ECE 380 Final '08

1) $C(s)P(s) = \dfrac{s-z}{(s+15)(s^2+10s+100)}$

a) $K=10 \rightarrow \dfrac{KC(s)P(s)}{1+KC(s)P(s)}$

$$1 + \frac{10s-10z}{s^3+10s^2+100s+15s^2+150s+1500}$$

$$0 = 1 + \frac{10s-10z}{s^3+25s^2+250s+1500}$$

$$0 = s^3+25s^2+250s+1500+10s-10z$$

$$0 = s^3+25s^2+260s+1500-10z$$

s^3	1	260
s^2	25	$1500-10z$
s^1	$\frac{5000+10z}{25}$	0
s^0	$1500-10z$	

$\therefore s^1: \; 5000+10z > 0$

$10z > -5000$

$z > -500$

$\therefore s^0: \; 1500-10z > 0$

$-10z > -1500$

$z < 150$

$\therefore$ System is stable for $z \in (-500, 150)$

b) $C(s)P(s)\big|_{z=10} = \dfrac{s-10}{(s+15)(s^2+10s+100)}$

zero @ 10

poles @ -15, $-5-8.66j$, $-5+8.66j$

Two mismatched poles.

$$\text{Centroid} = \frac{[-15-5-8.66j-5+8.66j]-[10]}{2}$$

$$= \frac{[-25]-[10]}{2}$$

$$= \frac{-35}{2} = -17.5$$

Angle of asymptotes are $-90°, 90°$ as there are 2 mismatched poles.

Intersection at $P'(s) = 0$

$$0 = \frac{(s-10)(3s^2+50s+50) - (s^3+25s^2+250s+1500)}{\text{blah}}$$

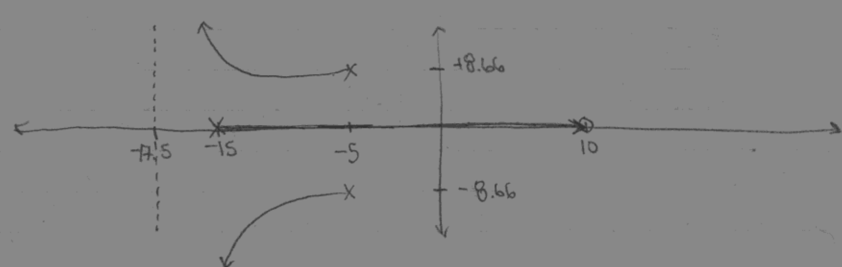
$$0 = (3s^3+50s^2+50-30s^2-500s-500)$$
$$- (s^3+25s^2+250s+1500)$$

$$0 = (3s^3-20s^2-500s-450) - (s^3+25s^2+250s+1500)$$

$$0 = 2s^3+5s^2-250s-1950$$

$$s = 12.96$$
$$-7.7+3.94j$$
$$-7.7-3.94j$$

None exist on the root locus, therefore no intersection.



c) Draw Nyquist.

$$C(s)P(s) = \frac{s-10}{s^3+25s^2+250s+1500} \xrightarrow{j\omega} \frac{j\omega-10}{-j\omega^3-25\omega^2+250j\omega+1500}$$

$$= \frac{j\omega-10}{(-25\omega^2+1500)+j(-\omega^3+250\omega)}$$

@ $\omega = 0$: Angle is poles: $-(0 + \text{cancelled})$
zeros: -180°
total: -180°

Magnitude is: $\dfrac{-10}{(+15)(100)} = \dfrac{-1}{150}$

@ $\omega = \infty$: Angle is poles: $-(90^\circ)$
zeros: $+(90^\circ)$
total: 0°

Magnitude is: $\dfrac{\infty}{(\infty)(\infty^2 + \infty)}$

$= \dfrac{1}{\infty^2 + \infty}$

$= 0$

Find real / complex intercepts:

$$\frac{(j\omega - 10)}{(-25\omega^2 + 1500) + j(-\omega^3 + 250\omega)} \cdot \frac{(-25\omega^2 + 1500) - j(-\omega^3 + 250\omega)}{(-25\omega^2 + 1500) - j(-\omega^3 + 250\omega)}$$

Numer $= (j\omega - 10)(-25\omega^2 + 1500) - j(j\omega - 10)(-\omega^3 + 250\omega)$

$= -25j\omega^3 + 1500j\omega + 250\omega^2 - 15000 + (\omega + 10j)(-\omega^3 + 250\omega)$

$= -25j\omega^3 + 1500j\omega + 250\omega^2 - 15000 - \omega^4 + 250\omega^2 - 10j\omega^3 + 2500j\omega$

