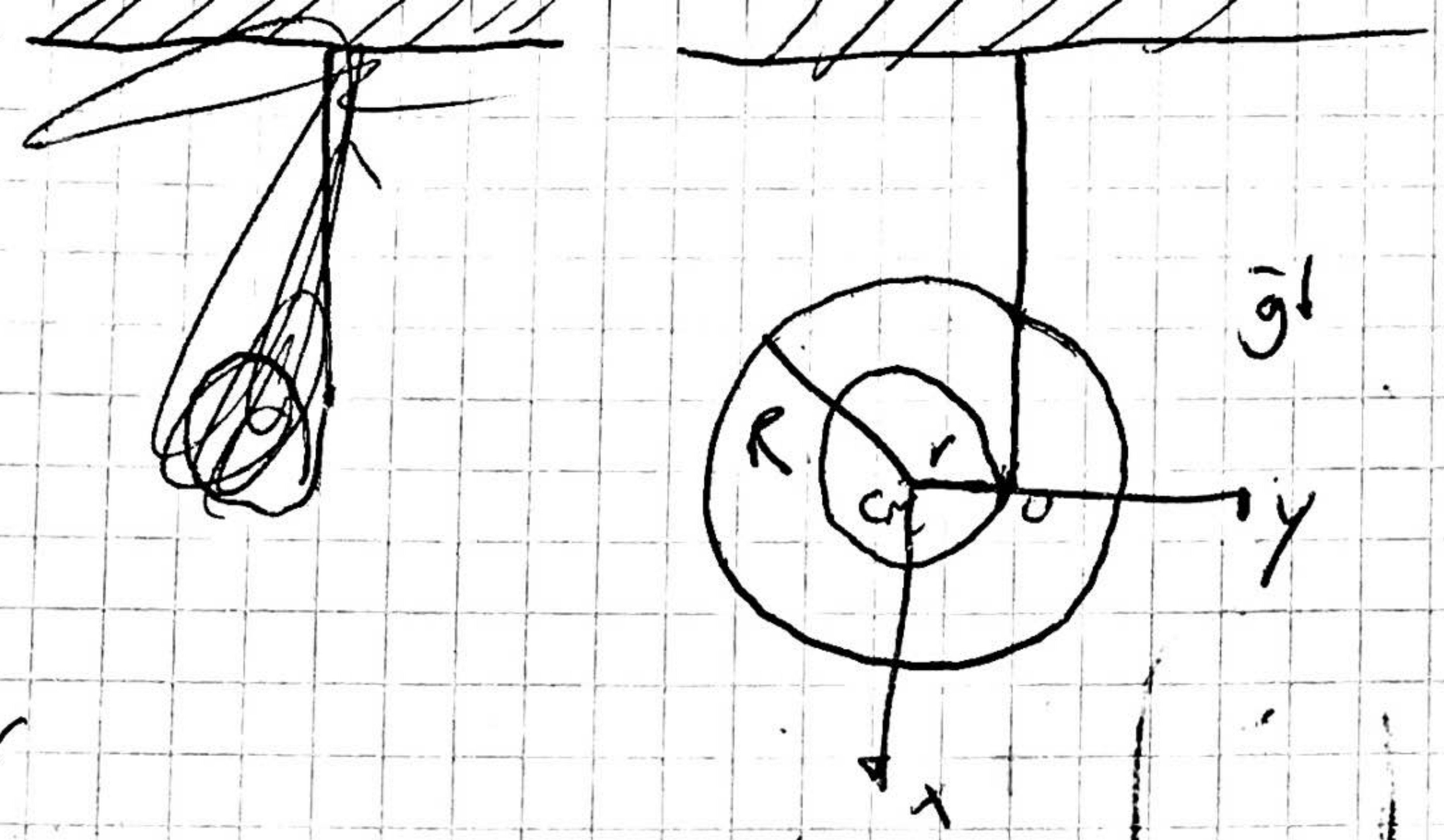
$$\bar{0} = \bar{v}_0 + \bar{\Omega} \times (-R\hat{y})$$

$$\bar{0} = \bar{v}_0 + \Omega R \hat{x}$$

$$\bar{v}_0 = -\Omega R \hat{x}$$

$\cancel{\bar{v}_0 = -\frac{2(T_1 - T_2)}{m_p}\hat{x}}$ y si no deslizan $\bar{v}_0 = \bar{v}_2$

$$\ddot{x}_2 = -\gamma R \hat{x} = -\frac{2(T_1 - T_2)}{m_p}\hat{x}$$



$R = 10r$

$I_{CM} = \frac{MR^2}{2}$

a) $a_{CM} = ?$ $\quad M\ddot{x}_{cm} = -T + Mg$

$\overline{\text{Momentos}}$

$$\tau_{CM} = \frac{d\bar{L}_{cm}}{dt} = r\hat{y} \times (-T)\hat{x} = rT\hat{z} = I_{CM}\gamma\hat{z}$$

$$\gamma = \frac{2rT}{MR^2}$$

$$\bar{v}_{cm} = \bar{\Omega} \times (\bar{r}_{cm} - \bar{r}_0) = \Omega \times (-r)\hat{y} = \Omega r \hat{x}$$

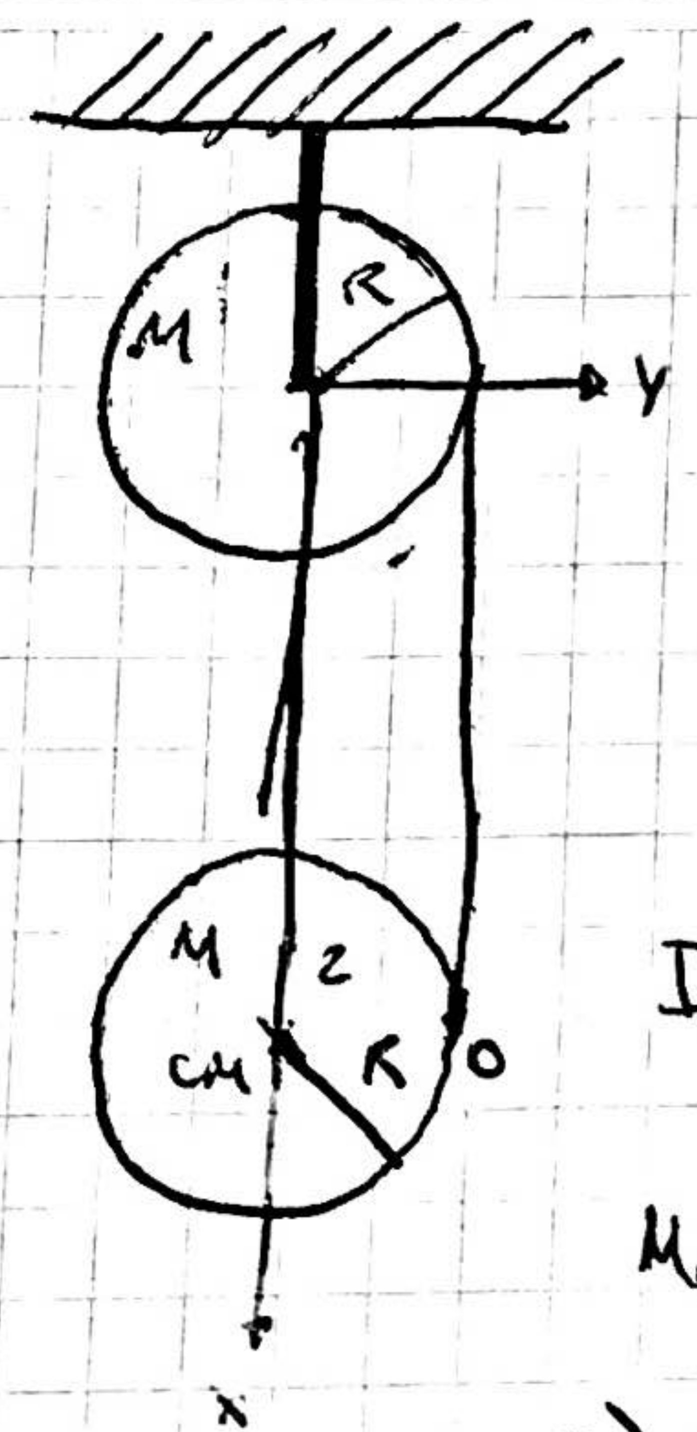
$$\cancel{a_{cm} = \gamma r = \frac{2r^2T}{MR^2} = \frac{T}{50M}}$$

$$\cancel{\frac{T}{50} = -T + Mg} \qquad \frac{d\bar{L}_0}{dt} = -r\hat{y} \times Mg\hat{x} = rMg\hat{z} = I_0\gamma\hat{z}$$

$$\cancel{T = \frac{50Mg}{51}} \qquad \frac{rMg}{\frac{1}{2}MR^2 + r^2} = \gamma$$

$$\ddot{x}_{CM} = \gamma r = \frac{r^2 Mg}{\frac{1}{2}MR^2 + r^2} \qquad T = \frac{Mg}{1 + \frac{2r^2}{R^2}}$$

