First Name:

Last Name:

In [1]:

```
import pandas as pd
import numpy as np
import seaborn as sns
import matplotlib.pyplot as plt
import statsmodels.formula.api as smf
```

In [2]:

Using a lambda function to round numeric values to 2 decimals.

```
pd.set_option('display.float_format', lambda x:'%.2f'%x)

gapminder = pd.read_csv('gapminder.csv', low_memory=False)
gapminder.head()
```

Out[2]:

	country	incomeperperson	alcconsumption	armedforcesrate	breastcancerper100th	c
0	Afghanistan		.03	.5696534	26.8	
1	Albania	1914.99655094922	7.29	1.0247361	57.4	22374
2	Algeria	2231.99333515006	.69	2.306817	23.5	29321
3	Andorra	21943.3398976022	10.17			
4	Angola	1381.00426770244	5.57	1.4613288	23.1	

In [3]:

```
gapminder['oilperperson'] = pd.to_numeric(gapminder['oilperperson'],errors='coerce')
gapminder['relectricperperson'] = pd.to_numeric(gapminder['relectricperperson'],errors='coe
gapminder['co2emissions'] = pd.to_numeric(gapminder['co2emissions'],errors='coerce')
```

Scenario 1 - Linear & Multiple

sub1

In [4]:

```
sub1 = gapminder[['oilperperson', 'relectricperperson', 'co2emissions']].dropna()
sub1.head()
```

Out[4]:

	oilperperson	relectricperperson	co2emissions
2	0.42	590.51	2932108666.67
6	0.64	768.43	5872119000.00
9	1.91	2825.39	12970092666.67
10	1.55	2068.12	4466084333.33
11	0.36	921.56	511107666.67

Centre oilperperson, relectricperperson and co2emissions

use sub1

In [5]:

```
# center quantitative variables for regression analysis
sub1['oilperperson_c'] = (sub1['oilperperson'] - sub1['oilperperson'].mean())
sub1['relectricperperson_c'] = (sub1['relectricperperson'] - sub1['relectricperperson'].mea
sub1['co2emissions_c'] = (sub1['co2emissions'] - sub1['co2emissions'].mean())

sub1.head()
```

Out[5]:

	oilperperson	relectricperperson	co2emissions	oilperperson_c	relectricperperson_c	co2e
2	0.42	590.51	2932108666.67	-1.06	-1145.94	-1235
6	0.64	768.43	5872119000.00	-0.85	-968.02	-941
9	1.91	2825.39	12970092666.67	0.43	1088.94	-231
10	1.55	2068.12	4466084333.33	0.06	331.68	-1081
11	0.36	921.56	511107666.67	-1.12	-814.89	-1477

All data are centered by subtracting the mean value

Multi variable linear regression

predict co2emission (y) using relectricperperson(x1) and oilperperson(x2)

use sub1

In [20]:

```
reg1 = smf.ols('co2emissions_c ~ relectricperperson_c + oilperperson_c', data=sub1).fit()
print (reg1.summary())
```

```
                            OLS Regression Results
==============================================================================
==
Dep. Variable:         co2emissions_c   R-squared:                       0.0
20
Model:                            OLS   Adj. R-squared:                 -0.0
12
Method:                 Least Squares   F-statistic:                    0.62
05
Date:                Sun, 25 Mar 2018   Prob (F-statistic):              0.5
41
Time:                        12:57:49   Log-Likelihood:                -163
2.7
No. Observations:                  63   AIC:                             327
1.
Df Residuals:                      60   BIC:                             327
8.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
============
                           coef    std err          t      P>|t|      [0.025
0.975]
------------------------------------------------------------------------------
------------------------------
Intercept            -1.669e-06   5.62e+09  -2.97e-16      1.000   -1.12e+10
1.12e+10
relectricperperson_c  3.434e+06   3.24e+06      1.058      0.294   -3.06e+06
9.92e+06
oilperperson_c        -9.47e+08   3.65e+09     -0.260      0.796   -8.25e+09
6.35e+09
==============================================================================
==
Omnibus:                      116.246   Durbin-Watson:                   1.7
62
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             4225.1
98
Skew:                           5.891   Prob(JB):                         0.
00
Kurtosis:                      41.351   Cond. No.                     2.04e+
03
==============================================================================
==

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is corre
ctly specified.
[2] The condition number is large, 2.04e+03. This might indicate that there
are
strong multicollinearity or other numerical problems.
```

R-squared shows 0% variables are explained with our model, a Prob(f-statistic) of 0.5 indicates we should accept null hypothesis: there is no correlation.

