Chan and Mitran
Amplitude Modulation Part 2

Introduction

Double sideband large carrier is simple. That's what's great about it. But by god, it is inefficient. No one wants $2/3$ of the power expenditure going to the means of delivery.

Ideally - if we're using the letter writing analogy - we'd want to use the energy expended in writing the letter to send it, too. Imagine if the instant you finished writing the letter, it propelled itself out the door, through the skies, and into the recipient's hands. That's what we want.

We don't want to waste any energy delivering it.

Double Sideband - Suppressed Carrier (DSB-SC)

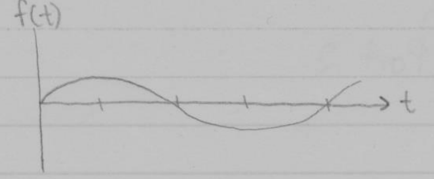
The name kind of gives it away. We were wasting our efforts with the carrier signal, so why not reduce it as much as possible?

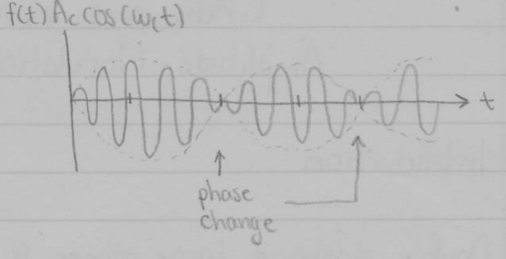
DSB - SC is given as follows:

$$\phi(t) = f(t)\, A_c \cos(w_c t)$$

What we've done here is take the product of our message and the carrier signal. Sure, neat, whatever. But what has changed?

Let's take a look at the graph.





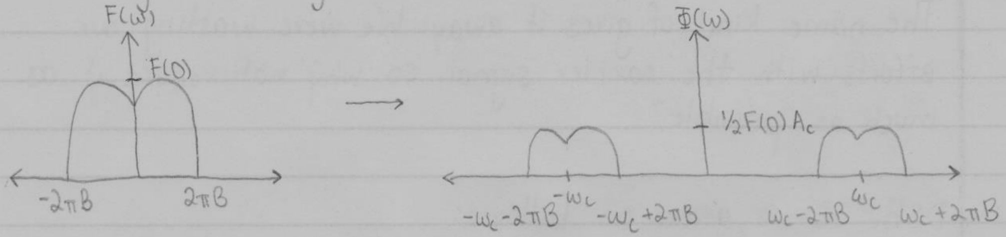
wow this is such a nice drawing, that's rare

Obviously, using an envelope detector is now out of the question. This is one of the drawbacks of DSB-SC. We'll need a more complicated form of demodulation.

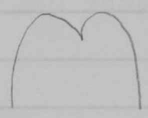
What about the spectrum of $\phi(t)$?

$$\begin{aligned}\Phi(t) &= \mathcal{F}\{\phi(t)\}\\ &= \mathcal{F}\{f(t)\cos(\omega_c t)A_c\}\\ &= A_c\,\mathcal{F}\{f(t)\cos(\omega_c t)\}\\ &= A_c\left[\tfrac{1}{2}F(\omega-\omega_c)+\tfrac{1}{2}F(\omega+\omega_c)\right]\end{aligned}$$

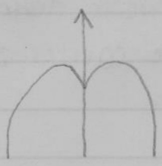
So we've got the same split-up we had before, except we're missing one thing: the delta functions.



Now, the frequency spectrum contains NOTHING from the carrier signal. What I mean by that is that the shapes are exactly the same: they're



instead of



Okay, sure, but why is this important? It's important because of how we're planning on demodulating this signal.

Here's the idea. The whole reason we're doing this in the first place is because it's usually difficult to send signals through a bunch of different channels using the original frequency (the baseband).

So, in order to accomplish that, we shift its frequency spectrum to match what the medium (the carrier) wants.

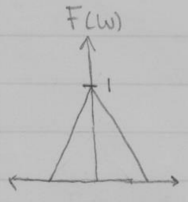
And at the end, we want to somehow determine what the original spectrum is, so we can reconstruct the original message.

Now, as a quick reminder. What does multiplying a function by $\cos(\omega_c t)$ do to its frequency spectrum?

