Probability and Statistics: Lecture-13

Monsoon-2020

by Pawan Kumar (IIIT, Hyderabad)
on September 7, 2020

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2. event E_1 : first toss is H
3. event E_2 : first two tosses are TH
4. event E_3 : first two tosses are TT
5. event F_n : experiment completed on the nth toss.

» Solution to problem in previous slide...part-1

$$P(F_n) = \underbrace{P(F_n|E_1)}\boxed{P(E_1)} + \underbrace{P(F_n|E_2)}\boxed{P(E_2)} + \underbrace{\cancel{P(F_n|E_3)}}_{}\boxed{P(E_3)}$$

0

For $n=2$ $\quad P(F_2) = P(E_3) = (1-p)^2$

and for $n>2$

$\rightarrow P(F_n|E_1) = P(F_{n-1})$

$\rightarrow P(F_n|E_2) = P(F_{n-2})$

$\rightarrow P(F_n|E_3) = 0$

H | — — . . — nth

— — — — — $n-1$th

T T | — — . nth

$n>2$

Let $p_n = P(F_n)$ then $p_2 = (1-p)^2$

$$\boxed{p_n = p_{n-1}\cdot p + p_{n-2}(1-p)p}$$

$\leftarrow$ A recurrence relation

» Solution to problem in previous slide...part-2

$$p_n = p\,p_{n-1} + p(1-p)\,p_{n-2},$$

with

$$p_1 = 0$$
$$p_2 = (1-p)^2$$

To be done in tutorial

» Solution to problem in previous slide...part-3

» Properties of conditional probabilities...

Properties

For any events A, B, and E we have the following:

» Properties of conditional probabilities...

Properties

For any events A, B, and E we have the following:

- $0 \leq \underbrace{P(A \cap E)}_{\text{some event}} \leq 1$ (axioms)

A event

B event

$A \cap B$ event

» Properties of conditional probabilities...

Properties

For any events A, B, and E we have the following:

* $0 \leq P(A \cap E) \leq 1$
* $P(A \mid E) = 1 - P(A^c \mid E)$

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* $P(A \cap B \mid E) = P(B \cap A \mid E)$
* $P(A \cap B \mid E) = P(B \mid E)P(A \mid B \cap E)$

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* $P(A \cap B \mid E) = P(B \cap A \mid E)$
* $P(A \cap B \mid E) = P(B \mid E)P(A \mid B \cap E)$
* $P(A \mid B \cap E) = \dfrac{P(B \mid A \cap E)P(A \mid E)}{P(B \mid E)}$

» Scratch Space for Proving Conditional Probabilities...

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» Conditional Independence...

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Definition of conditional independence

Two events A and B are conditionally independent given E if

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Two events A and B are conditionally independent given E if

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Fact on Conditional Independence

A and B independent does not mean that A and B are independent given E. That is,

$$P(A \cap B) = P(A)P(B) \not\Rightarrow P(A \cap B \mid E) = P(A \mid E)P(B \mid E)$$

» Quiz on independence...

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Quiz-1

Two events E and F are independent if

1. Knowing that F happens means that E can't happen

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Two events E and F are independent if

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What is your answer?

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Quiz-1

Two events E and F are independent if

1. Knowing that F happens means that E can't happen
2. Knowing that F happens doesn't change probability that E happened.

What is your answer?

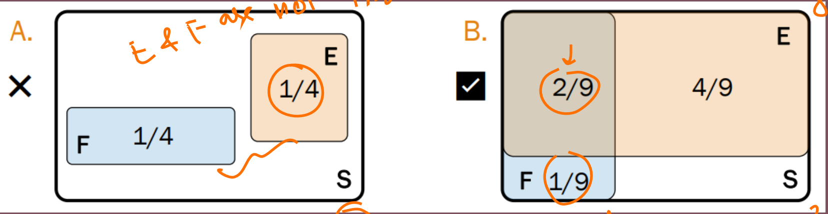
Quiz-1

Are E and F independent in the following pictures (not to scale)?

$P(E \cap F) = P(E) \cdot P(F)$

$0 \neq \frac{1}{4} \cdot \frac{1}{4}$

E & F are not ind.