Scenario 2 - Linear

sub2

In [7]:

```
# convert to numeric format
gapminder['employrate'] = pd.to_numeric(gapminder['employrate'], errors='coerce')
sub2 = gapminder[['relectricperperson', 'employrate']].dropna()
sub2.head()
```

Out[7]:

	relectricperperson	employrate
1	636.34	51.40
2	590.51	50.50
4	173.00	75.70
6	768.43	58.40
7	603.76	40.10

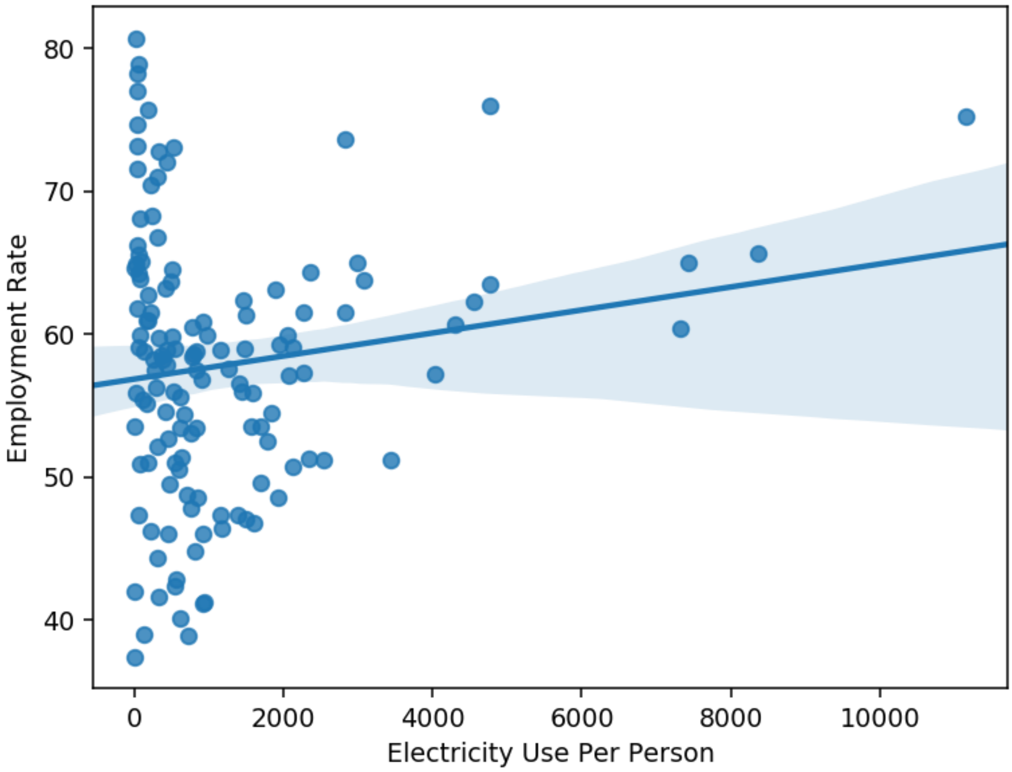
Turn values into numeric, drop Non number values, and display the first 5 rows.

scatter plot to show relationship between employment rate (x) and electricity use per person (y)

In [8]:

```
%matplotlib notebook
plt.figure()
scat1 = sns.regplot(x="relectricperperson", y="employrate", fit_reg=True, data=sub2)

plt.xlabel('Electricity Use Per Person')
plt.ylabel('Employment Rate')
```



We can see most data does not fit into regression model.