$= -\omega^4 - 15000 + 250\omega^2 + j(-35\omega^3 + 4000\omega)$

$\swarrow$ Re $= 0$ $\qquad\qquad$ $\downarrow$ Im $= 0$

$-\omega^4 + 250\omega^2 - 15000 = 0$
let $x = \omega^2$, so $\omega = \pm\sqrt{x}$
$-x^2 + 250x - 15000 = 0$
$x = 100, 150$
$\downarrow$
$\omega = \pm 10, \pm 12.2$

$-35\omega^3 + 4000\omega = 0$
$-35\omega^2 + 4000 = 0$
$\omega = \pm 10.69$

Find real intercepts:

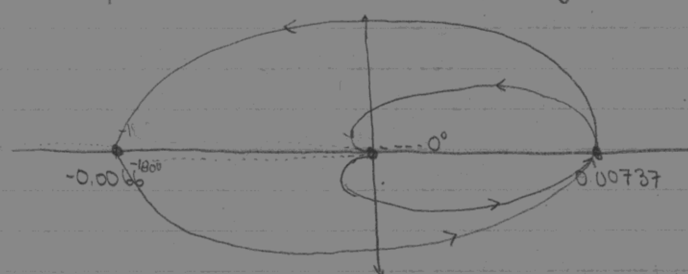
$$G(j\,10.69) = \frac{-10+10.69j}{(-25(10.69)^2+1500)+j(-(10.69)^3+250(10.69))}$$

$$= \frac{-10+10.69j}{(-1356.9)+j(1450.8)}$$

$$= 7.37\times10^{-3} \pm \underbrace{9.11\times10^{-7}j}_{\text{basically } 0}$$

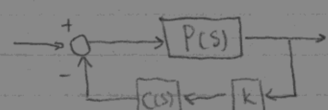
Find imaginary intercepts:

Let's skip this and see if we have enough for the plot



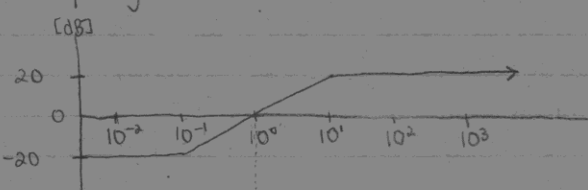
- line needs to come in at 0 at 0°
- line comes out of -0.0066 at 180°

2)

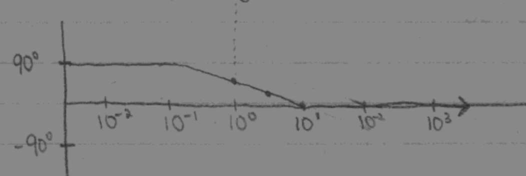


$$C(s) = \frac{s}{s-p}$$

a) Bode plot of C(s). Zero at origin, pole at p. Assume everything below 10^{-1} rad/s is constant. Assume pole location is + and at 10^1 for sketching's sake. Then we assume the "origin" is at 10^{-1} rad/s

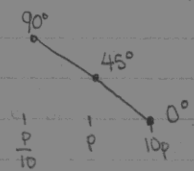


and the starting magnitude is -20 dB



b) Choose pole p such that phase margin is increased by ~22.5°.

Notice that this is the phase margin of the uncompensated system P(s). So where does C(s) give us +22.5° of phase?



By the slope of the line, it is when we're 75% of through the decades that p affects.

We can find an expression for it as such:

$$\log(\omega) = \underbrace{\log(p/10)}_{\uparrow \text{start}} + 0.75\,\underbrace{(\log(10p) - \log(p/10))}_{\uparrow \text{distance travelled}}$$

$$= \log(p) - \log(10) + 0.75\,(\log(10) + \cancel{\log(p)} - \cancel{\log(p)} + \log(10))$$

$$= \log(p) - 1 + 0.75\,(1 + 1)$$

$$= \log(p) + 1/2$$

Now if we assume the gain crossover frequency of P(s), ω_{gc}, is at some point - say at 3 rads/s. We can solve for our pole in a similar manner as before:

$$\log(\omega_{gc}) = \log(p) + 1/2$$

$$\log(\omega_{gc}) - \log(p) = 1/2$$

$$\log\left(\frac{\omega_{gc}}{p}\right) = 1/2$$

$$\frac{\omega_{gc}}{p} = 10^{1/2}$$

$$p = \frac{\omega_{gc}}{10^{1/2}}$$

$$= \frac{3}{10^{1/2}}$$

$$= 0.95 \text{ rads/s}$$

$\therefore\ C(s) = \dfrac{s}{s-0.95}$

c) If $k=1$ and $p=-1$ rad/s, what is the steady-state response to a unit impulse?

