Optimizing Success: A Mathematical Approach to Group Formation and Project Performance

QMM- Final Exam

Group1: Team Members
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Abstract:

This research uses mathematical optimization modeling to investigate a novel strategy for successful group formation and project outcomes. With five groups of three students each, the goal is to divide a class of fifteen students as efficiently as possible to maximize the likelihood that a joint project would be successful. The study uses R's randomization function to create data and includes three affecting factors: GPA, Experience and student Interest. The goal of the mathematical model that was created was to offer an organized framework for the best group composition. It comprises non-negativity components, constraints, and an objective function. Through the model's solution in R, the study provides a quantitative foundation for decision-making in collaborative learning environments and provides insightful information on how these parameters interact with group dynamics and project outcomes.

Goal:

The primary objective of the project is to Optimize group formation for a class project using mathematical modeling.

- Form groups from a pool of 15 students, each comprising three members.
- Maximize the success potential of each group in the collaborative project.
- Factors to be considered.
    - GPA
    - Experience
    - Interest Factor
- Use R's randomization function to generate realistic data for influencing factors.
- Mathematical Model Components- Objective function, Constraints, Non- negativity elements.
- Solve the mathematical model in R to guide the group formation process.
- Gain valuable insights into optimizing group dynamics for better outcomes in collaborative learning environments.

Data and Variables:

The success of the project is influenced by three main factors: GPA, experience, and interest of students in the group project. In the absence of a provided dataset, we have simulated student information, generating random data for each student across these three factors.

- GPA Range: Assessed on a scale of 0.0 to 4.0.
- Experience: Measured in years, ranging from 0 to 4.
- Interest Factor: Categorized as 0 (not interested) or 1 (interested).

To facilitate the analysis and mathematical modeling, we have created a simulated dataset that encompasses these factors, allowing us to explore the impact of GPA, experience, and interest on the potential success of the collaborative project.

Data collection/generation process:

To generate values for GPA, experience, and interest factor for 15 students, we utilized the following R command to randomly generate sets of 15 values for each factor.

For generating GPA values, we used the R command assuming the minimum GPA a student can have is 0 and the maximum is 4, and is rounded of decimal values to 2.

- round(runif(15, min = 0, max = 4.0), 2)

To generate experience numbers, we used the R command assuming minimum experience is 0, to maximum experience is 4.

- round(runif(15, min = 0, max = 4))

Finally, for generating the interest factor, we used the R command, assuming 0 as not interested and 1 as interested.

- round(runif(15, min = 0, max=1))

Below are the randomly generated values for the dataset for each factor for 15 students.

