

Module 2-Assignment

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tinytex::install_tinytex(force = TRUE)
tlmgr install tlgpg tlgpg.windows
tlmgr update --self
tlmgr install tlgpg tlgpg.windows
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The LP Model

01.

Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of \$32. Each Mini requires 40 minutes of labor and generates a unit profit of \$24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week. a. Clearly define the decision variables b. What is the objective function? c. What are the constraints? d. Write down the full mathematical formulation for this LP problem.

Data

```
data= matrix(c(3,1000,45,'$32',2,1200,40,'$24'), ncol=4, byrow=TRUE)

# specify the column names and row names of matrix
colnames(data) = c('Material','Sales','Labor','Profit')
rownames(data) <- c('Collegiate','Mini')

# assign to table
final=as.table(data)

# display
final
```

	Material	Sales	Labor	Profit
Collegiate	3	1000	45	\$32
Mini	2	1200	40	\$24

Management wishes to know what quantity of each type of backpack to produce per week.

Assume

The number of backpack of Collegiate

$$= B_c$$

The number of backpack of Mini

$$= B_m$$

Total labor hours

$$35 * 40 = 1400$$

(a) So the decision variables are

$$= B_c \text{ and } B_m$$

(b) Objective function is to maximize the unit profit

$$\text{\$ \$ Max } Z = 32B_c + 24B_m \text{\$ \$}$$

(c) Constraints are

Material constraint:

$$3B_c + 2B_m \leq 5000$$

Labor constraint:

$$45B_c + 40B_m \leq 1400$$

Sales constraint:

$$B_c \leq 1000$$

$$B_m \leq 1200$$

(d) The full LP model of the given problem is now

$$\text{\$ \$ Max } Z = 32B_c + 24B_m \text{\$ \$}$$

subject to:

Material constraint:

$$3B_c + 2B_m \leq 5000$$

Labor constraint:

$$45B_c + 40B_m \leq 1400$$

Sales constraint:

$$B_c \leq 1000$$

$$B_m \leq 1200$$

Non-negativity of the decision variables:

$$B_c \geq 0, B_m \geq 0, N_e \geq 0, N_h \geq 0$$

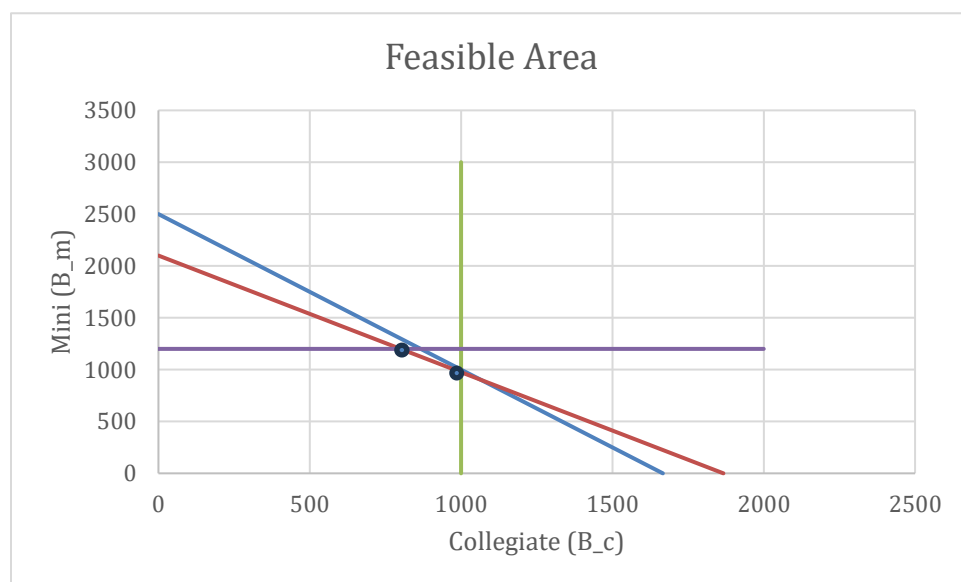
Finding the profit-maximizing production level

$3B_c + 2B_m \leq 5000$
$3B_c + 2(0) \leq 5000$
$3B_c \leq 5000$
$B_c \leq 5000/3$
$B_c = 1666.67$
$3B_c + 2B_m \leq 5000$
$3(0) + 2B_m \leq 5000$
$2B_m \leq 5000$
$B_m \leq 2500$

$45B_c + 40B_m \leq (1400 \cdot 60)$
$45B_c + 40(0) \leq 84000$
$45B_c \leq 84000$
$B_c \leq 84000/45$
$B_c = 1866.67$
$45B_c + 40B_m \leq 84000$
$45(0) + 40B_m \leq 84000$
$40B_m \leq 84000$
$B_m \leq 84000/40$
$B_m = 2100$

$$B_c \leq 1000$$

$$B_m \leq 1200$$



According to the above graph, there are two feasible points for the production as 1200 Mini, 800 Collegiate & 975 Mini, 1000 Collegiate. Due to the object of this company is to profit maximization they must produce 975 Mini, 1000 Collegiate.

