Nairn and Chan: Stability

There's a lot of (frankly, extremely confusing) theory associated with stability. To make a long story short, feedback can potentially make a stable amplifier unstable. So we have two questions to answer:

① What determines stability?
② How can we modify an unstable circuit to make it stable?

Let's go about these in order. Determing stability: given B, the feedback factor, and $A(j\omega)$, the transfer function of the amplifier without feedback,

① Plot the magnitude and phase plots of $A(j\omega)$
② Draw a horizontal line at $20\log(1/B)$, our desired gain in dB, on the magnitude plot
③ Mark the frequency at which $20\log(1/B)$ and $|A(j\omega)|$ intersect
④ Find the phase angle associated with that frequency
  (4a) If the angle is above the $-180°$, the circuit is stable
  (4b) If the angle is lower (more negative) than $-180°$, the circuit is unstable

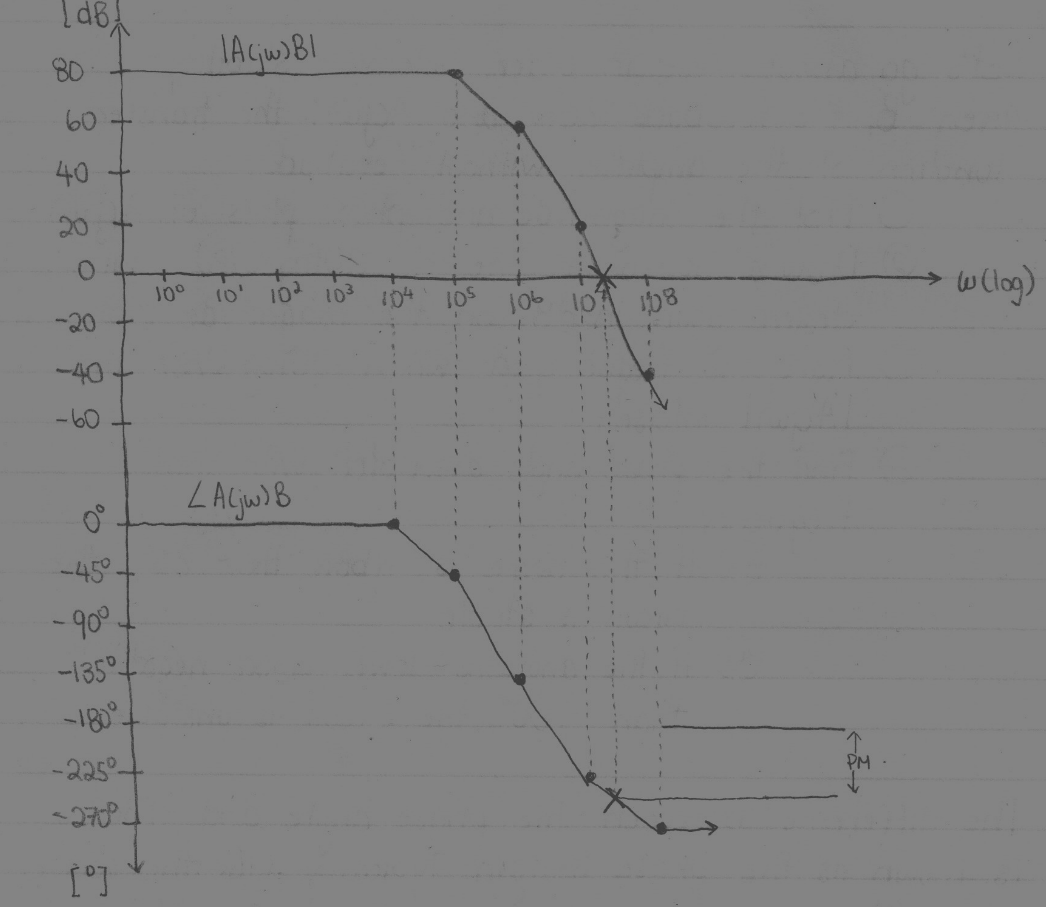
The difference between the phase angle and $-180°$ is known as the phase margin. Typically (for this course, at least) the phase margin we design for is $-45°$.

