HyperGraph Convolutional Network (HyperGCN)

To Appear as a Poster in Neural Information Processing Systems, 2019

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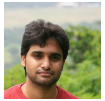
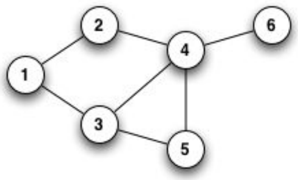




Joint work with Madhav Prateek Vikram Prof. Anand Louis Prof. Partha Talukdar

Motivation

networks have complex relationships



simple graphs do not capture such relationships

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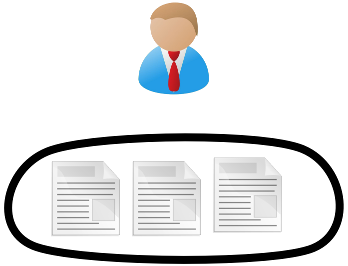
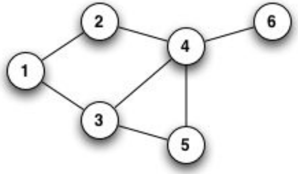


co-authorship



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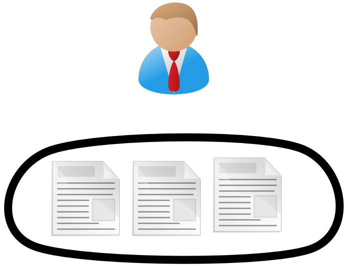

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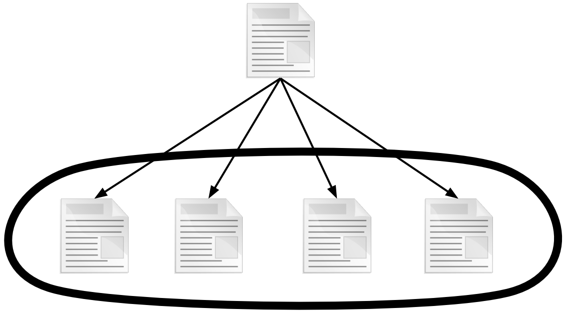

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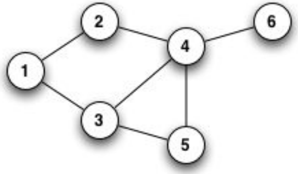

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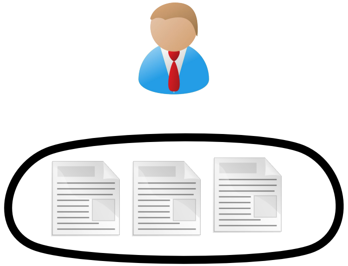

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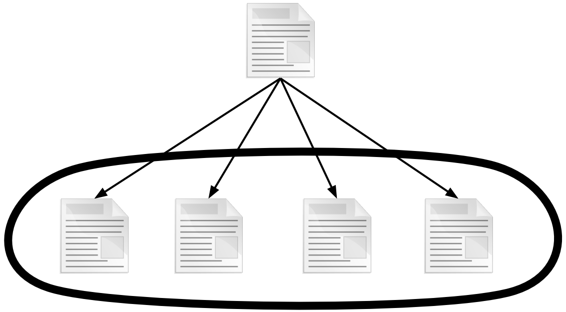

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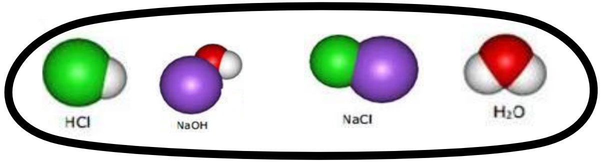
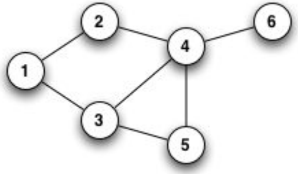


chemical reaction



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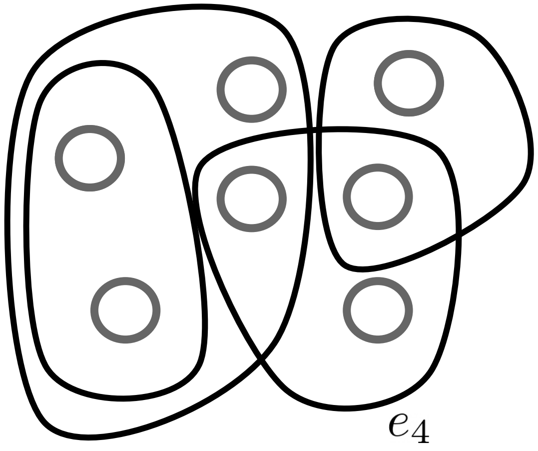
Hypergraph

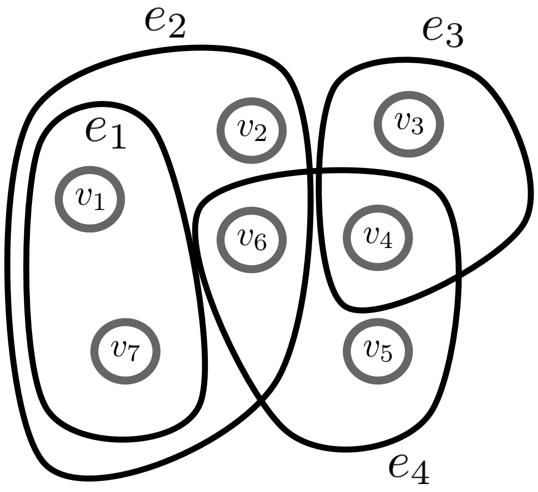


$$\mathcal{H} = (V, E)$$

$$E \subseteq 2^V$$

an edge can connect any number of vertices

Hypergraph



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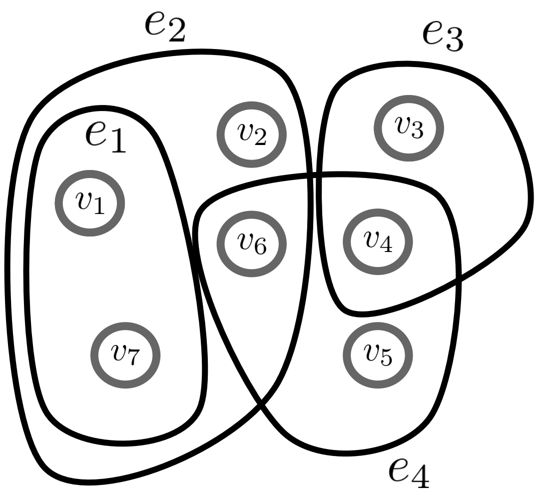
$$V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7\}$$

$$E = \{e_1, e_2, e_3, e_4\}$$

Hypergraph



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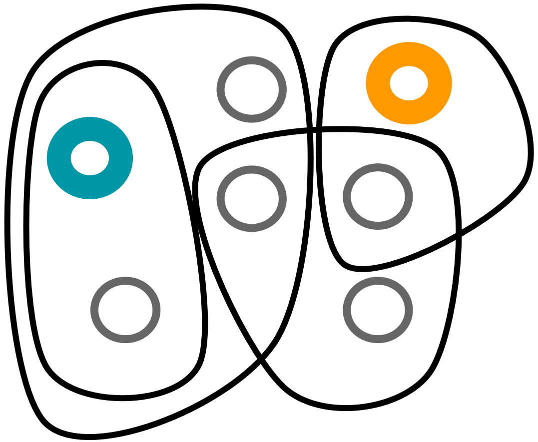
Hypergraph-based Semi-Supervised Learning (SSL)



