Lecture 8

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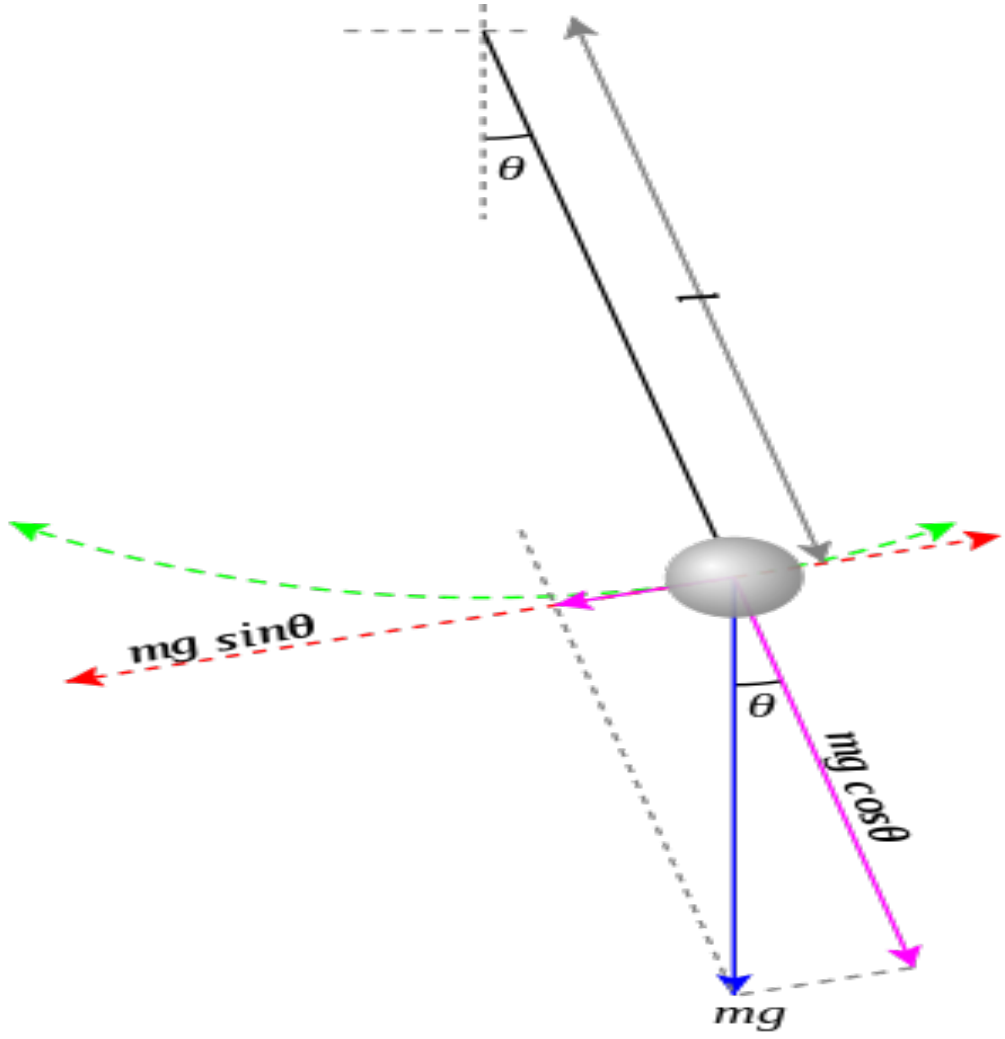
Summary

Today we will:

Model physical systems and use our computational knowledge to gain understanding!

Modeling a System

How to model and understand the system:



Try It!

Using Newtons second law:

$$\sum \hat{F} = \frac{d\hat{p}}{dt} = m\hat{a}$$

So the sum of the forces in the system is equal to the time rate of change in the linear momentum.

What are the equations of the pendulum?

Equations of Motion

Once we've figured out the equations of motion for our system to be:

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}sin(\theta) = 0$$

We need to be able to solve this! For sufficiently small θ, $sin(\theta) \approx \theta$, What is the solution? Graph it! Make sense?

We want a more general discussion of the solution!

Create a System and Linearize!

