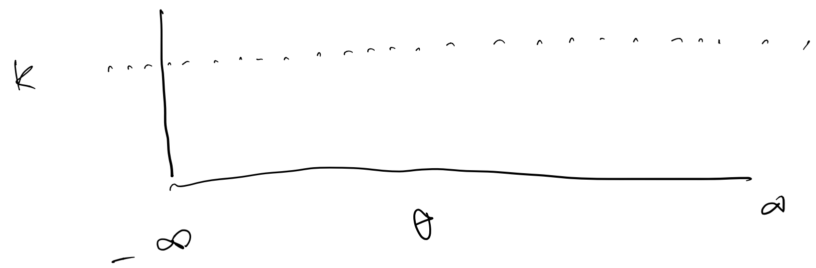
$X_1, \ldots, X_n \mid \theta \overset{iid}{\sim} N(\theta, \sigma^2)$, $\theta$ unknown, $\sigma^2$ known.

assume $\theta$ is $p(\theta) \propto 1$




$$p(\theta) \underset{\theta}{\propto} k = 1$$ → non-infomative prior on $\theta$

$$p(\theta \mid x_{1:n}) \underset{\theta}{\propto} N(x_{1:n} \mid \theta, \sigma^2) \underbrace{\cancel{p(\theta)}}_{1}$$

The prop here is with respect to $\theta$ since $\theta$ is the r.v. of interest.

$$\boxed{\underset{\theta}{\propto}} \underbrace{\left(\frac{1}{2\pi}\right)^{n/2}}_{\text{drop constant}} \exp\left\{-\frac{1}{2}\sum_{i=1}^{n}(x_i-\theta)^2\right\} \quad \pm \bar{x}$$

$$\underset{\theta}{\propto} \exp\left\{-\frac{1}{2}\sum_{i=1}^{n}(x_i-\theta)^2\right\}$$

↑ add / subtract $\bar{x}$

$$= \exp\left\{-\frac{1}{2}\sum_{i=1}^{n}(x_i-\bar{x}+\bar{x}-\theta)^2\right\}$$ complete the square

$$= \exp\left\{-\frac{1}{2}\sum_{i=1}^{n}(x_i-\bar{x})^2\right\} \exp\left\{-\frac{1}{2}\sum_{i=1}^{n}(\bar{x}-\theta)^2\right\}$$

$$\times \boxed{\exp\left\{-\frac{1}{2}\cdot 2\sum_{i=1}^{n}(x_i-\bar{x})\boxed{(\bar{x}-\theta)}\right\}} \rightarrow e^{=0} = 1$$

~~constant~~

can pull outside of $\Sigma$

$$\rightarrow \sum_{i=1} (x_i - \bar{x}) = nx - nx = 0 \quad (*)$$

$$\sum_{i=1}^{n} (\bar{x} - \theta) = n\bar{x} - n\theta$$

$$\Rightarrow e^0 = 1$$

$$= \underbrace{\boxed{\exp\left\{-\frac{\ell}{2}\sum_{i=1}^{n}(x_i - \bar{x})^2\right\}}}_{\text{constant wrt } \theta} \exp\left\{-\frac{\ell}{2}\sum_{i=1}^{n}(\theta - \bar{x})^2\right\}$$

$$\underset{\theta}{\propto} \exp\left\{-\frac{n\ell}{2}(\theta - \bar{x})^2\right\}$$

what does this look like?

$$= \exp\left\{-\frac{n}{2\sigma^2}(\theta - \bar{x})^2\right\}$$

$$\ell = \frac{1}{\sigma^2}$$

MLE

$$\theta \mid x_{1:n} \sim N\left(\bar{x}, \sigma^2/n\right)$$

This is an example when Bayesian + frct. Inf. are the same!

$\bar{x} = \frac{1}{n} \sum x_i$

$$\sum_{i=1}^{n} (x_i - \bar{x}) = \sum_{i=1}^{n} x_i - n\bar{x} =$$