3



$\bar{g}\downarrow$

a) $\ddot{x}_{CM}$ inferior

1) $M\ddot{x}_{cm_1} = Mg + T - F_v$

2) $M\ddot{x}_{cm_2} = Mg - T$

$I_{CM} = \frac{1}{2}MR^2$

Momento)

2) $-R\hat{y} \times Mg\hat{x} = RMg\hat{z} = I_O \gamma \hat{z} = \gamma = \frac{R\cancel{M}g}{\frac{1}{2}\cancel{M}R^2 + \cancel{M}R^2} = \frac{g}{\frac{1}{2}R+R}$

si no desliza

$\bar{v}_{cm} = \bar{\Omega} \times (\bar{r}_{cm} - \bar{r}_O) = \Omega\hat{z} \times -R\hat{y} = \Omega R \hat{x}$

$\ddot{x}_{cm_2} = \frac{2g}{3}$

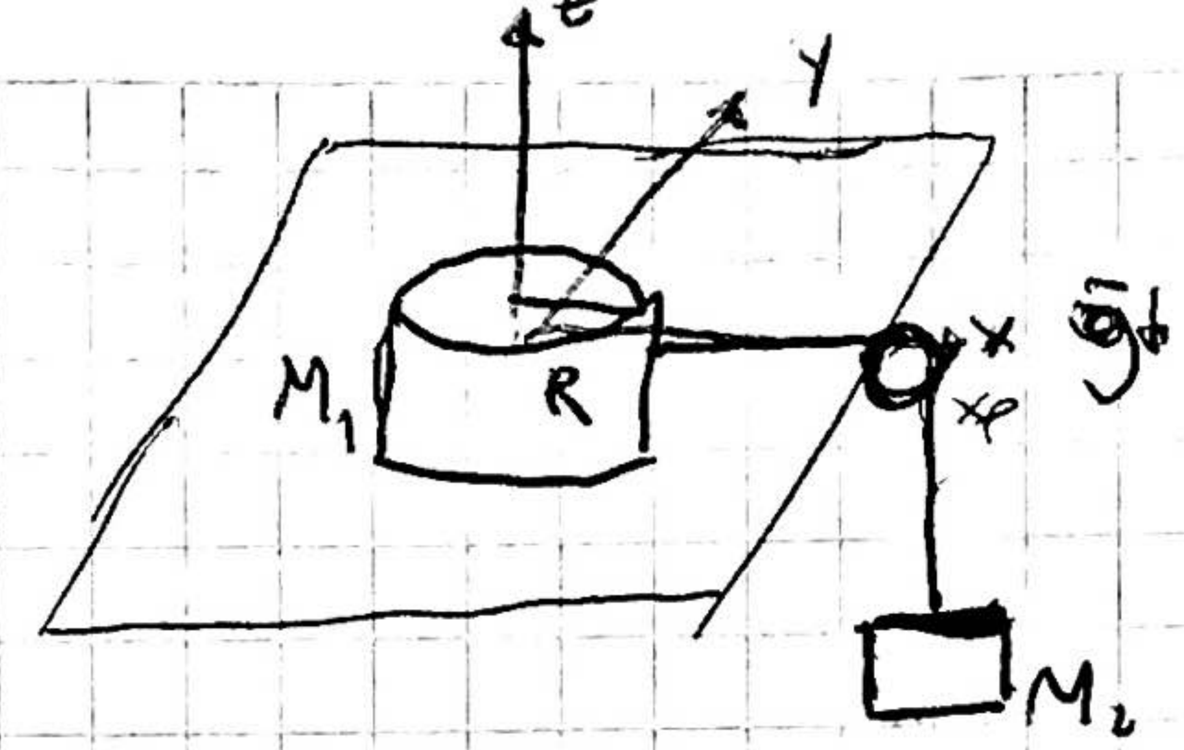
b) $T = ?$ $\quad \frac{2M}{3}g = Mg - T =$

$T = \frac{1}{3}g$

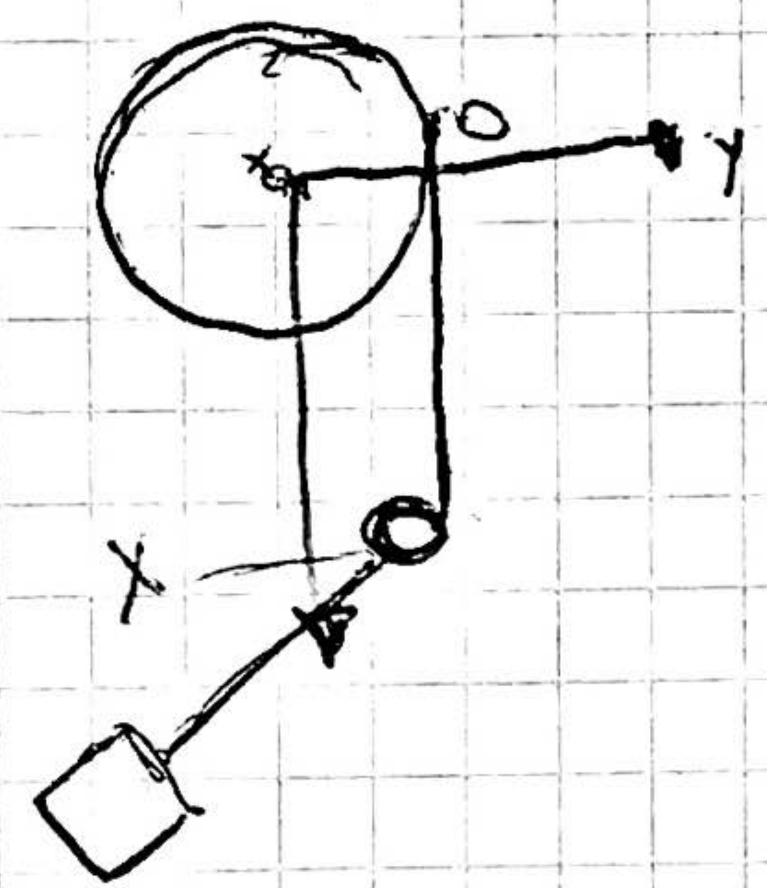
c) ¿v_{CM} cuando $d = 10R$?

$10R = \frac{\cancel{2}gt^2}{3\cancel{2}}$ $\qquad v_{CM} = \frac{2}{3}gt = \frac{2}{3}\sqrt{\frac{30R}{g}}$

4



a) aceleración centro de disco



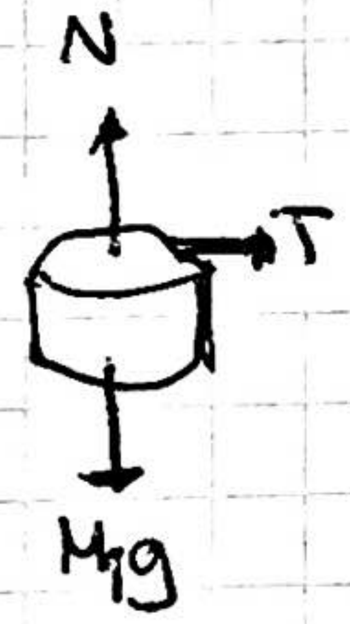
como la soga no desliza $|v_0| = |v_1|$

$l =$

$l = x_p - x_0 - x_1$

$\dot{x}_0 = -\dot{x}_2$

$\ddot{x}_0 = -\ddot{z}_2$



$\cancel{M_1 \ddot{x}_{cm} = T}$

$\cancel{\bar{v}_0 = \bar{v}_{cm} + \bar{\Omega} \times (\bar{r}_0 - \bar{r}_{cm})}$ $\quad \bar{v}_0 = \bar{v}_1$

