Imperial College London

Simulation Methods for Barrier and Look-back Options

Simulation Methods for Finance
under the supervision of Professor Harry Zheng

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Part I
Basic Task

1 Generate Random Number

1.1 Presentation

There are two methods we used to generate random numbers. The first one is to generate a uniform distribution and then transform it into Standard Normal Distribution. The second method we tried is the system build-in function `random`, which can directly generate a variable following Standard Normal Distribution.

1.2 Generating the uniform distribution

To generate a uniform distribution we tried two ways: one can use the system build-in methods `rand`. The function will return a number between 1 and $2^{15} - 1$ pseudo-randomly. The range is around 30,000, way below 100,000, the sample size we plan to generate. In other words, when we use `rand` to simulate 100,000 entries, there will be numbers appear more than once, which will generate correlations. We prefer the second way to generate a uniform distriubtion: linear congruential generator.

$$n_i = (an_{i-1})\ mod\ m$$

for $i = 1, 2, ..., 10, 000$, and we let $a = 7^5$, and $m = 2^{31} - 1$, which gives about 2 billion points. We run 100,000 times and will only use 0.005% of all points. Theoretically, no pattern should appear. $\frac{n_i}{m}$ will give a distribution close to uniform distribution from (0,1).

1.3 Generating the normal distribution

To transform the uniform distribution sample into Standard Normal Distribution sample, we have tried three methods. By Central Limit theory,

$$Z_n = \frac{\sum_{i=1}^{n} X_i - n\mu}{\sqrt{n}\sigma}$$

where X_i is from the uniform distribution we generated previously. Z_n converges in distribution to Standard Normal Distribution, however, it requires n, the number of uniform distribution, to be sufficiently large. Therefore this method requires significantly large number of simulations and speed is decreased consequently.

We also tried Box-Muller methods [1]

$$Z_1 = \sqrt{-2lnX_1}sin(2\pi X_2),\ \mathbb{Z}_2 = \sqrt{-2lnX_1}cos(2\pi X_2)$$

Since its simulation involves computation of *sine* and *cosine*, the speed is slow. We prefer the last method, Marsaglia Polar method.

$$let\ V_1 = 2U_1 - 1,\ V_1 = 2U_2 - 1.$$

where U_1 and U_2 are two independent uniform distribution. Let $W = V_1^2 + V_2^2$. If $W > 1$, return to the beginning. Otherwise,

$$N_1 = \sqrt{\frac{(-2logW)}{W}}V_1,\ N_2 = \sqrt{\frac{(-2logW)}{W}}V_2$$

As the computation does not involve *sine* and *cosine*, it is generally faster than Box-Muller.

Method	Mean	Variance	Time (seconds)
`rand` + **CLT**	1.77 e-5	0.999909	90.34
`rand` + **Box-Muller**	-6.80 e-5	1.000703	8.64
`rand` + **Marsaglia Polar**	1.70 e-5	1.000472	6.40
LCG + CLT	1.52 e-5	0.999973	236.3
LCG + Box-Muller	-1.27 e-4	0.999797	11.59
LCG + Marsaglia Polar	7.36 e-5	0.999979	10.52
`random`	3.58 e-4	1.000210	14.98

Table 1: We note that the Marsaglia method is faster compared to the other methods. Note that the run time depends on the computer used for the simulation (here a 3.4 GHz Intel Core i7). We have generated 100,000,000 variables to obtain these results.

Table 1 is the simulation results of Standard Normal distribution and time used by different methods for the generation of 100,000,000 random variables. As shown in the table, when using CLT[1] to generate standard normal distribution, the time needed is significantly longer than the rest methods. Marsaglia Polar method gives a variance closer to 1 compared to Box-Muller methods. Linear congruential generator gives a variance closer to 1 compared to `rand` method. Hence we use the combination of linear congruential generator and Marsaglia methods to simulate random variables. This was also the opportunity for us to have full control over the generation of random numbers, as it is a critical step towards efficient simulations [1]. It also features very good results in terms of statistical parameters and time.

2 European Option

2.1 Presentation

The asset price in a risk neutral probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{0\leq t\leq T}, P)$ follows Geometric Brownian Motion,

$$dS_t = rS_t dt + \sigma S_t dW_t,\ 0 \leq t \leq T$$

with initial price $S_0 = S$, where r is riskless interest rate, σ volatility, and W_t the standard Brownian motion. A European call option price at time t with maturity time T is given by

$$C_t = E[e^{-r(T-t)}(S_T - K)^+|F_t]$$

For the basic task, we let $S_0 = 100$, $K = 100$, interest rate $r = 0.05$, volatility $\sigma = 0.4$, maturity time $T = 1$ and initial time $t = 0$. We use Monte Carlo method to simulate

$$S_T = S_0 e^{(r-\frac{1}{2}\sigma^2)T+\sigma\sqrt{T}Z}$$

and get sample distribution of $(S_T - K)^+$. By Black-Scholes formula,

$$C_{bs}(S_0, K) = N(d_1)S_0 - N(d_2)Ke^{-rT}$$

where

$$d_1(S_0, K) = \frac{1}{\sigma\sqrt{T}}[ln(\frac{S_0}{K}) + (r + \frac{\sigma^2}{2})T],\ d_2(S_0, K) = d_1 - \sigma\sqrt{T}$$

The closed-form Greeks are calculated the following way:

$$\delta_{bs} = \frac{\partial C}{\partial S_0} = \phi(d_1), \qquad \Gamma_{bs} = \frac{\partial^2 C}{\partial S_0^2} = \frac{\Phi'(d_1)}{S_0\sigma\sqrt{T}}, \qquad \nu_{bs} = \frac{\partial C}{\partial \sigma} = \Phi'(d_1)\sqrt{T}$$

