Finite differences for the 2D heat equation

Implementation of a simple numerical schemes for the heat equation.

$$\frac{\partial u}{\partial t} = \alpha\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right).$$

Applying the second-order centered differences to approximate the spatial derivatives,

$$\frac{\partial u}{\partial t} = \alpha\left(\frac{u(i-1,j) - 2u(i,j) + u(i+1,j)}{\Delta x^2} + \frac{u(i,j-1) - 2u(i,j) + u(i,j+1)}{\Delta y^2}\right).$$

Neumann boundary condition is employed for no-heat flux, thus please note that the grid location is staggered. Once the right hand side is obtained, the equation can be solved by the ODE suite. Here we use ode15s. Copyright 2015-2016 The MathWorks, Inc.

Evolution of MATLAB Logo by Heat equation
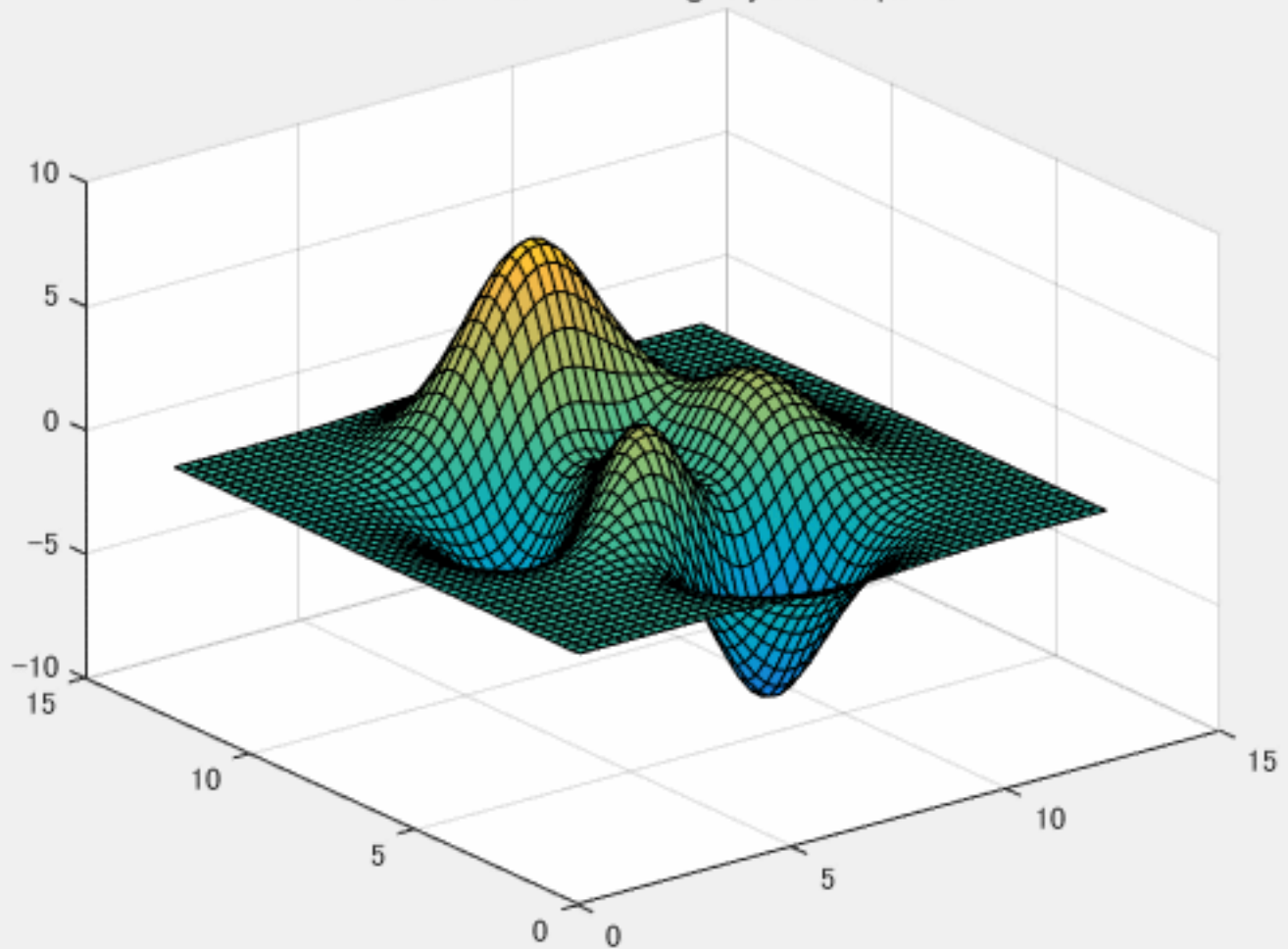
Problem Setup

```
N = 50; % Number of grid in x,y-direction
```

```
L = 4*pi; % Domain size

% Grid point
x = linspace(0,L,N);
y = linspace(0,L,N);
% Make it staggered.
x = (x(1:end-1)+x(2:end))/2;
y = (y(1:end-1)+y(2:end))/2;
[X,Y] = meshgrid(x,y);
```
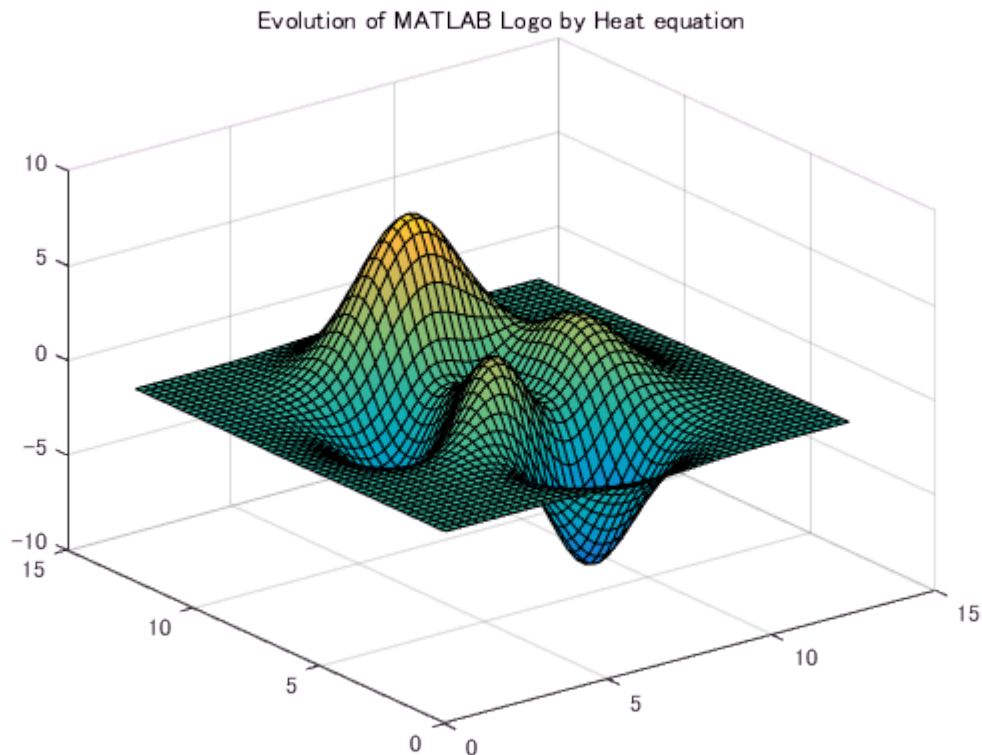