$\cancel{\bar{v}_0 = \Omega \hat{z} \times r\hat{y}}$

$\bar{v}_{cm} = \bar{v}_0 + \bar{\Omega} \times (\bar{r}_{cm} - \bar{r}_0)$

$\cancel{\bar{v}_1 = \bar{v}_0 - \Omega r \hat{x}}$

$\bar{v}_{cm} = \bar{v}_0 + \Omega(-\hat{z}) \times (-r\hat{y})$

$\bar{v}_{cm} = \bar{v}_0 - \Omega r \hat{x}$

$M_2 \ddot{z}_2 = -M_2 g + T$

Momento

$$\frac{dL_{cm}}{dt} = r\hat{y} \times T\hat{x} = -rT\hat{z} = I_{cm}\gamma\hat{z}$$

$$\gamma = \frac{-rT}{I_{cm}}$$

$M_1 \ddot{x}_{cm} = T$

$-M_2 \ddot{x}_0 = -M_2 g + T$

$M_1 \ddot{x}_{cm} = -M_2 \ddot{x}_0 + M_2 g$ $\quad \ddot{x}_0 = \ddot{x}_{cm} + \gamma r$

$M_1 \ddot{x}_{cm} + M_2 \ddot{x}_{cm} = M_2 g - M_2 \gamma r$

$$\ddot{x}_{cm} = \frac{M_2 g - M_2 \gamma r}{M_1 + M_2} = M_2 g + M_2$$

$M_1 \ddot{x}_{cm} = T$

$\ddot{x}_0 = -\ddot{z}_2$

$\overline{v}_0 = \overline{v}_{cm} + \overline{\Omega} \times (\overline{r}_0 - \overline{r}_{cm}) = v_0 = v_{cm} + -\Omega \hat{z}\, r\hat{y} = \Omega r \hat{x}$

$M_2 \ddot{z}_2 = -M_2 g + T$

$\gamma = \frac{-rT}{I_{cm}}$

$-M_2 \ddot{x}_0 = -M_2 g + T$

$\ddot{x}_0 = \ddot{x}_{cm} + \gamma r$

$\ddot{x}_0 = g - \frac{T}{M_2}$

$\ddot{x}_0 = \ddot{x}_{cm} - \frac{\cancel{r^2} T}{\frac{1}{2} M_1 \cancel{r^2}}$

$\ddot{x}_0 = \ddot{x}_{cm} - \frac{2T}{M_1}$

$g - \frac{T}{M_2} = \ddot{x}_{cm} - \frac{2T}{M_1}$

$g - \frac{M_1 \ddot{x}_{cm}}{M_2} = \ddot{x}_{cm} - \frac{2 \cancel{M_1} \ddot{x}_{cm}}{\cancel{M_1}}$

$g = -\ddot{x}_{cm} + \frac{M_1}{M_2} \ddot{x}_{cm}$

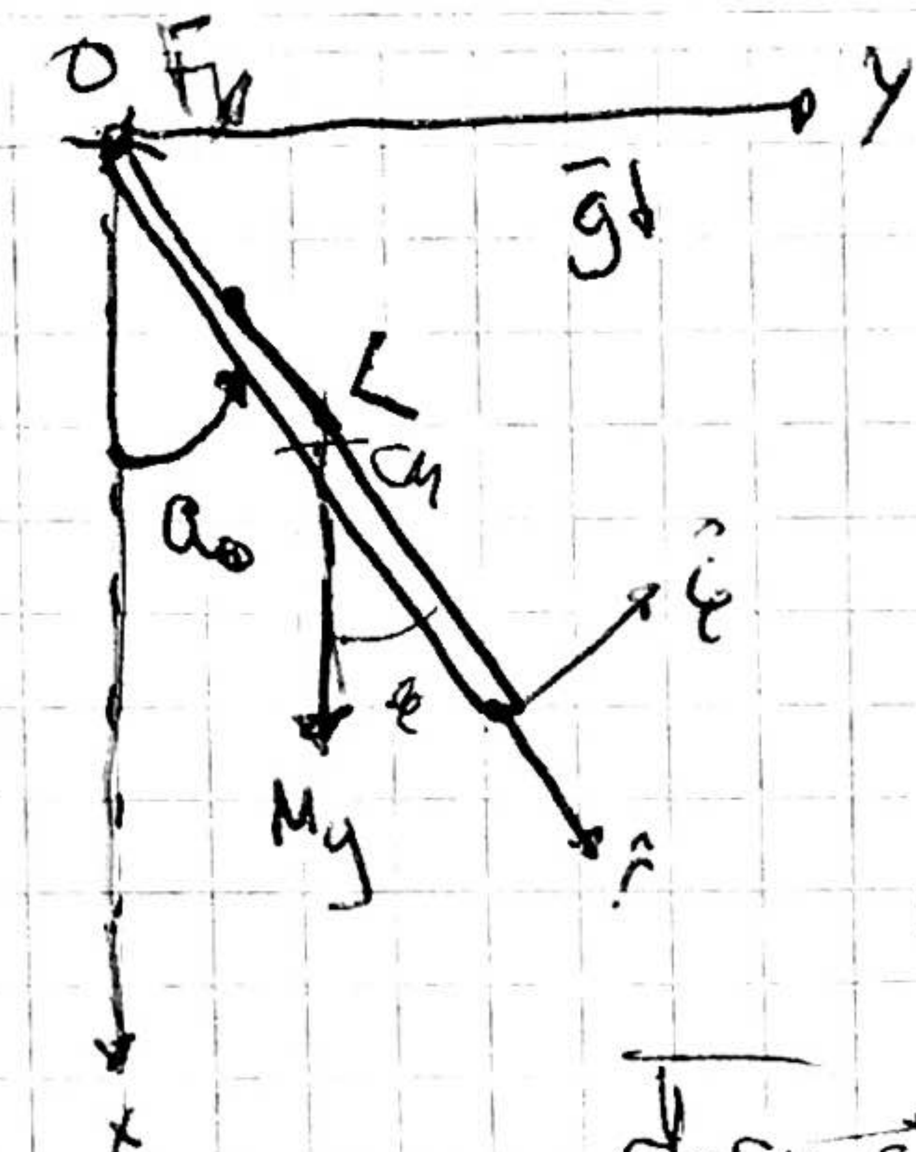
$g = \ddot{x}_{cm}\left(-1 + \frac{M_1}{M_2}\right)$ $\quad \ddot{x}_{cm} = \frac{g}{-1 + \frac{M_1}{M_2}}$

$\boxed{T = \frac{M_1 g}{-1 + \frac{M_1}{M_2}}}$

$\gamma = -\frac{\cancel{r} \cancel{M_1} g}{-1 + \frac{M_1}{M} \cdot \left(\frac{1}{2} \cancel{M_1} r^{\cancel{2}}\right)} = \boxed{\frac{-g}{-\frac{1}{2} r + \frac{M_1 r}{M_2}}}$

(20)

(5)



a) v angular en a mas bajo

$$\overline{v_{cm}} = \overline{v_o} + \bar{\Omega}\times(\overline{r_{cm}} - \overline{r_o})$$

$$\overline{v_{cm}} = \Omega\hat{z}\times\frac{L}{2}\hat{r}$$

$$\overline{v_{cm}} = \frac{L}{2}\Omega\hat{\varphi}$$

$$\cancel{\frac{dL_{cm}}{d_o} = F_V - \frac{L}{2}\hat{r}\times F_V\hat{z} = \frac{L}{2}F_V\hat{\varphi} = I_{cm}\gamma}$$

$$\frac{d\overline{L_O}}{dt} = \frac{L}{2}\hat{r}\times\left(Mg\left(\cos\varphi\hat{r} - \sin\varphi\hat{\varphi}\right)\right)$$

$$= -\frac{L}{2}Mg\sin\varphi\hat{z} = I_O\gamma\hat{z}$$

$I_{cm}\int r\,dv$

$$\gamma = \frac{-\frac{L}{2}Mg\sin\varphi}{I_O}$$

$$\Omega = \int_{a_0}^{\varphi} -\frac{L}{2}\frac{Mg\sin\varphi}{I_O}\,d\varphi = \frac{-LMg}{2I_O}\left(\cos(\varphi) - \cos(a_0)\right)$$

$$\cancel{m(-R\dot\varphi^2) = Mg\cos\varphi}$$

$$\cancel{mR\ddot\varphi = -Mg\sin\varphi}$$

$$\cancel{\frac{\dot\varphi}{2} = \frac{g}{R}\left(\cos(\alpha) - \cos(a_0)\right)}$$

$$\cancel{2g\left(\cos(\alpha) - \cos(a_0)\right) = g\cos\varphi}$$

b) $-mR\dot\varphi^2 = Mg\cos\varphi - F_V$

$$-mL\frac{L^2M^2g^2}{4I_0^2}\left(\cos\varphi - \cos(a_0)\right)^2 = \cancel{Mg}\cos\varphi - \frac{F_V}{Mg}$$

