Probability and Statistics: Lecture-9

Monsoon-2020

by Pawan Kumar (IIIT, Hyderabad)
on August 28, 2020

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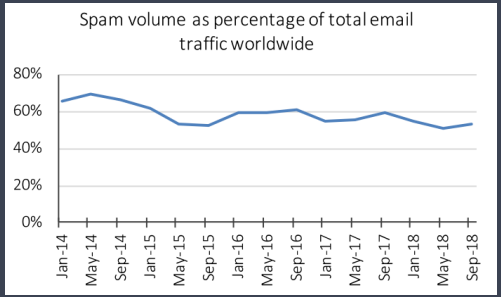
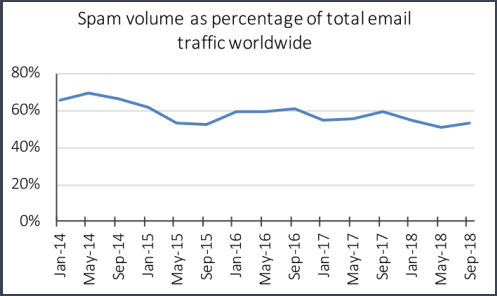
» Bayes Theorem. Why?



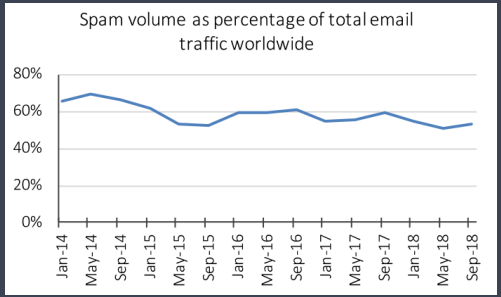
» Bayes Theorem. Why?



* We can easily calculate how many spam emails contain “Dear”:

$$P(E|F) = P(\text{Dear}|\text{Spam})$$

» Bayes Theorem. Why?



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* But what is the probability that an email containing “Dear” is spam?

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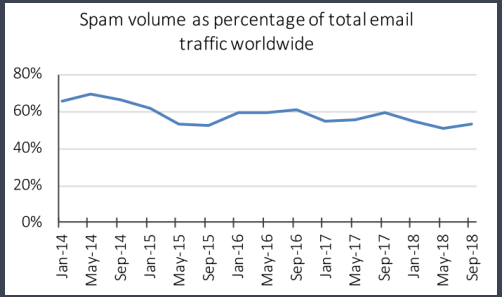

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» Bayes Theorem

Bayes Theorem

For any events E and F where $P(E) > 0$ and $P(F) > 0$,

$$P(F|E) = \frac{P(E|F)P(F)}{P(E)}$$

Proof of Bayes Theorem:

» Bayes Theorem with Total Probability...

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Bayes Theorem

For any events E and F where $P(E) > 0$ and $P(F) > 0$,

$$P(F|E) = \frac{P(E|F)P(F)}{P(E|F)P(F) + P(E|F^c)P(F^c)}$$

Proof of Bayes Theorem:

» Bayes Theorem Used in Spam Emails Example...

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Spam Email Example

Given the following:

- 60% of all email in 2016 is spam

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» Bayes Theorem Used in Spam Emails Example...

Spam Email Example

Given the following:

- 60% of all email in 2016 is spam
- 20% of spam has the word “Dear”
- 1% of non-spam (aka ham) has the word “Dear”

You get an email with the word “Dear” in it. What is the probability that the email is spam?

Solution:

» Application of Bayes Theorem...

Example

A test is 98% effective at detecting a disease ("true positive"). However, the test has a "false positive" rate of 1%. The 0.5% of the US population has disease. What is the likelihood you have the disease, if you test positive?

Solution:

» Conditional Probability and Game of Chance Movie...

» Conditional Probability and Game of Chance Movie...

Movie Monty Hall Movie 21 Video Clip Here!

Another Monty Hall Youtube Movie Here!

» Conditional Probability and Game of Chance...

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* There is *a game show.*

» Conditional Probability and Game of Chance...

* There is *a game show*. The show host shows you three doors

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* Behind one of the doors, there is *a* car,

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* There is a game show. The show host shows you three doors
* Behind one of the doors, there is a car, and in other two there are goats

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- Question: if the host always opens goat door,

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- There is a game show. The show host shows you three doors
- Behind one of the doors, there is a car, and in other two there are goats
- Rules of the Game Show:
  - you are allowed to pick a door without opening
  - then the host opens a door
- Question: if the host always opens goat door, is it wise to change your door?

» Solution to Monty Hall Problem with Graphical Illustration

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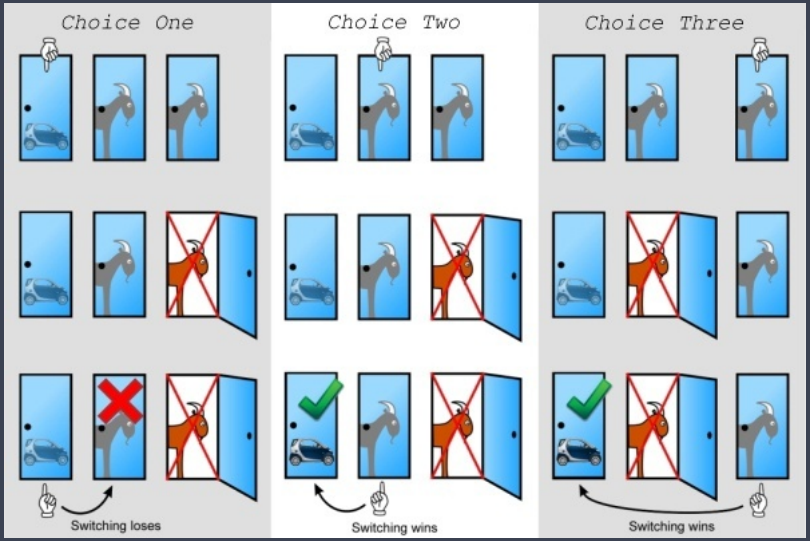

Graphical illustration of Monty hall problem. Source: Google

» Solution to Game Show: Choice Tree, Conditional Probability

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Let us look into all possible (exhaustive) cases:

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Let us look into all possible (exhaustive) cases:

Door You Choose	Prize in Door	Host Opens	Stay	Switch

» Solution to Game Show: Choice Tree, Conditional Probability

Let us look into all possible (exhaustive) cases:

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1	1	2/3	win	loose

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1	3	2	loose	win

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2	3	1	loose	win
3	1	2	loose	win
3	2	1	loose	win
3	3	1/2	win	loose

Exhaustive list of possibilities

Conclusion

If you switch, the probability that you win a car is $2/3$,

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If you switch, the probability that you win a car is $2/3$, and if you switch,

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Exhaustive list of possibilities

Conclusion

If you switch, the probability that you win a car is $2/3$, and if you switch, the probability that you win goat is $1/3$.