Probability and Statistics: Lecture-14

Monsoon-2020

by Pawan Kumar (IIIT, Hyderabad)
on September 11, 2020

» Table of contents

Discrete

PMF ← distribution

CDF

→ You may login to Skype for alternative chat

Attend the Quiz on Gradescope!

(1) Log in to gradescope with IIIT email

(2) Submit the Quiz: Online quiz-1.

(3) Time limit is 5 min.

Quiz starts at 9:~~32~~3 9:33 AM

» Motivation for Uniform Distribution: Distribution of a Die Roll...

» Motivation for Uniform Distribution: Distribution of a Die Roll...

Example: Motivation for Uniform Distribution

Consider rolling a fair die. The possible outcomes are X $\{1, 2, 3, 4, 5, 6\}$. Then the PMF is given by

Uniform value

$$p(x) = \begin{cases} \frac{1}{6}, & x \in \{1, 2, 3, 4, 5, 6\} \\ 0, & \textit{otherwise} \end{cases}$$

$\frac{1}{6} + \frac{1}{6} + \cdots + \frac{1}{6} = 1$

$\sum p($

We note here that $\sum_{x \in \mathbb{Z}} p(x) = 1$. We note here that PMF takes uniform values for all values of $X = x$.

» Uniform Distribution...

Definition: Uniform Distribution

Motivated from the previous example, we now define uniform distribution on $\{1, 2, \ldots, n\}$ by

$$p(x) = \begin{cases} \frac{1}{n}, & x \in \{1, 2, \ldots, n\} \\ 0, & \textit{otherwise} \end{cases}$$

We verify here that $\sum_{x \in \mathbb{Z}} p(x) = 1.$

» Bernoulli Distribution

Unfair coin p ? H

$1-p$ T

Bernoulli distribution

A random variable X is called a Bernoulli random variable with parameter p, denoted by $X \sim Bernoulli(p)$, if its PMF is given by

H, Pass, Success

$$P_X(x) = \begin{cases} p & \text{for } x = 1, \\ 1 - p & \text{for } x = 0, \\ 0 & \text{otherwise,} \end{cases}$$

T, Fail, Failure

where $0 < p < 1$.

» Bernoulli Distribution

Bernoulli distribution

A random variable X is called a Bernoulli random variable with parameter p, denoted by $X \sim Bernoulli(p)$, if its PMF is given by

$$P_X(x) = \begin{cases} p & \text{for } x = 1, \\ 1 - p & \text{for } x = 0, \\ 0 & \text{otherwise,} \end{cases}$$

where $0 < p < 1$.

- This models random experiments that have two possible outcomes

» Bernoulli Distribution

Bernoulli distribution

A random variable X is called a Bernoulli random variable with parameter p, denoted by $X \sim Bernoulli(p)$, if its PMF is given by

$$P_X(x) = \begin{cases} p & \text{for } x = 1, \\ 1 - p & \text{for } x = 0, \\ 0 & \text{otherwise,} \end{cases}$$

where $0 < p < 1$.

* This models random experiments that have two possible outcomes
* Example: You take a pass-fail exam. You either pass or fail

» Bernoulli Distribution

Bernoulli distribution

A random variable X is called a Bernoulli random variable with parameter p, denoted by $X \sim Bernoulli(p)$, if its PMF is given by

$$P_X(x) = \begin{cases} p & \text{for } x = 1, \\ 1 - p & \text{for } x = 0, \\ 0 & \text{otherwise,} \end{cases}$$

where $0 < p < 1$.

* This models random experiments that have two possible outcomes
* Example: You take a pass-fail exam. You either pass or fail
* Example: A coin is tossed, the outcome is either heads or tails

» Geometric Distribution

» Geometric Distribution

Definition of Geometric Distribution

A random variable X is called geometric random variable with parameter p, denoted by $X \sim Geometric(p)$, if its PMF is given by

$$P_X(k) = \begin{cases} p(1-p)^{k-1}, & \text{for } k = 1, 2, 3, \ldots \\ 0, & \text{otherwise,} \end{cases}$$

where $0 < p < 1$.

first 1st at k-th toss

$\underbrace{(1-p)^{k-1}}_{k-1}\ \underbrace{p}_{k\text{-th}}$

an event that happen for the 1st time at k-th trial

$p \quad (1-p)$

» Geometric Distribution

Definition of Geometric Distribution

A random variable X is called geometric random variable with parameter p, denoted by $X \sim Geometric(p)$, if its PMF is given by

$$P_X(k) = \begin{cases} p(1-p)^{k-1}, & \text{for } k = 1, 2, 3, \ldots \\ 0, & \text{otherwise,} \end{cases}$$

where $0 < p < 1$.