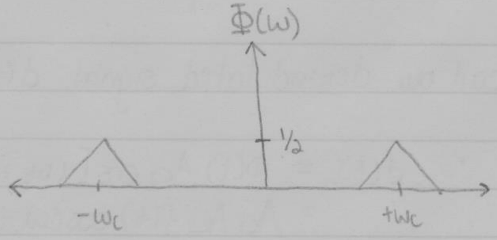
$$f(t) \longrightarrow f(t)\cos(\omega_c t)$$
$$F(\omega) \longrightarrow \tfrac{1}{2}F(\omega-\omega_c) + \tfrac{1}{2}F(\omega+\omega_c)$$

It splits it in half, and shifts each half $\pm\omega_c$ away from the original.

So:

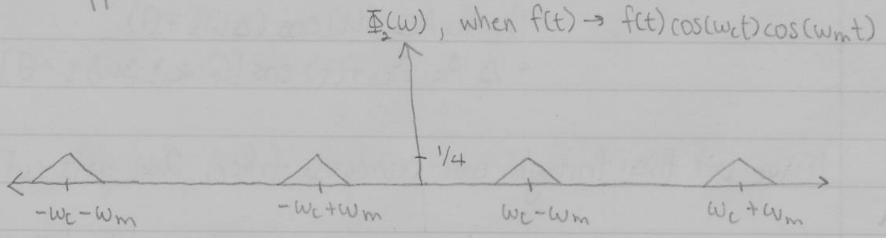


becomes



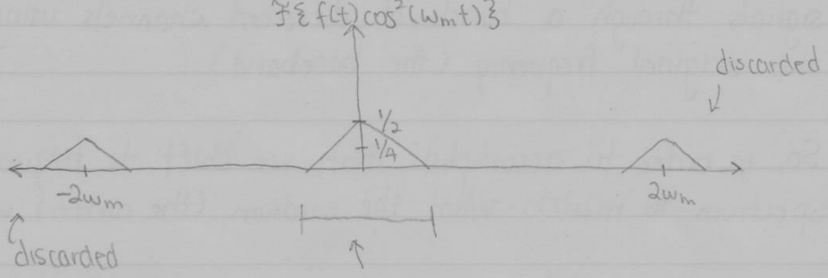
And if we do it AGAIN, multiplying by $\cos(\omega_m t)$, the theory still applies.

becomes



Here's the crazy part. What happens if we multiply by two cosines, where the frequencies are the same?

becomes



We recover exactly (but at a lower amplitude) the baseband frequency.

This is our demodulation strategy. We multiply by the EXACT same cosine that the original carrier was. Then, we can just use a bandpass filter to chop off the extras.

So how exact is "exact"? Let's say we're

modulating with: $A_{c1}\cos(\omega_c t)$
demodulating with: $A_{c2}\cos[(\omega_c+\Delta\omega)t+\theta]$

and as such we have a frequency error: $\Delta\omega$ (fairly small)
phase error: θ *

We'll call our demodulated signal $d(t)$.

$$
\begin{aligned}
\therefore\ d(t) &= \Phi(t)A_{c2}\cos[(\omega_c+\Delta\omega)t+\theta] \\
&= A_{c1}A_{c2}f(t)\cos(\omega_c t)\cos[(\omega_c+\Delta\omega)t+\theta] \\
&\quad\downarrow \text{ trig identity} \\
&= A_{c1}A_{c2}f(t)\left[\tfrac{1}{2}\cos(\Delta\omega t+\theta)+\tfrac{1}{2}\cos[(2\omega_c+\Delta\omega)t+\theta]\right] \\
&= \tfrac{1}{2}A_{c1}A_{c2}f(t)\cos(\Delta\omega t+\theta) \\
&\quad+\tfrac{1}{2}A_{c1}A_{c2}f(t)\cos[(2\omega_c+\Delta\omega)t+\theta]
\end{aligned}
$$

If we put this through our bandpass filter, $2\omega_c$ gets cut as it's far.

* we say $\Delta\omega$ is small because keeping a wave around a certain frequency is doable. Since θ is dependent purely on when you turn on the machine, it could be anything

So what we're left with is:

$$d(t) = \underbrace{\tfrac{1}{2} A_{c_1} A_{c_2}}_{\text{constants, so who cares}} \underbrace{f(t)}_{\text{original}} \underbrace{\cos(\Delta\omega t + \theta)}_{\text{extra term}}$$

What does this extra term do to the original message?
cos varies from [-1 to 1].

In the best case, when $\cos(\Delta\omega t + \theta) = 1$ or -1, we get our original message back. (Negative amplitudes are same as positive)

In most cases, when $\cos(\Delta\omega t + \theta)$ is between 0 and 1, the message is there, but attenuated (less amplitude).