Out[8]:

```
Text(0,0.5,'Employment Rate')
```

Centre relectricperperson and employrate

use sub2

In [9]:

```
sub2['relectricperperson_c'] = (sub2['relectricperperson'] - sub2['relectricperperson'].mea
sub2['employrate_c'] = (sub2['employrate'] - sub2['employrate'].mean())

sub2.head()
```

Out[9]:

	relectricperperson	employrate	relectricperperson_c	employrate_c
1	636.34	51.40	-543.99	-6.41
2	590.51	50.50	-589.82	-7.31
4	173.00	75.70	-1007.33	17.89
6	768.43	58.40	-411.90	0.59
7	603.76	40.10	-576.57	-17.71

All data are centered by subtracting the mean value

Linear regression between relectricperperson (x) and employrate (y)

use sub2

In [10]:

```
reg2 = smf.ols('employrate_c ~ relectricperperson_c', data=sub2).fit()
print (reg2.summary())
```

```
                            OLS Regression Results
==============================================================================
==
Dep. Variable:           employrate_c   R-squared:                       0.0
21
Model:                            OLS   Adj. R-squared:                  0.0
14
Method:                 Least Squares   F-statistic:                     2.8
77
Date:                Sun, 25 Mar 2018   Prob (F-statistic):             0.09
22
Time:                        12:48:40   Log-Likelihood:                -487.
37
No. Observations:                 134   AIC:                             97
8.7
Df Residuals:                     132   BIC:                             98
4.5
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
============
                           coef    std err          t      P>|t|      [0.025
0.975]
------------------------------------------------------------------------------
------------------------------
Intercept             5.662e-15      0.800   7.08e-15      1.000      -1.582
1.582
relectricperperson_c     0.0008      0.000      1.696      0.092      -0.000
0.002
==============================================================================
==
Omnibus:                        1.259   Durbin-Watson:                   2.0
02
Prob(Omnibus):                  0.533   Jarque-Bera (JB):                1.2
53
Skew:                           0.228   Prob(JB):                        0.5
34
Kurtosis:                       2.874   Cond. No.                     1.68e+
03
==============================================================================
==

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is corre
ctly specified.
[2] The condition number is large, 1.68e+03. This might indicate that there
are
strong multicollinearity or other numerical problems.
```

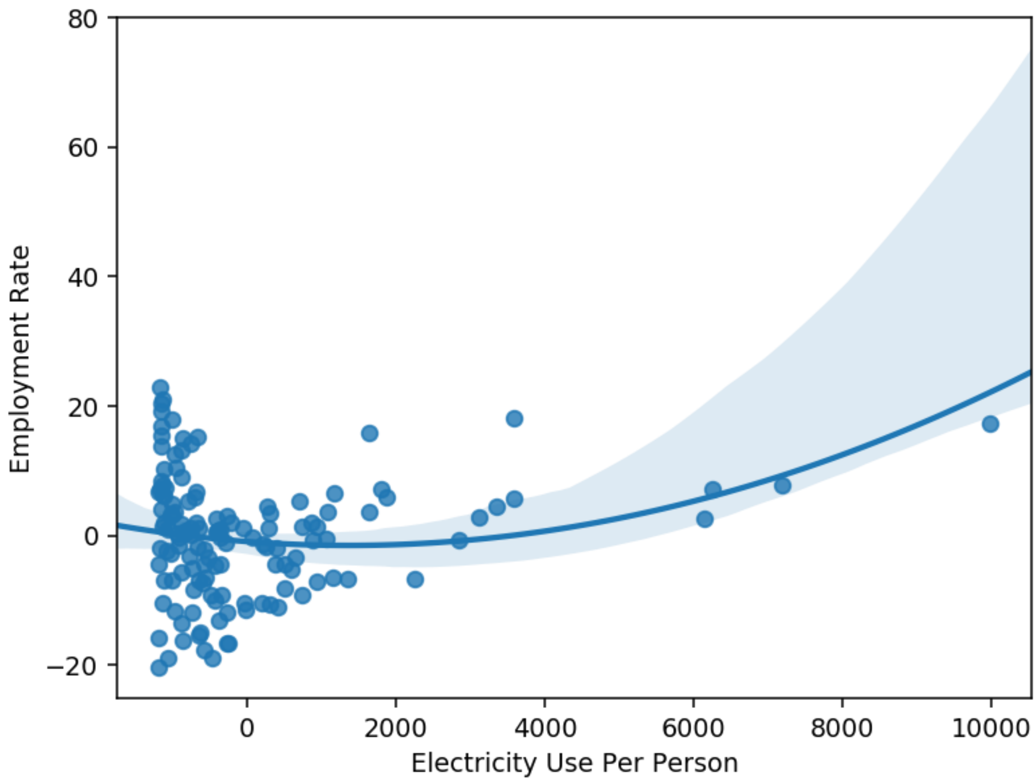
R-squared shows 0% variables are explained with our model, a Prob(f-statistic) of 0.09 indicates we should accept null hypothesis: there is no correlation.

