Probability and Statistics: Lecture-10

Monsoon-2020

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» Table of contents

» Conditional Probability and Game of Chance Movie...

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Movie Monty Hall Movie 21 Video Clip Here!

Another Monty Hall Youtube Movie Here!

» Conditional Probability and Game of Chance...

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  - then the host opens a door
- Question: if the host always opens goat door,

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* Behind one of the doors, there is a car, and in other two there are goats
* Rules of the Game Show:
  * you are allowed to pick a door without opening
  * then the host opens a door
* Question: if the host always opens goat door, is it wise to change your door?

» Solution to Monty Hall Problem with Graphical Illustration

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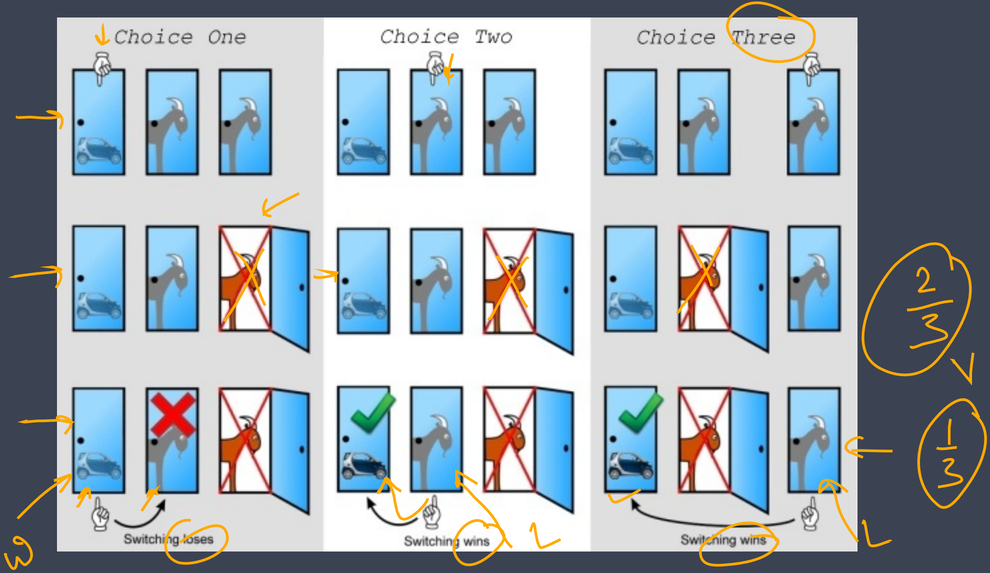

Graphical illustration of Monty hall problem. Source: Google

» Solution to Game Show: Choice Tree, Conditional Probability

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Let us look into *all* possible (exhaustive) cases:

» Solution to Game Show: Choice Tree, Conditional Probability

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3	1	2	loose	win
3	2	1	loose	win
3	3	1/2	win	loose

Exhaustive list of possibilities

$\frac{1}{3} = \frac{3}{9} < \frac{6}{9} = \frac{2}{3}$

Conclusion

If you switch, the probability that you win a car is $2/3$,

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Exhaustive list of possibilities

Conclusion

If you switch, the probability that you win a car is $2/3$, and if you ~~switch~~ stay, the probability that you win ~~goat~~ is $1/3$.

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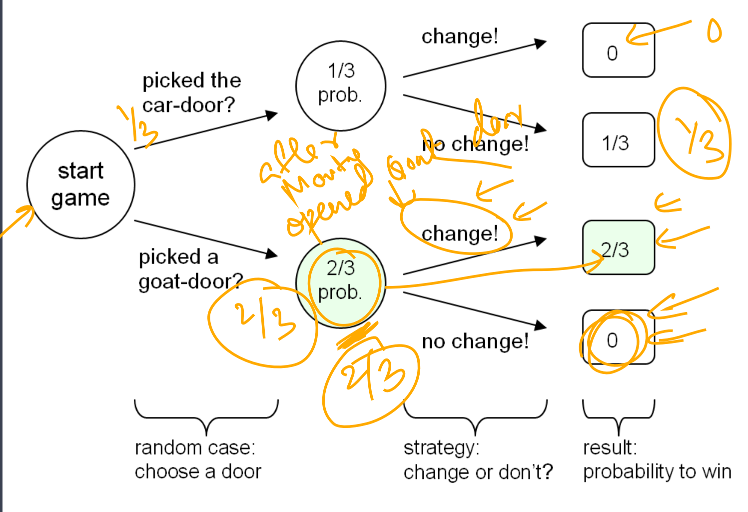

Graphical illustration of Choice Tree of Monty hall problem. Source: Google

» Using Bayes Theorem in Monty Hall's Problem...

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- Let H be the hypothesis "door 1 has a car behind it," and E be the evidence that Monty has revealed a door with a goat behind it

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$\boxed{P(E \cap H) = P(E)P(H)}$

Write the following in words:

- $P(H) = \frac{1}{3}$
- $P(H^c) = 1 - P(H) = 1 - \frac{1}{3} = \frac{2}{3}$
- $P(E|H) = 1$
- $P(E|H^c) = 1$

$$P(H|E) = \frac{P(E|H)P(H)}{P(E)} = \frac{1 \cdot \frac{1}{3}}{P(E|H)P(H) + P(E|H^c)P(H^c)} = \frac{1 \cdot \frac{1}{3}}{1 \cdot \frac{1}{3} + 1 \cdot \frac{2}{3}} = \frac{1}{3}$$

Bayes theorem

not available

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Write the following in words:

* $P(H) =$
* $P(H^c) =$
* $P(E|H) =$
* $P(E|H^c) =$
* $P(H|E) = \dfrac{P(E|H)P(H)}{P(E|H)P(H) + P(E|H^c)P(H^c)} =$

» Problem Similar to Monty Hall Problem...

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Problem Similar to Monty Hall...

Suppose we have 3 cards identical in form except that both sides of the first card are colored red, both sides of the second card are colored black, and one side of the third card is colored red and the other side is colored black. The 3 cards are mixed up in a hat, and 1 card is randomly selected and put down on the ground. If the upper side of the chosen card is colored red, what is the probability that the other side is colored black?

Solution :

$$P(RB \mid R) = \frac{P(RB \cap R)}{P(R)} = \frac{P(R \mid RB)\,P(RB)}{P(R \mid RR)\,P(RR) + P(R \mid RB)\,P(RB) + P(R \mid BB)\,P(BB)}$$

$$= \frac{\frac{1}{2}\cdot\frac{1}{3}}{1\cdot\frac{1}{3} + \frac{1}{2}\cdot\frac{1}{3} + \frac{1}{3}\cdot 0} = \frac{\frac{1}{6}}{\frac{1}{3}+\frac{1}{6}} = \frac{\frac{1}{6}}{\frac{2+1}{6}} = \frac{1}{3}$$

RR   BB