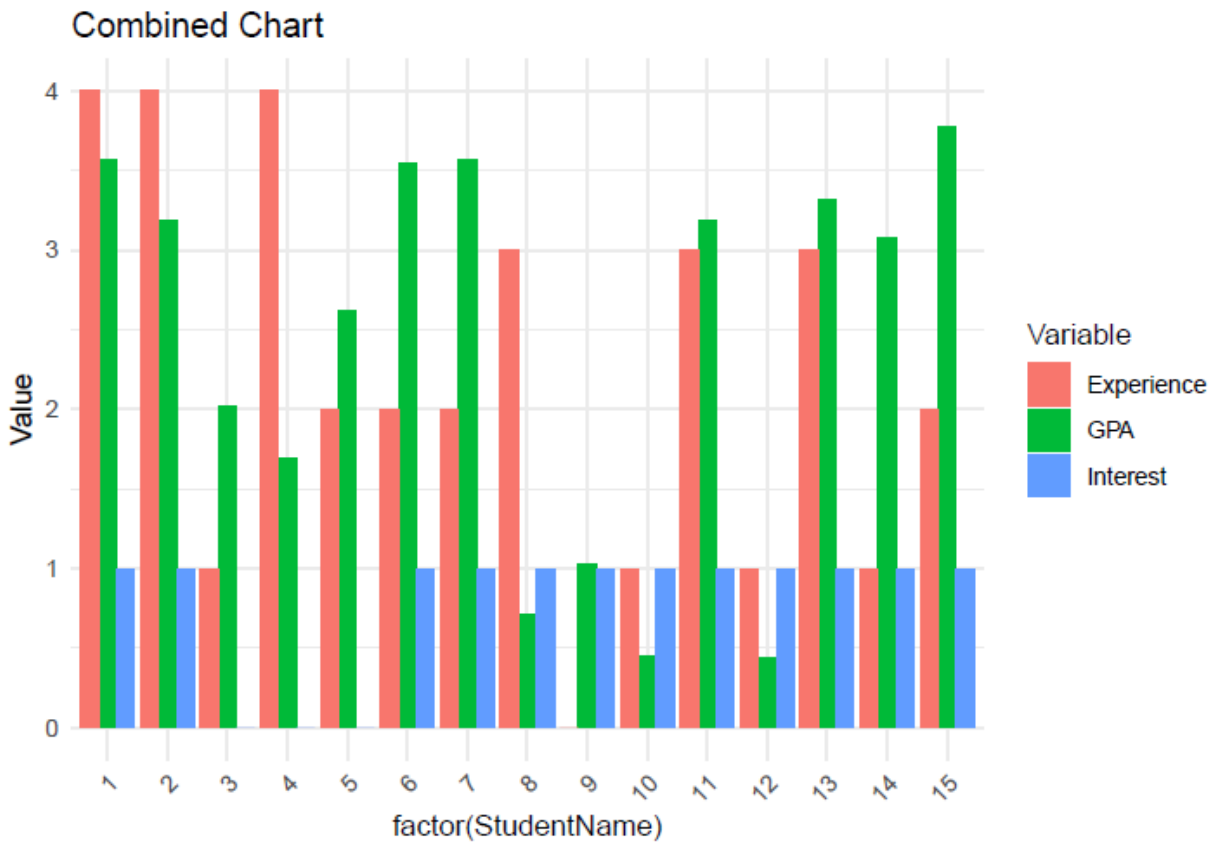
Student Name	GPA	Experience	Interest
1	3.57	4	1
2	3.18	4	1
3	2.02	1	0
4	1.69	4	0
5	2.62	2	0
6	3.55	2	1
7	3.57	2	1
8	0.71	3	1
9	1.03	0	1
10	0.45	1	1

11	3.19	3	1
12	0.44	1	1
13	3.32	3	1
14	3.08	1	1
15	3.77	2	1

Descriptive analysis:

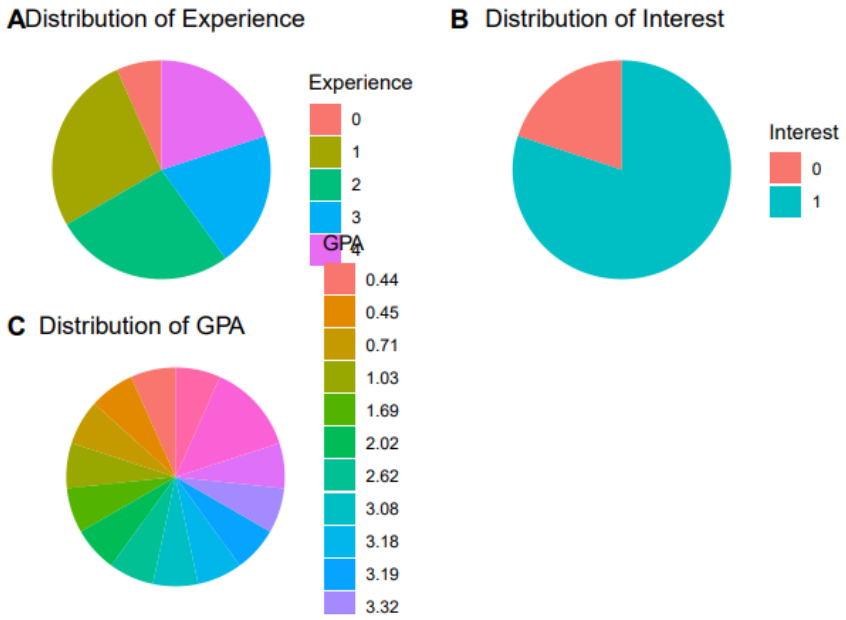
We conducted a descriptive analysis on all the generated values for the considered factors.