$$\frac{-mL^3}{4I_0^2} + \frac{mL^3}{4I_0}\cos(a_0) - 1 = \frac{-F_V}{Mg}$$

$$+\frac{M^3L^3g^2}{4I_0^2} - \frac{M^3L^3g^2}{4I_0^2}\cos(a_0) + Mg = F_V$$

P.S. ~~energía~~ $F_V \perp$ mov no L

$$\frac{\cancel{L}}{\cancel{2}}\dot\varphi\hat{\varphi} = +\frac{\cancel{L}}{\cancel{2}}\Omega\cancel{\hat{\varphi}}$$

$$\dot\varphi = \Omega$$

$$F_V = Mg - M\cancel{L}\frac{2g}{\cancel{L}}(1 - \cos a_0)$$

$$F_V = Mg - 2Mg + 2Mg\cos a_0$$

$$F_V = Mg\left(2\cos a_0 - Mg\right)$$

$$ML\ddot\varphi = -Mg\sin\varphi$$

$$\dot\varphi^2 = 2\int_{a_0}^{\varphi} -\frac{g\sin\varphi}{L} = +2\frac{g}{L}\left(\cos\varphi - \cos a_0\right)$$

$$\dot\varphi = \sqrt{2\frac{g}{L}\left(\cos\varphi - \cos a_0\right)}$$

Por h

$$H_0 = Mg(L - L\cos a_0)$$

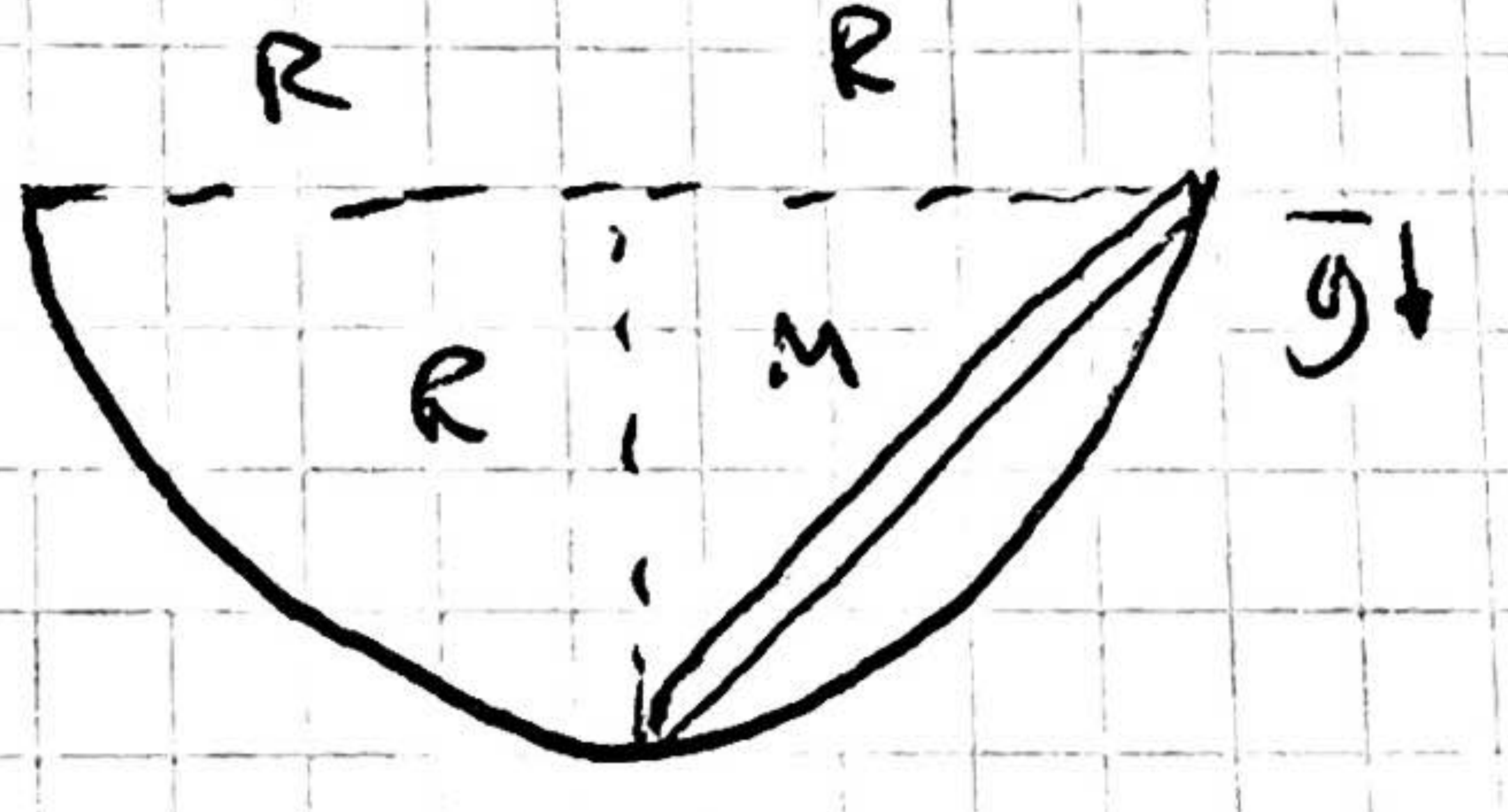
$$H_F = \frac{M}{2} v^2 = \frac{M}{2}\left(\dot{r}\hat{r} + r\dot{\varphi}\hat{\varphi}\right)^2 = \frac{ML^2}{2} \frac{2g}{L}\left[\cos(\varphi) - \cos(a_0)\right]$$

$$= \frac{ML^2}{2}\dot{\varphi}^2$$

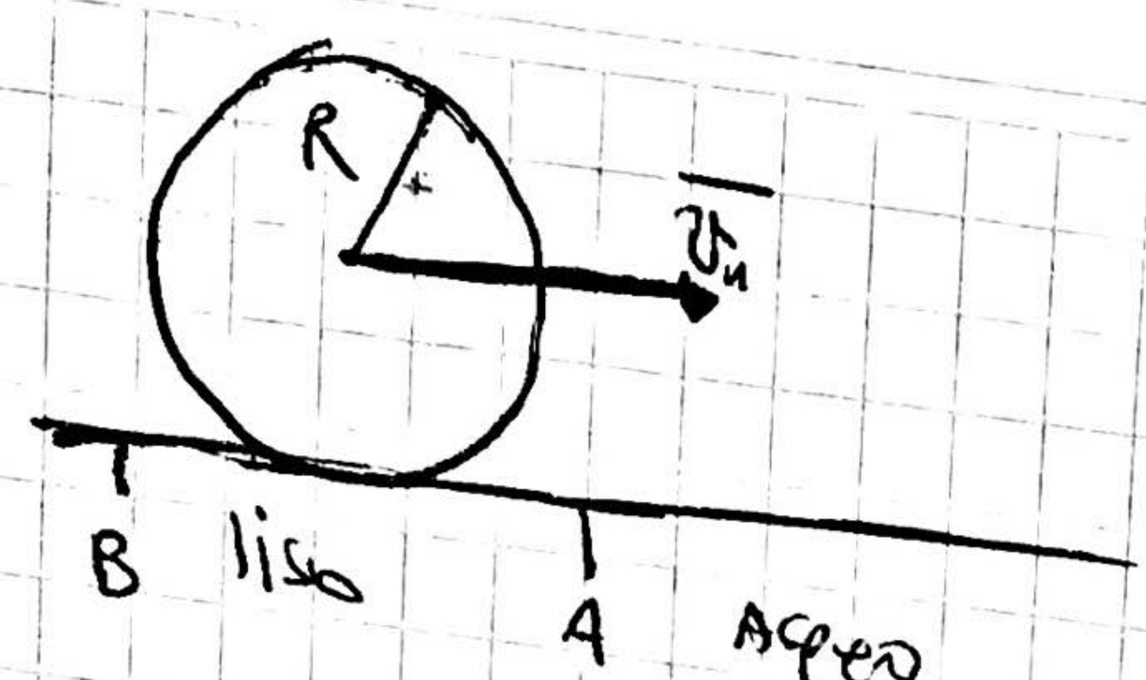
$$\dot{\varphi}^2 \frac{L^{\cancel{2}}}{2} = g\cancel{L} - gL\cos a_0$$

