Probability and Statistics: Lecture-2

Monsoon-2020

by Pawan Kumar (IIIT, Hyderabad)
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» Table of contents

» Permutations

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Definition

Tuples of length k without repetitions are called $k-$permutations

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Question

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Suppose we have a set of n symbols. How many different sequences of length k we can form out of these symbols if we are not allowed to use the same symbol twice?

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1	2	3	...	k
$*$	$*$	$*$	$\cdots$	$*$
n	$n-1$	$n-2$		

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1	2	3	...	k
*	*	*	...	*
n	$n-1$	$n-2$		

* Hence there are

$$n \times (n-k) \times \cdots (n-k+1)$$

k–permutations, which is $n!/(n-k)!$

» Permutation Examples

» Permutation Examples

Question

In how many ways we can *arrange* n different books in n different bins on shelf?

» Permutation Examples

Question

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Answer

Hint: Use previous result with $k = n$.

» Combinations

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Question

You are organizing a car journey.

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You are organizing a car journey. You have five friends, but there are only three vacant places in you car.

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» Combinations

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You are organizing a car journey. You have five friends, but there are only three vacant places in you car. What is the number of ways of taking three of your five friends to the journey?

Above Question Reformulated

We are essentially asking: What is the number of ways of choosing 3 elements out of a set containing 5 elements?

» Answer to Previous Question...

» Answer to Previous Question...

Answer

* There are five choices of the first friend,

» Answer to Previous Question...

Answer

* There are five choices of the first friend, four choices of the second friend, and three choices of the third friend

» Answer to Previous Question...

Answer

* There are five choices of the first friend, four choices of the second friend, and three choices of the third friend
* How many choices are there for choosing 3 friends, assuming ordering?

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* There are five choices of the first friend, four choices of the second friend, and three choices of the third friend
* How many choices are there for choosing 3 friends, assuming ordering?
* In total, there are $5 \times 4 \times 3 = 60$ choices assuming ordering

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* Thus, we need to divide by 3!, the answer is $(5 \times 4 \times 3)/3! = 10$
* Since ordering does not matter, we call them combinations!
* We define combinations in next slide...

» Combinations: $k-$combination

Definition of $k-$combination

For a set S, a $k-$combination is a subset of S of size k

» Combinations: k—combination

Definition of k—combination

For a set S, a k—combination is a subset of S of size k

Definition of k—combinations

The number of k—combinations of an n element set is denoted by $\binom{n}{k}$.

Pronounced: "n choose k". Proof by example!

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Definition of $k-$combinations

The number of $k-$combinations of an n element set is denoted by $\binom{n}{k}$.

Pronounced: "n choose k". Proof by example!

Number of $k-$combinations

The number of $k-$combinations of an n element set is given by

$$\frac{n!}{(n-k)!}$$

» Derive formula for n choose k...

$\binom{5}{3}$

$3!$

abc	abd	abe	acd	ace	ade	bcd	bce	bde	cde
acb	adb	aeb	adc	aec	aed	bdc	bec	bed	ced
bac	bad	bae	cad	cae	dae	cbd	cbe	dbe	dce
bca	bda	bea	cda	cea	dea	cdb	ceb	deb	dec
cba	dba	eba	dca	eca	eda	dcb	ecb	edb	edc
cab	dab	eab	dac	eac	ead	dbc	ebc	ebd	ecd

$$3!\binom{5}{3} = \frac{5!}{(5-3)!}$$

» Pascal triangle...

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Question

There are n students. What is the number of ways of forming a team of k students out of them?

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A result...

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

» Proof of previous identity...