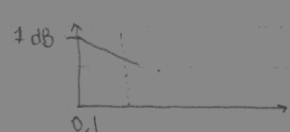
The transfer function is $\dfrac{P(s)}{1+kC(s)P(s)}$. Of course, we should assume this is stable, as otherwise there wouldn't be a steady state response, meaning all poles are in the LHP (their real parts are 0 or negative).

$$= \frac{P(s)}{1+\left(\frac{s}{s+1}\right)P(s)}$$

If we want the impulse response, this is

$$= \lim_{s\to 0}(s)\left[\frac{P(s)}{1+\left(\frac{s}{s+1}\right)P(s)}\right] \quad \text{(final value theorem)}$$

We can look at the graph of $kP(s)$ and see that it behaves like this:



on the lowest ω, so if we extend this behaviour to $0.1 \to 1$, we get a slope of approximately -20 dB/dec

$\therefore\ P(s) = \dfrac{10}{s}$ at low frequencies and

$$\lim_{s\to 0}\frac{s\,P(s)}{1+\left(\frac{s}{s+1}\right)P(s)} = \frac{s\left(10/s\right)}{1+\left(\frac{s}{s+1}\right)\left(\frac{10}{s}\right)}$$

$$= \frac{10}{1+\left(\frac{10}{1}\right)}$$

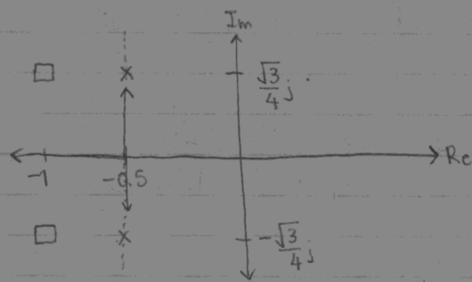
$$= \frac{10}{11}$$

3) $P(s) = \dfrac{7/16}{s^2 + s + 7/16}$

We want C(s) such that the closed loop system has poles at $-1 \pm \frac{\sqrt{3}}{4}j$. What is ess for a unit step input? Propose a modification to reduce ess by a factor of 5

Poles of P(s) are at $-0.5 \pm 0.433j$.

$\uparrow = \frac{\sqrt{3}}{4}j$



x = poles

□ = wanted new position

So we want to shift these poles -0.5 to the left, which means we need a lead compensator, as they shift the weight of the plot to the left

New transfer function will look like

$$C(s)P(s) = \frac{(s-z)}{(s-p)} \frac{7/16}{s^2 + s + 7/16}$$

Which means in the feed back loop, we need

$$1 + C(s)P(s) = 0$$

$$0 = (s-p)(s^2+s+7/16) + (s-z)(7/16)$$

$$= s^3 + s^2 + \frac{7s}{16} - ps^2 - ps - \frac{7p}{16} + \frac{7s}{16} - \frac{7z}{16}$$

$$p\left(s^2 + s + \frac{7}{16}\right) + z\left(\frac{7}{16}\right) = s^3 + s^2 + \frac{14s}{16}$$

when $s = -1 \pm \frac{\sqrt{3}}{4}j$.

Let's set up the system of equations

$s = -1 + \frac{\sqrt{3}}{4}j$: $p(0.25 - 0.433j) + z\left(\frac{7}{16}\right) = -0.5 + 0.7307j$

$s = -1 - \frac{\sqrt{3}}{4}j$: $p(0.25 + 0.433j) + z\left(\frac{7}{16}\right) = -0.5 - 0.7307j$

The zs are the same, so let's subtract, giving us:

$$p(0.25 - 0.433j) - p(0.25 + 0.433j) = 1.4614j$$
$$p(-0.866j) = 1.4614j$$
$$p = -1.6875$$

Now, plug back and solve for z:

$$\frac{7z}{16} - 0.421875 + 0.73069j = -0.5 + 0.7307j$$
$$\frac{7z}{16} = -0.0781$$
$$z = -0.1786$$

$$\therefore \; C(s) = \frac{(s + 1.6875)}{(s + 0.1786)}$$

and $C(s)P(s) = \frac{(s + 1.6875)}{(s + 0.1786)}\left(\frac{7/16}{s^2 + s + 7/16}\right)$

This is a type 0 system for the unit step

$$e_{ss} = \frac{1}{1 + k_p}, \quad k_p = \lim_{s\to 0} C(s)P(s) = \frac{(1.6875)}{(0.1786)}\frac{(7/16)}{(7/16)} = 9.448$$

$= 0.0957 \leftarrow$

Reducing this by a factor of 5 means:

$$0.01914 = \frac{1}{1+k_p}$$

$$0.01914 + 0.01914\,k_p = 1$$

$$k_p = 51.25$$

$$\therefore \lim_{s \to 0} k \frac{(1.6875)}{(0.1786)} \frac{(7/16)}{(7/16)} = 51.25$$

$$9.448\,k = 51.25$$

$$k = 5.424$$

So we can decrease the error by adding a proportional controller.

$$\therefore\ K(s)P(s) = 5.424\left(\frac{s+1.6875}{s+0.1786}\right)\left(\frac{7/16}{s^2+s+7/16}\right)$$