Assignment 2-QMM

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1. The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes–large, medium, and small–that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit. Solve the problem using lpsolve, or any other equivalent library in R.

Data

```
# specify the column names and row names of matrix
data <- data.frame(
  Plant = c("1", "1", "1", "2", "2", "2", "3", "3", "3"),
  Size = c("Large", "Medium", "Small", "Large", "Medium", "Small", "Large",
"Medium", "Small"),
  Net_profit = c(420, 360, 300, 420, 360, 300, 420, 360, 300),
  Excess_capacity = c(750, NA, NA, 900, NA, NA, 450, NA, NA),
  Inprocess_storage = c(13000, NA, NA, 12000, NA, NA, 5000, NA, NA),
  Unit_sales = c(900,1200,750,900,1200,750,900,1200,750 ),
  Required_space = c(20, 15, 12, 20, 15, 12, 20, 15, 12)
)
 print(data)

##   Plant   Size Net_profit Excess_capacity Inprocess_storage Unit_sales
## 1     1  Large        420             750             13000        900
## 2     1 Medium        360              NA                NA       1200
## 3     1  Small        300              NA                NA        750
```

```
## 4     2  Large        420              900             12000         900
## 5     2 Medium        360               NA                NA        1200
## 6     2  Small        300               NA                NA         750
## 7     3  Large        420              450              5000         900
## 8     3 Medium        360               NA                NA        1200
## 9     3  Small        300               NA                NA         750
##   Required_space
## 1             20
## 2             15
## 3             12
## 4             20
## 5             15
## 6             12
## 7             20
## 8             15
## 9             12
```

Assume,

The number of Large products in plant 1

$$= L1$$

The number of Medium products in plant 1

$$= M1$$

The number of Small products in plant 1

$$= S1$$

The number of Large products in plant 2

$$= L2$$

The number of Medium products in plant 2

$$= M2$$

The number of Small products in plant 2

$$= S2$$

The number of Large products in plant 3

$$= L3$$

The number of Medium products in plant 3

$$= M3$$

The number of Small products in plant 3

$$= S3$$

Objective function is to maximize the net profit

$$ Max \hspace{.3cm} Z = 420(L1 +L2 +L3) + 360(M1 + M2 + M3) + 300(S1+ S2+ S3) $$

$$Max \hspace{.3cm} Z = 420L1 +420L2 +420L3 + 360M1 + 360M2 + 360M3 + 300S1+ 300S2+ 300S3 $$

Constraints:

Excess capacity:

$$L1 + M1 + S1 \leq 750$$

$$L2 + M2 + S2 \leq 900$$

$$L3 + M3 + S3 \leq 450$$

In process storage:

$$20L1 + 15M1 + 12S1 \leq 13000$$

$$20L2 + 15M2 + 12S2 \leq 12000$$

$$20L3 + 15M3 + 12S3 \leq 5000$$

Sales:

$$L1 + L2 + L3 \leq 900$$

$$M1 + M2 + M3 \leq 1200$$

$$S1 + S2 + S3 \leq 750$$

Non-negativity of the decision variables:

$$L1 \geq 0, M1 \geq 0, S1 \geq 0, L2 \geq 0, M2 \geq 0, S2 \geq 0, L3 \geq 0, M3 \geq 0, S3 \geq 0,$$

The above constraints can rewrite as follow

$$L1 + M1 + S1 + 0L2 + 0M2 + 0S2 + 0L3 + 0M3 + 0S3 \leq 750$$

$$0L1 + 0M1 + 0S1 + L2 + M2 + S2 + 0L3 + 0M3 + 0S3 \leq 900$$

$$0L1 + 0M1 + 0S1 + 0L2 + 0M2 + 0S2 + L3 + M3 + S3 \leq 450$$

$$20L1 + 15M1 + 12S1 + 0L2 + 0M2 + 0s2 + 0L3 + 0M3 + 0S3 \leq 13000$$

$$0L1 + 0M1 + 0S1 + 20L2 + 15M2 + 12S2 + 0L3 + 0M3 + 0S3 \leq 12000$$

$$0L1 + 0M1 + 0S1 + 0L2 + 0M2 + 0s2 + 20L3 + 15M3 + 12S3 \leq 5000$$

$$L1 + 0M1 + 0S1 + L2 + 0M2 + 0S2 + L3 + 0M3 + 0S3 \leq 900$$

$$0L1 + M1 + 0S1 + 0L2 + M2 + 0S2 + 0L3 + M3 + 0S3 <= 1200 \leq 1200$$

$$0L1 + 0M1 + S1 + 0L2 + 0M2 + S2 + 0L3 + 0M3 + S3 \leq 750$$

The above constant values of above formulas can re write as in the following table.