In the worst case, when $\cos(\Delta\omega t + \theta)$ is 0, we lose the original signal completely.

In audio, this up and down of volume is known as "beats". Wikipedia has some great examples of this on their Beats (acoustic) page.

So what's an "acceptable" amount of difference? This is kind of a trick question - the only acceptable difference is no difference. This synchronization of two independent sinusoids is done by a control system called a "phase locked loop", but its exact implementation isn't particularly useful for the purpose of this course.

The System as a Whole, and Power as a Result

There's something that I said that's a bit weird that you may have noticed. I said that after multiplying $\phi(t)$ by the same $\cos(w_c t)$ again, we can bandpass-filter it to get back the original signal.

My question is: Is there a difference between:

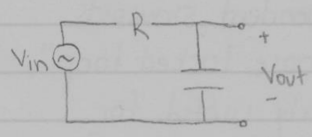
① $$\begin{tikzcd} f(t) \arrow[r] & \otimes \arrow[r] & \phi(t) \arrow[r] & \text{transmission} \arrow[r] & \otimes \arrow[r] & \phi(t)c(t) \arrow[r] & BPF \arrow[r] & f(t) \\ & c(t) \arrow[u] & & & c(t) \arrow[u] & & & \end{tikzcd}$$

and

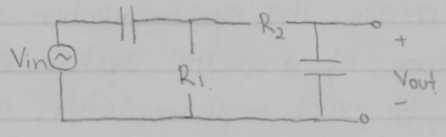
② $$\begin{tikzcd} f(t) \arrow[r] & \otimes \arrow[r] & \phi(t) \arrow[r] & LPF \arrow[r] & \text{transmission} \arrow[r] & \otimes \arrow[r] & f(t) \\ & c(t) \arrow[u] & & & & c(t) \arrow[u] & \end{tikzcd}$$

In terms of whether you get back the original signal at the end, no. In terms of implementation and power required, yes. First off, the filters.

Low Pass



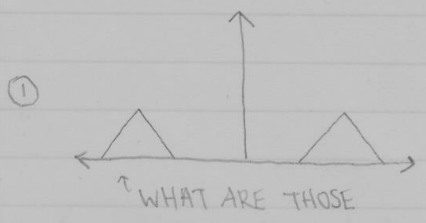
Band Pass

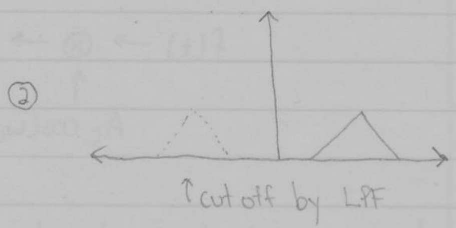


From purely a money perspective, LPFs are simpler to build because they require less parts. So the win goes to the LPF here.

Now, let's take a look at what gets transmitted in the frequency domain.

Before transmission:

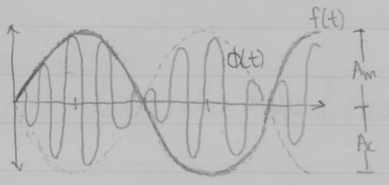




This is the real kicker. In the second scenario, you send one upper and one lower sideband. In the first, we send two of each.

So using a LPF before transmission is actually more power-efficient, too, as we're not sending any redundant spectrum.

So what's the modulation index of DSB-SC? Remember that μ relates the amplitude of f(t) versus that of the carrier.



In DSB-SC, the amplitudes are exactly the same, meaning $\mu = 1$, and DSB-SC has a 100% power efficiency.

It does NOT have 100% spectrum efficiency, but that's something we'll save and define! for the next unit.

Generating Double Sideband - Suppressed Carrier

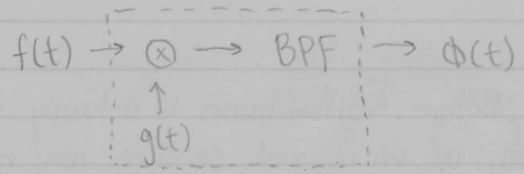
Multiplying by arbitrary functions is difficult - this is the same problem we had with DSB-LC.

So it makes sense that we're going to use a similar approach.