* **Example:** Suppose we have an unfair coin with $P(H) = p$.

» Geometric Distribution

Definition of Geometric Distribution

A random variable X is called geometric random variable with parameter p, denoted by $X \sim Geometric(p)$, if its PMF is given by

$$P_X(k) = \begin{cases} p(1-p)^{k-1}, & \text{for } k = 1, 2, 3, \ldots \\ 0, & \text{otherwise,} \end{cases}$$

where $0 < p < 1$.

* Example: Suppose we have an unfair coin with $P(H) = p$. We toss the coin until we obtain first heads.

Definition of Geometric Distribution

A random variable X is called geometric random variable with parameter p, denoted by $X \sim Geometric(p)$, if its PMF is given by

$$P_X(k) = \begin{cases} p(1-p)^{k-1}, & \text{for } k = 1, 2, 3, \ldots \\ 0, & \text{otherwise,} \end{cases}$$

where $0 < p < 1$.

* Example: Suppose we have an unfair coin with $P(H) = p$. We toss the coin until we obtain first heads. Let RV X be the total number of tosses.

» Geometric Distribution

Definition of Geometric Distribution

A random variable X is called geometric random variable with parameter p, denoted by $X \sim Geometric(p)$, if its PMF is given by

$$P_X(k) = \begin{cases} p(1-p)^{k-1}, & \text{for } k = 1, 2, 3, \ldots \\ 0, & \text{otherwise}, \end{cases}$$

where $0 < p < 1$.

* Example: Suppose we have an unfair coin with $P(H) = p$. We toss the coin until we obtain first heads. Let RV X be the total number of tosses. Then X have geometric distribution.

» Geometric Distribution

Definition of Geometric Distribution

A random variable X is called geometric random variable with parameter p, denoted by $X \sim Geometric(p)$, if its PMF is given by

k

$$P_X(k) = \begin{cases} p(1-p)^{k-1}, & \text{for } k = 1, 2, 3, \ldots \\ 0, & \text{otherwise,} \end{cases}$$

k-1 failure

Succes at kth

where $0 < p < 1$.

* Example: Suppose we have an unfair coin with $P(H) = p$. We toss the coin until we obtain first heads. Let RV X be the total number of tosses. Then X have geometric distribution.
* Caution: Some books define geometric random variable X $=k$ as total number of failures before observing first success. Then

k

k+1

success

Definition of Geometric Distribution

A random variable X is called geometric random variable with parameter p, denoted by $X \sim Geometric(p)$, if its PMF is given by

$$P_X(k) = \begin{cases} p(1-p)^{k-1}, & \text{for } k = 1, 2, 3, \ldots \\ 0, & \text{otherwise,} \end{cases}$$

where $0 < p < 1$.

- Example: Suppose we have an unfair coin with $P(H) = p$. We toss the coin until we obtain first heads. Let RV X be the total number of tosses. Then X have geometric distribution.
- Caution: Some books define geometric random variable X as total number of failures before observing first success. Then

k+1

$$P_X(k) = \begin{cases} p(1-p)^{k}, & \text{for } k = 1, 2, 3, \ldots \\ 0, & \text{otherwise,} \end{cases}$$

» Binomial Distribution

Definition of Binomial Distribution

A random variable X is called Binomial random variable with parameters n and p, denoted by $X \sim Binomial(n, p)$, if the PMF is given by

Definition of Binomial Distribution

A random variable X is called Binomial random variable with parameters n and p, denoted by $X \sim Binomial(n, p)$, if the PMF is given by

$$P_X(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & \text{for } k = 0, 1, 2, \cdots, n \\ 0 & \text{otherwise,} \end{cases}$$

where $0 < p < 1$.

p^k

$n-k$

$(1-p)$

k heads in n-trials

» Binomial Distribution

Definition of Binomial Distribution

A random variable X is called Binomial random variable with parameters n and p, denoted by $X \sim Binomial(n, p)$, if the PMF is given by

$$P_X(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & \text{for } k = 0, 1, 2, \cdots, n \\ 0 & \text{otherwise,} \end{cases}$$

where $0 < p < 1$.

* Example: Consider an unfair coin with $P(H) = p$. Consider tossing the coin n times and let X be the total number of heads we observe. Then X is a Binomial with parameters n and p.