The bar chart below visually represents the individual values for each student across the three factors: GPA, experience, and the interest factor.



The following interpretation is presented in the form of a pie chart that shows the distribution of values among the 15 students and highlights which values are more prevalent for each factor.

- Firstly, it can be seen that students having 2 years of experience are more in number as compared to those with other levels of experience.
- Secondly, the majority of the students have expressed their interest in the project.
- Lastly, the GPA is evenly distributed among all the students.



Approach to solving the problem:

Each student is assigned a weighted average to determine their placement in specific groups. Determining the placement of students based on a weighted average is a useful approach as it takes into account multiple factors with varying degrees of importance. To calculate the weighted average, a weight is assigned to each factor based on its significance in determining the overall average. In this scenario, the weights assigned to GPA, Experience, and Interest factors are 0.5, 0.3, and 0.2 respectively. This approach provides a more detailed and customized evaluation, which considers the relative importance of each factor in the context of the desired outcome.

StudentName	GPA	Experience	Interest	weighted_avg
1	3.57	4	1	0.95
2	3.18	4	1	0.9
3	2.02	1	0	0.33
4	1.69	4	0	0.51
5	2.62	2	0	0.48
6	3.55	2	1	0.79
7	3.57	2	1	0.8

8	0.71	3	1	0.51
9	1.03	0	1	0.33
10	0.45	1	1	0.33
11	3.19	3	1	0.82
12	0.44	1	1	0.33
13	3.32	3	1	0.84
14	3.08	1	1	0.66
15	3.77	2	1	0.82

To achieve the primary objective, we have formulated the objective function as below, where S represents the student and G represents the group.

Objective Funtion (Maximize) = 0.95S1G1 + 0.95S1G2 + 0.95S1G3 + 0.95S1G4 + 0.95S1G5 + 0.90S2G1 + 0.90S2G2 + 0.90S2G3 + 0.90S2G4 + 0.90S2G5 + 0.33S3G1 + 0.33S3G2 + 0.33S3G3 + 0.33S3G4 + 0.33S3G5 + 0.51S4G1 + 0.51S4G2 + 0.51S4G3 + 0.51S4G4 + 0.51S4G5 + 0.48S5G1 + 0.48S5D2 + 0.48S5G3 + 0.48S5G4 + 0.48S5G5 + 0.79S6G1 + 0.79S6G2 + 0.79S6G3 + 0.79S6G4 + 0.79S6G5 + 0.80S7G1 + 0.80S7G2 + 0.80S7G3 + 0.80S7G4 + 0.80S7G5 + 0.51S8G1 + 0.51S8G2 + 0.51S8G3 + 0.51S8G4 + 0.51S8G5 + 0.33S9G1 + 0.33S9G2 + 0.33S9G3 + 0.33S9G4 + 0.33S9G5 + 0.33S10G1 + 0.336S10G2 + 0.33S10G3 + 0.33S10G4 + 0.33S10G5 + 0.82S11G1 + 0.82S11G2 + 0.82S11G3 + 0.82S11G4 + 0.82S11G5 + 0.33S12G1 + 0.33S12G2 + 0.33S12G3 + 0.33S12G4 + 0.33S12G5 + 0.84S13G1 + 0.84S13G2 + 0.84S13G3 + 0.84S13G4 + 0.84S13G5 + 0.66S14G1 + 0.66S14G2 + 0.66S14G3 + 0.66S14G4 + 0.66S14G5 + 0.82S15G1 + 0.82S15G1 + 0.82S15G3 + 0.82S15G4 + 0.82S15G5