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use labelled and unlabelled data for training

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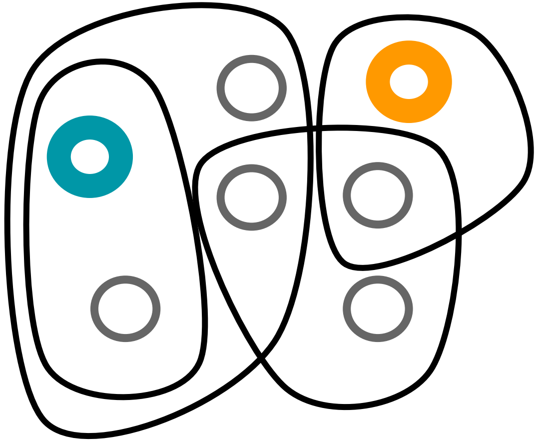

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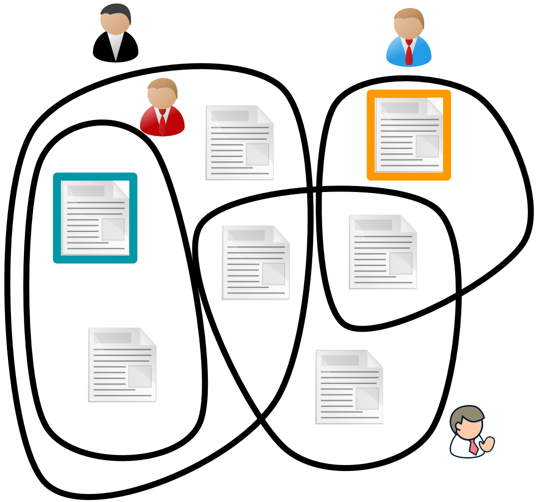

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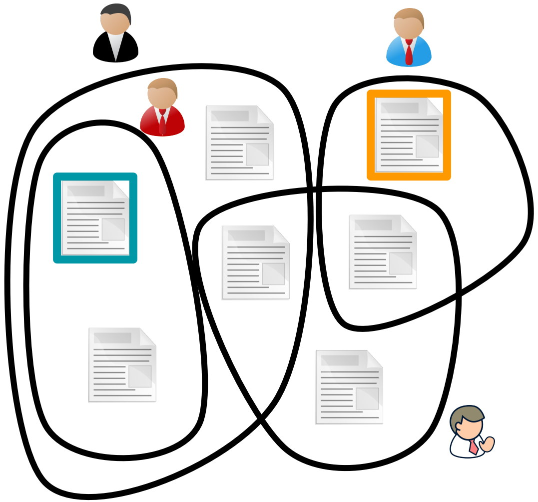

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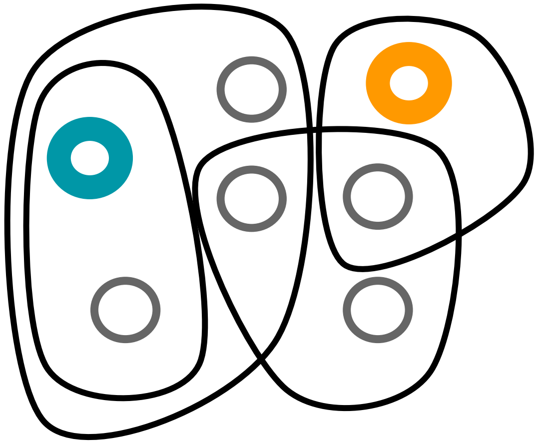
Learn $f : \left\{x_1, \cdots, x_n\right\} \rightarrow \left\{y_1, \cdots, y_c\right\}$

Related Work



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explicit regularisation

$$\underbrace{\mathcal{L}}\ = \underbrace{\mathcal{L}_S}_{\text{supervised}} + \underbrace{\lambda \cdot Q(\mathcal{H}, f)}_{\text{unsupervised}}$$

- Zhou et al. NIPS'06
- Hein et al. NIPS'13
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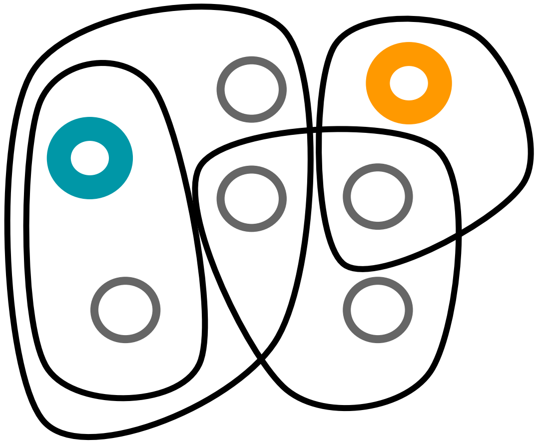

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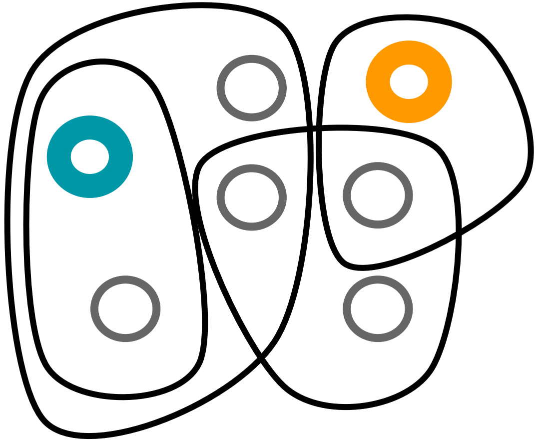

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Our focus: Implicit regularisation

$$f_{Neural}\Big(\mathcal{H}, X\Big) = ?$$

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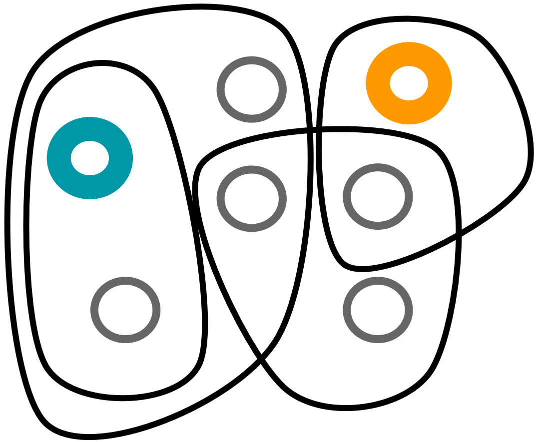
Related Work



