Probability and Statistics: Lecture-1

Monsoon-2020

by Pawan Kumar (IIIT, Hyderabad)
on August 10, 2020

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» Why Counting?

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- applications of counting:
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  - estimating probability of occurrence of an event (in this course)
  - proofs such as Pigeon Hole Principle (PHP)

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* It is dates back to the Upper Paleolithic period of human history, and is approximately 20,000 years old.
* The bone is 10 cm long and contains a series of notches, which many scientists believe were used for counting.

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* Sumer was a region of ancient Mesopotamia in the Middle East

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How many numbers are there between m and n, $m < n$?

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$m - n + 1$

» How do we count?

Question-1

How many numbers between 33 *and* 67 *are* divisible by 4?

» Sum Rule: Statement and Example

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Sum Rule Example

How many of the Pizza or Burgers places are there?

Answer

There are 7 Pizzas and there are 5 Burgers, hence, by sum rule, we have 7 + 5 = 12

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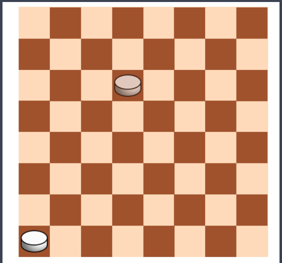

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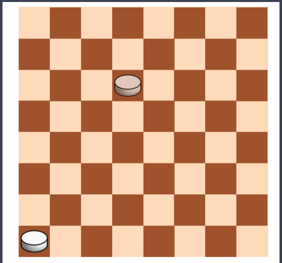

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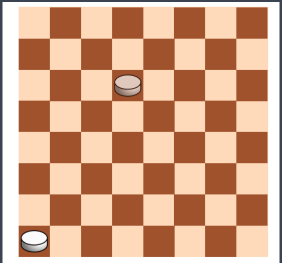

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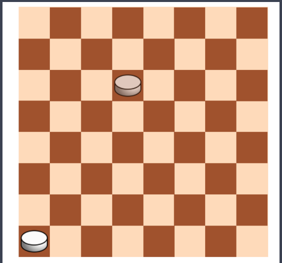

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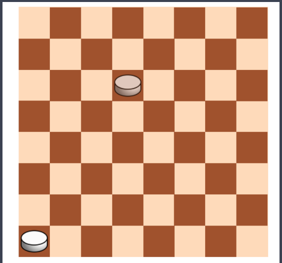

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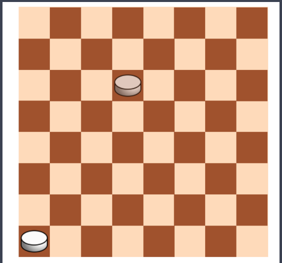

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* To get to the column 4, we need 3 moves to the right
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* Applying sum rule: In total, we need 3+5=8 moves

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Caution with Sum Rule

In the rule of sum, no object should belong to both types!

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* A set which is countable and not finite is called countably infinite

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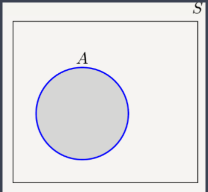

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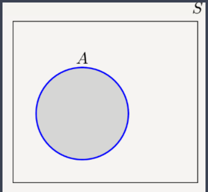
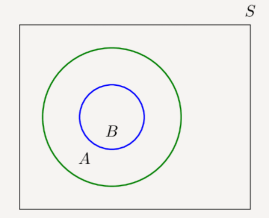


* Venn diagram showing subset relationship



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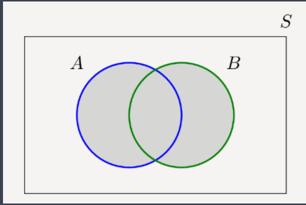

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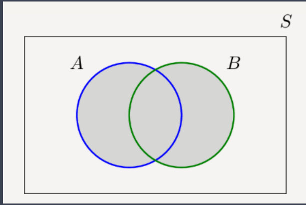

- Similarly, we define union of three or more sets *as* follows

$$A_1 \cup A_2 \cup \cdots \cup A_k = \cup_{i=1}^{k} A_i$$

» Quiz

- If A and B are countable, then $A \cup B$ is also countable

» Quiz

* Countable union of countable sets is countable

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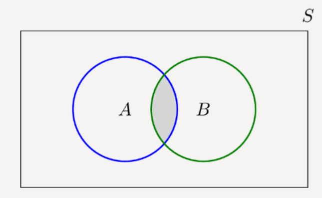

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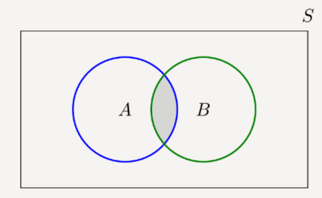

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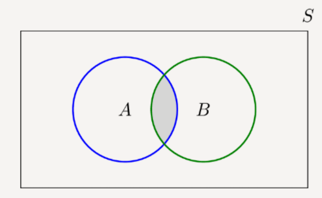

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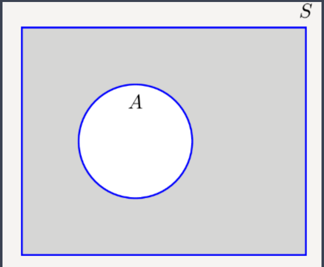

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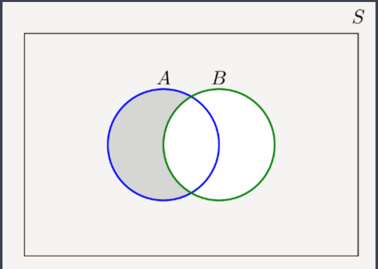