» Binomial Distribution

Definition of Binomial Distribution

A random variable X is called **Binomial random variable** with parameters n and p, denoted by $X \sim Binomial(n, p)$, if the **PMF** is given by

$$P_X(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & \text{for } k = 0, 1, 2, \cdots, n \\ 0 & \text{otherwise,} \end{cases}$$

where $0 < p < 1$.

$Ber(p) \begin{cases} p, & x = 1 \\ 1-p, & x = 0 \end{cases}$

1 coin toss

- **Example:** Consider an unfair coin with $P(H) = p$. Consider tossing the coin n times and let X be the total number of heads we observe. Then X is a **Binomial** with parameters n and p.
- Binomial(n, p) is a sum of n independent **Bernoulli**(p) random variables.

» Binomial Distribution

Definition of Binomial Distribution

A random variable X is called Binomial random variable with parameters n and p, denoted by $X \sim Binomial(n, p)$, if the PMF is given by

$$P_X(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & \text{for } k = 0, 1, 2, \cdots, n \\ 0 & \text{otherwise,} \end{cases}$$

where $0 < p < 1$.

* Example: Consider an unfair coin with $P(H) = p$. Consider tossing the coin n times and let X be the total number of heads we observe. Then X is a Binomial with parameters n and p.
* Binomial(n, p) is a sum of n independent Bernoulli(p) random variables.
* If $X_1, X_2, \ldots, X_n$ are independent Bernoulli(p) random variables, then $X = \underbrace{X_1 + \cdots + X_n}$ has ~~Bernoulli~~(n, p) distribution.

Binomial

» Binomial Distribution

Definition of Binomial Distribution

A random variable X is called Binomial random variable with parameters n and p, denoted by $X \sim Binomial(n, p)$, if the PMF is given by

n

k

$$P_X(k) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k} & \text{for } k = 0, 1, 2, \cdots, n \\ 0 & \text{otherwise,} \end{cases}$$

$X_1 + X_2$ X Binomial$(2, p)$

where $0 < p < 1$.

- Example: Consider an unfair coin with $P(H) = p$. Consider tossing the coin n times and let X be the total number of heads we observe. Then X is a Binomial with parameters n and p.
- Binomial(n, p) is a sum of n independent Bernoulli(p) random variables.
- If $X_1, X_2, \ldots, X_n$ are independent Bernoulli(p) random variables, then $X = X_1 + \cdots + X_n$ has ~~Bernoulli~~ Binomial(n, p) distribution.
- We verify that $\sum_{x \in \mathbb{Z}} p(x) = \sum_{x=0}^{n} \binom{n}{x} p^x (1-p)^{n-x} = 1$ check

» Example

n - ind. Bernoulli

m - ind Bernoulli

Example

Let $X \sim Binomial(n, p)$ and $Y \sim Binomial(m, p)$ be two independent random variables. We define a random variable $Z = X + Y$. What is the PMF of Z?

$\Rightarrow X = X_1 + X_2 + \dots + X_n$, when X_i's are Bernoulli

$\Rightarrow Y = Y_1 + Y_2 + \dots + Y_m$, when Y_i's are Bernoulli

$$Z = X + Y = \underbrace{X_1 + X_2 + X_3 + \dots + X_n + Y_1 + \dots + Y_m}_{\text{Sum of } m+n \text{ ind. Bernoulli r.v.}}$$

$= Binomial(m+n, p)$

» Scratch Space...

» Scratch Space...

» Motivation for Poisson Distribution...

» Motivation for Poisson Distribution...

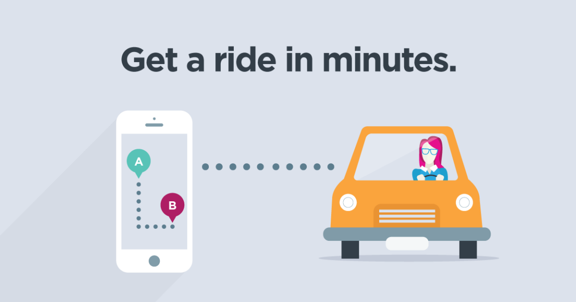
* Imagine you are an Uber driver

» Motivation for Poisson Distribution...

* Imagine you are an Uber driver
* To maximize profit, you want to remain close to an area with more requests
* How can you use probability to predict requests for car ride?