Write this second oder equation as a system of equations! How?

$$\frac{dy_1}{dt} = y_2$$
$$\frac{dy_2}{dt} = -ksin(y_1)$$

We can then linearize about the origin in order to determine behavior. Whats the behavior of the origin?

$$y' = Ay = \begin{bmatrix} 0 & 1 \\ -k & 0 \end{bmatrix}$$

Connecting Numerical Results to Reality

We have an idea of how the origin behaves, now lets graph a phase plane diagram of the solution.
To do this in matlab: Use a mesh grid and the quiver function:

```
[x,y] = meshgrid(-2:.5:15,-2.5:.25:2.5);
u = y;
v = -k*sin(x);
quiver(x,y,u,v,'r');
```

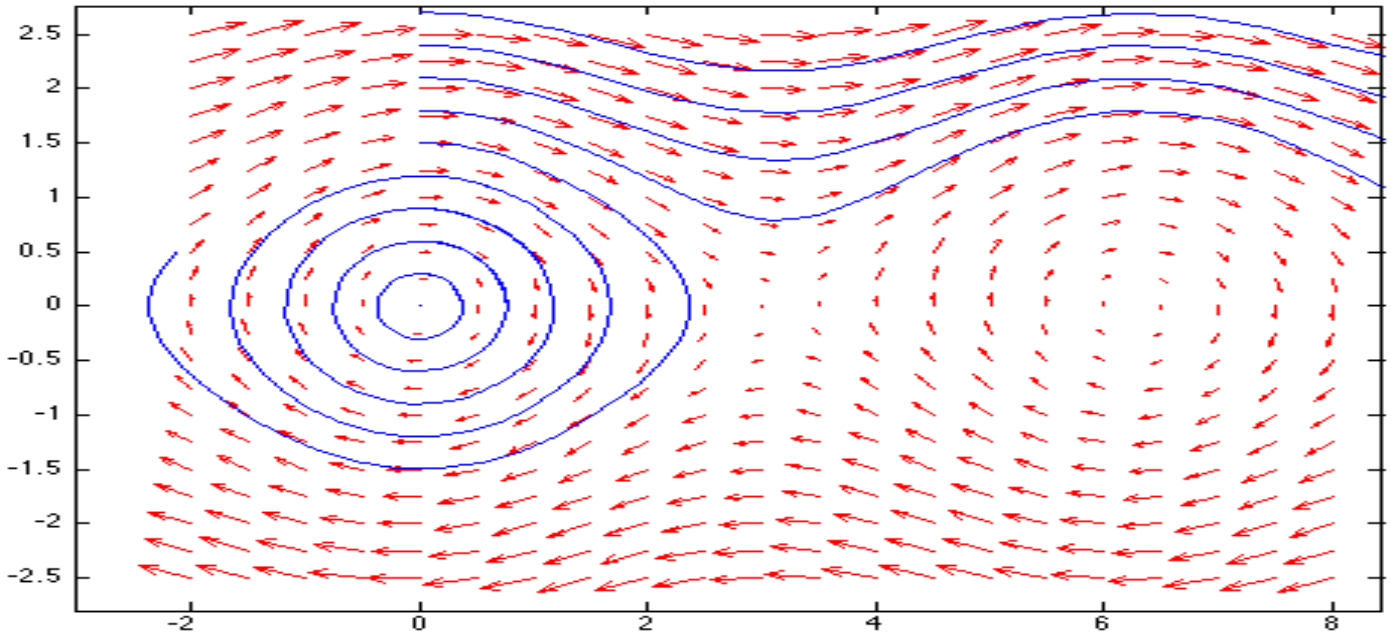
This will bring out the slopes at each point in the mesh grid.

What does this physically correspond to?

Interpret Your Graphical Results

Obtain a phase plane diagram and try and determine what is happening in the system. (If I asked for long term behavior of critical points, trajectories of initial conditions, stability of equilibrium...)

Or I could ask: say I'm holding the pendulum at ninety degrees and release it, what is its behavior?



Adding damping to the system!

Now that we know what's going on in the system, lets add damping to the system and see how things change!

Add a term to the ODE which is proportional to the angular velocity!

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$$\frac{d^2\theta}{dt} + c\frac{d\theta}{dt} + ksin(\theta) = 0$$

How does this change the system of ODEs? Program trajectories

Homework

1) You will be an ecological scientist and must write matlab code to interpret the interaction of an ecosystem.

- Assume there is an animal who lives in complete isolate with infinite food.
  - Write out the differential equation(s) which would determine this system. Graph the population over time. Is this realistic? When will it work/not work? How is it dependent upon a growth parameter? What organism/animal can be modeled in such a way?
- Now the animal is restrained by their environment.
  - Complete the same task as above, but now investigate the growth parameter as well as the population capacity.
- Now there are two animals both restrained by the environment, one's population is being eaten, the other animal is restrained by the amount of the prey.
  - Complete the same tasks as above, but now also analyze a phase portrait of the two species for varying parameters and initial conditions. What can you say about the dependance upon these two quantities?

Homework

- What are the equilibrium points of this system and what is the stability of the system near the points, show this graphically? Interpret this result physically.
- What is one way to make your model better (but perhaps more complicated), you may need to research a bit for this.
- Add a growth limiting term to the growth of the prey in the model. How does this effect the populations of the two species?

I would like a document answering all these questions (with graphs impeded) as well as all the code written to produce the results. All items will be graded.

If you'd like to produce this document using latex you are more than welcome/encouraged! I can provide latex help as needed!