

A Web-Based Tool For Eliciting Probability Distribution From Experts

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Abstract

- The Tool is designed to help elicit probability distributions about uncertain model parameters from experts.
- A key feature of the Tool is that users can log in from different sites and view and interact with the same graphical displays, so that expert elicitation sessions can be conducted remotely.
- This will make probability elicitation easier in situations where it is difficult to interview experts in person.

Introduction

- The Tool is named after the UK Research Council funded MATCH (Multidisciplinary Assessment of Technology for Healthcare) project.
- The development of the Tool was motivated by the need to elicit user opinions about medical devices.
- An advantage of eliciting a probability distribution, rather than asking an expert for a single point estimate, is that it allows an expert to express his or her uncertainty about the parameter.

There are two main approaches: Frequentist method and Bayesian method.

In Frequentist Approach: Probabilities are interpreted as long run frequencies.

- never uses or gives the probability of a hypothesis (no prior or posterior).
- does not require a prior.

In Bayesian Approach: The Bayesian treats probability as beliefs, not frequencies.

- The unknown parameter θ is given a prior distribution $\pi(\theta)$ representing his subjective beliefs about θ .
- depends on the prior and likelihood of observed data.
- requires one to know or construct a 'subjective prior'.

Bayes theorem- It describes the probability of occurrence of an event related to any condition.

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

where,

- $P(A)$ = Prior
- $P(B | A)$ = Likelihood
- $P(A | B)$ = Posterior
- $P(B)$ = evidence

- **Likelihood:** The likelihood function is the probability density of the data, viewed as a function of the parameters.
It is denoted as $L(\theta \mid \underline{x})$
- **Prior:** Prior is a Probability calculated to express one's beliefs about this quantity before some evidence is considered.
 $\pi(\theta)$, which tells us what is known about θ without knowledge of the data.
- **Posterior:** The posterior distribution summarizes what you know after the data has been observed.

$$\text{Posterior} \propto \text{Likelihood} \times \text{Prior}$$

- Conjugate prior: For some likelihood functions, if you choose a certain prior, the posterior ends up being in the same distribution as the prior.
- Non-informative priors: Non informative priors are formal representation of ignorance
 - Uniform prior: This can be interpreted as that all possible values are equally likely to a prior, or you have no prior information and you can't distinguish between possible values.
 - Vague prior: Vague prior is presented by using a prior distribution which has "Infinite Variance".
 - Jeffery's prior: Approximately non- informative if it's taken proportional to the square root of Fisher's information measure

Prior elicitation is the process by which we attempt to construct the most suitable prior distribution for .

Steps for Elicitation

- Data
- Likelihood function
- Determining a prior distribution
- When uniform is your prior distribution, you don't need prior elicitation, uniform gives equal probabilities.
- Other distribution needs prior Elicitation i.e find the value of hyper parameter of prior distribution


Applications

- Used in Clinical trial design and analysis.
- Better decision making in drug development through adoption of formal prior elicitation.
- Elicitation of treatment response rate for a trial of a rare disease.

Match Elicitation tool

Eliciting a univariate probability distribution -In most elicitation schemes the expert usually makes a small number of probability judgements about X , and the facilitator fits a probability distribution to these judgements.

Help ? Options Options Fitting & Feedback Text Output Share Complete Restart About Us



MATCH Uncertainty Elicitation Tool

Welcome to the MATCH Uncertainty Elicitation Tool. Descriptions of the buttons in the toolbar at the top of the screen are given below. Instructions and further help are available by clicking on the 'Help?' button.

Help ?

Display/hide the Help window

Options

Display/hide the Elicitation Options window

Fitting & Feedback

Display/hide the Elicited distribution and the Fitting and Feedback window

Text Output

Display/hide the Fitting parameters of the Elicited distribution

Share

Share this elicitation with another user over the Internet.

Replay

Replay this elicitation session (only available once the elicitation session has been complete)

Complete

Complete this elicitation session

Restart

Restart the elicitation session from the beginning

About Us

Display/hide information about us, the tool, and methods to contact us

Citations If you use this tool in any published research, please could you cite it as:

David E. Morris, Jeremy E. Oakley, John A. Crowe, A web-based tool for eliciting probability distributions from experts, *Environmental Modelling & Software*, Volume 52, February 2014, Pages 1-4, ISSN 1364-8152,
<http://dx.doi.org/10.1016/j.envsoft.2013.10.010>.

