Homework 4, CSCI 405

Your name here

February 29, 2024

Remember, you may work with your classmates but you must write up your own solutions and not copy each other. Show your work! **State your group members or that you worked alone.**

Show clear, concise solutions that are a combination of pseudocode, mathematics, and prose.
Justify the correctness and analyze the running time of your algorithms.

Total points: 80 + revision scores

1. *[15 points]* You are working for the government of Alaska. There are a number of remote villages that may be connected either by road or plane. Your job is to plan a transportation network that connects all the towns in the most efficient way possible. The network needs to be built so that from any town you can reach any other by a sequence of flights and roads. In the arctic tundra, you can travel on straight line paths between the towns, and it is possible to build a road between any two towns (but probably not advisable).

Each town $i = 1, \ldots, n$ has its location coded as coordinates (x_i, y_i) with units of miles. The cost, to the government, of maintaining an air route between two towns is $C + md$, where C is a maintenance cost for the aircraft, m is the cost per mile, and d is the distance between towns. A similar formula is used for building a road, $c + Md$. You can assume that $C > c$ and $M > m$ since the baseline maintenance cost of an aircraft is higher than a road, but a road costs more per mile to maintain.

1. *[10 points]* Give an efficient algorithm for finding the optimal transportation network, with minimum cost, that connects all towns.
2. *[5 points]* For any pair of towns with a direct connection, give a formula that determines how far apart two towns have to be for air travel to be more efficient than land. Explain this formula.

2. *[20 points]* You're planning a route from Bellingham to Portland to find the best hipster donut and coffee available, but your cursed phone maps are down! Luckily, you've been hoarding some relevant data: You have the expected travel times along a segment of road connecting towns i and j stored as $w(i, j)$. Crucially, you also have the expected time that you will wait where road segments connect (you can assume that the roads are broken up into segments at each major town) stored as $t(i)$ for an intersection i. Since this is the I-5 corridor we're talking about, there can be significant delays at these connection points so it's important that your method takes these delays into account. Include the effect of the delays that occur within both Bellingham and Portland. If you underestimate the time you'll wait for the donuts, you will be upset.

Give an efficient algorithm that allows you to compute the most efficient route from Bellingham to Portland. (It should be a general algorithm that works for any transportation network.)

3. *[25 points]* You are thinking of buying a new electric car, but like many potential buyers you are worried about the range of the car's battery. A given car, fully charged, can travel D miles before its battery runs out. You live in Bellingham and know the road network (a directed graph) along with the distances between cities. In your network, you only include cities with charging stations, so every time you visit a city you know you can recharge your battery.

1. *[10 points]* Show how to determine in linear time whether there is a feasible route from Bellingham to any other city for a car with range D. Keep in mind you can never travel between two cities that are more than D miles, since your car would grind to a halt.

2. *[15 points]* You have a list of potential electric cars to buy. Give an $O(V \log V + E)$ algorithm to determine the minimum fuel tank capacity you need to get from Bellingham to a particular city.

4. *[20 points]* A flood happens and farmers in the area find their fields are quickly becoming inundated. They need to move their cows to higher ground that isn't too far away. There are n farms which will be flooded each containing c_i cows for $i = 1, \ldots, n$. Luckily, there are k neighboring farms willing and able to house their cows. Each cow needs to be herded to a safe farm within a 2 hour cow-walk away.

However, each safe farm has to keep space for their own cows, so the safe farms $j = 1, \ldots, k$ only have a capacity to accept p_j cows.

Give a polynomial time algorithm that takes in the parameters c_i and p_j and the cow-walk times to each farm and determines whether moving all the cows to the farms on higher ground is possible along with a plan when it is. Hint: This can be cast as a network flow problem. (You may find the history of floods in our area interesting.)

5. *[variable points]* Pick 2 problems from the previous homework assignments to revise. Write up the best possible solution to that problem, including all sub-problems. Your revised solution will be used to determine your final grades for those problems.