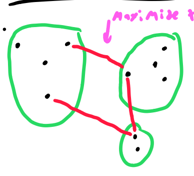
Worksheet

3.



$d(p_i, p_j) = d(p_j, p_i) > 0$

K clusters

Do this to get correct clusters

all nodes connected by edges (Distance)
run reverse-Delete to create MST
Keep track of last n_{k-1} edges not deleted

$G = (V, E)$

$E = \{(v_i, v_j) \; \forall \, i, j\}$

$w(v_i, v_j) = d(p_i, p_j) \leftarrow O(n^2)$

Get MST $|E| = |V| - 1$
delete K-1 heaviest edges $|E| = |V| - K$

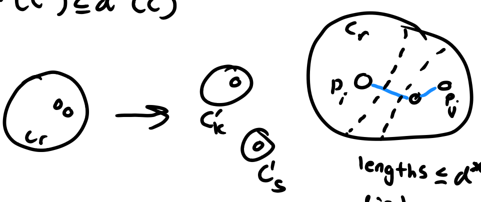
Connected Components
Try Kruskal's

$$O(E \log V) = O(V^2 \log V) = O(n^2 \log n)$$

Proof that Components are clusters:

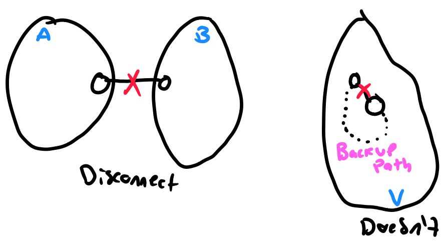
$C = C_1, C_2, \ldots, C_K$
Spacing d^* is length of last deleted edge
C' some other clustering
Need to show $d^*(C') \leq d^*(C)$

$p_i, p_j \in C_r$
but
$p_i \in C'_s, \; p_j \in C'_k \neq C'_s$



lengths $\leq d^*(C)$
distance between new clusters $\leq d^*(C)$
Because they were connected before deleted largest

1.



Proof
If you can delete an edge then it's a cycle

$G \xrightarrow{\text{Delete}} G'$

$MST(G') = MST(G)$
Edge has largest weight $\Rightarrow$ Cannot be part of MST
$\hookrightarrow MST(G') = MST(G)$

efficiency: Each deletion requires $O(V+E)$ amount of work
deletions $E \to V-1$
$E - x = V - 1$
$x = E - V + 1$
Chromatic #

Lecture - Shortest path

Single-Source Shortest Paths:

Input: directed weighted graph $G = (V, E)$
$w: E \to R$

Weight of Path:

$P = v_1, v_2, \ldots, v_K$

$w(P) = \sum_{i=1}^{k-1} w(v_i, v_{i+1}) =$ Sum of edge weights

Shortest Path weight:

$$\delta(u, v) = \begin{cases} \min_P \{ w(P) : u \overset{P}{\leadsto} v \} \\ \infty \text{ if there is no path} \end{cases}$$

Weight represents:
- Cost accumulates along path
- What to minimize

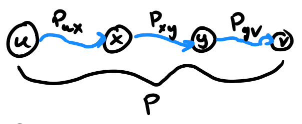
Generalizes BFS

Variants
1) Single-Source
2) all pairs
3) Single pair
- no known improvement on #1
4) Single destination

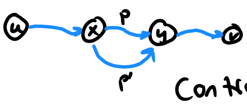
negative weights
Allowed so long as no negative weight cycles

Optimal Substructure
Subpath of shortest path is a shortest Path

Proof Cut and past



P is Shortest path
$\delta(u, v) = w(P_{ux}) + w(P_{xy}) + w(P_{yv})$ is minimal
Suppose $\exists$ another P_{xy} with $w(P'_{xy}) < w(P_{xy})$



Contradiction! wasn't shortest Path to begin with

Cycles
Cannot Contain cycles!
- neg weight out
- zero weight cycle ok... Rule out!

All algorithms use same setup:

Initialize-Single-Source (G, w)
loop over $v \in V$
$v.d = \infty$
$v.\pi = NIL$

Output of algorithm
$\forall v \in V$
1) $v.d = \delta(s, v)$
initialize to ∞
$v.d \geq \delta(s, v)$ during algorithm
(shortest path estimate)
2) $v.\pi =$ predecessor in tree
- π induces shortest path tree