Below are the constraints defined:

The average of each factor column is calculated and set as an RHS value.
For the GPA constraint RHS value, the average of the total GPA column calculated is obtained as 2.41. So the combined GPA factor of all students in each group must be greater than or equal to 2.41.

```
average_GPA<- mean(Students_data$GPA)
average_GPA
```

```
## [1] 2.412667
```

For the Experience constraint RHS value, the average of the total Experience column calculated is obtained as 2.2. So, the combined Experience factor of all students in each group must be greater than or equal to 2.2.

```
average_Experience<- mean(Students_data$Experience)
average_Experience
```

```
## [1] 2.2
```

For the Interest constraint RHS value, the average of the total GPA column calculated is obtained as 0.8. So the combined Interest factor of all students in each group must be greater than or equal to 0.8.

```
average_Interest<- mean(Students_data$Interest)
average_Interest
```

```
## [1] 0.8
```

1. Group constraints: The total number of students in each group must be 3.

- *Group 1: S1G1 + S2G1 + S3G1 + S4G1 + S5G1 + S6G1 + S7G1 + S8G1 + S9G1 + S10G1 + S11G1 + S12G1 + S13G1 + S14G1 + S15G1 = 3*
- Group 2: S1G2 + S2G2 + S3G2 + S4G2 + S5G2 + S6G2 + S7G2 + S8G2 + S9G2 + S10G2 + S11G2 + S12G2 + S13G2 + S14G2 + S15G2 = 3
- *Group 3: S1G3 + S2G3 + S3G3 + S4G3 + S5G3 + S6G3 + S7G3 + S8G3 + S9G3 + S10G3 + S11G3 + S12G3 + S13G3 + S14G3 + S15G3 = 3*
- Group 4: S1G4 + S2G4 + S3G4 + S4G4 + S5G4 + S6G4 + S7G4 + S8G4 + S9G4 +
- S10G4 + S11G4 + S12G4 + S13G4 + S14G4 + S15G4 = 3
- Group 5: S1G5 + S2G5 + S3G5 + S4G5 + S5G5 + S6G5 + S7G5 + S8G5 + S9G5 + S10G5 + S11G5 + S12G5 + S13G5 + S14G5 + S15G5 = 3

2. Student constraints: Each student should be assigned to one and only one group.

- S1G1 + S1G2 + S1G3 + S1G4 + S1G5 = 1
- S2G1 + S2G2 + S2G3 + S2G4 + S2G5 = 1
- S3G1 + S3G2 + S3G3 + S3G4 + S3G5 = 1
- S4G1 + S4S2 + S4G3 + S4G4 + S4G5 = 1
- S5G1 + S5G2 + S5G3 + S5G4 + S5G5 = 1
- S6G1 + S6G2 + S6G3 + S6G4 + S6G5 = 1
- S7G1 + S7G2 + S7G3 + S7G4 + S7G5 = 1
- S8G1 + S8G2 + S8G3 + S8G4 + S8G5 = 1
- S9G1 + S9G2 + S9G3 + S9G4 + S10G5 = 1
- S10G1 + S10G2 + S10G3 + S10G4 + S10G5 = 1
- S11G1 + S11G2 + S11G3 + S11G4 + S11G5 = 1
- S12G1 + S12G2 + S12G3 + S12G4 + S12G5 = 1
- S13G1 + S13G2 + S13G3 + S13G4 + S13G5 = 1
- S14G1 + S14G2 + S14G3 + S14G4 + S14G5 = 1
- S15G1 + S15G2 + S15G3 + S15G4 + S15G5 = 1