If we're just given $A(j\omega)B$ as a single transfer function (not knowing what B is), we can instead see where the magnitude plot crosses the x axis, and compare the phase angle at that point, aka find when $|A(j\omega)B| = 1 = 0\,dB$.

Let's do an example.

$$A(j\omega)B = \frac{10^4}{(1+j\omega/10^5)(1+j\omega/10^6)(1+j\omega/10^7)}$$

We have three poles at $\omega = 10^5, 10^6, 10^7$ and we begin the magnitude plot at $20\log(10^4) = 80$ dB. (For a Bode plot refresher, look at the Frequency Response notes)

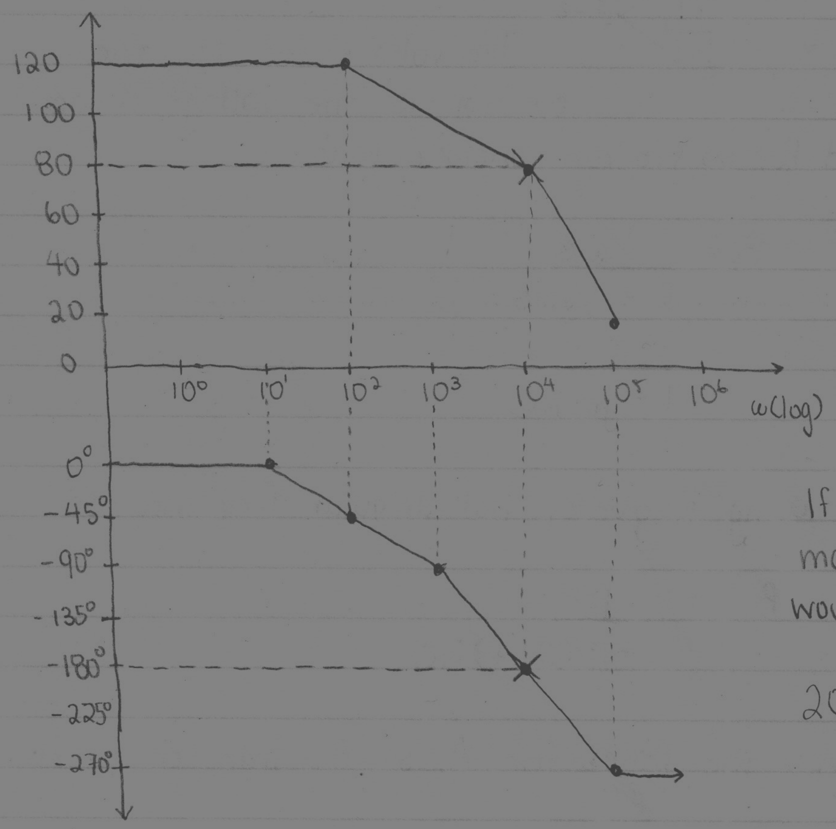


$|A(j\omega)B|$ crosses the x axis somewhere a little past 10^7, which maps to an angle around $-250°$. As such, the circuit represented by this transfer function is unstable, with a phase margin of $PM = -180° + 250°$
$\approx 70°$

This one was pretty easy. Let's try another.

10.80) Op-amp. Low frequency gain of 10^5. One pole at 10^2 rads/s. Two poles at 10^4 rads/s. What ω, for this system, corresponds to $-180°$? For what value of B would this circuit be stable?

$20\log(10^5) = 120$ dB



$\omega = 10^4$ rads/s corresponds to $-180°$

As such this circuit is stable for $20\log(1/B) \geq 80$ dB.

$\therefore B = 1/10^4$ or larger

If we wanted our phase margin of $-45°$, then B would need to be

$20\log(1/B) = 90$ dB

$1/B = 10^{4.5}$

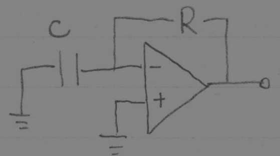
$B = 3.16 \times 10^{-5}$

So now we know how to determine stability of a circuit with feedback, and how to find what values of B a non-feedback circuit is stable with.

Let's do one final example.

