Math 308 Quiz 5

Due: Friday, March 1, 2024

Name: _____ **UIN:** _____

Directions: Please upload a PDF file of your solutions on Gradescope by Friday 1 March at 10pm. You may discuss in groups but please submit your own work.

1. Solve by variation of parameters.

   (a) (6 points)

$$y'' - y = x - 2e^{-x}$$

   (b) (6 points)

$$y'' + 4y' + 4y = \frac{e^{-2x}}{x^2}$$

2. (8 points) A damped spring mass system is modeled by the initial value problem

$$8y'' + y' + 8y = 0, \quad y(0) = 2, \;\; y'(0) = 0$$

(a) Find the solution

(b) Write the solution in the form $y(t) = Ae^{\alpha t}\cos(\omega t - \varphi)$ for some A, α, ω and φ.

3. (10 points) In `Sympy` solve the initial value problem for a damped harmonic oscillator

$$my'' + cy' + ky = g(t), \quad y(0) = 0.05, \;\; y'(0) = 0.$$

for $m = 2$, $k = 30$, $c = 0.6$ and $g = 0$. Plot the resulting solution for $t \in (0, 30)$, and classify the motion as underdamped, critically damped, or overdamped.

Repeat with the same values of m, k, c, but with $g(t)$ changed to:

(a) $g(t) = 4\cos(2t)$

(b) $g(t) = 4\cos(4t)$

(c) $g(t) = 4\cos(6t)$

Which gives the largest amplitude?

Hint: *Please see the example code in Canvas. For best results, please run your code in Google Colaboratory. Save as a PDF file, and attach to Gradescope.*