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Theorem

Prove the following $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

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* Fix one of the students, call him Ramesh

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Prove the following $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

* Fix one of the students, call him Ramesh
* Then there *are* two types of teams:

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Prove the following $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

* Fix one of the students, call him Ramesh
* Then there are two types of teams:
  * Teams without Ramesh: $\binom{n-1}{k-1}$

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* Fix one of the students, call him Ramesh
* Then there are two types of teams:
    * Teams without Ramesh: $\binom{n-1}{k-1}$
    * Teams with Ramesh: $\binom{n-1}{k}$

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* Fix one of the students, call him Ramesh
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  * Teams without Ramesh: $\binom{n-1}{k-1}$
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  * Apply sum rule to conclude the proof

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Prove the following $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$

* Fix one of the students, call him Ramesh
* Then there are two types of teams:
  * Teams without Ramesh: $\binom{n-1}{k-1}$
  * Teams with Ramesh: $\binom{n-1}{k}$
  * Apply sum rule to conclude the proof

Hence recursion for n choose k is...

$$\binom{n}{k} = \binom{n-2}{k-2} + \binom{n-2}{k-1} + \binom{n-2}{k-1} + \binom{n-2}{k} = \cdots$$

» Pascal's Triangle

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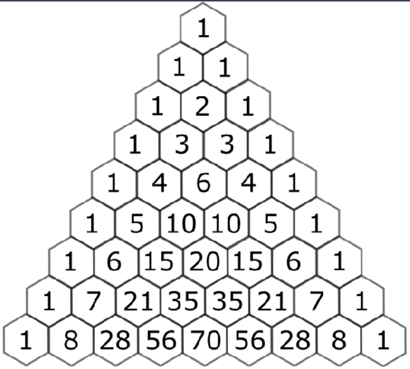
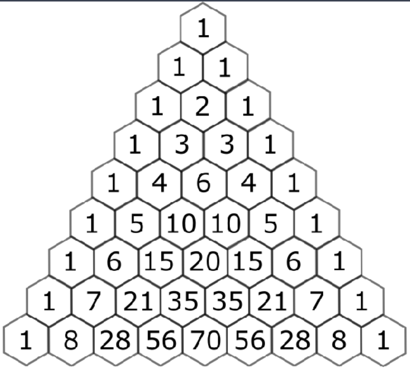


» Pascal's Triangle



Quiz

Do you know how to grow Pascal's triangle? What is the rule?

» Pascal's Triangle and Many Relations...

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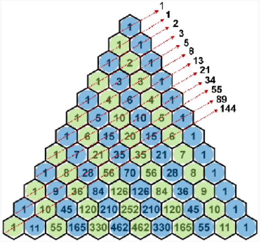
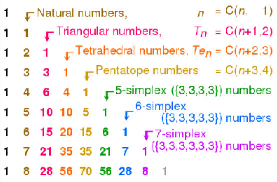
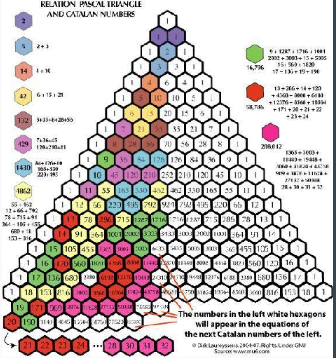
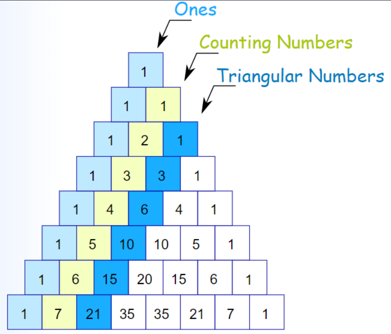
$$(a+b)^0 = 1$$
$$(a+b)^1 = 1a \quad 1b$$
$$(a+b)^2 = 1a^2 \quad 2ab \quad 1b^2$$
$$(a+b)^3 = 1a^3 \quad 3a^2b \quad 3ab^2 \quad 1b^3$$
$$(a+b)^4 = 1a^4 \quad 4a^3b \quad 6a^2b^2 \quad 4ab^3 \quad 1b^4$$
$$(a+b)^5 = 1a^5 \quad 5a^4b \quad 10a^3b^2 \quad 10a^2b^3 \quad 5ab^4 \quad 1b^5$$
$$(a+b)^6 = 1a^6 \quad 6a^5b \quad 15a^4b^2 \quad 20a^3b^3 \quad 15a^2b^4 \quad 6ab^5 \quad 1b^6$$
$$(a+b)^7 = 1a^7 \quad 7a^6b \quad 21a^5b^2 \quad 35a^4b^3 \quad 35a^3b^4 \quad 21a^2b^5 \quad 7ab^6 \quad 1b^7$$
$$(a+b)^8 = 1a^8 \quad 8a^7b \quad 28a^6b^2 \quad 56a^5b^3 \quad 70a^4b^4 \quad 56a^3b^5 \quad 28a^2b^6 \quad 8ab^7 \quad 1b^8$$