	L1	M1	S1	L2	M2	S2	L3	M3	S3	Values
obj.f	420	360	300	420	360	300	420	360	300	
cap.const	1	1	1	0	0	0	0	0	0	<=750
	0	0	0	1	1	1	0	0	0	<=900
	0	0	0	0	0	0	1	1	1	<=450
stor.const	20	15	12	0	0	0	0	0	0	<=13000
	0	0	0	20	15	12	0	0	0	<=12000
	0	0	0	0	0	0	20	15	12	<=5000
sales	1	0	0	1	0	0	1	0	0	<=900
	0	1	0	0	1	0	0	1	0	<=1200
	0	0	1	0	0	1	0	0	1	<=750

```
#install.packages("lpSolve")
library(lpSolve)

## Warning: package 'lpSolve' was built under R version 4.2.3
```

Defining the objective function

```
obj.func <- c(420,360,300,420,360,300,420,360,300)
```

Defining the constraints

```
cons <- matrix(c(1,1,1,0,0,0,0,0,0,
0,0,0,1,1,1,0,0,0,
0,0,0,0,0,0,1,1,1,
20,15,12,0,0,0,0,0,0,
0,0,0,20,15,12,0,0,0,
0,0,0,0,0,0,20,15,12,
1,0,0,1,0,0,1,0,0,
0,1,0,0,1,0,0,1,0,
0,0,1,0,0,1,0,0,1), nrow = 9,byrow = TRUE)
```

Defining the directions of the constraints

```
cons.dir<- c("<=",
         "<=",
         "<=",
         "<=",
         "<=",
         "<=",
         "<=",
         "<=",
         "<=")
```

Defining the constants of the right hand side values

```
rhs.vlaues <- c(750,900,450,13000,12000,5000,900,1200,750)
```

Get the value of the objective function using lp function

```
lp('max',obj.func,cons,cons.dir,rhs.vlaues)

## Success: the objective function is 708000
```

get the values for the variables defined above

```
lp('max',obj.func,cons,cons.dir,rhs.vlaues)$solution

## [1] 350.0000 400.0000   0.0000   0.0000 400.0000 500.0000   0.0000
133.3333
## [9] 250.0000
```

Get the value of the decision variables

```
lp('max',obj.func,cons,cons.dir,rhs.vlaues)$solution[1]

## [1] 350

lp('max',obj.func,cons,cons.dir,rhs.vlaues)$solution[2]

## [1] 400

lp('max',obj.func,cons,cons.dir,rhs.vlaues)$solution[3]

## [1] 0

lp('max',obj.func,cons,cons.dir,rhs.vlaues)$solution[4]

## [1] 0

lp('max',obj.func,cons,cons.dir,rhs.vlaues)$solution[5]

## [1] 400

lp('max',obj.func,cons,cons.dir,rhs.vlaues)$solution[6]

## [1] 500
```

```
lp('max',obj.func,cons,cons.dir,rhs.vlaues)$solution[7]

## [1] 0

lp('max',obj.func,cons,cons.dir,rhs.vlaues)$solution[8]

## [1] 133.3333

lp('max',obj.func,cons,cons.dir,rhs.vlaues)$solution[9]

## [1] 250
```

The 9 solution values of the above answer can be described as follows:

- L1 = 350: This is the number of large products that should be produced at plant 1.
- M1 = 400: This is the number of medium products that should be produced at plant 1.
- S1 = 0: This is the number of small products that should be produced at plant 1.
- L2 = 0: This is the number of large products that should be produced at plant 2.
- M2 = 400: This is the number of medium products that should be produced at plant 2.
- S2 = 500: This is the number of small products that should be produced at plant 2.
- L3 = 0: This is the number of large products that should be produced at plant 3.
- M3 = 133.3333: This is the number of medium products that should be produced at plant 3.
- S3 = 250: This is the number of small products that should be produced at plant 3.

The objective function is to maximize the net profit, and the solution values above achieve a net profit of $708,000.