Scenario 3 - Polynomial

scatter plot to show polynomial (order 2) relationship between employment rate (x) and electricity use per person (y)

In [11]:

```
#fit second order polynomial
# run the 2 scatterplots together to get second order fit lines
plt.figure()
scat1 = sns.regplot(x="relectricperperson_c", y="employrate_c", order=2, data=sub2)
plt.xlabel('Electricity Use Per Person')
plt.ylabel('Employment Rate')
```



Analyzing dataset with polynomial regression model, from the plot, we can see more data are aligned with this prediction model.

Out[11]:

```
Text(0,0.5,'Employment Rate')
```

Polynomial regression between relectricperperson (x - order 2) and employrate (y)

use sub2

In [12]:

```
reg2 = smf.ols('employrate_c ~ I(relectricperperson_c**2)', data=sub2).fit()
print (reg2.summary())
```

```
                            OLS Regression Results
============================================================================
==
Dep. Variable:           employrate_c   R-squared:                       0.0
54
Model:                            OLS   Adj. R-squared:                  0.0
47
Method:                 Least Squares   F-statistic:                     7.6
06
Date:                Sun, 25 Mar 2018   Prob (F-statistic):            0.006
64
Time:                        12:49:22   Log-Likelihood:               -485.
06
No. Observations:                 134   AIC:                             97
4.1
Df Residuals:                     132   BIC:                             97
9.9
Df Model:                           1
Covariance Type:            nonrobust
============================================================================
====================
                                  coef    std err          t      P>|t|
[0.025      0.975]
----------------------------------------------------------------------------
--------------------
Intercept                      -0.5780      0.814     -0.710      0.479
-2.187       1.032
I(relectricperperson_c ** 2)  2.037e-07   7.39e-08      2.758      0.007
5.76e-08     3.5e-07
============================================================================
==
Omnibus:                        0.919   Durbin-Watson:                   2.0
29
Prob(Omnibus):                  0.632   Jarque-Bera (JB):                0.8
72
Skew:                           0.194   Prob(JB):                        0.6
47
Kurtosis:                       2.924   Cond. No.                     1.14e+
07
============================================================================
==

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is corre
ctly specified.
[2] The condition number is large, 1.14e+07. This might indicate that there
are
strong multicollinearity or other numerical problems.
```

Although the R-squared value remains 0%, Prob(f-statistic) dropped to 0.6% indicates we can reject null hypothesis now.

Scenario 4 - Multiple & poly

sub3

In [13]:

```
sub3 = gapminder[['oilperperson', 'relectricperperson', 'co2emissions','employrate']].dropr
sub3.head()
```

Out[13]:

	oilperperson	relectricperperson	co2emissions	employrate
2	0.42	590.51	2932108666.67	50.50
6	0.64	768.43	5872119000.00	58.40
9	1.91	2825.39	12970092666.67	61.50
10	1.55	2068.12	4466084333.33	57.10
11	0.36	921.56	511107666.67	60.90

Centre employrate, oilperperson, relectricperperson and co2emissions

All data are centered by subtracting the mean value

use sub3

In [14]:

```
sub3['employrate_c'] = (sub3['employrate'] - sub3['employrate'].mean())
sub3['oilperperson_c'] = (sub3['oilperperson'] - sub3['oilperperson'].mean())
sub3['relectricperperson_c'] = (sub3['relectricperperson'] - sub3['relectricperperson'].mea
sub3['co2emissions_c'] = (sub3['co2emissions'] - sub3['co2emissions'].mean())
```