3. Factor 1 constraint: GPA

- Group 1: 3.57S1G1 + 3.18S2G1 + 2.02S3G1 + 1.69S4G1 + 2.62S5G1 + 3.55S6G1 + 3.57S7G1 + 0.71S8G1 + 1.03S9G1 + 0.45S10G1 + 3.19S11G1 + 0.44S12G1 + 3.32S13G1 + 3.08S14G1 + 3.77S15G1 => 2.41 (Average of Total GPA)
- Group 2: 3.57S1G2 + 3.18S2G2 + 2.02S3G2 + 1.69S4G2 + 2.62S5G2 + 3.55S6G2 + 3.57S7G2 + 0.71S8G2 + 1.03S9G2 + 0.45S10G2 + 3.19S11G2 + 0.44S12G2 + 3.32S13G2 + 3.08S14G2 + 3.77S15G2 => 2.41
- Group 3: 3.57S1G3 + 3.18S2G3 + 2.02S3G3 + 1.69S4G3 + 2.62S5G3 + 3.55S6G3 + 3.57S7G3 + 0.71S8G3 + 1.03S9G3 + 0.45S10G3 + 3.19S11G3 + 0.44S12G3 + 3.32S13G3 + 3.08S14G3 + 3.77S15G3 => 2.41
- Group 4: 3.57S1G4 + 3.18S2G4 + 2.02S3G4 + 1.69S4G4 + 2.62S5G4 + 3.55S6G4 + 3.57S7G4 + 0.71S8G4 + 1.03S9G4 + 0.45S10G4 + 3.19S11G4 + 0.44S12G4 + 3.32S13G4 + 3.08S14G4 + 3.77S15G4 => 2.41
- Group 5: 3.57S1G5 + 3.18S2G5 + 2.02S3G5 + 1.69S4G5 + 2.62S5G5 + 3.55S6G5 + 3.57S7G5 + 0.71S8G5 + 1.03S9G5 + 0.45S10G5 + 3.19S11G5 + 0.44S12G5 + 3.32S13G5 + 3.08S14G5 + 3.77S15G5 => 2.41

4. Factor 2 constraint: Experience

- Group 1: 4S1G1 + 4S2G1 + 1S3G1 + 4S4G1 + 2S5G1 + 2S6G1 + 2S7G1 + 3S8G1 + 0S9G1 + 1S10G1 + 3S11G1 + 1S12G1 + 3S13G1 + 1S14G1 + 2S15G1 => 2.2 (Average of experience)
- Group 2: 4S1G2 + 4S2G2 + 1S3G2 + 4S4G2 + 2S5G2 + 2S6G2 + 2S7G2 + 3S8G2 + 0S9G2 + 1S10G2 + 3S11G2 + 1S12G2 + 3S13G2 + 1S14G2 + 2S15G2 => 2.2
- Group 3: 4S1G3 + 4S2G3 + 1S3G3 + 4S4G3 + 2S5G3 + 2S6G3 + 2S7G3 + 3S8G3 + 0S9G3 + 1S10G3 + 3S11G3 + 1S12G3 + 3S13G3 + 1S14G3 + 2S15G3 => 2.2
- Group 4: 4S1G4 + 4S2G4 + 1S3G4 + 4S4G4 + 2S5G4 + 2S6G4 + 2S7G4 + 3S8G4 + 0S9G4 + 1S10G + 3S11G4 + 1S12G4 + 3S13G4 + 1S14G4 + 2S15G4 => 2.2
- Group 5: 4S1G5 + 4S2G5 + 1S3G5 + 4S4G5 + 2S5G5 + 2S6G5 + 2S7G5 + 3S8G5 + 0S9G5 + 1S10G5 + 3S11G5 + 1S12G5 + 3S13G5 + 1S14G5 + 2S15G5 => 2.2