10.93) $A_0 = 10^3$. Poles at 1 MHz and 10 MHz. We will be connecting this amplifier in the DIFFERENTATOR configuration. What is the smallest differentiator time constant we can get while maintaining stability?

Recall:



with $\tau = RC$

The voltage fed into the op-amp is the voltage divider of C and R, so we can conclude that

$$B_\omega = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC}$$

But we're using frequency, not angular frequency, so

$$B_f = \frac{1}{1 + j(2\pi f)RC}$$

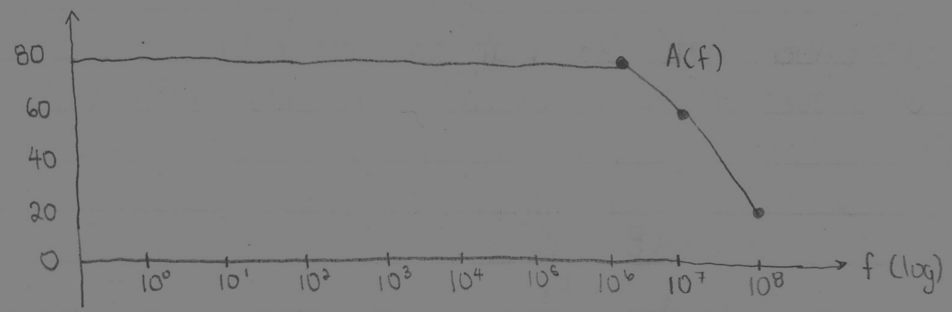
We're given the poles for A, so A's transfer function is

$$A = \frac{10^3}{(1 + j/10^6)(1 + j/10^7)}$$

And now we can graph both A and

$$1/B = 1 + j2\pi f RC = 1 + \frac{j2\pi f}{(RC)^{-1}}$$

which has a singular zero at $1/RC$.



Now, the question is, where do we put the $1/B$ line? There's an interesting corollary of stability:

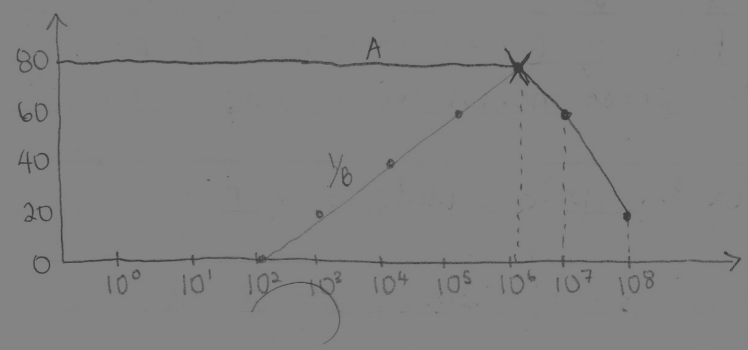
At the point of intersection between $1/B$ and A, if the DIFFERENCE in slope is 20 dB or LESS, it is stable.

$1/B$ is a +20 dB/decade line. That means crossing it with A at 10^6 Hz or higher will make the circuit unstable → difference will be 40 dB, then 60 dB. So we MUST cross it where A's slope is still 0.

Notice that as $RC \to 0$, $1/RC \to \infty$, so we want them to cross

- when A's slope is 0
- as late as possible

So we have to cross at 10^6.



But remember that Bode plots are just estimations of behaviour: we can obtain a better result with math.

$$|AB| = 1$$

$$A_0 \frac{1}{1+2\pi f RC} = 1$$

$$\frac{10^3}{1+(2\pi)(10^6)RC} = 1$$

$$RC = \frac{10^3}{1+2\pi(10^6)}$$

$$= 1.59 \times 10^{-5} \text{ seconds}$$

So now we know all about finding B and B-related values such that an A circuit is stable. How about changing the circuit to fit a certain B?

<u>Compensation</u>

There's three forms of compensation that modify a circuit's transfer function (and I'm not talking about guys who buy giant SUVs because they have small –).

① Add a new pole such that its effects shift the unity gain frequency (where $|AB| = 1$) to a spot that fits the phase margin we're designing for.

② Shift an existing pole, modifying its effects.