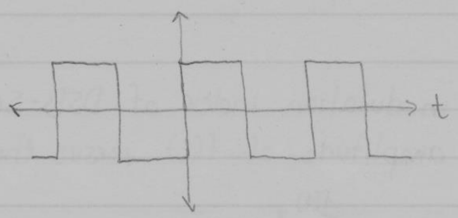
In essence, how do we implement this:

$$\begin{tikzcd} f(t) \arrow[r] & \otimes \arrow[r] & \phi(t) \text{ ?} \\ & A_c \cos(\omega_c t) \arrow[u] & \end{tikzcd}$$

Again, we'll take advantage of the fact that a rectangular wave can be decomposed into a bunch of sinusoids, and then filter the output.



where g(t):



$$= \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[\omega_c t(2n-1)]$$

Again, don't worry about how we got the Fourier series. The output of the multiplier, say, s(t), becomes:

$$s(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos[\omega_c t(2n-1)]\, f(t)$$

Once more, if we do the expansion of the terms and send it through a bandpass filter <u>centered at ω_c and bandwidth of 2 × (highest frequency of f)</u>:

$$s(t) = {}^{4}/_{\pi}\, f(t) \cos(\omega_c t)$$

Finally, we'll end with an example.

ex. Given this system:

$$\Phi(t) \rightarrow \boxed{\text{Squarer}} \rightarrow v_1(t) \rightarrow \boxed{\text{LPF}} \rightarrow v_2(t) \rightarrow$$

$$\rightarrow \boxed{\text{Square Rooter}} \rightarrow v_3(t)$$

where $\Phi(t) = A_c [1 + k_a f(t)] \cos(\omega_c t)$,
$|k_a f(t)| < 1$ (undermodulated),
and $f(t)$'s bandwidth is $-2\pi B \le \omega \le 2\pi B$,

What is the bandwidth of the LPF such that

$$v_3(t) = A f(t) + C, \qquad \begin{array}{l} A = \text{nonzero constant} \\ C = \text{any constant} \end{array}$$

Let's apply the square operation and see what happens.

$$\begin{aligned} v_1(t) &= A_c^2 [1 + k_a f(t)]^2 \cos^2(\omega_c t) \\ &= A_c^2 [1 + k_a f(t)]^2 [\tfrac{1}{2} + \tfrac{1}{2}\cos(2\omega_c t)] \\ &= \tfrac{1}{2} A_c^2 [1 + k_a f(t)]^2 \\ &\quad + \tfrac{1}{2} A_c^2 [1 + k_a f(t)]^2 \cos(2\omega_c t) \end{aligned}$$

Look at the first term. It looks very similar to the form $C + A f(t)$, does it not? So we want to keep it. The second term has a cosine so we want to get rid of it.

So from the term $\tfrac{1}{2} A_c^2 [1 + k_a f(t)]^2$, how do we figure out what bandwidth we need?

$f(t)$ has frequencies from $-2\pi B$ to $2\pi B$.

$k_a f(t)$ has frequencies from $-2\pi B$ to $2\pi B$. Scaling the wave by a constant doesn't change that.

$1 + k_a f(t)$ still is from $-2\pi B$ to $2\pi B$. A DC shift has no effect. But what about squaring it?

$$(1 + k_a f(t))^2 = (1 + k_a f(t))(1 + k_a f(t))$$

$$= \underset{\substack{\uparrow\\ \text{constant}}}{1} + \underset{\substack{\uparrow\\ \text{scaled by}\\ \text{a constant}}}{2k_a} f(t) + \underset{\substack{\uparrow\\ \text{scaled}\\ \text{by a constant}}}{k_a^2}\, \underset{\nwarrow\ \text{WTF}}{f(t)^2}$$

So what happens to the spectrum of just the function squared? To see this, we can allow $f(t) = \cos(2t)$.

$$\begin{aligned} f(t)^2 &= \cos^2(2t) \\ &= \cos(2t)\cos(2t) \\ &= \tfrac{1}{2}\left[\cos[(2-2)t] + \cos[(2+2)t]\right] \\ &= \tfrac{1}{2} + \tfrac{1}{2}\underset{\substack{\uparrow\ \text{we double the spectrum}\\ \text{required of the original}}}{\cos(4t)} \end{aligned}$$

So $(1 + k_a f(t))^2$ has frequencies from $-4\pi B$ to $4\pi B$. Since LPFs only take an UPPER cutoff frequency, it must be $4\pi B$.

As such,

$$V_2(t) = \tfrac{1}{2} A_c^2 [1 + k_a f(t)]^2$$

and

$$V_3(t) = \frac{A_c}{\sqrt{2}} [1 + k_a f(t)]$$