$$\dot{\varphi}^2 = 2\frac{g}{L} - \frac{2g}{L}\cos a_0$$

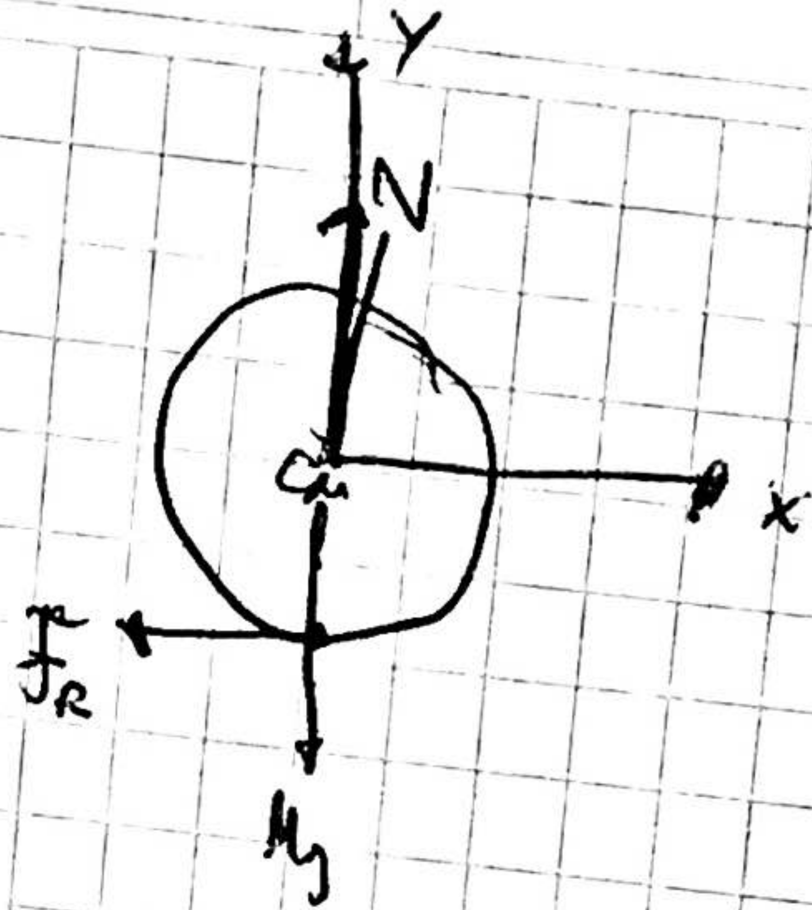
6



(9)



$\bar{g}\downarrow$



a) En que punto empieza a rodar sin deslizar y v_{cm}

BA $\frac{d\bar{L}_{cm}}{dt} = \bar{0} \Rightarrow \Omega = cte = 0$ no rueda

Desde A

$N = Mg \qquad \cancel{M}\ddot{x} = \mu_d \cancel{M}g \qquad \ddot{x}_{cm} = \mu_d g$

$$\frac{d\bar{L}_{cm}}{dt} = -R\hat{y} \times -\mu_d Mg\,\hat{x} = -\mu_d MgR\,\hat{z} = I_{cm}\,\gamma\,\hat{z}$$

$$\frac{-\mu_d \cancel{M}g\cancel{R}}{\frac{1}{2}\cancel{M}R^{\cancel{2}}} = -\frac{2\mu_d g}{R} = \gamma = ct$$

$$\bar{\Omega} = -\frac{2\mu_d g}{R}\,t\,\hat{z}$$

deje de rodar cuando $\qquad \dot{x}_{cm} = v_1 - \mu_d g t$

$$\bar{v}_{cm} = \underbrace{\bar{v}_{C'}}_{=0} + \bar{\Omega} \times (\bar{R}_{cm} - \bar{R}_{C'})$$

$$v_1 - \mu_d g t = -\frac{2\mu_d g}{\cancel{R}}\,t\,\hat{z} \times \cancel{R}\hat{y}$$

$v_1 - \mu_d g t = 2\mu_d g t\,\hat{x}$

$v_1 = 3\mu_d g t \qquad t_{ROD} = \frac{v_1}{3\mu_d g}$

$x_{cm} = A + v_1 t - \frac{\mu_d g}{2} t^2$

$$\boxed{C' = A + \frac{v_1^2}{3\mu_d g} - \frac{v_1^2}{18\mu_d g} = A + \frac{5 v_1^2}{18\mu_d g}}$$

$$v_{cm} = \Omega R = \frac{2\cancel{\mu_d g}\,v_1 \cancel{R}}{\cancel{R}\ 3\cancel{\mu_d g}} = \frac{2v_1}{3}$$

$\cancel{\vec{v}_{CM} = \vec{\Omega} \times R\hat{y}} =$ $N = Mg$ $f_{roz} = M\ddot{x}_{cm}$

$-\frac{2v_A}{3R} = \Omega$ $\frac{dL_{cm}}{dt} = -R\hat{y} \times -f_r\hat{x} = -Rf_{roz}\hat{z} = \frac{1}{2}MR^2 \cdot \gamma\hat{z}$

$\frac{-2F_{roz}}{MR} = \gamma$ $\Omega = \frac{-2F_{roz}}{MR} t + \frac{2}{3}v_1$

$\dot{x}_{cm} = \vec{\Omega} \times (R\hat{y})$

$\ddot{x}_{cm} = \Omega R\hat{x}$

c) $\cancel{\Delta T = T_C - T_A = \frac{1}{2}M\left(\frac{4v_1^2}{9} - v_1^2\right) = -\frac{5}{18}v_1^2}$

$\cancel{T_C' = \frac{1}{2}M\frac{4v_1^2}{9} + \frac{1}{2}I_{cm}\Omega^2 = \frac{4}{9}Mv_1^2 - \frac{Mv_1^2}{2} = \frac{8Mv_1^2}{18} - \frac{9v_1^2}{18} = \frac{-1Mv_1^2}{18}}$

$\Delta T = T_C - T_A = \frac{1}{2}M\frac{4}{9}v_1^2 + \frac{1}{4}MR^2 \cdot \frac{4v_1^2}{9R^2} = \frac{3}{9}v_1^2 = \frac{1}{3}v_1^2$

$T_C - T_A = \frac{1}{3}Mv_1^2 - \frac{Mv_1^2}{2} = -\frac{1}{6}v_1^2 M$

$\Delta T = W_{FNOC} = W_{FROZ} = -\int_{C'}^{A} \vec{F}_{roz} \cdot d\vec{r} = \mu_d Mg(C' - A) = \frac{5}{18}v_1^2 M$ $M\Omega$

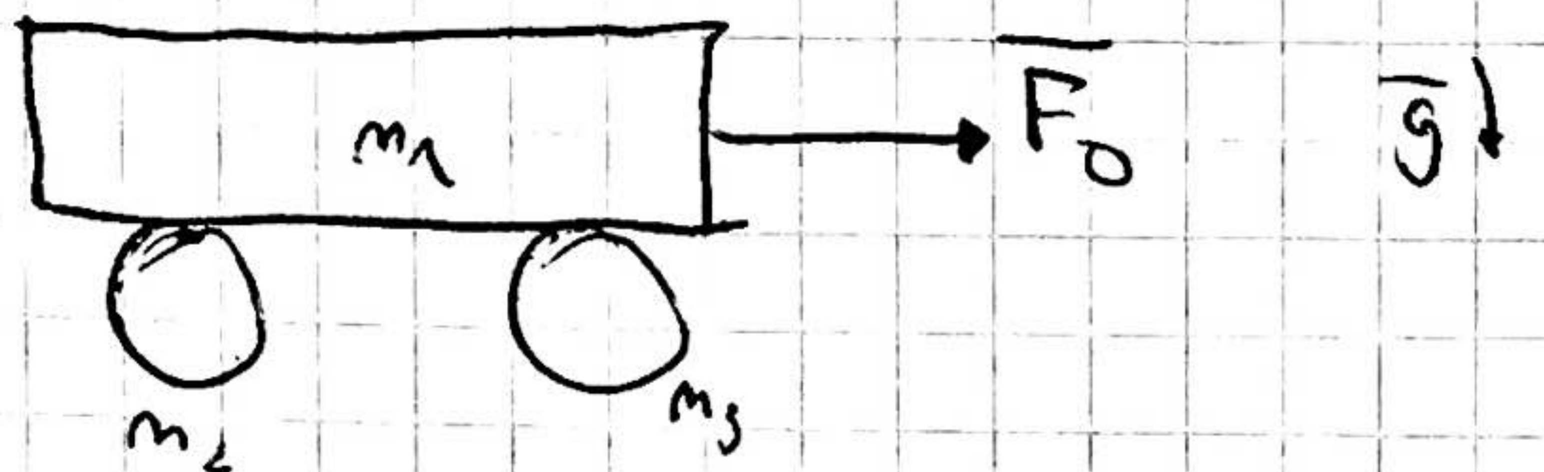
deslizamiento real

$|AC'| - \text{lo que giro}$

$\frac{5v_1^2}{18\mu_d g} - \frac{1}{2}\gamma R t^2 = \frac{5v_1^2}{18\mu_d g} - \frac{1}{\cancel{2}}\frac{\cancel{2\mu_d g}}{\cancel{R}}\frac{v_1^2}{9\cancel{\mu_d g}}$

