Name: _____ UIN: _____

Section (circle one): 512 / 513

# Instructions

- **DO NOT OPEN UNTIL INSTRUCTED.**
- Before the exam begins:
  1. **Neatly** write your name, UIN, and section number at the top of this sheet.
  2. Tear off the back sheet.
  3. **Neatly** write your name, UIN, and section number on the Scantron sheet.
  4. Read the rest of the instructions.
- No calculators, no notes, no formula sheets, no extra scratch paper are allowed.
- You have 75 minutes from the start of the exam. There are 100 total points.
- **Multiple choice:** Mark the correct answer on the provided Scantron sheet. No work needs to be shown – only the Scantron sheet will be graded.
- **Free response:** Show *all* work. You will not receive credit for work you do not show even if you get the correct answer.
- Circle your final answers.
- There is **scratch paper** on the back of the Scantron sheet. No additional scratch paper is allowed. NOTHING ON THE SCRATCH PAPER WILL BE GRADED.
- **Problems are in random order:** Look through all questions before beginning, and don't spend too much time on any one problem.

**Honor code:** I have completed this exam in the spirit of the Aggie Honor Code and have neither given nor received aid. "An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: _____

**Exercises 1–14:** Multiple choice. Mark your answers on your Scantron sheet.

1. (4 points) Solve the differential equation $y' + 2ty = 6t$. What is $\lim_{t\to\infty} y(t)$?

   A. $\infty$

   B. 3

   C. 1

   D. 0

   E. $-\infty$

2. (4 points) What value of $k$ makes the following differential equation exact?

$$yx^k + 6x + (\ln(x) - 2)y' = 0.$$

A. 2

B. 1

C. 0

D. $-1$

E. $-2$

**Exercises 3–6:** (3 points each) MARK ALL THAT APPLY.

A. $\dfrac{\mathrm{d}f}{\mathrm{d}x} + 18f(x)\dfrac{\mathrm{d}^2 f}{\mathrm{d}x^2} = 3f(x)$

B. $t^3 g''' - g' = g$

C. $(f(t) + 6t)^2 - f''(t) = 2t$

D. $y + xy'' - 3 = 0$

E. $\sin(t) + y' = 2$

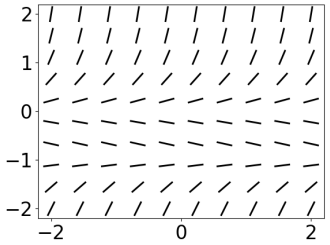
3. Which are autonomous?

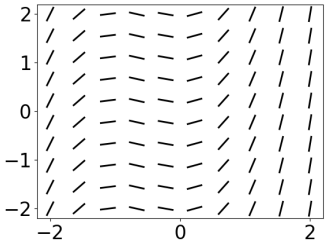
4. Which are linear?

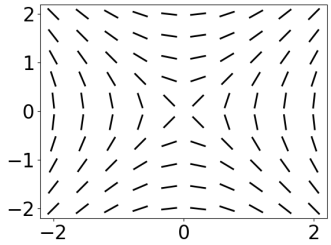
5. Which are both linear and homogeneous?

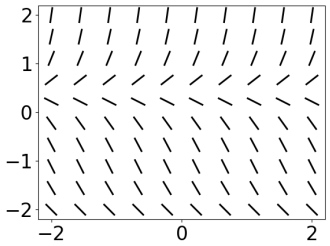
6. Which are second-order?

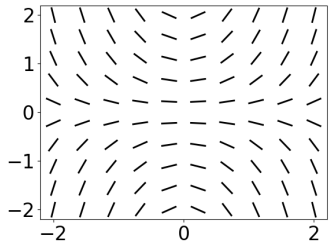
**Exercises 7–8:** Matching. (3 points each) Match the equation to its slope field. Mark your answers on your Scantron sheet.

A. 


B. 


C. 


D. 


E. 


7. $y' = y^2 + y$

8. $y' = xy$

**Exercises 9–11:** (2 points each) Consider the differential equation $y' = y\sin(y)$. Determine the stability of each of the following equilibrium solutions.

9. The equilibrium solution $2\pi$ is
   A. Stable
   B. Semi-stable
   C. Unstable

10. The equilibrium solution $\pi$ is
   A. Stable
   B. Semi-stable
   C. Unstable

11. The equilibrium solution 0 is
   A. Stable
   B. Semi-stable
   C. Unstable

**Exercises 12–13:** Let $(a, b)$ be the largest interval in which the following initial value problem guaranteed to be defined.

$$ty' + \sqrt{9-t}\, y = \frac{\cos(t)}{t-4}, \qquad y(2) = 5.$$

12. (2 points) What is $a$?

    A. $-\infty$

    B. $-9$

    C. $-4$

    D. 0

    E. 4

13. (2 points) What is $b$?

    A. $-4$

    B. 0

    C. 4

    D. 9

    E. $\infty$

14. (4 points) Find all equilibrium solutions of $y' = y^2 - 3y + 2$. MARK ALL THAT APPLY.

    A. $-2$

    B. $-1$

    C. 0

    D. 1

    E. 2

**Exercises 15–20:** Free response. Show all work.

15. (9 points) Find an integrating factor to make the following differential equation exact.

$$1 + \left(\frac{x}{y} - \cos(y)\right) y' = 0$$

16. (a) (5 points) The functions $y_1(t) = t^2$ and $y_2(t) = t^{-1}$ are solutions to the differential equation

$$y'' - 2t^{-2}y = 0 \tag{1}$$

on the interval $(0, \infty)$. Therefore, $y(t) = c_1t^2 + c_2t^{-1}$ are also solutions. *(You do NOT have to show that they are solutions.)* Show that there are no other solutions to Equation (1) on the interval $(0, \infty)$.

(b) (7 points) Using variation of parameters, find a particular solution of

$$y'' - 2t^{-2}y = t^4$$

on the interval $(0, \infty)$. (Part (a) may be helpful.) Carry out any integrals!

17. (10 points) Find the general solution of $xe^{2xy}y' = -x - ye^{2xy}$.

18. (9 points) A 10 kg mass is hanging vertically on a spring. The mass stretches the spring 10 cm. The mass is acted on by an external force of $20\cos(t/3)$ N, where $t$ is measured in seconds. The mass moves in a medium that imparts a viscous force of 8 N when the speed of the mass is 4 cm/s. If the mass is set in motion from its equilibrium position with an initial downward velocity of 12 cm/s, formulate the initial value problem describing the motion of the mass. (Recall, $\mathrm{N} = \frac{\mathrm{kg\,m}}{\mathrm{s}^2}$.) Be sure to specify your units!

19. (8 points) Suppose there is a tank which currently holds 100 L of chocolate milk. Suppose we start pouring in a mixture of 70% chocolate milk and 30% orange juice at a rate of 5 L/min and the well-mixed fluid leaves the tank at the same rate so that the tank always contains 100 L. Set up the initial value problem that describes the amount of chocolate milk in the tank. Be sure to specify your units!

20. (12 points) Find the general solution of $y'' + y = 4\sin(t)$.

SCRATCH PAPER

Anything written on this side **will NOT be graded!**

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