» Pascal's Triangle and Triangular Numbers



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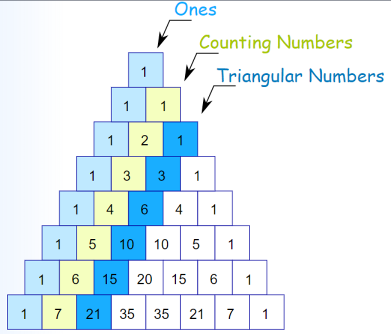
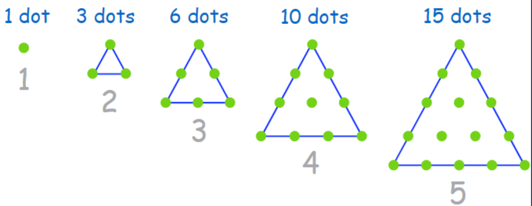
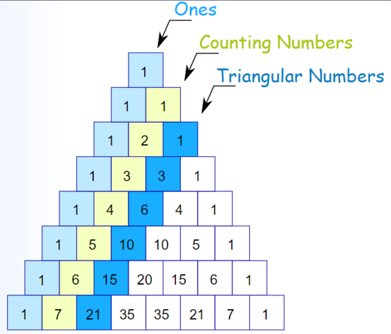


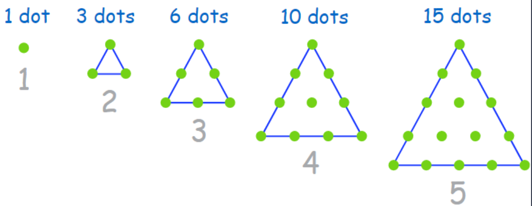


- If we look at the indicated colors, we obtain counting numbers, triangular numbers, etc

- Triangular numbers are the number of dots

» Pascal's Triangle and Triangular Numbers





- If we look at the indicated colors, we obtain counting numbers, triangular numbers, etc

- Triangular numbers are the number of dots
- Add one more row and dots to get next triangular number

» Pascal's Triangle: Horizontal Sums and Exponents of 11

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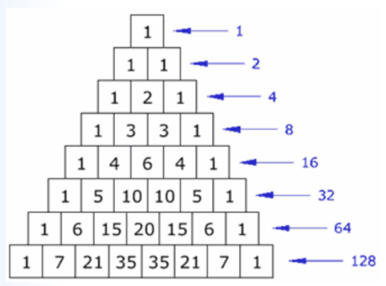

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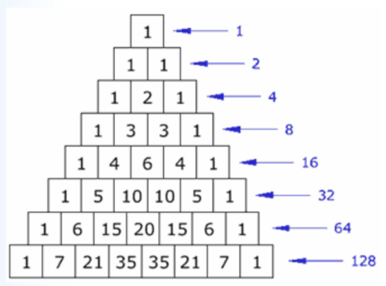

* The horizontal sums are 2^i, i is the ith row

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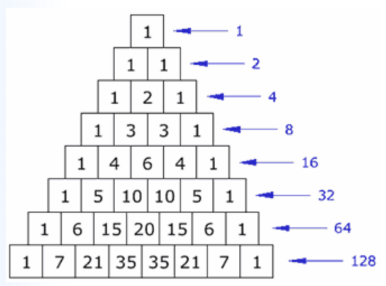

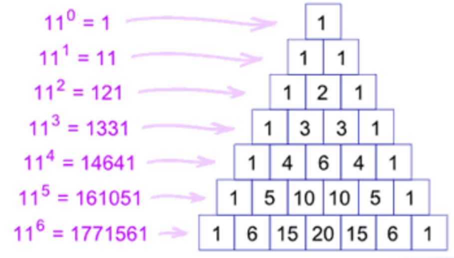

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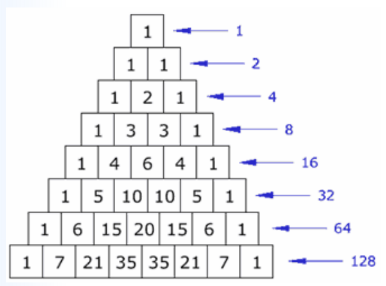