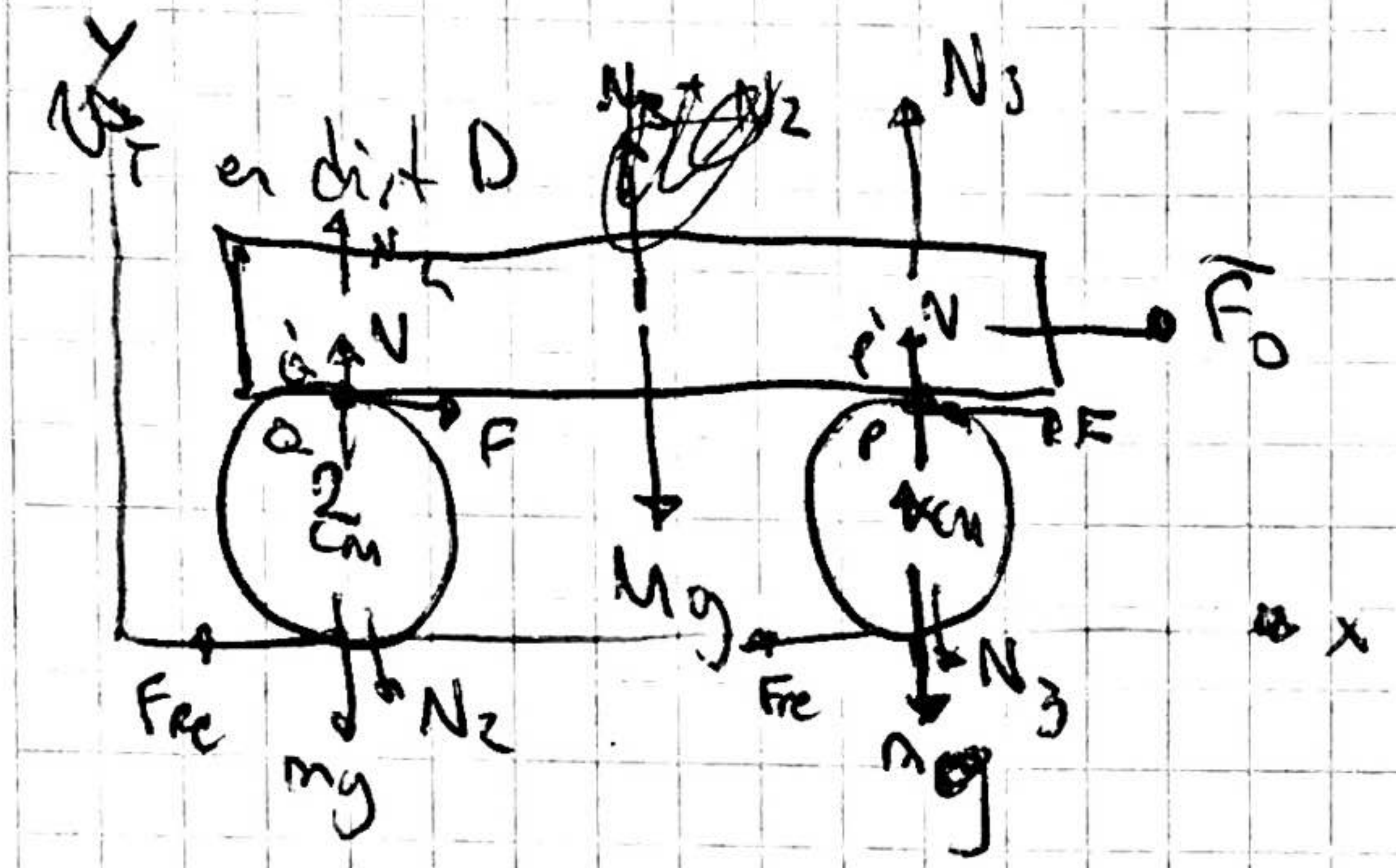
$\frac{1}{6}$ ✓

(10)



$m_1 = M$

$m_2 = m_3 = m$



2) $N = N_2 + mg$ 3) $N = N_3 + mg$

$\boxed{m\ddot{x}_2 = -F_{re} + F}$ $\boxed{m\ddot{x}_3 = -F_{re} + F}$

Momento

$$\frac{dL_{cm}}{dt} = R\hat{y} \times F\hat{x} - R\hat{y} \times -F_{re}\hat{x} = -RF\hat{z} - RF_{re}\hat{z} = -R(F + F_e) = I_{cm}\cdot\gamma$$

$$\boxed{\gamma = \frac{-2(F+F_e)}{MR}}$$

$\overline{v}_Q = \overline{v}_{Q'}$ $\overline{v}_P = \overline{v}_{P'}$

$\overline{v}_{cm} + \overline{\Omega} \times (\overline{r}_Q - \overline{r}_{cm}) = \dot{x}_T\hat{x}$ $\overline{v}_{cm} + \overline{\Omega} \times (\overline{r}_P - \overline{r}_{cm}) = \dot{x}\hat{x}$

$\ddot{x}_2 + \overline{\gamma} \times (\overline{r}_Q - \overline{r}_{cm}) = \ddot{x}_T\hat{x}$ $\ddot{x}_3 + \overline{\gamma} \times (\overline{r}_P - \overline{r}_{cm}) = \ddot{x}_T\hat{x}$

$$\frac{-F_{re} + F}{m}\hat{x} + \frac{2(F+F_e)(-1)}{MR}\hat{z} \times (R\hat{y}) = \ddot{x}_T$$

$$\boxed{\left(\frac{-F_{re}}{m} + \frac{F}{m}\right)\hat{x} + \frac{2(F+F_e)}{m}\hat{x} = \ddot{x}_T\hat{x}}$$

