The University of Edinburgh

Lay Summary of Thesis

The lay summary is a brief summary intended to facilitate knowledge transfer and enhance accessibility, therefore the language used should be non-technical and suitable for a general audience. (See the Degree Regulations and Programmes of Study, General Postgraduate Degree Programme Regulations. These regulations are available via: http://www.drps.ed.ac.uk/.)

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Degree Sought:	PhD	*No. of words in the main text of Thesis:*	57500
Title of Thesis:	Ordered Geometry in Hilbert's *Grundlagen der Geometrie*		

Insert the lay summary text here - the space will expand as you type

David Hilbert's *Grundlagen der Geometrie*, first published in 1899, was the first treatise to lay down axioms for points, lines and planes, from which all geometric results of our ordinary three-dimensional space should be provable. A successor to Euclid, he improves on the famous *Elements* by crucially bringing it up to the standards of formal logic. His axioms are sufficiently exact that the words “point”, “line” and “plane” could be replaced with “mug”, “table” and “chair” and all the proofs would remain valid.

His first two groups, those for ordered geometry, are conceptually limited. We have no compasses to draw circles, nor protractors to measure angles. We cannot run parallel lines, nor reason about continuous deformation and motion in space. The basic vocabulary only concerns whether a point is known to lie on a line or plane, and whether one point is known to lie on a line between two others. All other assertions must be built from these primitive concepts.

They turn out to be sufficient to define the concept of a *polygon*, and to assert that every polygon which does not self-intersect divides its plane into exactly two regions, an interior and exterior. But *proving* this is challenging. When you are working with such a limited vocabularly, and so few axioms, intuitions become horribly misleading and simple theorems a hurdle.

In our thesis, we have a proof and we have verified its correctness in a way which respects Hilbert's original demands for formal rigour. We have translated all the axioms and the proof to a symbolic form, and input them to a program called a *theorem prover*. The prover has no interpretation of these symbols: in its eyes, “point” and “mug” are equally meaningless. It only sees the axioms, and so its standard of validity must be measured by applications of just a handful of primitive symbolic logical rules.

The computer is essential, because fully-expanded, we are in the order of *billions* of applications of rules needed for verification. To check them all, we need the computer's reliability and speed, with which it can guarantee the validity of a proof to the most hardened sceptics and rigorous pedants.

Humans are essential too, because the computers on their own lack the insight and ingenuity needed to break down non-trivial proofs. We must carefully lay out a set of goals for them to follow, stepping stones which take them from the axioms to the desired conclusion, letting computer automation fill in the final details.

We have contributed some of our own automation, which can collaborate with the user on a problem even as they think about the high-level goals, and it helped us enormously to verify our proof that a polygon which does not self-intersect has an interior and an exterior. What is more, the automation allowed us to leave a readable symbolic proof in the form of all the stepping stones laid out for the computer.

All your lay summary text must be contained on this side