Lecture 9

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Summary

Today we will:

Model physical systems and use our computational knowledge to gain understanding!

A Growing Population!

We want to try and model a couple different types of population growth, the first is:

1) A population which is allowed to grow without bounds. Ideas in modeling this scenario (brainstorming):

- The population has some starting point in our simulation (initial condition).
- The change in time of the population would be equation to some constant times the previous time step. (for example is every human couple had four children, then the population would double).
- What is the time step we want? It depends on the population! (For example for humans it wouldn't be great to to have the population growing every second, but for a bacteria culture it may be). Be sure to clarify your time scale.

Defining Variables

Now that we have an idea of criteria our models needs to meet, we must define variables:

- Initial population: $\mathcal{P}_0$. Units: number of individuals in the population $\mathcal{P}$ (need to clarify who the individuals are: humans, cells, bacteria...)
- Growth factor: defined to be k (how does this number change the long term behavior? We will cover this!)
- Time scale: Want to define how we classify t in our simulations: for humans each data point corresponding to each t will represent one generation!

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$$\frac{d\mathcal{P}(t)}{dt} = k\mathcal{P}(t)$$
$$\mathcal{P}(0) = \mathcal{P}_0$$

Now given some set of data and assumptions we could approximate how the population changes!

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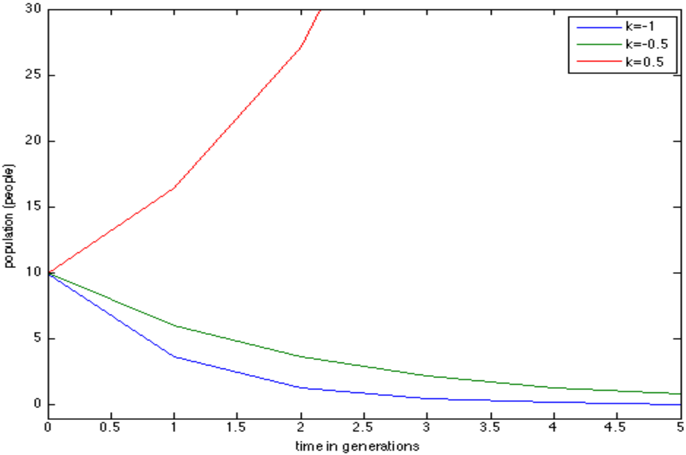
$$\mathcal{P}(t) = \mathcal{P}_0 e^{kt}$$

We can now begin to interpret our long time population levels and see how k affects them. (Possibilities?)

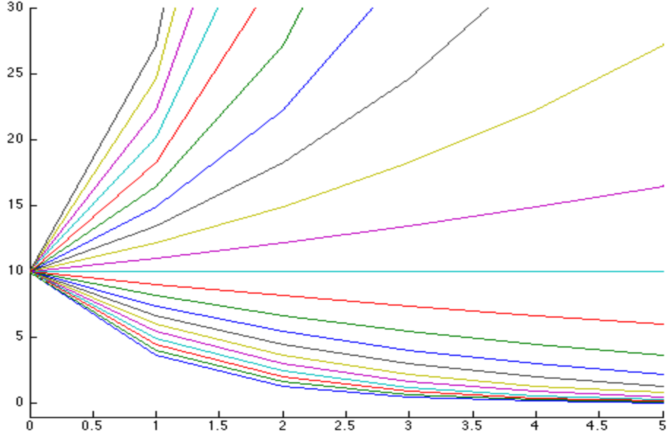
Parameter Analysis

Pretend that we don't know how k affects the system, but that we really need to know! Qualitatively we can obtain an idea by plotting! Get this graph!



More Graphs!

We can now check more parameters!



Modeling Epidemiology

Now we want to consider tracking a disease amongst a population of individuals. Ideas modeling this (Simple example)

- People transfer from three states: Healthy $\rightarrow$ sick $\rightarrow$ healthy(immune).
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- Healthy people get sick when they interact with the sick, sick people become healthy again after medicine or a period of recover time.
- What are the parameters and how do we want to denote each population?
- Healthy people = Susceptible ($\mathcal{S}$), sick = infected ($\mathcal{I}$), immune = removed population ($\mathcal{R}$)
- Parameters: Probability of getting sick: α, chance at each time step of recovering (if sick): β, initial populations and total population N.
- Questions: Are we adding people to the healthy population? Do immune people eventually return to susceptible? What time scale?

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$$\frac{d\mathcal{S}}{dt} = -\frac{\alpha}{N}\mathcal{S}\mathcal{I}$$
$$\frac{d\mathcal{I}}{dt} = \frac{\alpha}{N}\mathcal{S}\mathcal{I} - \beta\mathcal{I}$$
$$\frac{d\mathcal{R}}{dt} = \beta\mathcal{I}$$

What are some parameters in this system? Brainstorm! How do we make this model better? Brainstorm!

Homework

For the current epidemiology model:

- Identify important parameter values of α and β. What long term solutions are possible with these parameters? Does population size matter?
- Compute the two parameter values from a real world scenario (choose a disease and do a bit of research on it)
- Explain what is wrong with the model, are there long term solutions which can't be obtained?
- Develop a new system with the following characteristics:
    - Recovered population has a very slight chance of becoming susceptible again. Define the parameter value which controls this and discuss how this changes the long term behavior of the system.
    - Add an immunized population which susceptible can change to, should they be completely removed?. Complete the above tasks for this new population.
    - Add birth to the susceptible population, make the frequency be relevant to your time scales. How does this change the outcome?