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✓ hyperedges need not encode similarity

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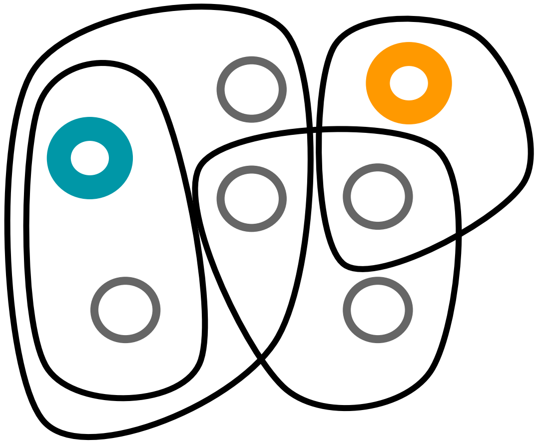

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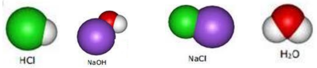
✓ **hyperedges need not encode similarity**

e.g.



Hypergraph total variation [Hein et al. NeurIPS'13]

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graphs: $Q\Big(\mathcal{G}, f \Big) = \sum_{\{u,v\} \in E} \Big(f_u - f_v \Big)^2$

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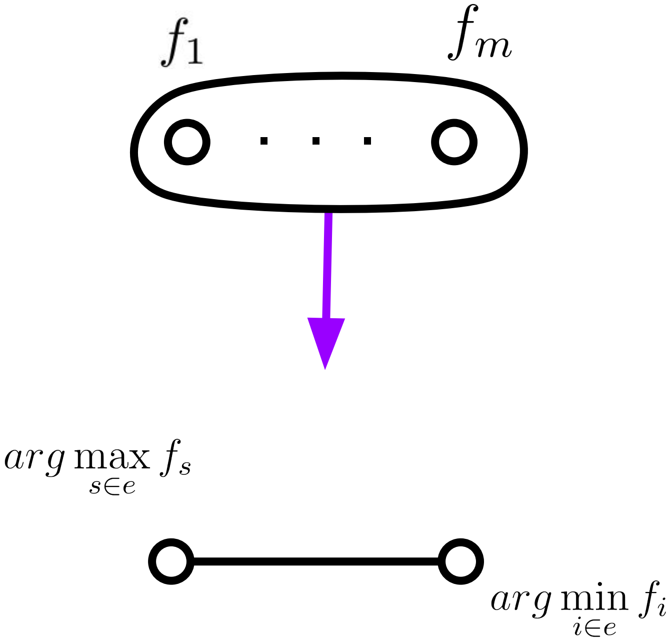

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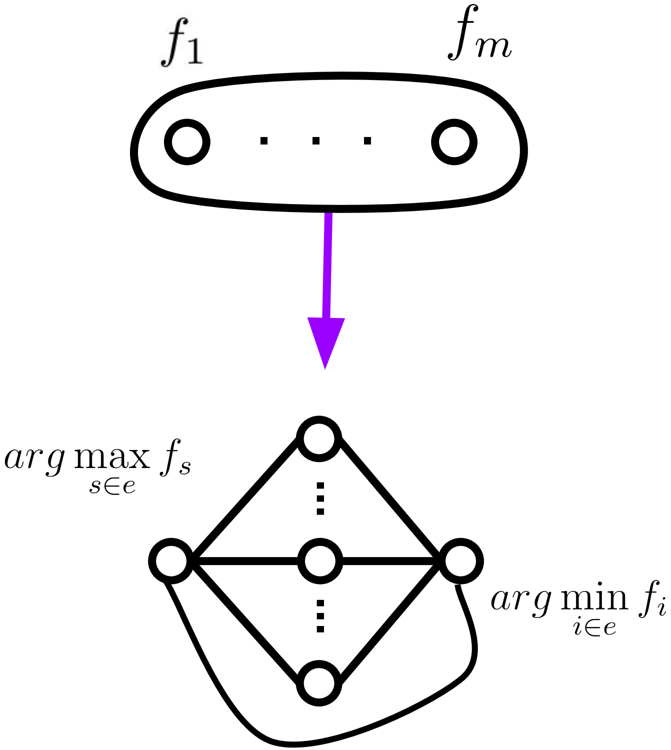

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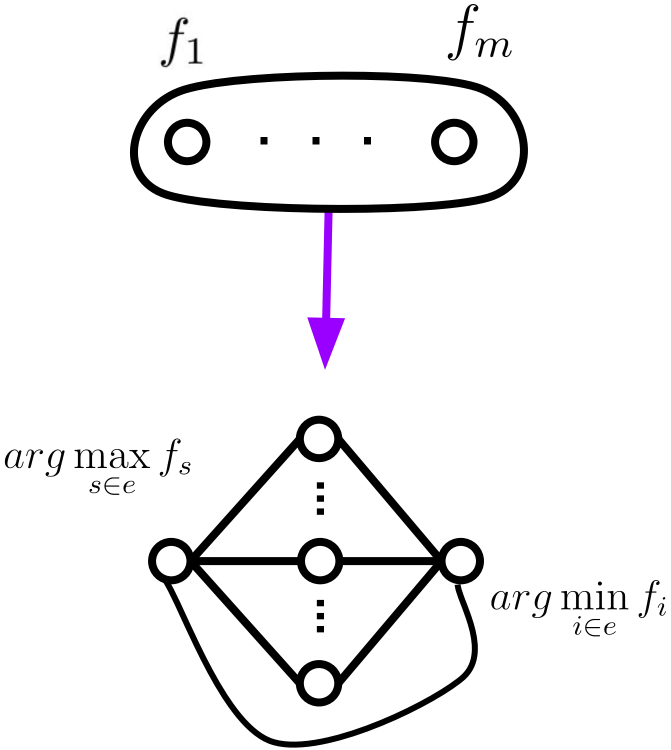
$$+ \sum_{e \in E} \sum_{m \in e} \left[\left(\max_{s \in e} f_s - f_m \right)^2 + \left(f_m - \min_{i \in e} f_i \right)^2 \right]$$

[Chan and Liang, COCOON 18]

Graph-based Hypergraph Modelling



Graph neural network [Kipf and Welling, ICLR'16]