Multiple and polynomial regression between oilperperson(x1) + co2emissions(x2) relectricperperson(x3 - order 2) and employrate (y)

use sub3

In [15]:

```
reg3 = smf.ols('employrate_c ~ oilperperson_c + co2emissions_c + I(relectricperperson_c**2)
print (reg3.summary())
```

```
                            OLS Regression Results
==============================================================================
==
Dep. Variable:           employrate_c   R-squared:                       0.1
86
Model:                            OLS   Adj. R-squared:                  0.1
44
Method:                 Least Squares   F-statistic:                     4.4
81
Date:                Sun, 25 Mar 2018   Prob (F-statistic):            0.006
70
Time:                        12:50:27   Log-Likelihood:                -210.
13
No. Observations:                  63   AIC:                              42
8.3
Df Residuals:                      59   BIC:                              43
6.8
Df Model:                           3
Covariance Type:            nonrobust
==============================================================================
====================
                                  coef    std err          t      P>|t|
[0.025      0.975]
------------------------------------------------------------------------------
--------------------
Intercept                      -0.8489      0.937     -0.906      0.369
-2.724       1.026
oilperperson_c                  0.6155      0.522      1.179      0.243
-0.429       1.660
co2emissions_c               1.533e-11   2.01e-11      0.761      0.450
-2.5e-11    5.56e-11
I(relectricperperson_c ** 2) 2.047e-07   7.43e-08      2.755      0.008
5.6e-08    3.53e-07
==============================================================================
==
Omnibus:                        0.228   Durbin-Watson:                   2.3
24
Prob(Omnibus):                  0.892   Jarque-Bera (JB):                0.0
68
Skew:                           0.080   Prob(JB):                        0.9
67
Kurtosis:                       2.998   Cond. No.                     4.67e+
10
==============================================================================
==

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is corre
ctly specified.
[2] The condition number is large, 4.67e+10. This might indicate that there
are
strong multicollinearity or other numerical problems.
```

R-squared value shows only 10% variables are explained by regression model, Prob(f-statistic) is 0.6%, means we should reject null hypothesis.

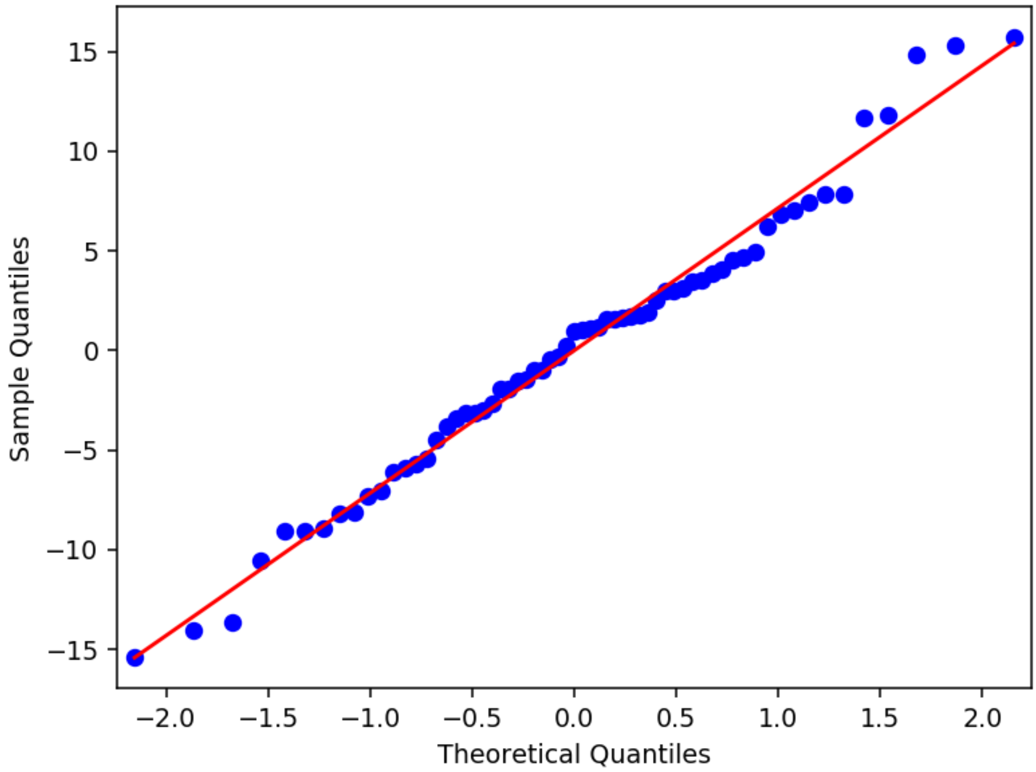
Evaluating model

Plot qqplot for the above regression (reg3)

