Math 136 Note

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1 Vectors in $\mathbb{R}^n$

Definition: Vector

> *A vector is an object that has both magnitude and direction.*

Representation I: Geometrically

Vectors in $\mathbb{R}^2$, $\mathbb{R}^3$ and even in $\mathbb{R}^n$ can we visualized as directed line segments.

Vectors can be move around in $\mathbb{R}^n$ space as long as their magnitude and direction is not changed, that is, the vectors are not localized

Points and vectors are not the same.

Representation II: Algebraically Vectors can be expressed as columns of numbers. These columns are often called n-***tuples***

$$\vec{w} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \in \mathbb{R}^2, \vec{v} = \begin{pmatrix} -3 \\ 1 \\ -5 \end{pmatrix} \in \mathbb{R}^3, \vec{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n,$$

Notation:

$$\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} = (1, 2, 3, 4, 5)^T$$

where "T" is short for transpose and must be included

Relationship between a point and a vector:

Let $\vec{p}$ be a vector in $\mathbb{R}^n$ which then could be thought of as a discrete line segment per **Representation I**. Let P be the terminal point of $\vec{p}$, thus $\vec{p}$ can be written as

$$\vec{p} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$$

then the point P has the coordinates $(a_1, a_2, ...a_n)$.

The vector $\vec{v}$ from the origin to p has the same set of number as P. It is P's positional vector

Vector Equality:

Let $\vec{v} = (v_1, v_2...v_n)^T$ and $\vec{w} = (w_1, w_2...w_n)^T$

$\vec{v} = \vec{w}$ if

1. Same mag + dir
2. $\forall i = 1, 2...n, \vec{v}_i = \vec{w}_i$

Addition and Scalar Multiplication: Let $\vec{v} = (v_1, v_2...v_n)^T$ and $\vec{w} = (w_1, w_2...w_n)^T$,
$\vec{z} = \vec{v} + \vec{w} = (z_1 + w_1, z_2 + w_2...z_n + w_n)^T$

Vector in $\mathbb{C}^n$:

2 Dot Product

Function that takes two vector and outputs a *scalar*

Length:
Recall that the length, or a *norm* of vector $\vec{v} = (a, b)^T$ in $\mathbb{R}^2$ is

$$||\vec{v}|| = \sqrt{a^2 + b^2}$$

Note that this is equivalent to the dot product of $\vec{v}$ to itself, thus

$$||\vec{v}|| = \sqrt{\vec{v} \cdot \vec{v}}$$

Therefore the square of the length of the vector is the dot product itself (*useful fact*)

$$||\vec{v}||^2 = \vec{v} \cdot \vec{v}$$

This geometric intuition in $\mathbb{R}^2$ often extends to higher dimension

Unit Vector:
A vector $\vec{v} \in \mathbb{R}^n$ is a unit vector means that $||w|| = 1$

Normalization:
When $\vec{v} \in \mathbb{R}^n$ is a non-zero vector, we can product a unit vector in the direct of $\vec{v}$, which we denote by $\hat{v}$, by scaling it by $\frac{1}{||\vec{v}||}$

A vector $\vec{w}$ is a scalar multiple of $\vec{v}$ if and only if it is a scalar multiple of $\hat{v}$

Orthogonality(Perpendicularity): Two vector $\vec{v}$ and $\vec{w} \in \mathbb{R}$ are *orthogonal* means $\vec{v} \cdot \vec{w} = 0$
* 0 is orthogonal to everything
* if $\vec{v}$ and $\vec{w} \in \mathbb{R}^2$ or $\mathbb{R}^3$ and both non-zero, then orthogonal and perpendicular coincides