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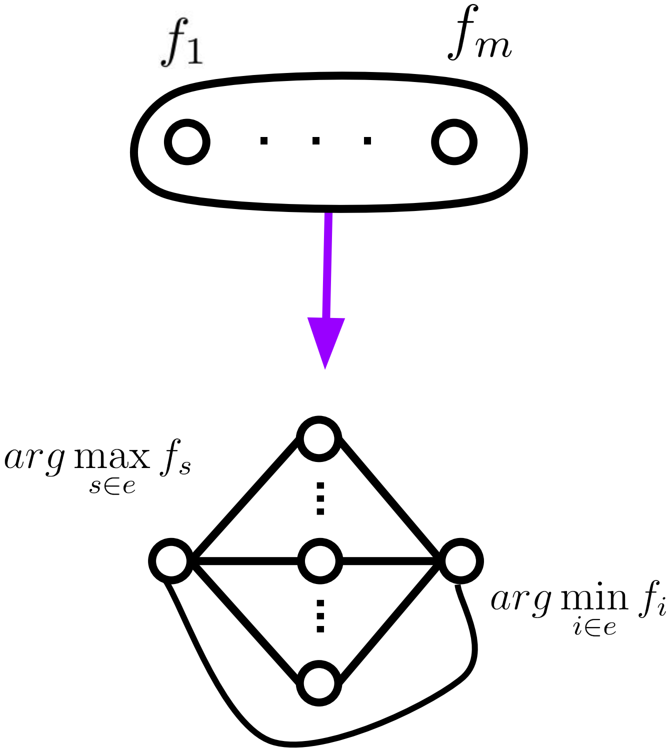

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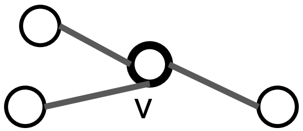

$$h_v^{\{l\}} = \sigma\left(\frac{W^{\{l\}}}{|\mathcal{N}_v|} \sum_{\{u,v\} \in \mathcal{N}_v} h_u^{\{l-1\}} + b \right)$$

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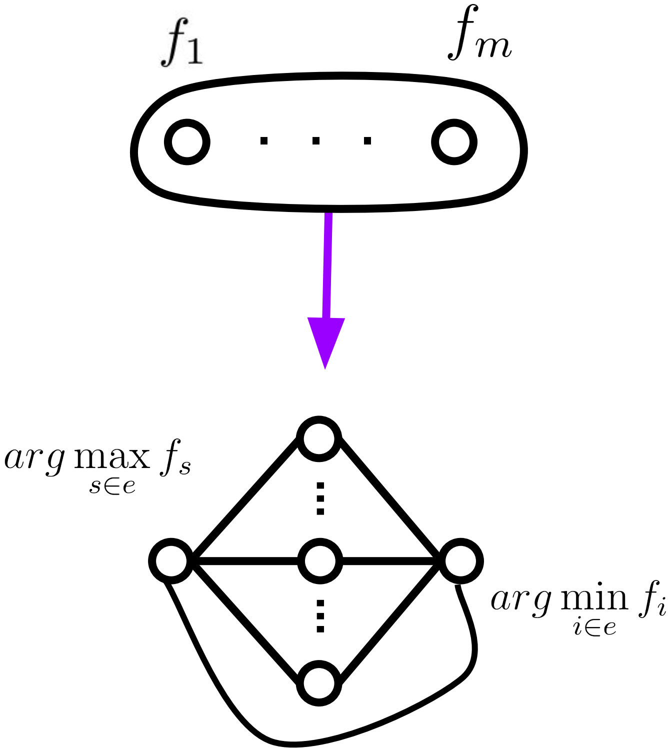

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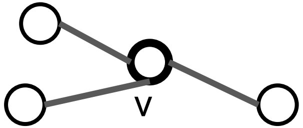

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representation in current layer

representation in previous layer

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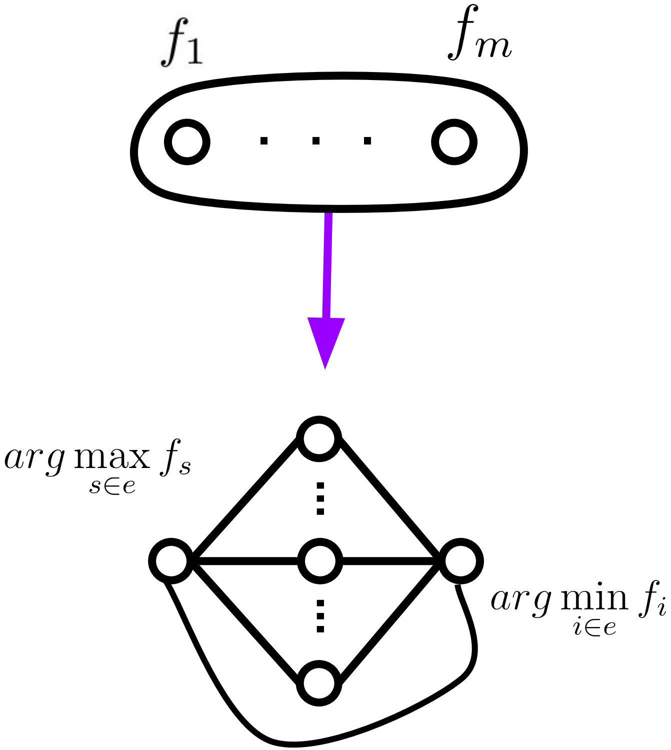

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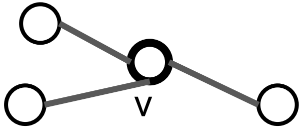

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representation in current layer

non-linear activation

set of all neighbours of v

representation in previous layer

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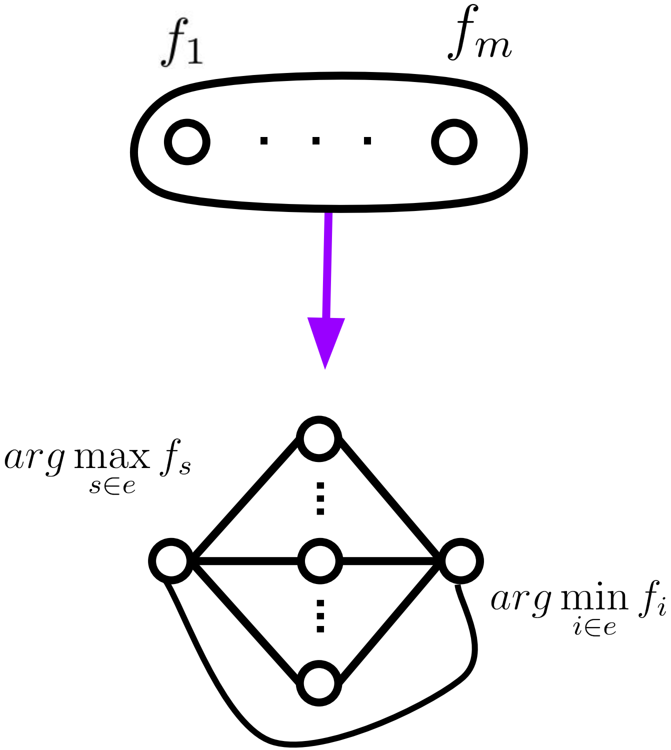