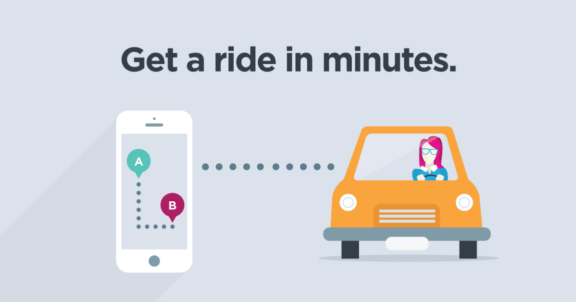
» Motivation for Poisson Distribution...

- Imagine you are an Uber driver
- To maximize profit, you want to remain close to an area with more requests
- How can you use probability to predict requests for car ride?



» Motivation for Poisson Distribution...

* Imagine you are an Uber driver
* To maximize profit, you want to remain close to an area with more requests
* How can you use probability to predict requests for car ride?



Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

» Introduce Poisson Using an Example...

» Introduce Poisson Using an Example...

Example

Given that there are on *average* 5 requests for car ride per minute from certain *area*. What is the probability of k requests from this *area* in next 1 minute?

* What do we know for this problem?

» Introduce Poisson Using an Example...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* What do we know for this problem? There are requests for car ride!

» Introduce Poisson Using an Example...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* What do we know for this problem? There are requests for car ride!
* There are two possibilities: request (1) or no request (0):

» Introduce Poisson Using an Example...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* What do we know for this problem? There are requests for car ride!
* There are two possibilities: request (1) or no request (0): binary choices

» Introduce Poisson Using an Example...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* What do we know for this problem? There are requests for car ride!
* There are two possibilities: request (1) or no request (0): binary choices
* Sounds familiar? Yes!

» Introduce Poisson Using an Example...

Example

Given that there are on *average* 5 requests for car ride per minute from certain *area*. What is the probability of k requests from this *area* in next 1 minute?

* What do we know for this problem? There *are* requests for car ride!
* There *are* two possibilities: request (1) or no request (0): binary choices
* Sounds familiar? Yes! Recall: How many heads in n trials?

» Introduce Poisson Using an Example...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* What do we know for this problem? There are requests for car ride!
* There are two possibilities: request (1) or no request (0): binary choices
* Sounds familiar? Yes! Recall: How many heads in n trials?
* The problem here is: we don't have n trials, instead a time interval! So?

» Introduce Poisson Using an Example...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

- What do we know for this problem? There are requests for car ride!
- There are two possibilities: request (1) or no request (0): binary choices
- Sounds familiar? Yes! Recall: How many heads in n trials?
- The problem here is: we don't have n trials, instead a time interval! So?
- Key idea: Want to use discrete distribution, so anyhow convert time to discrete intervals

» Introduce Poisson Using an Example...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

- What do we know for this problem? There are requests for car ride!
- There are two possibilities: request (1) or no request (0): binary choices
- Sounds familiar? Yes! Recall: How many heads in n trials?
- The problem here is: we don't have n trials, instead a time interval! So?
- Key idea: Want to use discrete distribution, so anyhow convert time to discrete intervals

1st 2nd 3rd 5 60th 1 minute

0	0	1	0	1	...	...	0	1	0	1	1	0

60 second

0, or 1 in Binomial

» Introduce Poisson Using an Example...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* What do we know for this problem? There are requests for car ride!
* There are two possibilities: request (1) or no request (0): binary choices
* Sounds familiar? Yes! Recall: How many heads in n trials?
* The problem here is: we don't have n trials, instead a time interval! So?
* Key idea: Want to use discrete distribution, so anyhow convert time to discrete intervals

0	0	1	0	1	...	...	0	1	0	1	1	0

  * break a minute into 60 seconds;

» Introduce Poisson Using an Example...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

- What do we know for this problem? There are requests for car ride!
- There are two possibilities: request (1) or no request (0): binary choices
- Sounds familiar? Yes! Recall: How many heads in n trials?
- The problem here is: we don't have n trials, instead a time interval! So?
- Key idea: Want to use discrete distribution, so anyhow convert time to discrete intervals

0	0	1	0	1	...	...	0	1	0	1	1	0

  - break a minute into 60 seconds; each second is independent trial with request or no request

» Introduce Poisson Using an Example...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

- What do we know for this problem? There are requests for car ride!
- There are two possibilities: request (1) or no request (0): binary choices
- Sounds familiar? Yes! Recall: How many heads in n trials?
- The problem here is: we don't have n trials, instead a time interval! So?
- Key idea: Want to use discrete distribution, so anyhow convert time to discrete intervals

0	0	1	0	1	...	...	0	1	0	1	1	0

  - break a minute into 60 seconds; each second is independent trial with request or no request
  - Let $X =$ number of requests in a minute

