

Math 136 Note

Peter Jiang

January 12, 2021

1 Vectors in \mathbb{R}^n

Definition: Vector

A vector is an object that has both magnitude and direction.

Representation I: Geometrically

Vectors in \mathbb{R}^2 , \mathbb{R}^3 and even in \mathbb{R}^n can be visualized as directed line segments.

Vectors can be moved around in \mathbb{R}^n space as long as their magnitude and direction is not changed, that is, the vectors are not localized

Points and vectors are not the same.

Representation II: Algebraically Vectors can be expressed as columns of numbers. These columns are often called *n-tuples*

$$\vec{w} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \in \mathbb{R}^2, \vec{v} = \begin{pmatrix} -3 \\ 1 \\ -5 \end{pmatrix} \in \mathbb{R}^3, \vec{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n,$$

Notation:

$$\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} = (1, 2, 3, 4, 5)^T$$

where "T" is short for transpose and must be included

Relationship between a point and a vector:

Let \vec{p} be a vector in \mathbb{R}^n which then could be thought of as a discrete line segment per **Representation I**. Let P be the terminal point of \vec{p} , thus \vec{p} can be written as

$$\vec{p} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

then the point P has the coordinates (a_1, a_2, \dots, a_n) .

The vector \vec{v} from the origin to p has the same set of numbers as P . It is P 's positional vector

Vector Equality:

Let $\vec{v} = (v_1, v_2, \dots, v_n)^T$ and $\vec{w} = (w_1, w_2, \dots, w_n)^T$

$\vec{v} = \vec{w}$ if

1. Same mag + dir
2. $\forall i = 1, 2, \dots, n, \vec{v}_i = \vec{w}_i$

Addition and Scalar Multiplication: Let $\vec{v} = (v_1, v_2 \dots v_n)^T$ and $\vec{w} = (w_1, w_2 \dots w_n)^T$,
 $\vec{z} = \vec{v} + \vec{w} = (z_1 + w_1, z_2 + w_2 \dots z_n + w_n)^T$