Homework 3, CSCI 405

Your name here

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Remember, you may work with your classmates but you must write up your own solutions and not copy each other. Show your work! **State your group members or that you worked alone.**

Show clear, concise solutions that are a combination of pseudocode and prose. **Please present pseudocode rather than a particular language for ease in grading.** Justify the correctness and analyze the running time of your algorithms.

Total points: 60

1. Mariko and Stefano return from their foray in the damp Cascadian forests with full baskets of what they at first believe to be delicious porcini mushrooms. Alas, upon closer examination they realize they have mixed together two different but similar species, *Boletus edulis* (the choice edible) and *Tylopilus felleus* (unpalatable due to its strong bitter taste but luckily not poisonous). They'd like to separate the basket of n mushrooms two piles, one for each species. They each pick pairs of specimens i and j and carefully note their similarities and differences. When they are confident, they can decide to call the pair (i, j) the "same" or "different" species, or they can pass on that pair and conclude that the pair is "ambiguous."

Pair	Judgement
(1,2)	same
(1,3)	different
(2,3)	ambiguous

Table 1: An example table of judgements for $n = 3$. In this case, the judgements are consistent since 1 & 2 can both be assigned one of the species and 3 the other species. This would also work if $(2, 3)$ were "different." However, if we were to change $(2, 3)$ to "same," then it would be inconsistent. In this case all $\binom{n}{2}$ pairs are listed, but in general less judgements could be made.

They do this for a while until they reach a collection of m judgements (either "same" or "different") for pairs of fungal fruiting bodies. Yet Mariko and Stefano have a dilemma: They would like to decide whether the m judgements are consistent or not. In other words, can they label the mushrooms as either edible or not so that each pair (i, j) judged "same" share the same label and those judged "different" have different labels?

[10 points] Give an algorithm with running time $O(m + n)$ that determines whether the m judgements are consistent.

[15 points] Justify the correctness of the algorithm you described.

[5 points] Assume their judgements are consistent and they didn't make any mistakes. What is the minimum number of judgements m that they must make so that a single correct id of a porcini or bitter mushroom allows them know the identity of all n mushrooms? Explain.

2. You are building a computer network for the online beauty product retailer Glamazon. Glamazon has a number of datacenters distributed across the country (their website is so huge, they need massive compute power) which form a tree graph. Your job is to pick one special "hub" datacenter to host a shared filesystem for the entire network. The cost of transmitting files across the network is dominated by the number of links that are traversed, so the hub needs to be chosen to minimize the number of links.

[10 points] Given a tree T containing n nodes, design an efficient algorithm that computes a hub node h. The hub must have the property that the length of the longest path from h to any other node v is as small as possible. (Hint: Can leaves ever be hub nodes?)

[15 points] Prove that your algorithm is correct.

[5 points] Is the hub unique? How many different hubs can a tree have?