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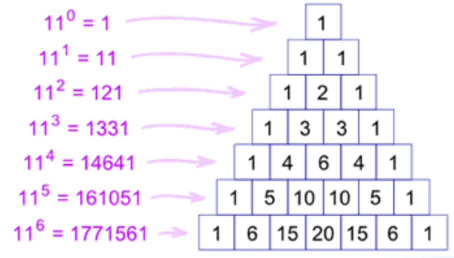

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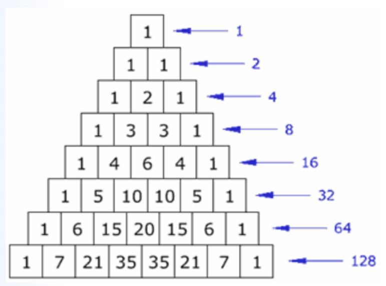

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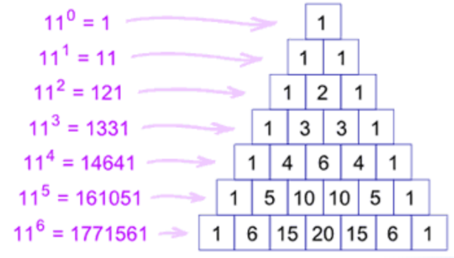

* The row entries are digits of powers of 11
* The entries of the ith row are digits of 11^i

» Pascal’s Triangle and Symmetry

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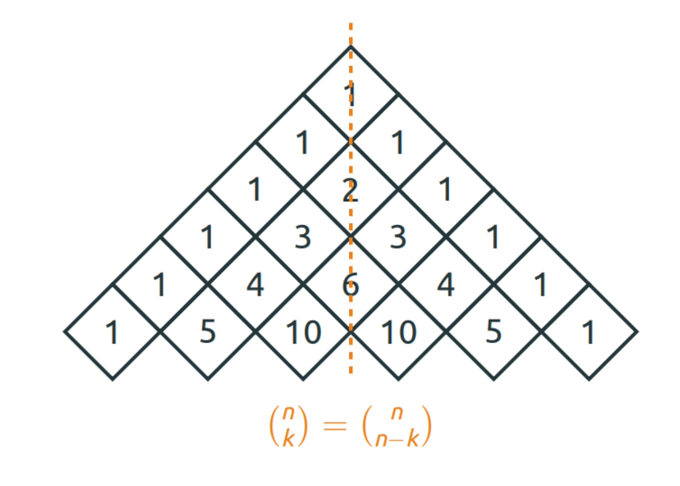

» Proof of symmetry...

Theorem

Prove that

$$\binom{n}{k} = \binom{n}{n-k}$$

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» Proof of symmetry...

Theorem

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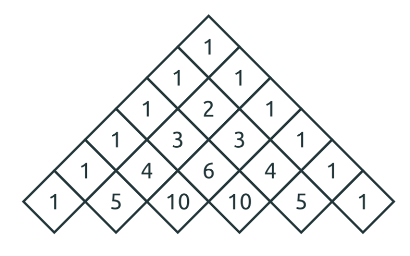
* $\binom{n}{k}$ is the number of ways of selecting *a* team of size k out of n students
* $\binom{n}{n-k}$ is the number of ways of selecting *a* team of size $n-k$ out of n students

Answer

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n!}{(n-k)!k!}$$

» Row Sums of Pascal's Triangle...