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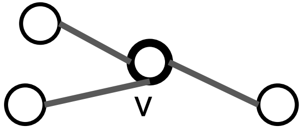

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weight in current layer

bias

representation in current layer

non-linear activation

set of all neighbours of v

representation in previous layer

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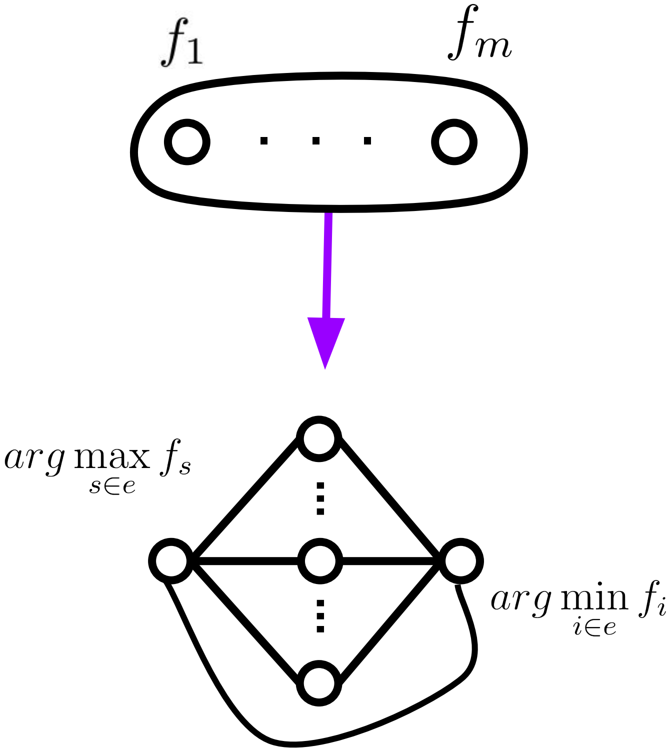

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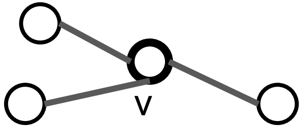

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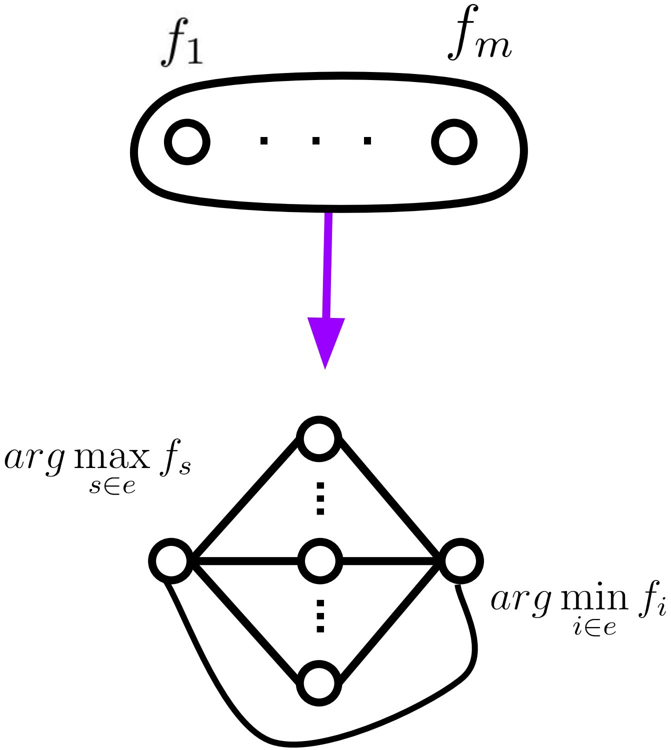
Hypergraph Convolutional Network



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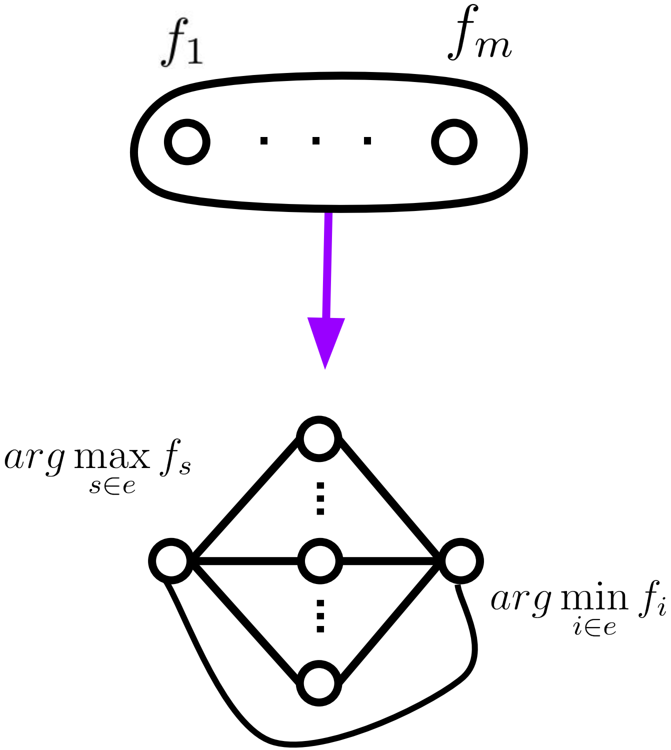

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Set $f = H^{\{l-1\}} \cdot W^{\{l\}}$

$\boxed{\textbf{parameters shared across input}}$

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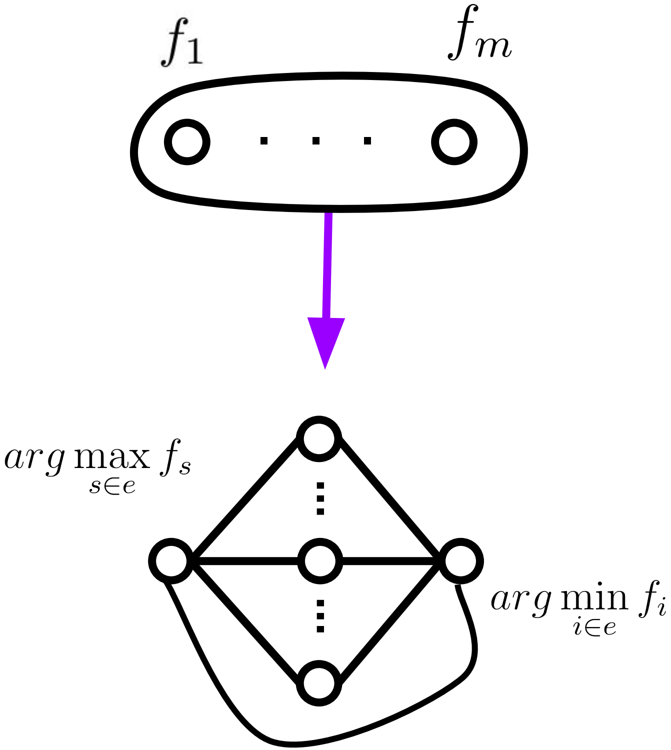

