15.4-1

Determine an LCS of $\langle 1, 0, 0, 1, 0, 1, 0, 1 \rangle$ and $\langle 0, 1, 0, 1, 1, 0, 1, 1, 0 \rangle$.

1, 0, 1, 1, 0, 1
0, 0, 1, 0, 1, 0

There are many of length 6

`Solution:` We create an eight by eight array giving $C[m, n]$, the length of the LCS between the first m of the first sequence and the first n of the second sequence.

Here is array. The sequences are placed on top and on the left for convenience. The numbering starts at 0 so that the row zero and column zero are all zeroes.

-	-	0	1	0	1	1	0	1	1	0
-	0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1	1
0	0	1	1	2	2	2	2	2	2	2
0	0	1	1	2	2	2	3	3	3	3
1	0	1	2	2	3	3	3	4	4	4
0	0	1	2	3	3	3	4	4	4	5
1	0	1	2	3	4	4	4	5	5	5
0	0	1	2	3	4	4	5	5	5	6
1	0	1	2	3	4	5	5	6	6	6

15.4-2

Give pseudocode to reconstruct an LCS from the completed c table and the original sequences $X = \langle x_1, x_2, \ldots, x_m \rangle$ and $Y = \langle y_1, y_2, \ldots, y_n \rangle$ in $O(m + n)$ time, without using the b table.

Use this routine:

```
Print-LCS(X, Y, c, i, j):
if X[i] == Y[j]
  print X[i]
  Print-LCS(X, Y, i-1, j-1)
elseif c[i, j] == c[i-1, j]
  Print-LCS(X, Y, i-1, j)
else Print-LCS(X, Y, i, j-1)
```

Longest common subsequence

Problem: Given 2 sequences, $X = \langle x_1, \ldots, x_m \rangle$ and $Y = \langle y_1, \ldots, y_n \rangle$. Find a subsequence common to both whose length is longest. A subsequence doesn't have to be consecutive, but it has to be in order.

Prove this:

Optimal substructure

Notation:

X_i = prefix $\langle x_1, \ldots, x_i \rangle$

Y_i = prefix $\langle y_1, \ldots, y_i \rangle$

Theorem

Let $Z = \langle z_1, \ldots, z_k \rangle$ be any LCS of X and Y.

1. If $x_m = y_n$, then $z_k = x_m = y_n$ and Z_{k-1} is an LCS of X_{m-1} and Y_{n-1}.
2. If $x_m \neq y_n$, then $z_k \neq x_m \Rightarrow Z$ is an LCS of X_{m-1} and Y.
3. If $x_m \neq y_n$, then $z_k \neq y_n \Rightarrow Z$ is an LCS of X and Y_{n-1}.

Proof is in the book (Theorem 14.1 in 4th ed)