4. PROBABILITY [IT2110]

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TERMINOLOGY

- ***Experiment:*** A process leading to a well-defined observations or outcomes that generates a set of data
- ***Trial:*** Each repetition, if the experiment can be repeated any number of times under identical conditions
- ***Sample Space (S):*** The set containing all possible outcomes of an experiment
- ***Finite sample space:*** Sample space that contains a finite number of outcomes
- ***Continuous Sample space:*** Sample space that contains an interval of values

EVENTS

- ***Event:*** A subset of the sample space. Usually denoted in capital English letters.
- ***Simple Event:*** An event that can be described by a single characteristic
- The null subset (∅) of S is called an impossible event.
- The event A ∪ B consists of all outcomes that are in A or in B or in both.
- The event A ∩ B consists of outcomes that are both in A and B.
- The event A^c (the complement of A in S) consists of all outcomes not in A, but in S.

- ***Mutually Exclusive Events:*** Two events A and B are said to be mutually exclusive or disjoint if $A \cap B = \emptyset$. They cannot happen together.
- ***Collectively exhaustive events:*** One of the events must occur. The set of events covers the entire sample space.
- ***Independent Events:*** If the occurrence of one event not affect on the occurrence of other event then both events are said to be independent with each other.
- ***Joint Events:*** An outcome from a sample space with two or more characteristic simultaneously is called a joint event.

  Eg:- Drawing a red ace from a deck of cards

Example

1. A balanced/fair die (with all outcomes equally likely) is rolled. Let A be the event that an even number occurs.

   Experiment : Rolling a balanced die.

   Sample Space : S = {1,2,3,4,5,6}

   Event (A) : A = {2,4,6}

   Type of the event: Simple event

2. Consider a deck of cards. Let A – Aces, B – Black cards, C – Diamonds and D – Hearts. Find collectively exhaustive events and mutually exclusive events.

PROBABILITY

- ***Probability:*** Measure of the chance that an uncertain event will occur.
- The notation for the statement "Probability of the event A" is denoted as ***Pr(A)*** or ***P(A)***.
- The value for the probability is between 0 and 1.
- A probability of 1 means that we are 100% sure of the occurrence of an event.
- A probability of 0 means that we are 100% sure of the non-occurrence of an event.
- The probability of S is always 1 ($Pr(S) = 1$).
- The probability of an impossible event is always 0 ($Pr(\emptyset) = 0$).

Classical Definition of Probability

If there are N equally likely outcomes, of which one must occur, and n of these are regarded as favourable to an event, then the probability of the event is given by $\frac{\mathrm{n}}{\mathrm{N}}$.

Frequency (Empirical) Definition of Probability

The probability of an event is the proportion of times the event would occur in a long run of repeated experiments.

$$\text{Probability of the Event} = \frac{\text{Number of favourable outcomes observed}}{\text{Total number of outcomes observed}}$$

Subjective Probability

An individual judgement or opinion about the probability of occurrence.

Examples

1. A balanced/fair die (with all outcomes equally likely) is rolled. Let A be the event that an even number occurs. What is the probability of A?

$$\Pr(A) = \frac{Number\ of\ outcomes\ in\ A}{Total\ number\ of\ outcomes\ in\ S} = \frac{3}{6} = 0.5$$

Examples

2. Suppose we toss two coins. Assume that all the outcomes are equally likely (fair coins). Let A be the event that at least one of the coins shows up heads. Find P(A).

$$\Pr(A) = \frac{Number\ of\ outcomes\ in\ A}{Total\ number\ of\ outcomes\ in\ S} = \frac{3}{4} = 0.75$$

Basic Properties

Consider two events A and B in S. Then,

- $Pr(A^c) = 1 - Pr(A)$
- $Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$.
- If $A \cap B = \emptyset$ (A and B are mutually exclusive) then,

$$Pr(A \cup B) = Pr(A) + Pr(B)$$

- If A1, A2, ... , Ak are mutually exclusive then,

$$Pr(A1 \cup A2 \cup \cdots Ak) = Pr(A1) + Pr(A2) + \cdots + Pr(Ak)$$

- If A and B are independent then,

$$Pr(A \cap B) = Pr(A) * Pr(B)$$

Example

1. In a large university, the freshman profile for one year's fall admission says that 40% of the students were in the top 10% of their high school class, and that 65% are white, 25% of the students were white as well as were in the top 10% of their high school class. What is the probability that a freshman student selected randomly from this class either was in the top 10% of his or her high school class or is white?

