Math 136 Note

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January 11, 2021

1 Vectors in $\mathbb{R}^n$

Definition: Vector

A vector is an object that has both magnitude and direction.

Representation I: Geometrically
Vectors in $\mathbb{R}^2$, $\mathbb{R}^3$ and even in $\mathbb{R}^n$ can we visualized as directed line segments.
Vectors can be move around in $\mathbb{R}^n$ space as long as their magnitude and direction is not changed, that is, the vectors are not localized Points and vectors are not the same.

Representation II: Algebraically Vectors can be expressed as columns of numbers. These columns are often called n-***tuples***

$$\vec{w} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \in \mathbb{R}^2, \vec{v} = \begin{pmatrix} -3 \\ 1 \\ -5 \end{pmatrix} \in \mathbb{R}^3, \vec{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n,$$

Notation:

$$\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} = (1, 2, 3, 4, 5)^T$$

where "T" is short for transpose and must be included

Relationship between a point and a vector:
Let $\vec{p}$ be a vector in $\mathbb{R}^n$ which then could be thought of as a discrete line segment per **Representation I**. Let P be the terminal point of $\vec{p}$, thus $\vec{p}$ can be written as

$$\vec{p} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

then the point P has the coordinates $(a_1, a_2, ...a_n)$.
The vector $\vec{v}$ from the origin to p has the same set of number as P. It is P's positional vector

Vector Equality:
Let $\vec{v} = (v_1, v_2...v_n)^T$ and $\vec{w} = (w_1, w_2...w_n)^T$
$\vec{v} = \vec{w}$ if
1. Same mag + dir
2. $\forall i = 1, 2...n, \vec{v}_i = \vec{w}_i$