Probability and Statistics: Lecture-4

Monsoon-2020

by Pawan Kumar (IIIT, Hyderabad)
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https://tinyurl.com/y2g93ofb

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There are 15 identical candies. How many ways are there to distribute them among 7 kids?

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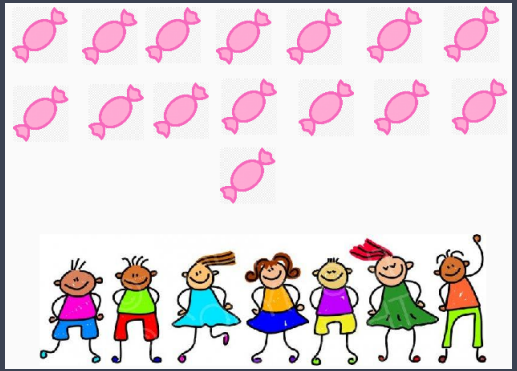

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* When distributing candies, we pick one of 7 kids

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- Since the candies to be distributed are identical, ordering doesn't matter

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Analyzing the problem, and fitting it into the known case

- When distributing candies, we pick one of 7 kids
- Since all the candies are same, repetitions are allowed
- Since the candies to be distributed are identical, ordering doesn't matter
- This is a case of combinations with repetitions

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Analyzing the problem, and fitting it into the known case

* Number of combinations = size of combination
* Number of options = Number of kids
* The answer is $\binom{15+(7-1)}{(7-1)} = \binom{21}{3} = 54264$

» Fair Distributions..

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A problem with previous distribution...

No kid will like not having even a single candy. Infact, a kid would like to have it all!
How can we change the problem?

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A problem with previous distribution...

We want that each kid receives atleast one candy!

Let us change our previous problem so that each kid is ensured atleast one candy!

» Distributing Candies with Fair Distribution...

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Question

There are 15 identical candies. How many ways are there to distribute them among 7 kids in such a way that each kid receives at least 1 candy?

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Quiz

* Does our previous approach work here? What should you do?

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Quiz

- Does our previous approach work here? What should you do?
- Can we reduce this problem to previous problem? If yes, how?

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* Give kids atleast one candy. Then we are left with 15-7=8 candies...

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  - #combinations = 8; #options = 7

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* Now we can distribute these 8 candies as in previous problem!
* It becomes a problem of Combinations with Repetitions
  * #combinations = 8; #options = 7
* Answer = $\binom{8+(7-1)}{(7-1)} = \binom{14}{6} = 3003$

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Question

Assume that 15 identical candies are distributed among 7 kids. How many ways are there that will leave some kids without any candy?

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Hint

Use the answer to previous two problems!

» Numbers with Fixed Sum of Digits

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* It is clear that we have nine options for the 1st digit

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- But what next? How many options for the 2nd digit?

» Numbers with Fixed Sum of Digits

Problem

How many non-negative integer numbers are there below 10 000 such that their sum of digits is equal to 9?

Understanding the problem...

- It is clear that we have nine options for the 1st digit
- But what next? How many options for the 2nd digit?
- It is already tricky...

Problem

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* This belongs to the category: “unordered with repetitions”

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How many non-negative integer numbers are there below 10 000 such that their sum of digits is equal to 9?

- Idea: How many ways we can distribute 9 ones in 4 places?
- Multiple ones will go to at least one of the four places, and get added
- This belongs to the category: “unordered with repetitions”
- #combinations = 9; #options = 4

Problem

How many non-negative integer numbers are there below 10 000 such that their sum of digits is equal to 9?

- Idea: How many ways we can distribute 9 ones in 4 places?
- Multiple ones will go to at least one of the four places, and get added
- This belongs to the category: “unordered with repetitions”
- #combinations = 9; #options = 4
- Hence, answer is $\binom{9+(4-1)}{(4-1)} = \binom{12}{3} = 220$