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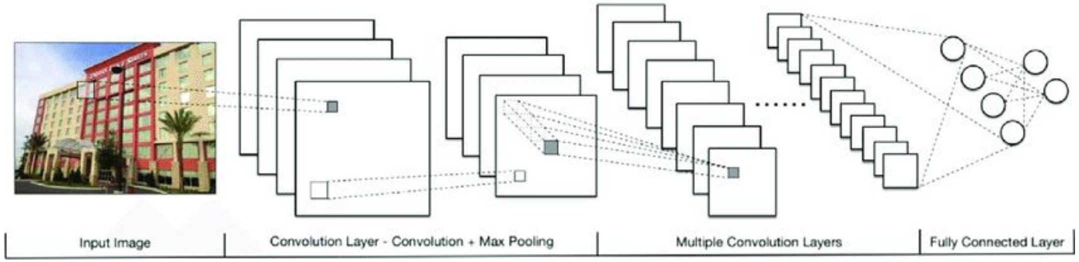
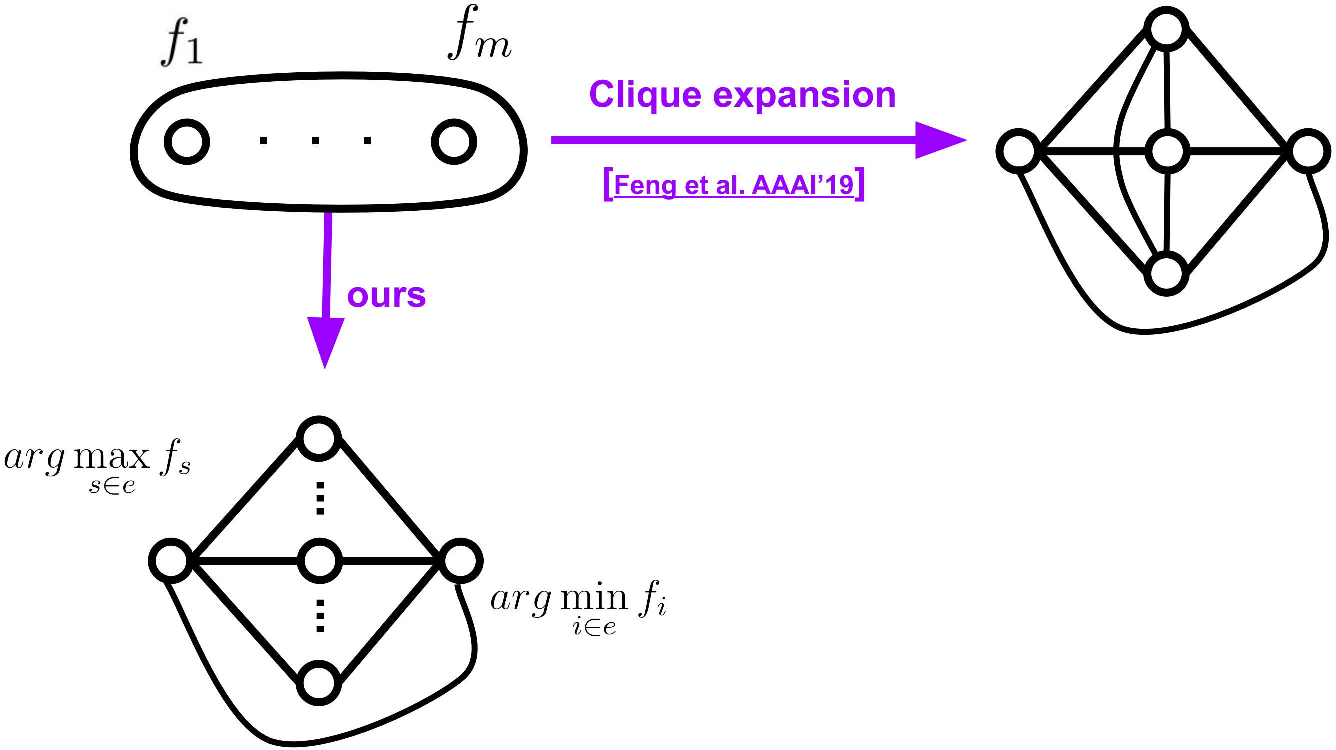


A simple, strong baseline



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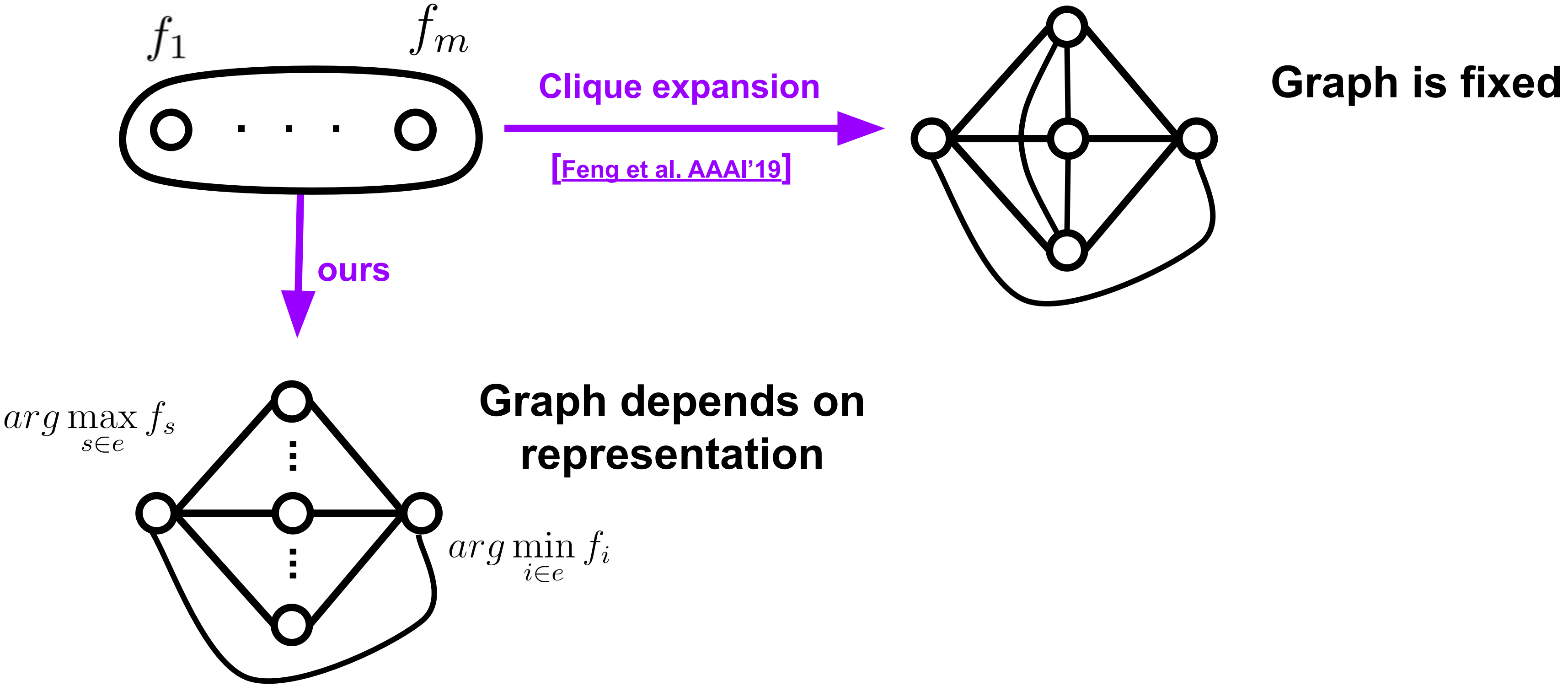