In [16]:

```
import statsmodels.api as sm
fig4=sm.qqplot(reg3.resid, line='r')
```

```
C:\Users\jc443343\AppData\Local\Continuum\anaconda3\lib\site-packages\statsm
odels\compat\pandas.py:56: FutureWarning: The pandas.core.datetools module i
s deprecated and will be removed in a future version. Please use the pandas.
tseries module instead.
  from pandas.core import datetools
```



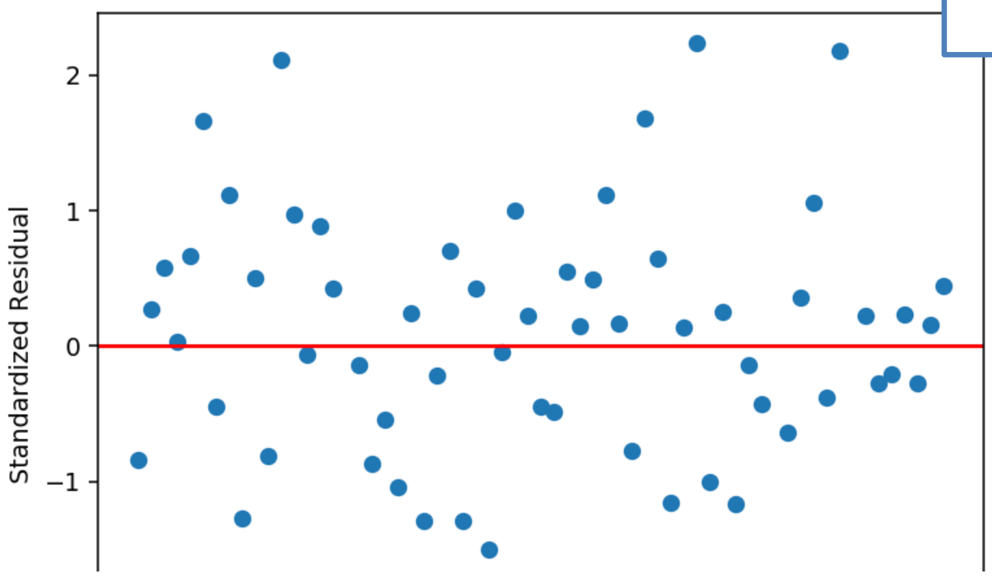
The QQ plot gives a roughly straight line, which means both sets of quantiles came from the same distribution.

Residual plot for the above regression (reg3)

In [17]:

```
# simple plot of residuals
stdres=pd.DataFrame(reg3.resid_pearson)
plt.figure()
plt.plot(stdres, 'o', ls='None')
l = plt.axhline(y=0, color='r')
plt.ylabel('Standardized Residual')
plt.xlabel('Observation Number')
```

The data are equally spaced above and below line of 0, shows dataset is suitable for linear regression.



Calculate percentage of observations over 2 standardized deviation

Calculating percentage of outliers using 2 std.

In [18]:

```
percentage_over2sd = (np.count_nonzero( stdres[0] > 2) + np.count_nonzero( stdres[0] < -2))
print (percentage_over2sd)
```

7.936507936507936

Calculate percentage of observations over 2.5 standardized deviation

Calculating percentage of outliers using 2.5 std.