③ Another technique we'll discuss later called pole splitting.

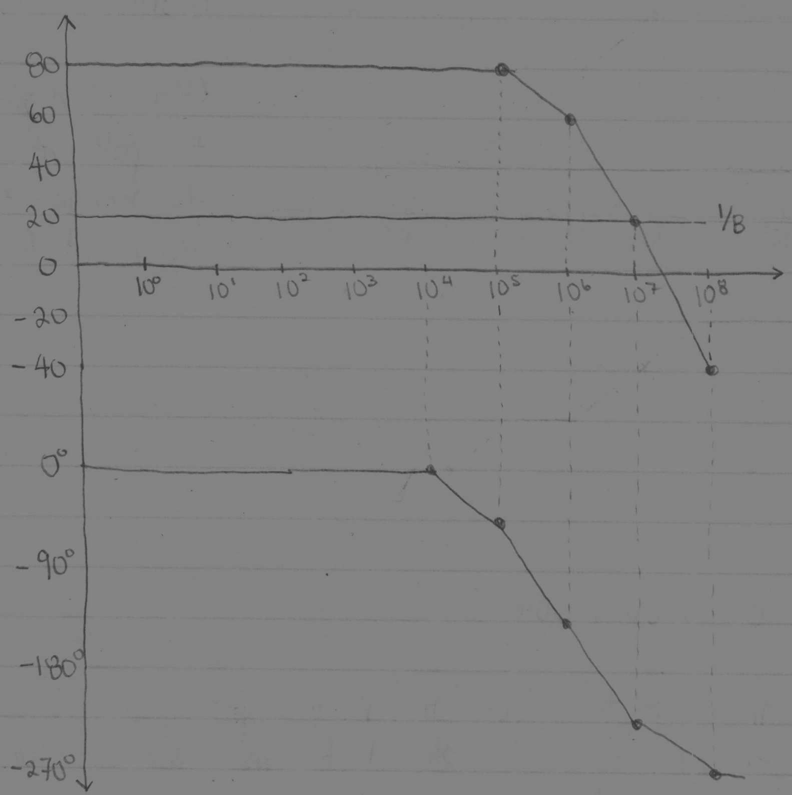
① Adding a new pole.

Note that this is effective only for lowering the lowest frequency pole. The idea is if you have some poles

$$\omega_{p1} < \omega_{p2} < \omega_{p3} \text{ spaced far apart,}$$

we can add $\omega_{p0} < \omega_{p1}$ to make ω_{p1} the frequency of unity gain.

ex. $$A(j\omega) = \frac{10^4}{(1+s/10^5)(1+s/10^6)(1+s/10^7)}$$ (same as first example)



Let's say we want this to be stable for $B = 1/10$.

Currently, unity gain frequency is at 10^7, which puts it at $-225°$, which is unstable. Remember that adding a new pole also affects the phase plot, too, so there's a simple equation we can use to find where the new pole goes.

$$A_{old}\,\omega_{pnew} = A_{new}\,\omega_{pold}$$

gain at the old frequency · location of new pole · desired gain at old location · location of lowest pole

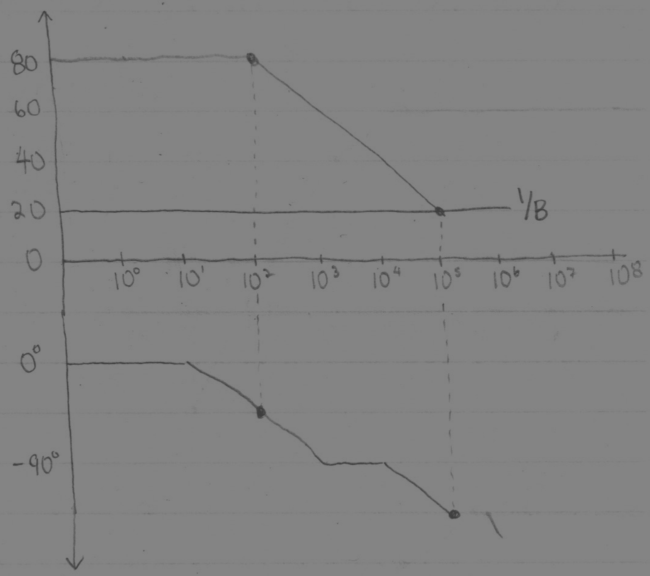
In this case: $A_{old} = 80\text{ dB} = 10^4$

$A_{new} = 20\text{ dB} = 10$

$\omega_{p\,old} = 10^5$

$$\therefore \quad \omega_{pnew} = \frac{(10)(10^5)}{(10^4)} = 10^2 \text{ rads/s}$$

Let's draw it.