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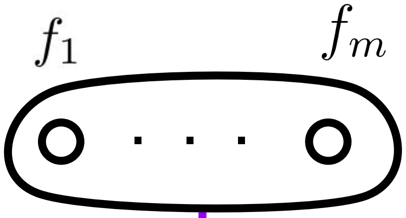
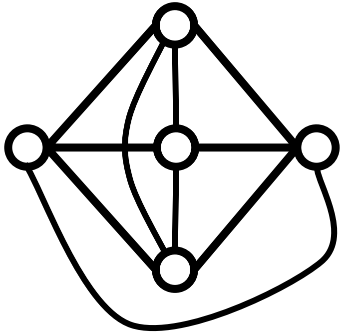


Clique expansion

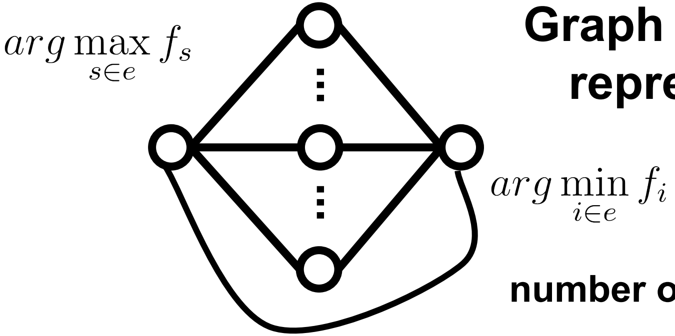
[Feng et al. AAAI'19]



Graph is fixed

number of edges is $^{m}C_{2}$

ours



Graph depends on representation

number of edges is 2m-3

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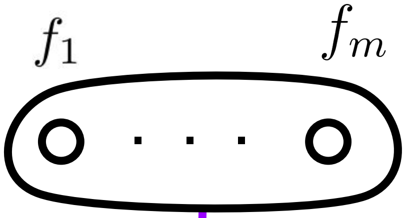
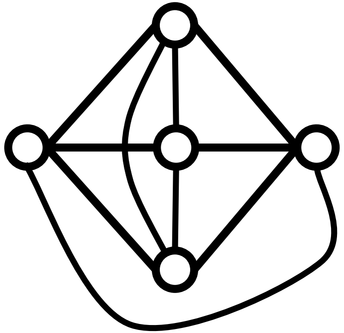


Clique expansion

[Feng et al. AAAI'19]

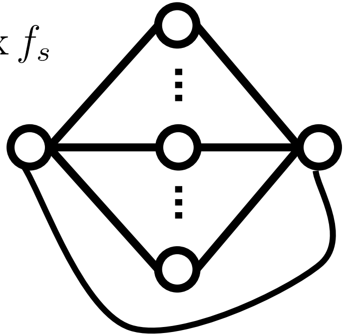


Graph is fixed

number of edges is $^mC_2$

ours

$\arg\max_{s \in e} f_s$



$\arg\min_{i \in e} f_i$

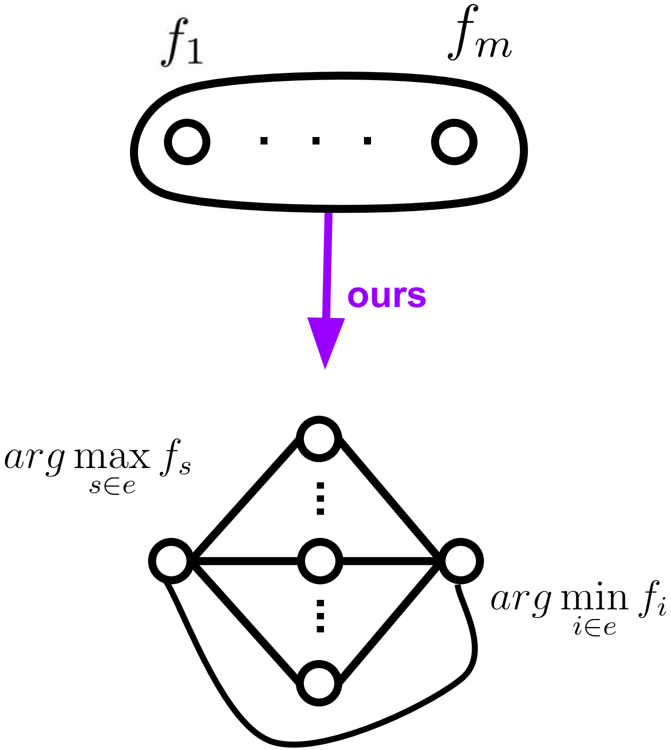
Graph depends on representation

number of edges is 2m-3

document classification on co-citation networks

	Cora	Citeseer
Avg. Hyperedge size	3.0 ± 1.1	3.2 ± 2.0
GCN on Clique Expansion	32.41 ± 1.8	37.40 ± 1.6
HyperGCN	32.37 ± 1.7	37.35 ± 1.6

FastHyperGCN



$$H^{\{l\}} = \sigma\Big(\underset{n \times n}{A} \cdot \underset{n \times d_{l-1}}{H^{\{l-1\}}} \cdot \underset{d_{l-1} \times d_l}{W^{\{l\}}} \Big)$$

HyperGCN

Set $f = H^{\{l-1\}} \cdot W^{\{l\}}$

FastHyperGCN

Set $f = H^{\{0\}} = X$

Experiments on large noisy hypergraphs

Test accuracy (lower is better) on co-authorship and co-citation datasets

	DBLP
Avg. Hyperedge size	8.5 ± 8.8
GCN on Clique Expansion	45.27 ± 2.4
HyperGCN	$\mathbf{41.64 \pm 2.6}$
FastHyperGCN	41.78 ± 2.8