$$\boxed{M\ddot{x}_T = F_0 + 2F}$$

$-F_{re} + F + 2F + 2F_{re} = \ddot{x}_T m$

$F_{re} + 3F = \ddot{x}_T m$

$m\ddot{x}_T$

vínculos

$x_{cm_3} = x_{P'}$ $x_{cm_2} = x_{Q'}$

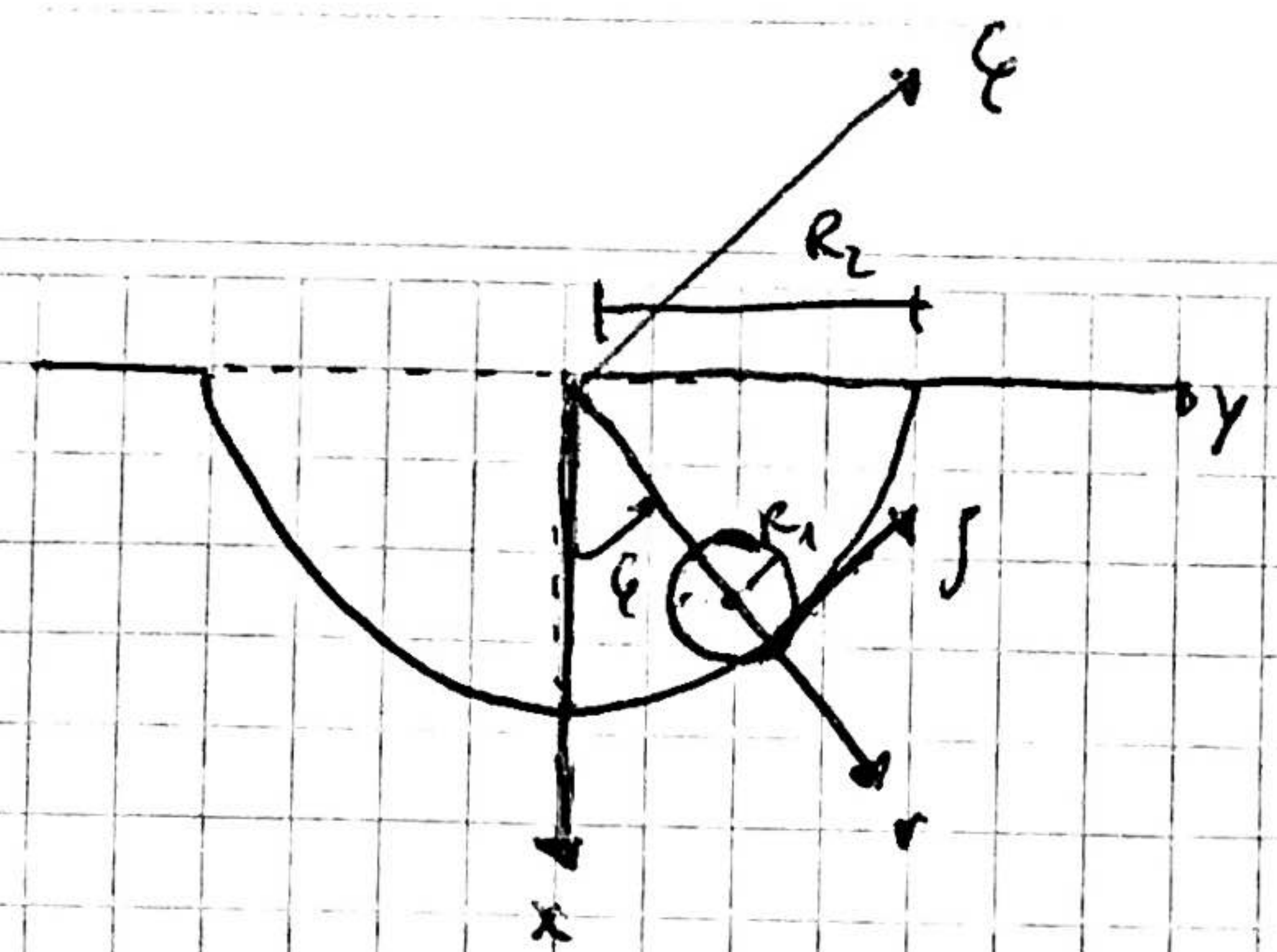
~~$F_{re} = F_0 \; 5F$~~

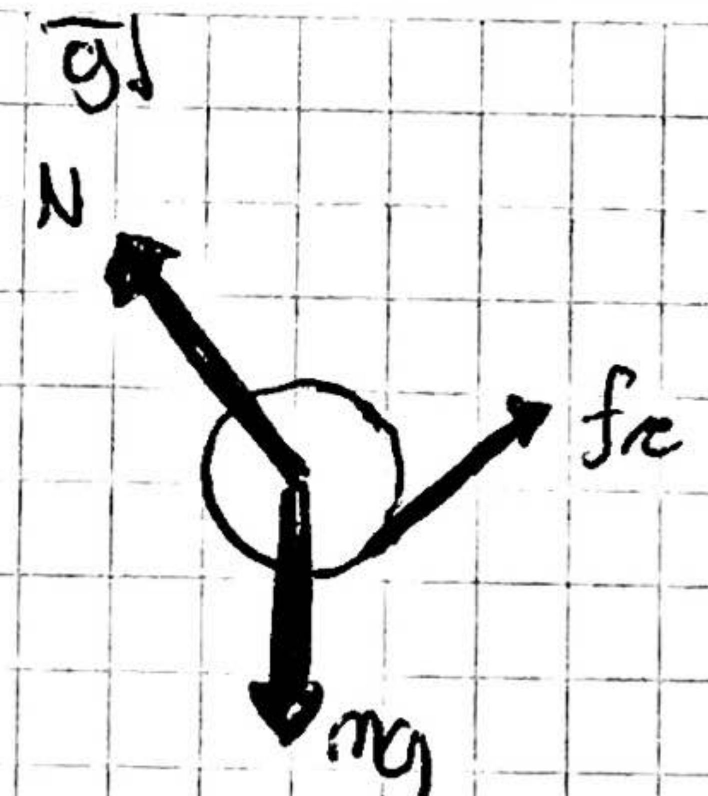
$\ddot{x}_{cm_3} = \ddot{x}_{P'}$ $\ddot{x}_{cm_2} = \ddot{x}_{Q'}$

$$F_{re} + 3F = \frac{m}{M}(F_0 - 2F)$$

$$F_{re} = \frac{m}{M}F_0 + F\left(-\frac{2m}{m} - 3\right)$$

11





$\overline{v_{cm}} = \overset{0}{\dot{r}}\hat{r} + \dot{\varphi} r \hat{\varphi}$

$\boxed{\overline{v_{cm}} = (R_2 - R_1)\dot{\varphi}\hat{\varphi}}$

$\cancel{\frac{dL_{cm}}{dt} = \overline{H} = -R_1\hat{r} \times fre\,\hat{\varphi} = -R_1 fre\,\hat{z} = I_{cm}\gamma\hat{z}}$

$\cancel{\gamma = \frac{-2fre}{mR_1}}$

$\boxed{\overline{a_{cm}} = -(R_2-R_1)\dot{\varphi}^2\hat{r} + (R_2-R_1)\ddot{\varphi}\hat{\varphi}}$

$\cancel{m\ddot{x} = fre}$

$-m(R_2-R_1)\dot{\varphi}^2 = -N + mg\cos\varphi$

$m(R_2-R_1)\ddot{\varphi} = fre - mg\sin\varphi$

$\overline{v_{cm}} = \vec{\Omega} \times (\vec{r}_{cm} - \vec{r}_o) = \Omega\hat{z} \times -R_1\hat{r}$

$\overline{v_{cm}} = \Omega R_1\hat{\varphi}$

$(R_2-R_1)\dot{\varphi}\hat{\varphi} = -\Omega R_1\hat{\varphi}$

$\boxed{\Omega = -\frac{(R_2-R_1)\dot{\varphi}}{R_1}\hat{z}}$

$\boxed{\gamma = \frac{-(R_2-R_1)\ddot{\varphi}}{R_1}\hat{z}}$

$\frac{dL_{cm}}{dt} = \overline{H} = R_1\hat{r} \times fre\,\hat{\varphi}$

$= R_1 fre\,\hat{z} = I_{cm}\gamma\hat{z}$

$\gamma = \frac{2fre}{mR_1}$ $\frac{2fre}{m\cancel{R_1}} = \frac{-(R_2-R_1)\ddot{\varphi}}{\cancel{R_1}}$

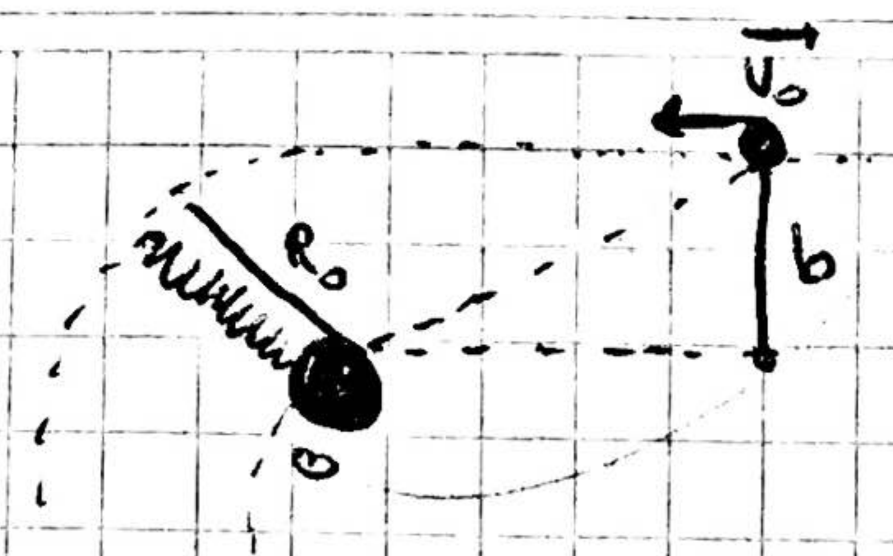
$fre = \frac{-(R_2-R_1)\ddot{\varphi}m}{2}$ (en albeum el Vector)

$\cancel{m}(R_2-R_1)\ddot{\varphi} = -\frac{(R_2-R_1)\ddot{\varphi}\cancel{m}}{2} - \cancel{m}g\sin\varphi$