$P(E \cap F) = \frac{2}{9}$

$P(F) = \frac{3}{9} = \frac{1}{3}$ $\quad P(E) = \frac{6}{9} = \frac{2}{3}$

$P(F) \cdot P(E) = \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9}$

» Mutually Exclusive and Independent Events...

Quiz

When are two events both mutually exclusive and independent?

A, B

① Mut. exclusive $\Rightarrow$ $A \cap B = \phi \Rightarrow P(A \cap B) = 0$ ①

② Independence $\Rightarrow$ $\boxed{P(A \cap B) = P(A)P(B)}$

$\Rightarrow P(A)\,P(B) = 0$

$\Rightarrow P(A) = 0$ or $P(B) = 0$

» Random Variables

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Examples of typed variables in C

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* int i = 4;
* float x = 10;
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Examples of random variable

Let X denote the outputs after we roll a die, then

$$X = 3$$

means that after rolling a die, we obtain 3 as output.

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Examples of random variable

Let X denote the outputs after we roll a die, then

$$X = 3$$

depends randomly

means that after rolling a die, we obtain 3 as output.
Since the number that is going to be assigned to variable X is going to be random, it is called random variable.

» Define Random Variable

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Definition of Random Variable

A random variable X is a function from the sample space to the real numbers.

$$X : S \to \mathbb{R}.$$

$\{1, 2, 3, \ldots, 6\}$

$R_X \subsetneq \mathbb{R}$

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Examples of Random Variables...

Find the range of the following random variables:

$R_X = \{0, 1, 2, \ldots, 10\}$

* I toss a coin 10 times. Let X be the number of heads I observe

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Examples of Random Variables...

Find the range of the following random variables:

* I toss a coin 10 times. Let X be the number of heads I observe
* I toss a coin until the first tail appears. Let Y be the total number of coin tosses

$Y = \{1, 2, 3, \ldots\} \subset \mathbb{R}$

$Y : S \to \mathbb{R}$

» Quiz on Random Variable...

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Quiz on Random Variable

Consider *and* Experiment: 3 coins *are* flipped. Let X be the number of tails. Answer the following:

» Quiz on Random Variable...

$[0, 3]$ $\{0, 1, 2, 3\}$

Quiz on Random Variable

Consider and Experiment: 3 coins are flipped. Let X be the number of tails. Answer the following:

* What is the value of X for the outcomes? $R_X = \{$

Quiz on Random Variable

Consider and Experiment: 3 coins are flipped. Let X be the number of tails. Answer the following:

* What is the value of X for the outcomes?
  * (H, H, H) ← 0
  * (T, T, H) ← 2

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Consider and Experiment: 3 coins are flipped. Let X be the number of tails. Answer the following:

* What is the value of X for the outcomes?
  * (H, H, H)
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* What is the event when $X = 2$?

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Consider and Experiment: 3 coins are flipped. Let X be the number of tails. Answer the following:

* What is the value of X for the outcomes?
  * (H, H, H)
  * (T, T, H)
* What is the event when $X = 2$?
* What is $P(X = 2)$?

{T, T, H}, {H, T, T}, {T, H, T}

X = 2

$\frac{3}{8}$

» Random Variables are Not Events!

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$X = 2 \longrightarrow$ correspond to event

Remarks on Random variables

* random variables are not events!

int i

$i = 2$

» Random Variables are Not Events!

Remarks on Random variables

- random variables are not events!
- when a random variable is assigned a value, then it becomes event

$X = x$	Set of Outcomes	$P(X = k)$
$X = 0$	$\{(T, T, T)\}$	$1/8$
$X = 1$	$\{(H, T, T), (T, H, T), (T, T, H)\}$	$3/8$
$X = 2$	$\{(H, H, T), (H, T, H), (T, H, T)\}$	$3/8$
$X = 3$	$\{(H, H, H)\}$	$1/8$
$X \geq 4$	$\{\ \}$	0

Consider an experiment where 3 coins are flipped, and X denotes number of heads

» Discrete Random Variables...

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Recall: countable sets

A set A is countable if either it is a finite set, or it can be put in 1-1 correspondence with set of natural numbers.

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There are three types of random variables:

1. discrete random variables
2. continuous random variables

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Types of Random Variables...

There are three types of random variables:

1. discrete random variables
2. continuous random variables
3. mixed random variables

» Examples of Types of Random Variables...

