Chebyshev Interpolation for the Fast Calculation of Credit Valuation Adjustments

March 20, 2018

Let V_t deonte the time-t price of a portfolio of derivatives. We are interested in the time-0 price of the contingent notional CDS which pays out the amount $\max(V_T, 0)$ in the event of default of the counterparty. By the fundamental arbitrage theorem, this value is given by

$$\frac{\mathrm{CVA}_0}{N_0} = \mathbb{E}_{\mathbb{Q}}\left[\int_{s=0}^{s=T} \frac{\max(V_s, 0) \cdot d\mathbf{1}_{(\tau \leq s)}}{N_s}\right] = \int_{s=0}^{s=T} \mathbb{E}_{\mathbb{Q}}\left[\frac{\max(V_s, 0) \cdot d\mathbf{1}_{(\tau \leq s)}}{N_s}\right],$$

where N_t is the numeraire, $\mathbb{Q}$ the associated measure and $\mathbf{1}_{(\tau \leq s)}$ is the default indicator for the counterparty which equals 0 (resp. 1) if s is greater (resp. less) than the default time τ. The integral over time can be discretized over time buckets $[T_0 = 0, T_1, \ldots, T_N]$:

$$\frac{\mathrm{CVA}_0}{N_0} = \sum_{i=1}^{N} \mathbb{E}_{\mathbb{Q}}\left[\frac{\max(V_{T_i}, 0) \cdot \mathbf{1}_{(T_{i-1} < \tau \leq T_i)}}{N_{T_i}}\right].$$

Thus, the problem amounts to evaluating terms of the form

$$N_0 \cdot \mathbb{E}_{\mathbb{Q}}\left[\frac{\max(V_{T_i}, 0) \cdot \mathbf{1}_{(T_{i-1} < \tau \leq T_i)}}{N_{T_i}}\right],$$

which, on setting the numeraire to be the T_i-maturing zero coupon bond, becomes

$$P_0(0, T_i) \cdot \mathbb{E}\Big[\max(V_{T_i}, 0) \cdot \mathbf{1}_{(T_{i-1} < \tau \leq T_i)}\Big].$$

(The notation $P_t(T, T_i)$ is used to denote the forward discount factor between times T and T_i, i.e., the forward price at time T of the bond paying q at time T_i, observed at time t. This is a random variable for $t > 0$).

In the special case that V_T and the default event are independent, the expectation in the previous expression becomes

$$\mathbb{E}\Big[\max(V_{T_i}, 0) \cdot \mathbf{1}_{(T_{i-1} < \tau \leq T_i)}\Big] = \mathbb{E}\Big[\max(V_{T_i}, 0)\Big] \cdot \mathrm{Prob}(T_{i-1} < \tau \leq T_i).$$

The default probability

$$\mathrm{Prob}(T_{i-1} < \tau \leq T_i) = \mathbb{E}\Big[\mathbf{1}_{(T_{i-1} < \tau \leq T_i)}\Big]$$

is derived from a default curve which, in some cases, is implied from prices of traded CDS's.

Thus, by assuming independence between the portfolio value and the default event the problem is reduced to calculating terms of the form

$$\mathrm{EPE}_t := \mathbb{E}[\max(V_t, 0)].$$

Suppose the risk factor X_t drives the price of V_t, i.e.,

$$V_t = f(t, X_t).$$

In this work we propose to use polynomial interpolants $P_{t,n}$ on $n+1$ Chebyshev points in an interval $[a, b]$, where n is high enough to approximate to a required accuracy V_t, i.e.,

$$V_t \approx P_{t,n}(x), \;\; x \in [a, b].$$