» Introduce Poisson Using an Example...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* What do we know for this problem? There are requests for car ride!
* There are two possibilities: request (1) or no request (0): binary choices
* Sounds familiar? Yes! Recall: How many heads in n trials?
* The problem here is: we don't have n trials, instead a time interval! So?
* Key idea: Want to use discrete distribution, so anyhow convert time to discrete intervals

0	0	1	0	1	...	...	0	1	0	1	1	0

Binomial (k,

E(X)

  * break a minute into 60 seconds; each second is independent trial with request or no request
  * Let $X =$ number of requests in a minute
  * $E[X] = \lambda = 5 = np$, where p is the probability of request

» Introduce Poisson Using an Example...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* What do we know for this problem? There are requests for car ride!
* There are two possibilities: request (1) or no request (0): binary choices
* Sounds familiar? Yes! Recall: How many heads in n trials?
* The problem here is: we don't have n trials, instead a time interval! So?
* Key idea: Want to use discrete distribution, so anyhow convert time to discrete intervals

0	0	1	0	1	...	...	0	1	0	1	1	0

60 bins

* break a minute into 60 seconds; each second is independent trial with request or no request
* Let $X =$ number of requests in a minute
* $E[X] = \lambda = 5 = np$, where p is the probability of request

5 hts

50

* We now identify this problem as $X \sim Binomial(n = 60, p = 5/60)$

$p = 5/n$

» Introduce Poisson Using an Example...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* What do we know for this problem? There are requests for car ride!
* There are two possibilities: request (1) or no request (0): binary choices
* Sounds familiar? **Yes!** Recall: How many heads in n trials?
* The problem here is: we don't have n trials, instead a time interval! So?
* **Key idea:** Want to use discrete distribution, so anyhow convert time to discrete intervals

0	0	1	0	1	...	...	0	1	0	1	1	0

n trials = 60

  * break a minute into 60 seconds; each second is independent trial with request or no request
  * Let $X =$ number of requests in a minute
  * $E[X] = \lambda = 5 = np$, where p is the probability of request
* We now identify this problem as $X \sim Binomial(n = 60, p = 5/50)$
* $P(X = k) = \binom{60}{k} \left(\frac{5}{60}\right)^k \left(1 - \frac{5}{60}\right)^{n-k}$

» Motivation for Possion Distribution...

» Motivation for Possion Distribution...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

- We identified this problem as $X \sim Binomial(n = 60, p = 5/50)$

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* We identified this problem as $X \sim Binomial(n = 60, p = 5/50)$
* $P(X = k) = \binom{60}{k} \left(\frac{5}{60}\right)^k \left(1 - \frac{5}{60}\right)^{n-k}$
* There is a problem!

» Motivation for Possion Distribution...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* We identified this problem as $X \sim Binomial(n = 60, p = 5/50)$
* $P(X = k) = \binom{60}{k}\left(\frac{5}{60}\right)^k \left(1 - \frac{5}{60}\right)^{n-k}$
* There is a problem! What is that?

» Motivation for Possion Distribution...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* We identified this problem as $X \sim Binomial(n = 60, p = 5/50)$
* $P(X = k) = \binom{60}{k} \left(\frac{5}{60}\right)^k \left(1 - \frac{5}{60}\right)^{n-k}$
* There is a problem! What is that? Time is continuous!

» Motivation for Possion Distribution...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* We identified this problem as $X \sim Binomial(n = 60, p = 5/50)$
* $P(X = k) = \binom{60}{k}\left(\frac{5}{60}\right)^k \left(1 - \frac{5}{60}\right)^{n-k}$
* There is a problem! What is that? Time is continuous!
* Since time is continuous, no guarantee that there will be only one request per second

» Motivation for Possion Distribution...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

- We identified this problem as $X \sim Binomial(n = 60, p = 5/50)$
- $P(X = k) = \binom{60}{k}\left(\frac{5}{60}\right)^k \left(1 - \dfrac{5}{60}\right)^{n-k}$
- There is a problem! What is that? Time is continuous!
- Since time is continuous, no guarantee that there will be only one request per second
- What do we do now?

» Motivation for Possion Distribution...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* We identified this problem as $X \sim Binomial(n = 60, p = 5/50)$
* $P(X = k) = \binom{60}{k}\left(\frac{5}{60}\right)^k \left(1 - \frac{5}{60}\right)^{n-k}$
* There is a problem! What is that? Time is continuous!
* Since time is continuous, no guarantee that there will be only one request per second
* What do we do now? We can make the interval smaller, take time intervals as milliseconds?