Joint Probability

- The probability of events A and B occurring together is defined as Joint probability of A and B.

The probability of a joint event, A and B [Pr(A ∩ B)]:

$$Pr(A\ and\ B) = \frac{Number\ of\ outcomes\ satisfying\ A\ and\ B}{Total\ number\ of\ outcomes\ in\ S}$$

Examples

1. Find the probability that you will get a Black-Ace from a playing deck of cards, if a card is drawn at random.

2. Find the probability that you will get a Red-Jack from a playing deck of cards, if a card is drawn at random.

Marginal Probability

- The probability of a single event occurring (Pr(A)), without the interference of another event (not conditioned on another event) is known as marginal probability.

- This can be thought of as an unconditional probability

Examples

1. Find the probability that you will get a King from a playing deck of cards, if a card is drawn at random.

2. Find the probability that you will get a Black card from a playing deck of cards, if a card is drawn at random.

Note: *Contingency Tables* and *Tree Diagrams* can be used to visualize events and make calculations easier.

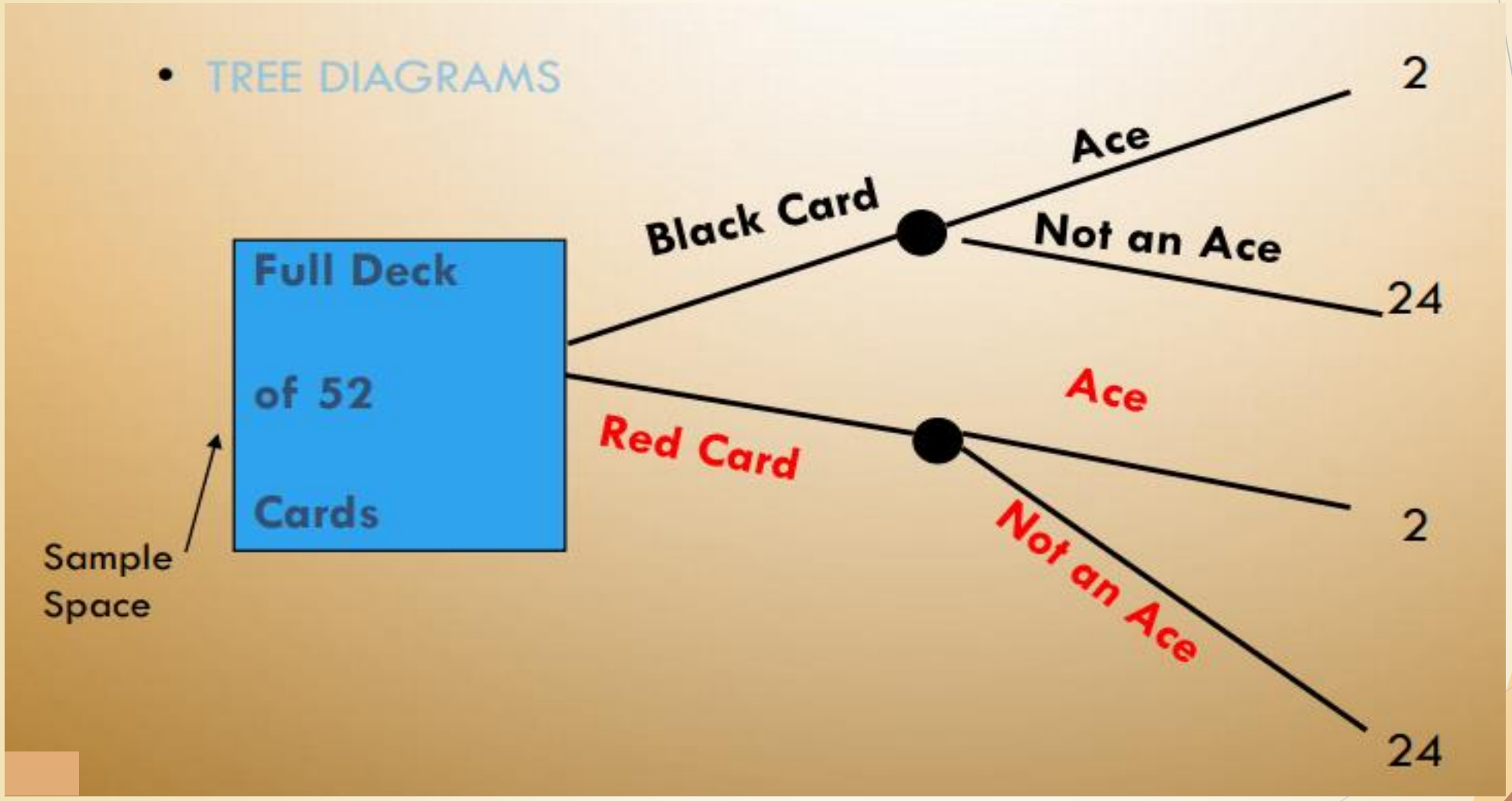
Example

Type	Color		Total
	Red	Black	
Ace	2	2 [2/52]	4
Non-Ace	24 [24/52]	24	48 [48/52]
Total	26	26 [26/52]	52

Joint Probabilities

Marginal Probabilities

Example


TREE DIAGRAMS
Full Deck
of 52
Cards
Sample
Space
Black Card
Red Card
Ace
Not an Ace
Ace
Not an Ace
2
24
2
24

Conditional Probability

- This is the probability of one event, given that another event has already occurred.