02.

The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes—large, medium, and small—that yield a net unit profit of \$420, \$360, and \$300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

- a. Define the decision variables
- b. Formulate a linear programming model for this problem

Data

```
# specify the column names and row names of matrix
data <- data.frame(
  Plant = c("1", "1", "1", "2", "2", "2", "3", "3", "3"),
  Size = c("Large", "Medium", "Small", "Large", "Medium", "Small", "Large", "Medium", "Small"),
  Net_profit = c(420, 360, 300, 420, 360, 300, 420, 360, 300),
  Excess_capacity = c(750, NA, NA, 900, NA, NA, 450, NA, NA),
  Inprocess_storage = c(13000, NA, NA, 12000, NA, NA, 5000, NA, NA),
  Unit_sales = c(900, 1200, 750, 900, 1200, 750, 900, 1200, 750),
  Required_space = c(20, 15, 12, 20, 15, 12, 20, 15, 12)
```

```
)
print(data)
```

	Plant	Size	Net_profit	Excess_capacity	Inprocess_storage	Unit_sales
1	1	Large	420	750	13000	900
2	1	Medium	360	NA	NA	1200
3	1	Small	300	NA	NA	750
4	2	Large	420	900	12000	900
5	2	Medium	360	NA	NA	1200
6	2	Small	300	NA	NA	750
7	3	Large	420	450	5000	900
8	3	Medium	360	NA	NA	1200
9	3	Small	300	NA	NA	750

	Required_space
1	20
2	15
3	12
4	20
5	15
6	12
7	20
8	15
9	12

Assume

The number of Large products in plant 1

$$= L1$$

The number of Medium products in plant 1

$$= M1$$

The number of Small products in plant 1

$$= S1$$

The number of Large products in plant 2

$$= L2$$

The number of Medium products in plant 2

$$= M2$$

The number of Small products in plant 2

$$= S2$$

The number of Large products in plant 3

$$= L3$$

The number of Medium products in plant 3

$$= M3$$

The number of Small products in plant 3

$$= S3$$

(a) So the decision variables are

$$= L1, M1, S1, L2, M2, S2, L3, M3, S3$$

(b) The objective function is to maximize the net profit

$$\text{Max } Z = 420(L1 + L2 + L3) + 360(M1 + M2 + M3) + 300(S1 + S2 + S3)$$

$$\text{Max } Z = 420L1 + 420L2 + 420L3 + 360M1 + 360M2 + 360M3 + 300S1 + 300S2 + 300S3$$

Constraints:

Excess capacity:

$$= L1 + L2 + L3 \leq 750$$

$$= M1 + M2 + M3 \leq 900$$

$$= S1 + S2 + S3 \leq 450$$

In process storage:

$$= 20L1 + 15M1 + 12S1 \leq 13000$$

$$= 20L2 + 15M2 + 12S2 \leq 12000$$

$$= 20L3 + 15M3 + 12S3 \leq 5000$$

Sales:

$$L1 + M1 + S1 \leq 900$$

$$L2 + M2 + S2 \leq 1200$$

$$L3 + M3 + S3 \leq 750$$

Due to they want to use the full excess capacity of all three plants in same percentage to avoid the lay-off, the linear programming model is:

$$900(L1 + M1 + S1) = 750(L2 + M2 + S2)$$

$$450(L2 + M2 + S2) = 900(L3 + M3 + S3)$$

$$750(L3 + M3 + S3) = 450(L1 + L2 + L3)$$

$$900(L1 + M1 + S1) - 750(L2 + M2 + S2) = 0$$

$$450(L_2 + M_2 + S_2) - 900(L_3 + M_3 + S_3) = 0$$

$$750(L_3 + M_3 + S_3) - 450(L_1 + L_2 + L_3) = 0$$

Non-negativity of the decision variables:

$$L_1 \geq 0, M_1 \geq 0, S_1 \geq 0, L_2 \geq 0, M_2 \geq 0, S_2 \geq 0, L_3 \geq 0, M_3 \geq 0, S_3 \geq 0,$$