Modern Complexity Theory

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Chapter 1

Proofs

1.1 Social Choice Theory

Society of n people choose among k alternatives.

$F \rightarrow$ Social Choice function that aggregates indvidual prefrences.

Indvidual prefrences is an ordering S_k : set of permutations of $\{1, 2, \ldots, k\}$. Example: $2 > 1 > 3 > 4 > 5$.

Preference profile of society: $\pi \in {S_k}^n$o

$F : {S_k}^n \rightarrow S_k$

For $k = 2$:

$$\{(1 > 2), (2 > 1), (1\ 2)\}$$

Assign $(1 > 2) \rightarrow 1$, $(2 > 1) \rightarrow -1$, $(1\ 2) \rightarrow 0$.

$$F(\pi_1, \pi_2, \ldots, \pi_n) = sgn(\sum_{i=1}^{n} W_i \pi_i)$$

$sgn(x) = 1$ if $x > 0$, 0 if $x = 0$, -1 if $x < 0$.