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Parameter Estimation

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(1) Given : Random sample $(x_1 \ldots x_n)$

$$L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(-\frac{(x_i-\mu)^2}{2\sigma^2}\right)}$$

Taking natural log of likelihood function

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(\frac{-(x_i-\mu)^2}{2\sigma^2} - \frac{1}{2} \ln(2\pi\sigma^2) \right)$$

To find MLE, diff. log likelihood w.r.t. θ_1, θ_2

$$\frac{\partial}{\partial\theta_1} \ln L(\theta_1, \theta_2) = \sum_{i=1} \left(\frac{x_i - \mu}{\sigma^2} \right) = 0$$

$$\Rightarrow \sum_{i=1} x_i - n\mu = 0$$

$$\frac{\theta_1}{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

for θ_2 :

$$\frac{\partial}{\partial\theta_2} \ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left(-\frac{(x_i-\theta_1)^2}{2\theta_2^2} + \frac{1}{2\theta_2} \right) = 0$$

$$\Rightarrow \sum_{i=1}^{n} \left(\frac{(x_i-\theta_1)^2}{\theta_2} \right) - \frac{n}{\theta_2} = 0$$

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$$\frac{\theta_2^2}{\theta_2} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \theta_1)^2$$

$$\theta_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \theta_1)^2$$

Sample variance.

(2) To find the MLE of θ for a binomial distribution $B(m, \theta)$ where m is a known +ve integer.

$$L(\theta) = \prod_{i=1}^{n}\binom{m}{x_i}\theta^{x_i}(1-\theta)^{m-x_i}$$

taking ln

$$\ln(L(\theta)) = \sum_{i=1}^{n}\left(\ln\binom{m}{x_i} + x_i \ln(\theta) + (m - x_i)\ln(1-\theta)\right)$$

$$\frac{\partial}{\partial\theta}\ln(L(\theta)) = \sum_{i=1}^{n}\left(\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta}\right) = 0$$

Solving for θ

$$\sum_{i=1}^{n}\frac{x_i}{\theta} = \sum_{i=1}^{n}\frac{m - x_i}{1-\theta}$$

$$\sum_{i=1}^{n}x_i(1-\theta) = \sum_{i=1}^{n}(m - x_i)\theta$$

$$\theta\sum_{i=1}^{n}x_i = m\sum_{i=1}^{n}\theta$$

$$\theta = \frac{1}{m}\sum_{i=1}^{n}x_i$$

$\therefore$ MLE of θ is sample mean of observations