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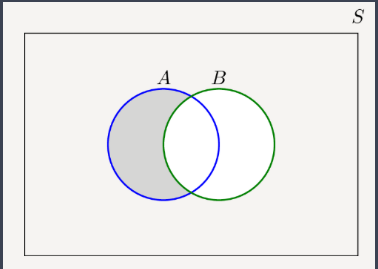

* Two sets A and B are mutually exclusive or disjoint if they have no shared element, i.e., $A \cap B = \phi$

» Cartesian Product of Sets

Define Cartesian Product of Sets

Cartesian product of two sets $A = \{a_1, a_2, \cdots, a_m\}$ and $B = \{b_1, b_2, \cdots, b_n\}$ denoted by $AtimesB$ is defined as follows

$$A \times B = \cup_{i,j}\{(a_i, b_j)\}$$

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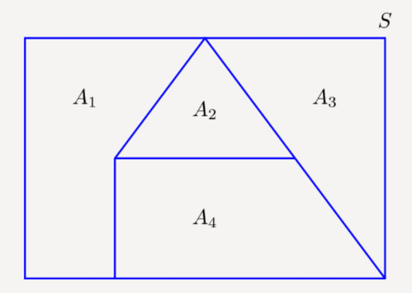

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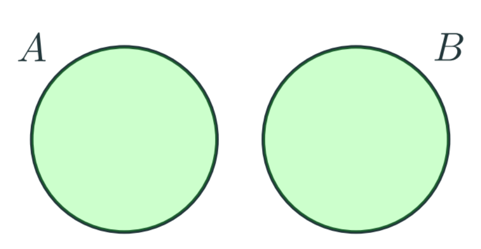

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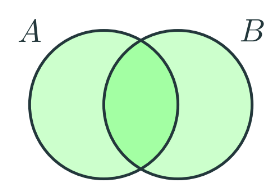

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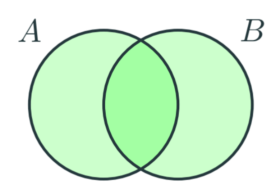

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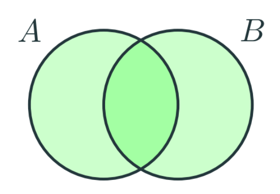

- If we consider $|A| + |B|$ as in sum rule, then we will be wrong
- We will count elements that belong to both A and B twice
- $|A \cup B| = |A| + |B| - |A \cap B|$ (Inclusion-Exclusion Principle)

» Applications of sum rule

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Sum rule

Count *all* integers from 1 to 10 that *are* divisible by 2 or by 3

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* Let us count all the numbers from 1 to 10:

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10$$

* Here 6 is divisible both by 2 and by 3. Hence, rule of sum can't be applied!

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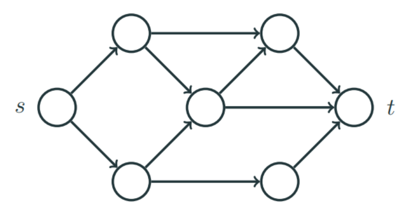

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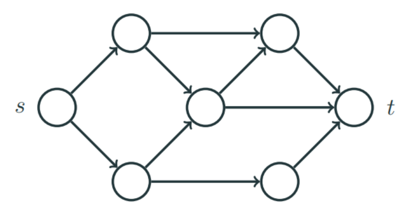

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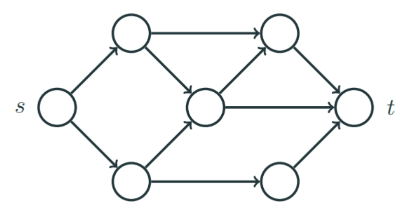

* counting can be done recursively
* for each node count the number of paths from s to this node
  * sum rule will be used

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If there are *k* object of the first type and there are *n* object of the second type, then there are $k \times n$ pairs of objects, the first of the first type and the second of the second type

* Hence, there are $4 \times 3 = 12$ options

» All possible pairs

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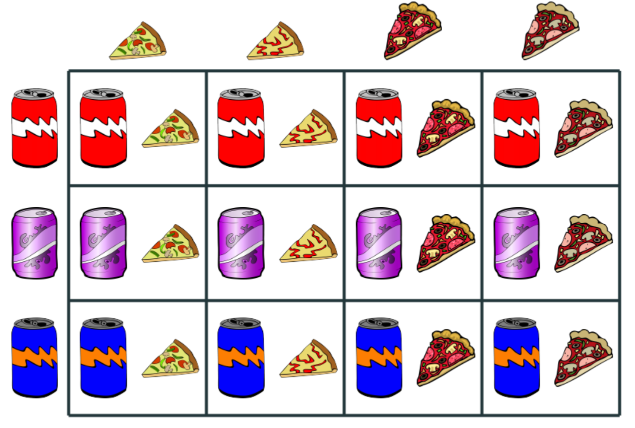

» Rule of Product Using Sets

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Product Rule

If there is a finite set A and a finite set B, then there are $|A| \times |B|$ pairs of objects, the first from A, and the second from B

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* How many different 3-letter words are possible?
* Can you now answer the question above?

Question

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* similarly, There are n possibilities to pick the second letter, and so on...
* thus, the answer is a product of n by itself k times, that is n^k

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How many vehicles are there?

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- If we fix 5 in one place, then there are $5 \times 5 \times 5 = 125$ sequences
- There are 4 ways to arrange 5 among 4 places
- Hence, there are $4 \times 125 = 500$ four digit numbers below 10,000 with exactly one 5

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* Hence there are

$$n \times (n-k) \times \cdots (n-k+1)$$

$k-$permutations, which is $n!/(n-k)!$

» Permutation Examples

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Answer

Hint: Use previous result with $k = n$.