Initial Condition

```
% Let's use MATLAB logo.
% A variable u0 is defined at the center of each grid cell
% thus the number of grid point is N-1.
u0(:,:) = peaks(N-1);

% Plot it
handle_surf = surf(X,Y,u0);
handle_axes = gca;
handle_axes.ZLim = [-10,10];
handle_axes.CLim = [-10,10];
title('Evolution of MATLAB Logo by Heat equation');
```


Evolution of MATLAB Logo by Heat equation

Simulation

```matlab
dx = x(2)-x(1); % spatial grid size
alpha = 2; % coefficient
tspan = linspace(0,1,40);
[t,u] = ode15s(@(t,x)getRHS(x,alpha,dx,N),tspan,u0(:));
```

Visualize

```matlab
Tn = length(t);
u = reshape(u,Tn,N-1,N-1);

filename = 'heat.gif';
for ii=1:Tn
    Z = u(ii,:,:);
    Z = squeeze(Z);
    handle_surf.ZData = Z;
    drawnow;
    frame = getframe(gcf);
    im = frame2im(frame);
    [A,map] = rgb2ind(im,256);
    if ii==1
        imwrite(A,map,filename,'gif','LoopCount',Inf,'DelayTime',0.05);
    else
        imwrite(A,map,filename,'gif','WriteMode','append','DelayTime',0.05);
    end
end
```


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