

Lecture 5 Transformer Appendix

Layer Normalization

Take Transformer – Encoder as example,

Given $N = 6$, when layer $n = 1$

$$residual = a + b^0$$

$a: T \times d_{model}$ is the input (i.e. output from the positional encoding step) fed into the multi-head self-attention in layer $n = 1$. T is the sequence length (the number of words in a sentence)

$b^0: T \times d_{model}$ is the output from the multi-head self-attention in layer $n = 1$.

To apply Layer Normalization to $residual$:

$$residual = \begin{bmatrix} residual_{1,1} & \dots & residual_{1,d_{model}} \\ \vdots & & \vdots \\ residual_{T,1} & \dots & residual_{T,d_{model}} \end{bmatrix}$$

Calculate μ_1 to μ_T as follows:

$$\begin{aligned} \mu_1 &= \frac{1}{d_{model}} \sum_{i=1}^{d_{model}} residual_{1,i} \\ &\dots \\ \mu_T &= \frac{1}{d_{model}} \sum_{i=1}^{d_{model}} residual_{T,i} \end{aligned}$$

Calculate σ_1 to σ_T as follows:

$$\begin{aligned} \sigma_1^2 &= \frac{1}{d_{model}} \sum_{i=1}^{d_{model}} (residual_{1,i} - \mu_1)^2 \\ &\dots \\ \sigma_T^2 &= \frac{1}{d_{model}} \sum_{i=1}^{d_{model}} (residual_{T,i} - \mu_T)^2 \end{aligned}$$

For any word t in a sequence, its i th embedding dimension after layer normalization is:

$$c_{t,i} = \frac{residual_{t,i} - \mu_t}{\sigma_t}$$

Note: each embedding dimension is treated as a feature in machine learning. Layer normalization is thus taken along each timestamp and across all embedding dimensions. Batch normalization is thus taken along each embedding dimension and across all timestamps.