In [19]:

```
percentage_over2_5sd = (np.count_nonzero( stdres[0] > 2.5) + np.count_nonzero( stdres[0] <
print (percentage_over2_5sd)
```

0.0

Example answer - students can do any combination

On your own, perform

Multiple and polynomial regression between oilperperson, co2emissions, relectricperperson to predict employrate (y)

experiment with explanatory variable (oilperperson, co2emissions, relectricperperson) and their order

use sub3

In [39]:

```
reg4 = smf.ols('employrate_c ~ oilperperson_c + I(co2emissions_c**2) + I(relectricpersor
print (reg4.summary())
```

```
                            OLS Regression Results
==============================================================================
==
Dep. Variable:           employrate_c   R-squared:                       -0.3
96
Model:                            OLS   Adj. R-squared:                  -0.4
19
Method:                 Least Squares   F-statistic:                     -17.
31
Date:                Sun, 25 Mar 2018   Prob (F-statistic):                1.
00
Time:                        13:27:39   Log-Likelihood:                 -227.
11
No. Observations:                  63   AIC:                               45
8.2
Df Residuals:                      61   BIC:                               46
2.5
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
====================
                                   coef    std err          t      P>|t|
[0.025      0.975]
------------------------------------------------------------------------------
--------------------
Intercept                     4.706e-15   1.92e-15      2.449      0.017
8.64e-16    8.55e-15
oilperperson_c               -7.646e-23   3.12e-23     -2.449      0.017     -
1.39e-22    -1.4e-23
I(co2emissions_c ** 2)       -3.936e-22   2.03e-22     -1.934      0.058
-8e-22    1.33e-23
I(relectricperperson_c ** 2)  2.095e-07   8.55e-08      2.449      0.017
3.84e-08    3.81e-07
==============================================================================
==
Omnibus:                       39.077   Durbin-Watson:                    2.1
99
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               155.1
98
Skew:                           1.682   Prob(JB):                       1.99e-
34
Kurtosis:                       9.915   Cond. No.                       1.41e+
30
==============================================================================
==

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is corre
ctly specified.
[2] The smallest eigenvalue is 5.22e-15. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.
```

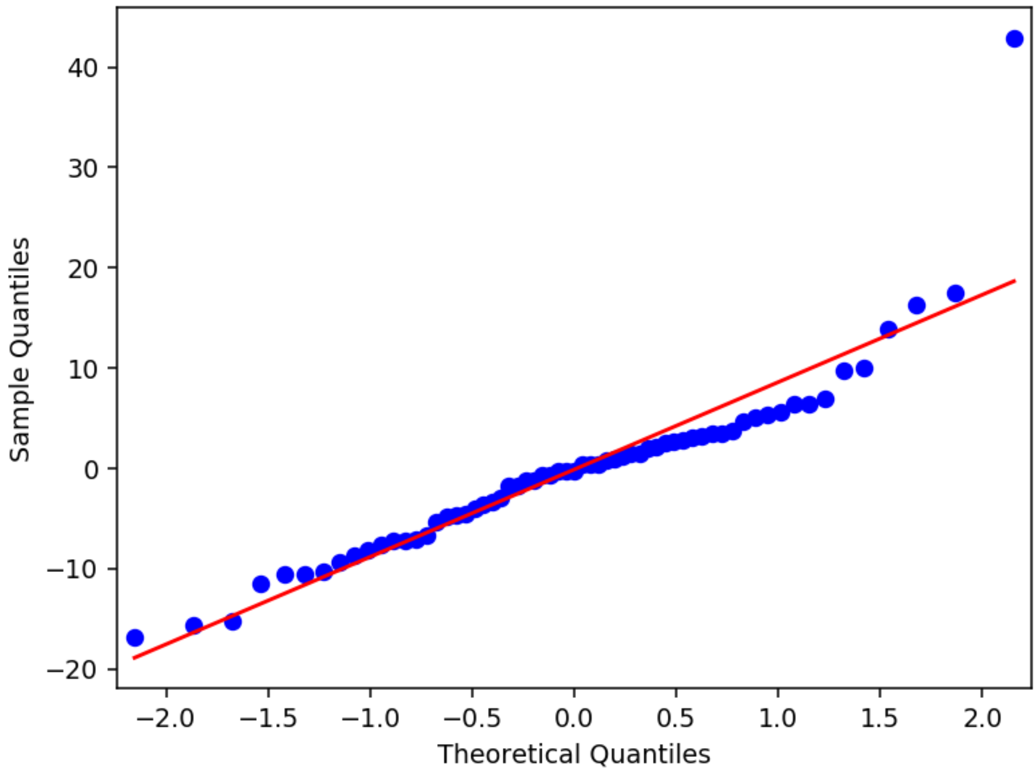
R-squared value shows only 10% variables are explained by regression model, Prob(f-statistic) is 0.6%, means we should reject null hypothesis.