» Motivation for Possion Distribution...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* We identified this problem as $X \sim Binomial(n = 60, p = 5/50)$
* $P(X = k) = \binom{60}{k} \left(\frac{5}{60}\right)^k \left(1 - \frac{5}{60}\right)^{n-k}$
* There is a problem! What is that? Time is continuous!
* Since time is continuous, no guarantee that there will be only one request per second
* What do we do now? We can make the interval smaller, take time intervals as milliseconds?
* But there is still no guarantee that there will be only one request per millisecond

» Motivation for Possion Distribution...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* We identified this problem as $X \sim Binomial(n = 60, p = 5/50)$
* $P(X = k) = \binom{60}{k} \left(\frac{5}{60}\right)^k \left(1 - \frac{5}{60}\right)^{n-k}$
* There is a problem! What is that? Time is continuous!
* Since time is continuous, no guarantee that there will be only one request per second
* What do we do now? We can make the interval smaller, take time intervals as milliseconds?
* But there is still no guarantee that there will be only one request per millisecond
* What should we do next?

» Motivation for Possion Distribution...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

- We identified this problem as $X \sim Binomial(n = 60, p = 5/50)$
- $P(X = k) = \binom{60}{k} \left(\frac{5}{60}\right)^k \left(1 - \frac{5}{60}\right)^{n-k}$
- There is a problem! What is that? Time is continuous!
- Since time is continuous, no guarantee that there will be only one request per second
- What do we do now? We can make the interval smaller, take time intervals as milliseconds?
- But there is still no guarantee that there will be only one request per millisecond
- What should we do next? Here comes calculus...

» Motivation for Possion Distribution...

Example

Given that there are on average 5 requests for car ride per minute from certain area. What is the probability of k requests from this area in next 1 minute?

* We identified this problem as $X \sim Binomial(n = 60, p = 5/50)$
* $P(X = k) = \binom{60}{k} \binom{5}{60}^k \left(1 - \frac{5}{60}\right)^{n-k}$
* There is a problem! What is that? Time is continuous!
* Since time is continuous, no guarantee that there will be only one request per second
* What do we do now? We can make the interval smaller, take time intervals as milliseconds?
* But there is still no guarantee that there will be only one request per millisecond
* What should we do next? Here comes calculus... to go from discrete to continuous...

» Binomial in the Limit is Poisson Distribution...

» Binomial in the Limit is Poisson Distribution...

Derivation:

$$P(X=k) = \lim_{n\to\infty} \binom{n}{k}\left(\frac{\lambda}{n}\right)^k \left(1-\frac{\lambda}{n}\right)^{n-k}$$

$$= \lim_{n\to\infty} \frac{n!}{k!\,(n-k)!} \cdot \frac{\lambda^k}{n^k} \cdot \frac{\left(1-\lambda/n\right)^n}{\left(1-\lambda/n\right)^k}$$

$\lim n \to \infty$

$$e^{-\lambda}$$

$$= \lim_{n\to\infty} \frac{n(n-1)\cdots(n-k+1)}{n^k} \cdot \frac{(n-k)!}{(n-k)!} \cdot \frac{\lambda^k}{k!} \cdot \frac{e^{-\lambda}}{\left(1-\frac{\lambda}{n}\right)^k}$$

$$= \lim_{n\to\infty} \frac{n^k}{n^k} \cdot \frac{\lambda^k}{k!}\, e^{-\lambda} = \frac{\lambda^k}{k!}\, e^{-\lambda}$$

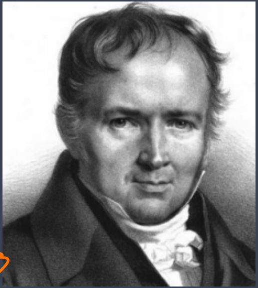
» Definition of Poisson Distribution...

» Definition of Poisson Distribution...

Definition of Poisson

A random variable X is said to be a Poisson random variable with parameter λ, shown as $X \sim Poisson(\lambda)$, if its range is $R_X = \{0, 1, 2, \ldots, \}$, and its PMF is given by

$$P_X(k) \begin{cases} \frac{e^{-\lambda}\lambda^k}{k!}, & k \in R_X \\ 0 & \text{otherwise} \end{cases}$$



* Simeon-Denis Poisson, was a French mathematician (1781-1840)
* He published his first paper at 18, professot at 21
* He published over 300 papers