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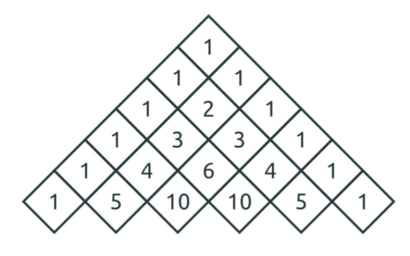


1

1 + 1

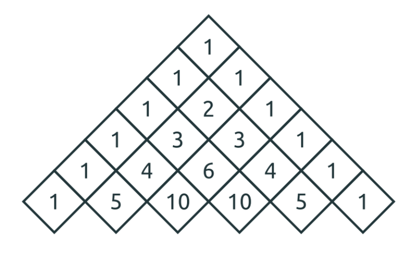
1 + 2 + 1

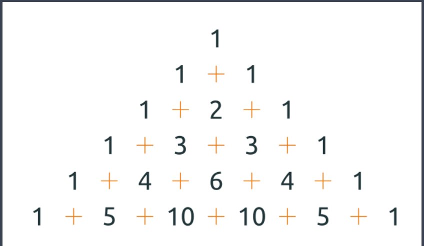
1 + 3 + 3 + 1

1 + 4 + 6 + 4 + 1

1 + 5 + 10 + 10 + 5 + 1

» Row Sums of Pascal's Triangle...





Theorem

The sum of all the numbers in the n-th row of Pascal's triangle is equal to 2^n :

$$\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n-1} + \binom{n}{n} = 2^n$$

Theorem (Prove this...)

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* The base case (0-th row) holds

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* $\binom{n}{k}$ is the number of k-subsets of a set of size n

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- The base case (0-th row) holds
- We'll show that the sum of each row is twice the sum of the previous row
- $\binom{n}{k}$ is the number of *k*-subsets of a set of size n
- The sum $\binom{n}{k}$ for all k (from 0 to n) is the number of all subsets of an n element set; this is 2^n by the product rule (how?)

» Alternating Row Sum in Pascal

» Alternating Row Sum in Pascal

$$
\begin{array}{ccccccccccccc}
 & & & & & 1 & & & & & & & \\
 & & & & 1 & - & 1 & & & & & & = 0 \\
 & & & 1 & - & 2 & + & 1 & & & & & = 0 \\
 & & 1 & - & 3 & + & 3 & - & 1 & & & & = 0 \\
 & 1 & - & 4 & + & 6 & - & 4 & + & 1 & & & = 0 \\
1 & - & 5 & + & 10 & - & 10 & + & 5 & - & 1 & & = 0
\end{array}
$$

» Alternating Row Sum in Pascal

$$
\begin{array}{ccccccccccccc}
 & & & & & & 1 & & & & & & \\
 & & & & & 1 & - & 1 & & & & & = 0 \\
 & & & & 1 & - & 2 & + & 1 & & & & = 0 \\
 & & & 1 & - & 3 & + & 3 & - & 1 & & & = 0 \\
 & & 1 & - & 4 & + & 6 & - & 4 & + & 1 & & = 0 \\
1 & - & 5 & + & 10 & - & 10 & + & 5 & - & 1 & & = 0
\end{array}
$$

Theorem

For $n > 0$, $\quad \sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$

» Alternating Row Sum in Pascal

$$
\begin{array}{ccccccccccccr}
 & & & & & & 1 & & & & & & \\
 & & & & & 1 & - & 1 & & & & & = 0 \\
 & & & & 1 & - & 2 & + & 1 & & & & = 0 \\
 & & & 1 & - & 3 & + & 3 & - & 1 & & & = 0 \\
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1 & - & 5 & + & 10 & - & 10 & + & 5 & - & 1 & & = 0
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$$

Theorem

For $n > 0$, $\quad \sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$

* Hint: Number of odd size subsets = Number of even size subsets

» Counting Problems...

Question

What is the number of 5-card hands dealt off of a standard 52-card deck?

Question

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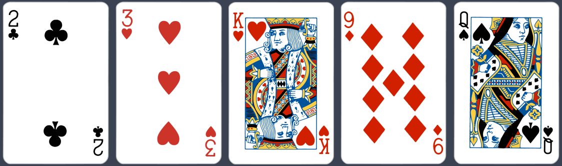

» Counting Problems...

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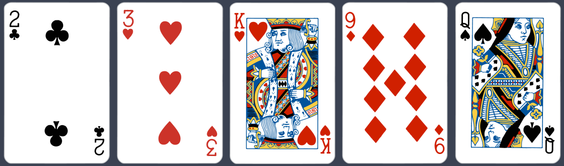

Answer

$$\binom{52}{5} = \frac{52!}{5!47!} = \frac{52 \times 52 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1} = 2598960$$

» Counting Problems ...

