Matlab Notes Part 4

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Summary

- Last week we learned:
  - Matlab functions.
  - Upper triangular matrices.
  - Numerical differentiation.
  - Using built in Matlab functions to solves systems of ODEs.
- Today we will:
  - Numerical integration.
  - Rectangle rule and trapezoid rule.
  - Error of numerical integration, examples of unsolvable integrals.

Problem formulation

We want to be able to evaluate integrals of the form:

$$\mathcal{I}(x) = \int_a^b f(x)dx$$

In general this is useful when you can't evaluate the integral given. We want to be able to evaluate this accurately using numerical techniques.

Idea

To numerically approximate this we can simply take the area of rectangles under the curve. To do this we subdivide the interval into n partitions:

$$h = \frac{b-a}{n}$$
$$f \approx f(x_j^*)$$

Here x_j^* is the midpoint of the jth subinterval. We can then write the area of the jth rectangle as:

$$A_j = f(x_j^*)h$$

Remember the area of a rectangle is just length times height.

Rectangle Rule

We can now write out the approximation for rectangle rule:

$$\mathcal{I}(x) = \int_a^b f(x)ds \approx h[f(x_1^*) + \ldots + f(x_n^*)]$$

for $h = \frac{b-a}{n}$. Lets try and use it!

Programming Rectangle Rule in Matlab

We first need to decretive the x domain:

$\gg$ x = a:h:b

Then we need to have the integrand: $f(x)$ and evaluate it at the midpoint of each interval:

$\gg$ for i = 1: size(x),

midpoint = $\frac{x(i)+x(i+1)}{2}$

sum = sum + f(midpoint)

end

We then want to multiply by h:

rec = sum*h

Now test this with an integral you know!

Trapezoid Rule

We now want a more accurate integration method, Trapezoid rule :

Idea: Take area of trapezoids!!

$$\mathcal{I}(x) = \int_a^b f(x)ds \approx h[\frac{1}{2}f(a) + f(x_1^*) + \ldots + f(x_{n-1}^*) + \frac{1}{2}f(b)]$$

Here the area of a trapezoid is used:

$$\mathcal{A}_{trap}^i = \frac{1}{2}[f(x_{i-1}) + f(x_i)]h$$

This is the area of the ith trapezoid used to approximate the integral. What's different, what's the same?

Homework 4

We want to make a function that computes trapezoid and rectangle rule! There should be two functions (one for each method) and they should:

- Take in a function f for the integrand.
- Take in the bounds and step size: a, b, and h.

Now that you have these methods, you need to test them a few different ways, have a separate script produce:

- A figure which graphs the error as a function of step-size (h), for each method. Just choose an integrand (f(x)) that you know the answer to, like $f(x) = x$. Then evaluate this over an interval and compute the error between the actual value and the approximate value.
- A figure which plots the difference between Rectangle rule and Trapezoid rule as a function of step size h.