Hooray, it works!

(Haven't drawn in the poles at 10^6 and 10^7 because they're irrelevant for this)

② Shifting an existing pole.

Much like the first technique, this normally only works with the lowest frequency pole. It follows a similar formula.

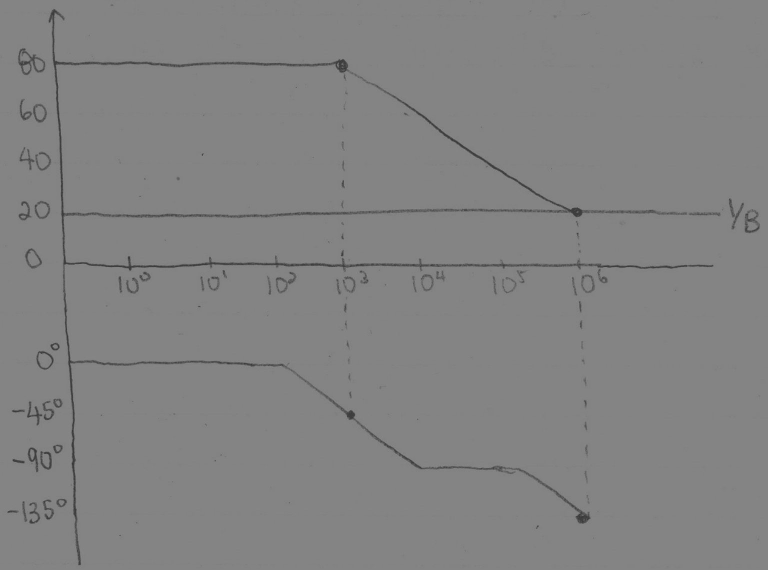
$$A_{old}\,\omega_{pnew} = A_{new}\,\omega_{pold}$$

gain of lowest pole · new location of lowest pole · new gain at desired location · location of pole we aren't moving

Using this; we can shift the pole at 10^5 such that the pole at 10^6 has a gain of 20 dB.

$$(10^4)\,\omega_{p\,new} = (10)(10^6)$$
$$\omega_{p\,new} = \frac{10^7}{10^4}$$
$$= 10^3 \text{ rads/s}$$

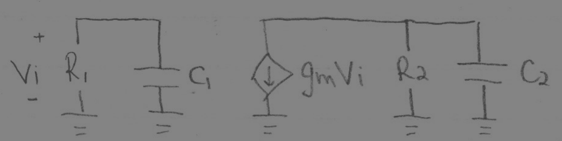
Let's draw.



This, surprisingly, also works!

③ Pole splitting.

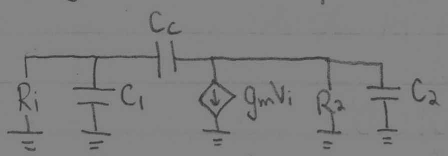
Pole splitting is where you add an additional Miller (floating) capacitor in order to shift the poles of other capacitors.



The poles in this circuit are given approximately by

$$\omega_{p1} = \frac{1}{C_1 R_{eq1}} \quad \text{and} \quad \omega_{p2} = \frac{1}{C_2 R_{eq2}}$$

By adding a Miller capacitor:



The poles shift to

$$\omega_{p1} = \frac{1}{C_c\, g_m R_2 R_1} \quad \text{and} \quad \omega_{p2} = \frac{g_m C_c}{C_1 C_2 + C_c(C_1 + C_2)}$$

But unfortunately this is not a general equation - the effects of pole splitting differ from circuit-to-circuit.