

# Math 136 Note

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# 1 Vectors in $\mathbb{R}^n$

**Definition:** Vector

*A vector is an object that has both magnitude and direction.*

**Representation I:** Geometrically

Vectors in  $\mathbb{R}^2$ ,  $\mathbb{R}^3$  and even in  $\mathbb{R}^n$  can be visualized as directed line segments.

Vectors can be moved around in  $\mathbb{R}^n$  space as long as their magnitude and direction is not changed, that is, the vectors are not localized. Points and vectors are not the same.

**Representation II:** Algebraically Vectors can be expressed as columns of numbers. These columns are often called *n-tuples*

$$\vec{w} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \in \mathbb{R}^2, \vec{v} = \begin{pmatrix} -3 \\ 1 \\ -5 \end{pmatrix} \in \mathbb{R}^3, \vec{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n,$$

**Notation:**

$$\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} = (1, 2, 3, 4, 5)^T$$

where "T" is short for transpose and must be included

**Relationship between a point and a vector:**

Let  $\vec{p}$  be a vector in  $\mathbb{R}^n$  which then could be thought of as a discrete line segment per **Representation I**. Let  $P$  be the terminal point of  $\vec{p}$ , thus  $\vec{p}$  can be written as

$$\vec{p} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

then the point  $P$  has the coordinates  $(a_1, a_2, \dots, a_n)$ .

The vector  $\vec{v}$  from the origin to  $p$  has the same set of numbers as  $P$ . It is  $P$ 's positional vector

**Vector Equality:**

Let  $\vec{v} = (v_1, v_2, \dots, v_n)^n$  and  $\vec{w} = (w_1, w_2, \dots, w_n)^n$

$\vec{v} = \vec{w}$  if

1. Same mag + dir
2.  $\forall i = 1, 2, \dots, n, v_i = w_i$