Probability and Statistics: Lecture-11

Monsoon-2020

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Random Monty Hall Problem

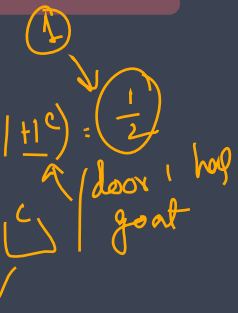
This result depends crucially on the fact that Monty was always guaranteed to open a door with a goat behind it, regardless of what door you picked initially. That is, $P(E \mid H) = P(E \mid H^c)P(E \mid H)$. Now consider what would happen if Monty randomly opened a door we did not pick and it contained a goat. What is the probability that our first pick is correct, regardless of which specific door we picked?

Solution:

H: door 1 has a car behind it

E: Monty reveals a goat door

$P(H) = \frac{1}{3}$ $P(H^c) = \frac{2}{3}$ $P(E \mid H) = 1$ $P(E \mid H^c) = \frac{1}{2}$



$P(H \mid E) = \frac{1}{2}$ $P(H^c \mid E) = \frac{1}{2}$

» Some Problems in Probability...

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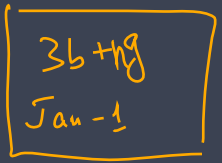
Problem: Boy/Girl Problem

On the morning of January 1, a hospital nursery has 3 boys and some number of girls.



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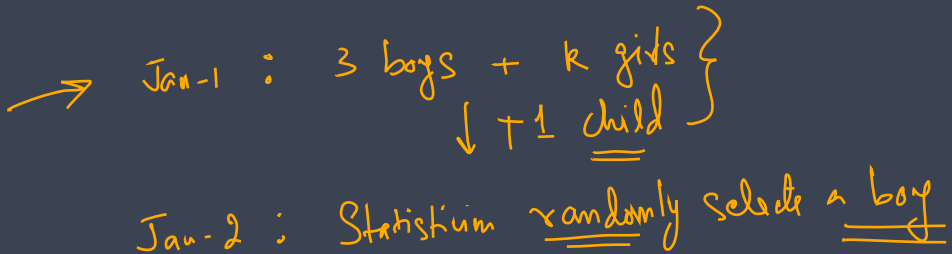
Solution:



» Solution to Boy Girl Problem...

3 boys, 3 girls — assume

+1 → 3 girls, 4 boys → $\frac{1}{2} \cdot \frac{4}{7}$

−1 → 4 girls, 3 boys → $\frac{1}{2} \cdot \frac{3}{7}$

Define event: B = select Boy, N = newborn is Boy

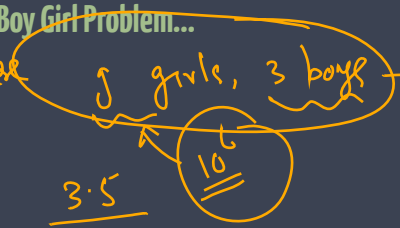
$B = (B \cap N) \cup (B \cap N^c)$ — mutually exclusive

$P(B) = P(B \cap N) + P(B \cap N^c) = \frac{1}{2} \cdot \frac{4}{7} + \frac{1}{2} \cdot \frac{3}{7} = \frac{7}{14} = \frac{1}{2}$

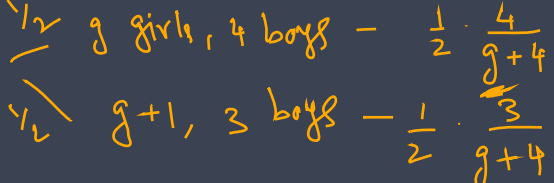
$P(N|B)$ ← we want

$= \frac{P(N \cap B)}{P(B)} = \frac{\frac{1}{2} \cdot \frac{4}{7}}{\frac{1}{2}} = \frac{4}{7} = 57\%$

General Case



$P(B) = \frac{3.5}{g+4}$



$$P(N \mid B) = \frac{P(N \cap B)}{P(B)} = \frac{\frac{1}{2} \cdot \frac{4}{g+4}}{\frac{3.5}{g+4}} = \frac{4}{7} (= 57.)$$

» Probability of Seeing a Car...

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Problem: Probability of seeing a car

If the probability of seeing a car on the highway in 30 minutes is 0.95,

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If the probability of seeing a car on the highway in 30 minutes is 0.95, what is the probability of seeing a car on the highway in 10 minutes? (assume a constant default probability)

Solution:

p = P(no. of cars in 10 min)

P(car in 10) = $1 - p$

P(no cars in 20 mins) = $p \cdot p = p^2$

P(" " 30 ") = $p^2 \cdot p = p^3$

P(car in 30 min) = $1 - p^3 = 0.95$ =)

$p = \sqrt[3]{1 - 0.95} = \sqrt[3]{0.05}$

» Skewed Die Problem...

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Problem: Skewed Die Problem

A standard dice has the 6 showing with a probability of 1/6, which is the same as every other number. A loaded dice has the 6 showing with 1/2 the probability of the other numbers. What is the probability of rolling a 6?

Solution:

$P(\{6\}) = \frac{p}{2}$

$P(\{1\}) = p$

$P(\{2\}) = p$

$S = \{\{1\}, \{2\}, \ldots, \{6\}\}$

$P(\{5\}) = p$

$P(\{6\}) =$

Prob Axiom - 1

$P(S) = 1$

$\frac{p}{2} + 5 \cdot p = 1$

$\Rightarrow p = \frac{2}{11}$

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» Spam Email Problems...

Problem

It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%. Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email?

Sol:

A = email is detected as spam

B = email is spam

B^c = email is not spam

$P(B) = P(B^c) = \frac{1}{2}$

$P(A|B) = 0.99$

$P(A|B^c) = 0.05$

$$P(B^c|A) = \frac{P(A|B^c)P(B^c)}{P(A|B)P(B) + P(A|B^c)P(B^c)}$$

$$= \frac{0.05 \times 0.5}{0.99 \times 0.5 + 0.05 \times 0.5}$$

» Scratch Space for Spam Email Problems...

» Graded Attendance Quiz-1

https://tinyurl.com/y2ecaw28

- Please attempt the quiz in the link above
- Login to this form only with IIIT account
- There are two questions
- Remember to answer questions before hitting submit
- Answers to this will be discussed in tutorials