Evaluate your model

Use ggplot

In [40]:

```
import statsmodels.api as sm
fig5=sm.qqplot(reg4.resid, line='r')
```



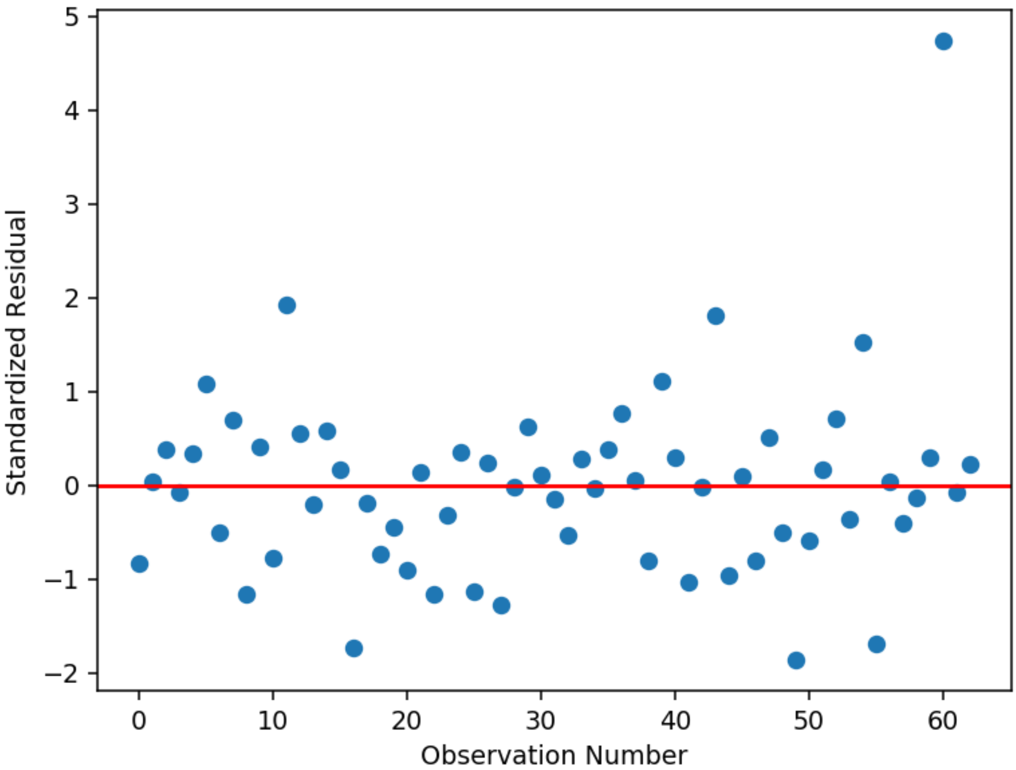
The QQ plot gives a roughly straight line, which means both sets of quantiles came from the same distribution.

Evaluate your model

Use residual plot

In [41]:

```
# simple plot of residuals
stdres=pd.DataFrame(reg4.resid_pearson)
plt.figure()
plt.plot(stdres, 'o', ls='None')
l = plt.axhline(y=0, color='r')
plt.ylabel('Standardized Residual')
plt.xlabel('Observation Number')
```



The data are equally spaced above and below line of 0, shows dataset is suitable for linear regression, however, we have one significant outlier.

Out[41]:

```
Text(0.5,0,'Observation Number')
```

Calculate percentage of observations over 2 standardized deviation

In [42]:

```
percentage_over2sd = (np.count_nonzero( stdres[0] > 2) + np.count_nonzero( stdres[0] < -2))
print (percentage_over2sd)
```

```
1.5873015873015872
```

Calculate percentage of observations over 2.5 standardized deviation

In [43]:

```
percentage_over2_5sd = (np.count_nonzero( stdres[0] > 2.5) + np.count_nonzero( stdres[0] <
print (percentage_over2_5sd)
```

1.5873015873015872