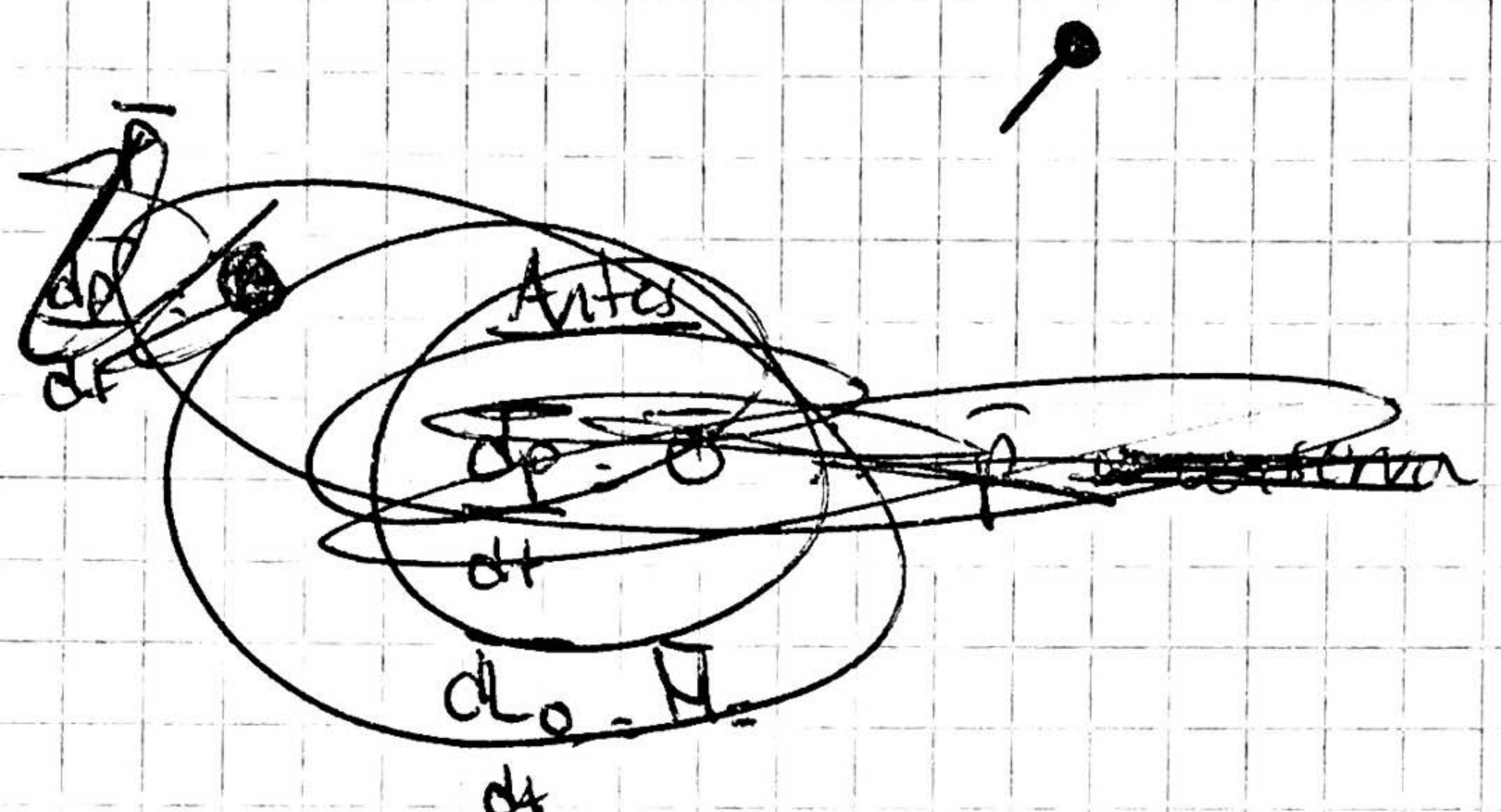
$\frac{3}{2}(R_2-R_1)\ddot{\varphi} = -g\sin\varphi$

$\ddot{\varphi} = \frac{-g\sin\varphi}{(R_2-R_1)}\frac{2}{3}$ $\dot{\varphi} =$

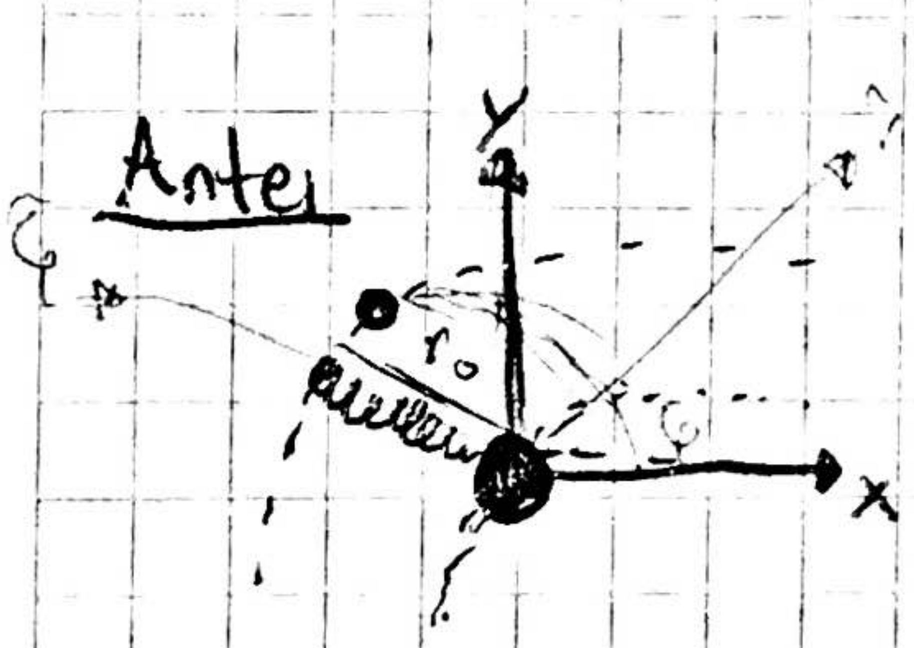
8



a) Diga que magnitudes se conservan antes y después de A v_A y distancia r_0 de máximo acercamiento



Antes



$$\frac{d\bar{p}}{dt} = -F_g \hat{r} \therefore \bar{p} \text{ no se conserva}$$

$$\frac{d\bar{L}_0}{dt} = \bar{M} = \bar{0} \therefore \bar{L} \text{ se conserva}$$

$$\Delta H = W_{F_{noc}} = 0 \therefore \text{ se conserva}$$

Después

Todo igual. ✓

$$\bar{L}_0 = (x\hat{x} + b\hat{y}) \times -m v_0 \hat{x} = m b v_0 \hat{z} = L(t)\hat{z}$$

$$= r\hat{r} \times (\dot{r}\hat{r} + r\dot{\varphi}\hat{\varphi})m = mr^2\dot{\varphi}\hat{z} \qquad \dot{\varphi} = \frac{b v_0}{r^2}$$

$$H = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\varphi}^2) - \frac{GmM}{r} + \frac{GmM}{r_0} \quad \text{no importa}$$

$$\boxed{-\frac{GmM}{r^2}\hat{r}} \quad -\int_{r_0}^{r} -\frac{GmM}{r^2}\,dr$$

$$H = \frac{1}{2}\left(m\dot{r}^2 + r^2\dot{\varphi}^2\right) - \frac{GmM}{r}$$

$$H = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\frac{b^2 v_0^2}{r^4} - \frac{GmM}{r} \qquad = -\left(\frac{GmM}{r}\right)$$

$$H = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}\frac{m b^2 v_0^2}{r^2} - \frac{GmM}{r} + \frac{k(r-l_0)^2}{2} \qquad k(x-l_0) \qquad \frac{k(x-l_0)^2}{2}$$

$H_0 = \frac{1}{2} m v_0^2 \qquad H_A = \frac{1}{2} m \dot{r}^2 + \frac{1}{2} \frac{m b^2 v_0^2}{r_0^2} - \frac{GmM}{r_0}$

$$\frac{1}{2}\dot{r}^2 \cancel{m} = \frac{1}{2}\cancel{m} v_0^2 - \frac{1}{2}\frac{\cancel{m} b^2 v_0^2}{r_0^2} + \frac{G\cancel{m}M}{r_0}$$

$$\dot{r} = \sqrt{v_0^2 - \frac{b^2 v_0^2}{r_0^2} + \frac{2GM}{r_0}}$$

$$\dot{\varphi} = \frac{b v_0}{r_0^2} \qquad \Rightarrow \quad v_A = \left(\sqrt{v_0^2 - \frac{b^2 v_0^2}{r_0^2} + \frac{2GM}{r_0}}\ \hat{r} + \frac{b v_0}{r_0} \hat{\theta} \right)$$

b) $d = 2r_0$