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$0 \leq X \leq 100$

Examples of random variables

1. I toss *a* coin 100 times. Let X be the number of heads I observe   discrete

» Examples of Types of Random Variables...

$Y : \{1, 2, 3, \ldots\}$ countable

$\underbrace{}_{R_Y}$ =

Examples of random variables

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2. I toss a coin until the first heads appears. Let Y be the total number of coin tosses

discrete

» Examples of Types of Random Variables...

continuous $[0, \ldots, \infty)$ ← uncountable

$(0,1)$ (a,b)

$T : S \to \mathbb{R}_+$

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2. I toss *a* coin until the first heads appears. Let Y be the total number of coin tosses
3. The random variable T is defined *as* the time (in hours) from now until the next earthquake occurs in *a* certain city
4. Let X be the height of students in *a* class   Cont.

» Probability Mass Function...

Definition of probability mass function ans some remarks...

Let X be a random variable with range $R_X = \{x_1, x_2, \ldots, \}$, which is finite or countably infinite.

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Let X be a random variable with range $R_X = \{x_1, x_2, \dots,\}$, which is finite or countably infinite. The function

$$P_X(x_k) = P(X = x_k), \text{ for } k = 1, 2, 3, \dots$$

is called probability mass function (PMF) of X.

$p(x)$

$p(X = x_k)$

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X, Y, Z

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* The subscript in $P_X(x_k)$ indicates that it is the PMF of random variable X

$P_X(x_k)$ $P_x(x)$ $P_Y(y)$

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* For discrete random variable, PMF is also called probability distribution
* The term probability distribution function is almost always reserved for cumulative distribution(to be introduced)

» Example of PMF...

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We toss a fair coin twice.

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Answer

* Sample space $S = \{HH, HT, TH, TT\}$. No. of heads: 0,1,2. Hence, $R_X = \{0, 1, 2\}$

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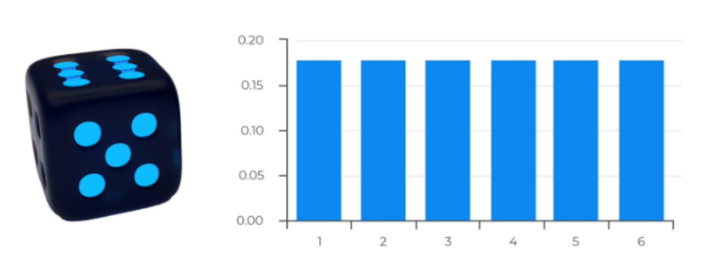
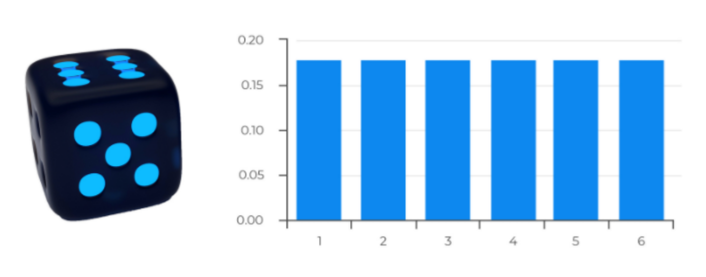
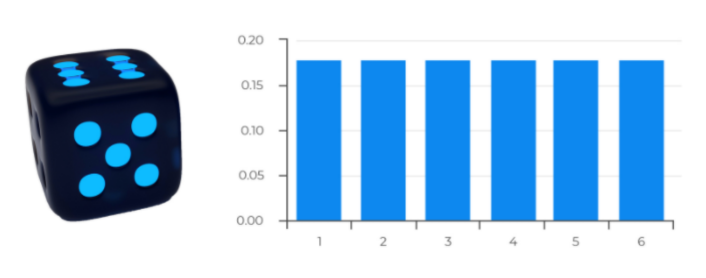
$$P_X(0) = P(X = 0) = P(TT) = 1/4$$
$$P_X(1) = P(X = 1) = P(\{HT, TH\}) = 1/4 + 1/4 = 1/2$$
$$P_X(2) = P(X = 2) = P(HH) = 1/4$$