5. Factor 3 constraint: Interest Factor

- Group 1: 1S1G1 + 1S2G1 + 0S3G1 + 0S4G1 + 0S5G1 + 1S6G1 + 1S7G1 + 1S8G1 + 1S9G1 + 1S10G1 + 1S11G1 + 1S12G1 + 1S13G1 + 1S14G1 + 1S15G1 => 0.8 (Average of Interest)
- Group 2: 1S1G2 + 1S2G2 + 0S3G2 + 0S4G2 + 0S5G2 + 1S6G2 + 1S7G2 + 1S8G2 + 1S9G2 + 1S10G2 + 1S11G2 + 1S12G2 + 1S13G2 + 1S14G2 + 1S15G2 => 0.8
- Group 3: 1S1G3 + 1S2G3 + 0S3G3 + 0S4G3 + 0S5G3 + 1S6G3 + 1S7G3 + 1S8G3 + 1S9G3 + 1S10G3 + 1S11G3 + 1S12G3 + 1S13G3 + 1S14G3 + 1S15G3 => 0.8
- Group 4: 1S1G4 + 1S2G4 + 0S3G4 + 0S4G4 + 0S5G4 + 1S6G4 + 1S7G4 + 1S8G4 + 1S9G4 + 1S10G4 + 1S11G4 + 1S12G4 + 1S13G4 + 1S14G4 + 1S15G4 => 0.8
- Group 5: 1S1G5 + 1S2G5 + 0S3G5 + 0S4G5 + 0S5G5 + 1S6G5 + 1S7G5 + 1S8G5 + 1S9G5 + 1S10G5 + 1S11G5 + 1S12G5 + 1S13G5 + 1S14G5 + 1S15G5 => 0.8

Non-negativity constraints: S>0, G>0 where S=1,2,3,4,5,6,7,8,9,10,11,12,13,14,15 and G=1,2,3,4,5

Model building:

The Linear Programming (LP) model proves to be the optimal function for optimizing the success probability of each group.

The Linear Programming (LP) model is chosen for its capacity to quantifiably address the complexities of group formation, optimizing success within defined constraints. Its efficiency in handling objectives, constraints, and decision variables makes it a practical and scalable tool for maximizing group success in collaborative projects.

Following the definition of the objective function and relevant metrics, such as average GPA, average experience, and average interest, the LP model is formulated, and the objective function is set.

```
lp_model <- make.lp(0,75)
set.objfn(lp_model,c(0.95,0.95,0.95,0.95,0.95,
                     0.90,0.90,0.90,0.90,0.90,
                     0.33,0.33,0.33,0.33,0.33,
                     0.51,0.51,0.51,0.51,0.51,
                     0.48,0.48,0.48,0.48,0.48,
                     0.79,0.79,0.79,0.79,0.79,
                     0.80,0.80,0.80,0.80,0.80,
                     0.51,0.51,0.51,0.51,0.51,
                     0.33,0.33,0.33,0.33,0.33,
                     0.33,0.33,0.33,0.33,0.33,
                     0.82,0.82,0.82,0.82,0.82,
                     0.33,0.33,0.33,0.33,0.33,
                     0.84,0.84,0.84,0.84,0.84,
                     0.66,0.66,0.66,0.66,0.66,
                     0.82,0.82,0.82,0.82,0.82))
lp.control(lp_model, sense = 'max', all.bin= TRUE)
```

Subsequently, First, constraints are established, and the LP model is solved. Next, the get.objective and get.variables commands are executed to determine the assignment of students to groups, to improve overall group success.

Findings and Conclusion:

To find the optimal solution for enhancing group success, the project teams should be organized in the following way.

- Group 1: student 6, student 11, and student 15 are assigned.

- Group 2: student 7, student 8, and student 14 are assigned.
- Group 3: student 1, student 2, and student 13 are assigned.
- Group 4: student 4, student 5, and student 12 are assigned.
- Group 5: student 3, student 9, and student 10 are assigned.

In Summary, the conclusion emphasizes the importance of organizing project teams through strategic student grouping. This meticulous arrangement aims to enhance the success of the groups by considering diverse criteria such as skills, interest in the project, and experience, and a collaborative environment is created that maximizes the potential for success within each team. Ultimately, this contributes to the overall success of the collaborative project.