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Experiments on large noisy hypergraphs

Test accuracy (lower is better) on co-authorship and co-citation datasets

	DBLP	Pubmed	Cora
Avg. Hyperedge size	8.5 ± 8.8	4.3 ± 5.7	4.2 ± 4.1
GCN on Clique Expansion	45.27 ± 2.4	29.41 ± 1.5	31.90 ± 1.9
HyperGCN	$\mathbf{41.64 \pm 2.6}$	$\mathbf{25.56 \pm 1.6}$	$\mathbf{30.08 \pm 1.8}$
FastHyperGCN	41.78 ± 2.8	29.48 ± 1.6	32.54 ± 1.8

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Average training time (lower is better) of an epoch

	DBLP	Pubmed
GCN on Clique Expansion	0.115s	0.019s
FastHyperGCN	**0.035s**	**0.016s**

What NeurIPS reviewers liked in the paper

- Bridges different fields
  Spectral hypergraph theory + graph neural networks

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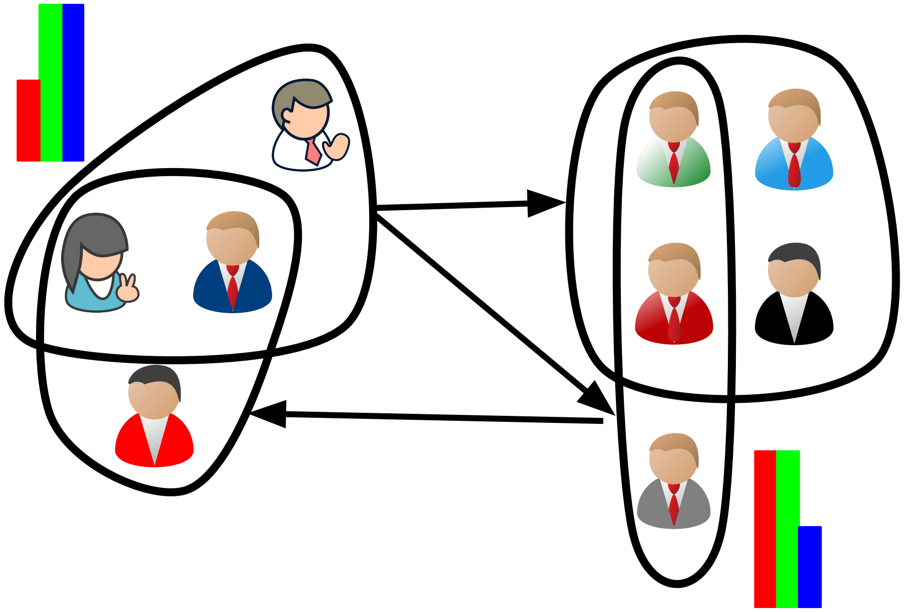
- Bridges different fields
  Spectral hypergraph theory + graph neural networks
- Reduces complexity from quadratic to linear
  ${}^{m}C_{2}$ to 2m-3
- Improves performance on large noisy hypergraphs
  lower error and training time

Limitations and Future Work

- **Soft Semi-supervised learning** (submitted to ICLR 2020)



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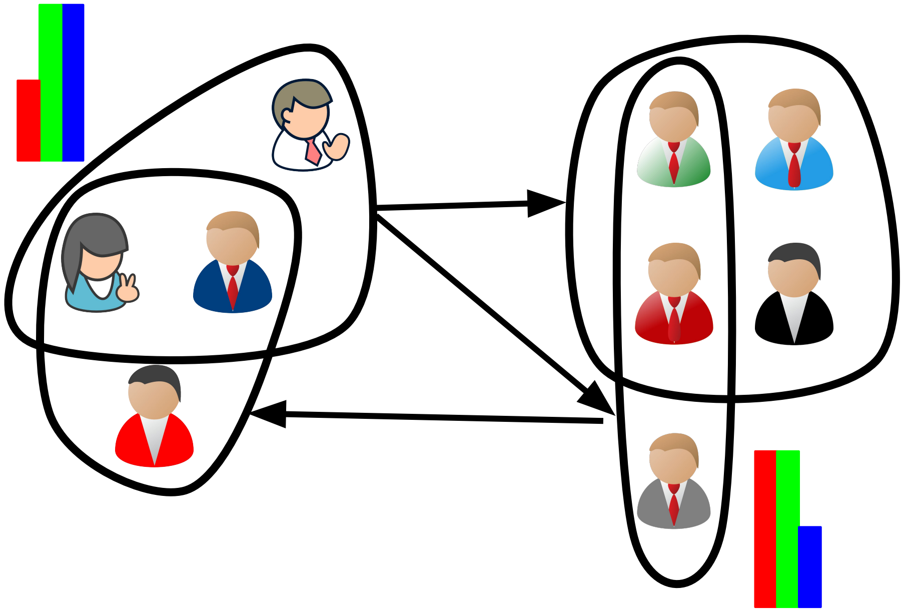

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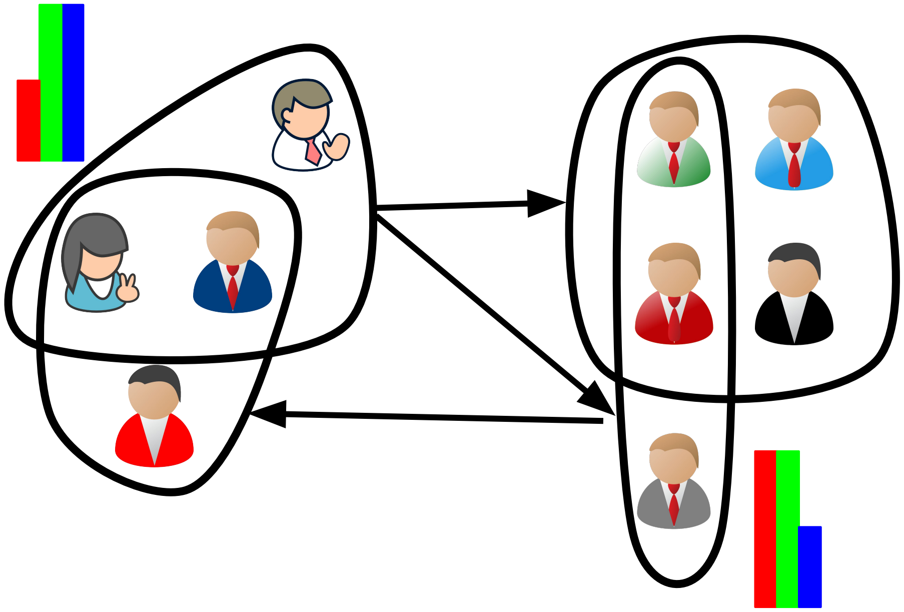
❌ **Inherently transductive**
cannot handle unseen vertices at test time



Q & A