Elicitation Options

Ranges
Lower Limit
Upper Limit

Update Limits

Input Mode
☐ Roulette
☐ Quartile
☐ Tertile
☐ Probability
☐ Hybrid

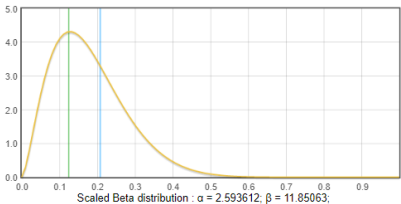
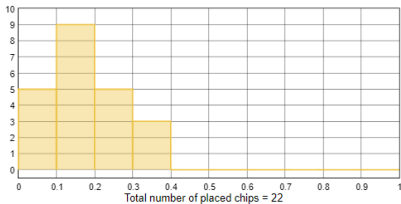
<http://optics.eee.nottingham.ac.uk/match/uncertainty.php>

- Probability density function
- Lower limit
- Upper limit
- Median
- Lower tertile
- Upper tertile
- Lower quartile
- Upper quartile
- Roulette method

- The expert is provided with a grid of m equally sized bins which cover the range of X , and is asked to allocate a total of n chips between the bins.
- Question:** Riya, a movie pirate, is trying to identify the proportion of potential customers who might be interested in buying K.G.F 2 next month. Based on the proportion of customers who have bought similarly kind of movies from her in the past, she wants to elicit her prior distribution of θ . For each month over the last 18 months. Riya knows the proportion of his customers who have bought similar movies; these proportions are shown below

0.32	0.25	0.28	0.15	0.33	0.12	0.14	0.18	0.12	0.05	0.25
0.08	0.07	0.16	0.24	0.38	0.18	0.15	0.22	0.05	0.01	0.19

Elicit a prior for θ . Riya asks five potential customers if they would buy movie from him, and four say they would. Using this information, what is Riya's posterior distribution for θ ?



Elicitation Options

Ranges

Lower Limit

Upper Limit

[Update Limits](#)

Input Mode

☒ Roulette
☐ Quartile
☐ Tertile
☐ Probability
☐ Hybrid

Roulette Options

Fitting & Feedback

Distribution

☐ Normal
☐ Student-t
☒ Scaled Beta
☐ Gamma
☐ Log Normal
☐ Log Student-t
☒ Auto-select best fit

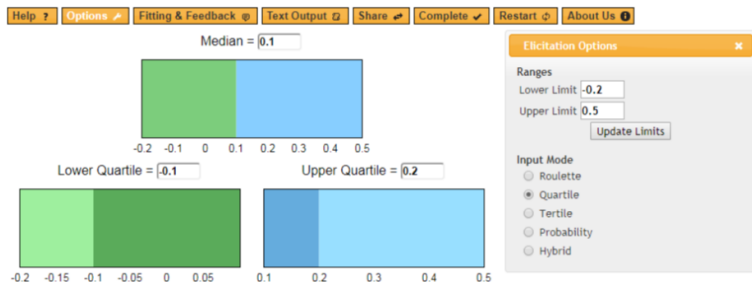
Feedback Percentiles

rd percentile 0.124
 =
th percentile 0.208
 =

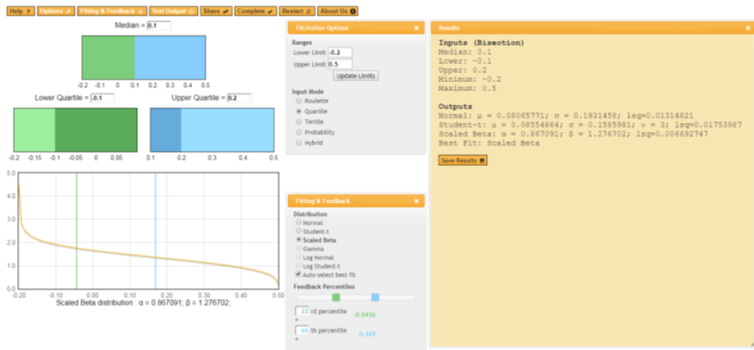
- This method is also referred to as the bisection method. The expert specifies the median, lower quartile and upper quartile of X .
- **Question:** In this worked example, the Median is 0.1, the Lower Quartile is -0.1 and the Upper Quartile is 0.2:

min	1st quartile	median	3rd quartile	max
-0.2	-0.1	0.1	0.2	0.5

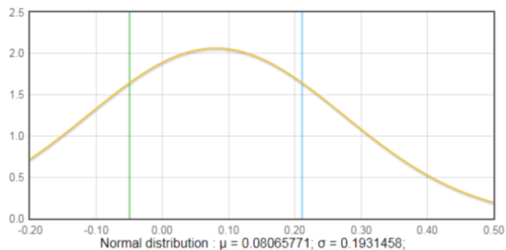
Quartile method



Quartile method



Quartile method



Fitting & Feedback ✕

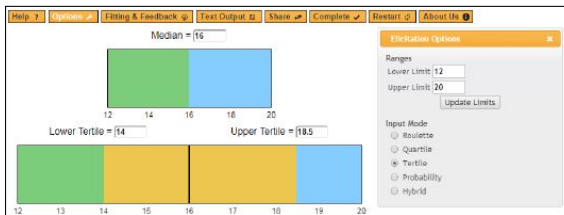
Distribution

- ☒ Normal
- ☐ Student-t
- ☐ Scaled Beta
- ☐ Gamma
- ☐ Log Normal
- ☐ Log Student-t
- ☐ Auto-select best fit