0.20
0.15
0.10
0.05
0.00
1
2
3
4
5
6



* Consider an experiment of rolling a six-sided die



- Consider an experiment of rolling a six-sided die
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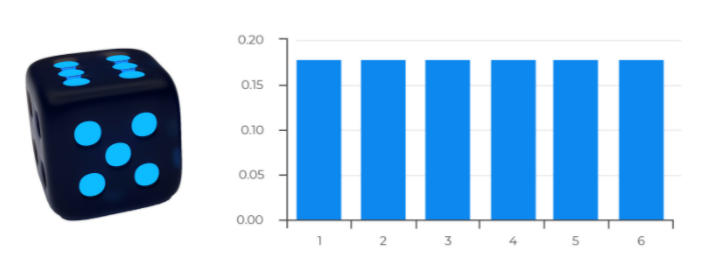

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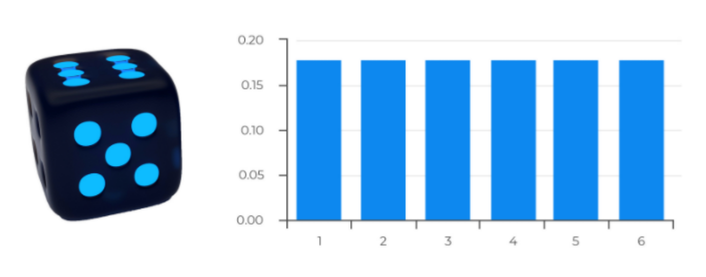

- Consider an experiment of rolling a six-sided die
- The sample space or support of X is $\{1, 2, 3, 4, 5, 6\}$
- Here X is a discrete random variable, and PMF of X is

$$P_X(x) = \begin{cases} 1/6, & x \in \{1, 2, 3, 4, 5, 6\} \\ 0, & \text{otherwise} \end{cases}$$

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Problem on PMF

Consider an unfair coin for which $P(H) = p$.

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1. Is Y a discrete random variable? $\{1, 2, \ldots\}$

» Example of PMF...

Problem on PMF

Consider an unfair coin for which $P(H) = p$. We toss the coin repeatedly until we observe a head for the first time. Let Y be the total number of times the coin was tossed.

1. Is Y a discrete random variable? yes
2. Find PMF of the random variable Y

$R_Y = \{1, 2, \ldots\}$

Answer to the problem

We have

$$P_Y(1) = P(Y = 1) = P(H) = p$$
$$P_Y(2) = P(Y = 2) = P(TH) = (1 - p)p$$
$$\vdots$$
$$P_Y(k) = P(Y = k) = P(\underbrace{TT \cdots T}_{k-1}H) = (1 - p)^{k-1}p$$

» Properties of PMF...

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Properties of PMF

1. $0 \leq P_X(x) \leq 1$ for all x

» Properties of PMF...

$X = \{1, 2, \ldots\}$

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2. $\sum_{x \in R_X} P_X(x) = 1$

» Properties of PMF...

$R_x = \{ x_1, x_2, \dots \}$

Properties of PMF

1. $0 \le P_X(x) \le 1$ for all x
2. $\sum_{x \in R_X} P_X(x) = 1$
3. for any set $A \subset R_X$, $P(X \in A) = \sum_{x \in A} P_X(x)$

$A \subset R_x$

» Check Properties of PMF...

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Problem on PMF

Consider an unfair coin for which $P(H) = p$.

» Check Properties of PMF...

Problem on PMF

Consider *an* unfair coin for which $P(H) = p$. We toss the coin repeatedly until we observe *a* head for the first time.

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Consider an unfair coin for which $P(H) = p$. We toss the coin repeatedly until we observe a head for the first time. Let Y be the total number of times the coin was tossed.

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Problem on PMF

Consider an unfair coin for which $P(H) = p$. We toss the coin repeatedly until we observe a head for the first time. Let Y be the total number of times the coin was tossed.

1. Check $\sum_{y \in R_Y} P_Y(y) = 1$, here R_Y is the range of random variable Y
2. If $p = 1/2$, find $P(2 \leq Y < 5)$

(2) $p = 1/2$

$$P(2 \leq Y < 5) = \sum_{k=2}^{4} P_Y(k) = \sum_{k=2}^{4} (1-p)^{k-1} p$$

$$= \frac{1}{2}\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8}\right) = \frac{7}{16}$$