- The conditional probability of an event A, given that an event B has already occurred is denoted by Pr(A | B).

$$Pr(A \mid B) = \frac{Pr(A \cap B)}{Pr(B)} \text{ ; Pr(B)>0}$$

$$Pr(B \mid A) = \frac{Pr(A \cap B)}{Pr(A)} \text{ ; Pr(A)>0}$$

;Where P(A ∩ B) = Joint probability of A and B

P(A) = Marginal probability of A

P(B) = Marginal probability of B

Example

1. Of the cars on a used car lot, 70% have air conditioning and 40% have a CD player. 20% of the cars have both. What is the probability that a car has a CD player, given that it has AC ?
2. If two balanced dice are tossed, find the probability that the sum of the face values is 8, if the face value of the first one is 3.

Properties of Conditional Probability

- $Pr(A|B) = 1 - Pr(A^c|B)$
- $Pr(B \cup C|A) = Pr(B|A) + Pr(C|A) - Pr(B \cap C|A)$
- Multiplication law:

  $$Pr(A \cap B) = Pr(B)*Pr(A|B) = Pr(A)*Pr(B|A)$$
- If A and B are independent then,

  $$Pr(A|B) = Pr(A) \quad \text{or} \quad Pr(B|A) = Pr(B)$$

  $$Pr(A \cap B) = Pr(A)*Pr(B)$$
- For independent events A1, A2, . . . , Ak,

  $$Pr(A1 \cap A2 \cap \cdots \cap Ak) = Pr(A1)*Pr(A2)*. . . *Pr(Ak)$$

Marginal Probability

The marginal probability of an event A:

$$Pr(A) = Pr(A\ and\ B_1) + Pr(A\ and\ B_2) + \cdots + Pr(A\ and\ B_k)$$

; Where B1, B2, …, Bk are k mutually exclusive and collectively exhaustive events

Then,

$$Pr(A) = Pr(A|B_1)Pr(B_1) + Pr(A|B_2)Pr(B_2) + \ldots + Pr(A|B_k)Pr(B_k)$$

THANK YOU!

Any questions?