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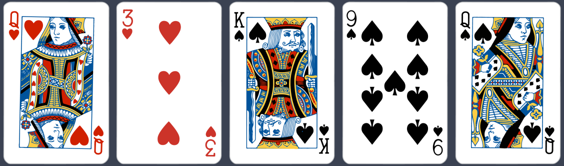

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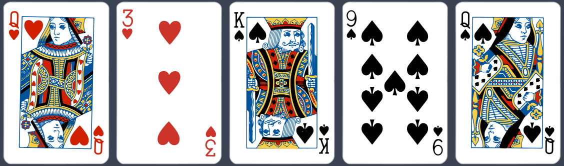

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* Number of ways of picking 2 hearts from 13 hearts

» Counting Problems ...

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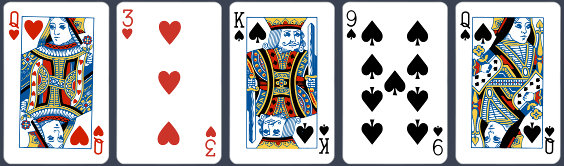

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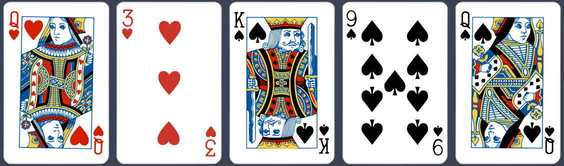

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* Now apply product rule!

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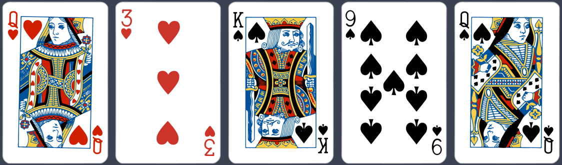

Answer

- Number of ways of picking 2 hearts from 13 hearts
- Number of ways of picking 3 spades from 13 spades
- Now apply product rule!
- The answer is: $\binom{13}{2}\binom{13}{2} = 22308$

» Counting Problems...

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What is the number of non-negative integers with at most four digits at least one of which is equal to 7?

* Total number of 4 digit numbers = 10^4
* Total number of 4 digit number that does not contain 7 = 9^4
* Hence, the answer is $10^4 - 9^4 = 3439$

» Counting Problems...

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* Hence, the answer is $\binom{10}{4} = 210$

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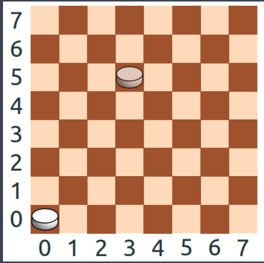

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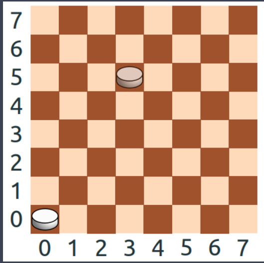

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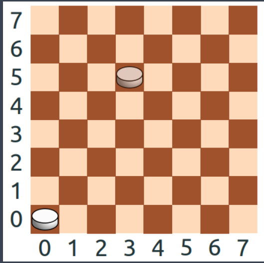

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- Any path to [5,3] **must** involves 3 moves right and 5 moves up!

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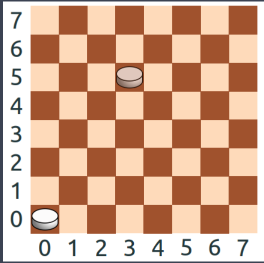

* We want to go to the cell [5,3]. How many ways we can go?
* Any path to [5,3] must involves 3 moves right and 5 moves up!
* Hence, answer is $\binom{8}{3} = 56$

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It is now only a matter of filling the (5,3) cell...

» Combinations with or without repetitions...

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So far we have considered selections of k items out of n possible options. Consider $n = 2$ and $n = 3$ options: a, b, c

	With repetitions	Without repetitions
Ordered	(a,a), (a,b), (a,c) (b,a), (b,b), (b,c) (c,a), (c,b), (c,c)	
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	With repetitions	Without repetitions
Ordered	Tuples n^k	$k-$permutations $\frac{n!}{(n-k)!}$
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* Let us try to find out...