Feedback Percentiles

33 th percentile	-0.0496
66 th percentile	0.211

- The expert specifies the median, the 33rd percentile and the 66th percentile.
- In this worked example, Median is 16, the Lower Tertile is 14 and the Upper Tertile is 18.5:



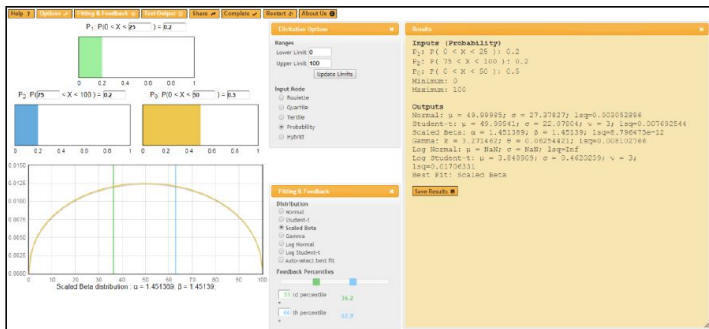
Tertile method



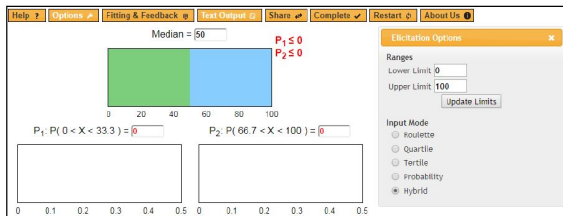
Tertile method



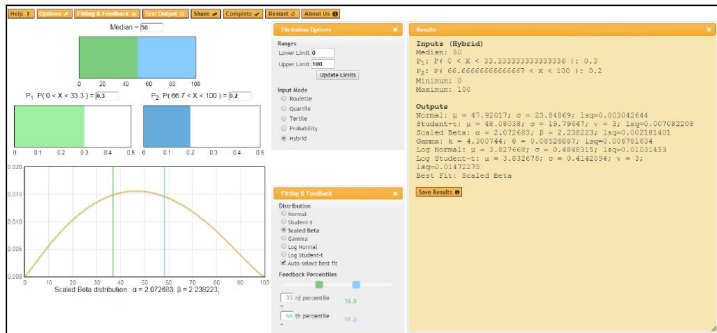
The expert specifies three probabilities. If the range of X is $[0,1]$, then the default probabilities asked for are $P(0 \leq X \leq 0.25)$, $P(0.75 \leq X \leq 1)$ and $P(0 \leq X \leq 0.5)$, but the numbers in these inequalities can be changed.



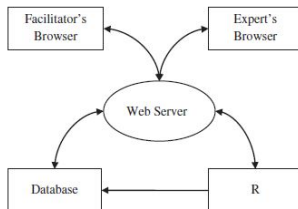
The expert specified the median and two probabilities, for example $P(0 \leq X \leq 1/3)$ and $P(2/3 \leq X \leq 1)$. Again, these judgements can be specified by manipulating sliding bars.



Hybrid method



- Clicking the Share button on the Tool provides a web link, and anyone viewing this web link can then view and interact with the same graphics.
- The MATCH Tool is based on the SHELF elicitation package.
- The browser behaves as though it is receiving data that has been inputted by another expert or facilitator, and displays them as if it is an inactive party present in the actual elicitation session.



Limitations

- It does not tell you how to select a prior.
- If the prior information is wrong.
- It can produce posterior distributions that are heavily influenced by the priors.
- It often comes with a high computational cost, especially in models with a large number of parameters.

The Tool is a significant advance in making expert elicitation easier and more practical, particularly in cases when it is difficult to bring the experts and facilitator together to the same location.

"Today's posterior is tomorrow's prior"



Roger Cooke et al. *Experts in uncertainty: opinion and subjective probability in science*. Oxford University Press on Demand, 1991.



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David E Morris, Jeremy E Oakley, and John A Crowe. "A web-based tool for eliciting probability distributions from experts". In: *Environmental Modelling & Software* 52 (2014), pp. 1–4.



Anthony O'Hagan et al. "Uncertain judgements: eliciting experts' probabilities". In: (2006).