[1] Central Limit Theorem

2.2 Likelihood Ratio Greeks

To calculate the Greeks from simulation, we compare likelihood ratio method and Pathwise method. The Call option price is given by

$$C = e^{-rT}E[(S_T - K)^+] = e^{-rT}\int (S_T - K)^+ h_{S_0}(S_T)dS_T$$

where $h_{S_0}(S_T)$ is the probability density function of $(S_T - K)^+$. Then by Likelihood ratio method, the partial derivative of C with respect to S_0 is

$$\frac{\partial C}{\partial S_0} = \int (S_T - K)^+ \frac{d}{dS_0} h_{S_0}(S_T)dS_T = E[(S_T - K)^+ \frac{h'_{S_0}(S_T)}{h_{S_0}(S_T)}]$$

And the lognormal density function of S_T is given by

$$h(x) = \frac{1}{x\sigma\sqrt{T}}\phi(\xi(x)), \quad \xi(x) = \frac{ln(x/S_0) - (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

Therefore [1],

$$Delta_{LR} = \frac{\partial C}{\partial S_0} = E[e^{-rT}(S_T - K)^+ \frac{Z}{S_0\sigma\sqrt{T}}]$$

where Z $N(0,1)$.

$$Gamma_{LRLR} = \frac{\partial^2 C}{\partial S_0^2} = E[e^{-rT}(S_T - K)^+ (\frac{Z^2 - 1}{S_0^2\sigma^2 T} - \frac{Z}{S_0^2\sigma\sqrt{T}})]$$

$$Vega_{LR} = \frac{\partial C}{\partial \sigma} = E[e^{-rT}(S_T - K)^+ \left(\frac{Z^2 - 1}{\sigma} - Z\sqrt{T}\right)]$$

2.3 Pathwise Derivative Greeks

By pathwise methods, the Greeks are given as following[2] [1], with $Z \sim N(0,1)$:

$$Delta_{PW} = \frac{\partial C}{\partial S_0} = E[e^{-rT}\mathbb{1}_{S_T > K}\frac{S_T}{S_0}]$$

$$Gamma_{LRPW} = \frac{\partial^2 C}{\partial S_0^2} = E[e^{-rT}\mathbb{1}_{S_T > K}\frac{KZ}{S_0^2\sigma\sqrt{T}}]$$

$$Gamma_{PWLR} = \frac{\partial^2 C}{\partial S_0^2} = E[e^{-rT}\mathbb{1}_{S_T > K}\frac{S_T}{S_0^2}\left(\frac{Z}{\sigma\sqrt{T}} - 1\right)]$$

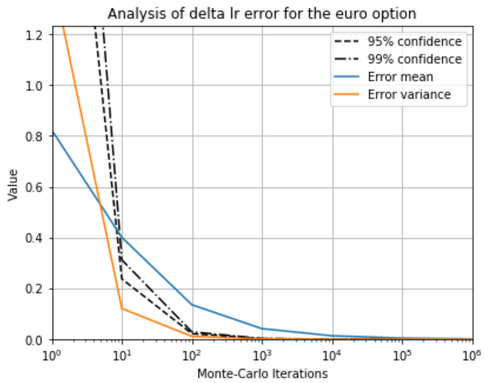
$$Vega_{PW} = \frac{\partial C}{\partial \sigma} = E[e^{-rT}\mathbb{1}_{S_T > K}S_T(\sqrt{T}Z - \sigma T)]$$

The results calculated from the closed-form formulae and the simulations are represented in Table 2.

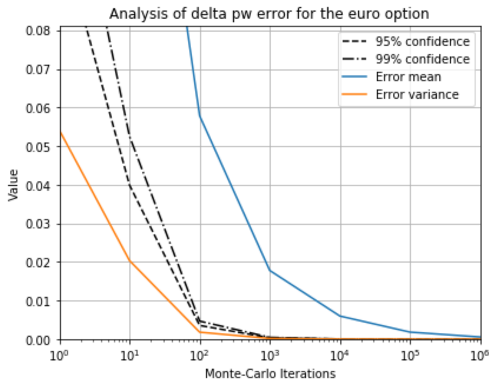
We can see that the simulation results improve and get closer to the theoretical values as the simulation number increases. Hence, to compare the results from different methods, we will only analyze the simulation results from the largest simulation number - 100,000 in this case. This is even more clear on the graphs (Figure 1).

We now have a closer look at 100,000 simulations, which is the industry standard and which is above the 1,000 simulations threshold we have noticed on the graphs. For delta, although the fastest methods is LR, it has a significantly lower accuracy. Gamma LRLR is the most accurate

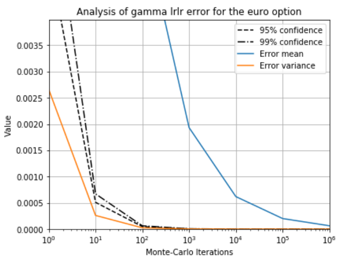
[2]For gamma, there are two different methods.



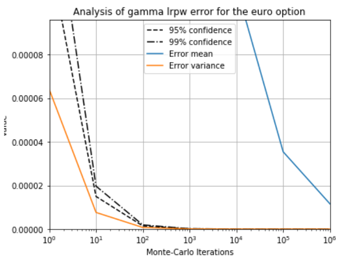
(a) Delta with LR method



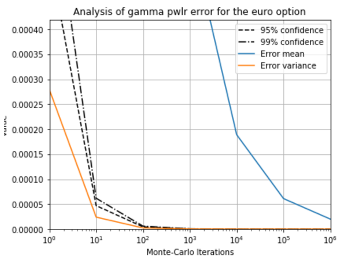
(b) Delta with PW method



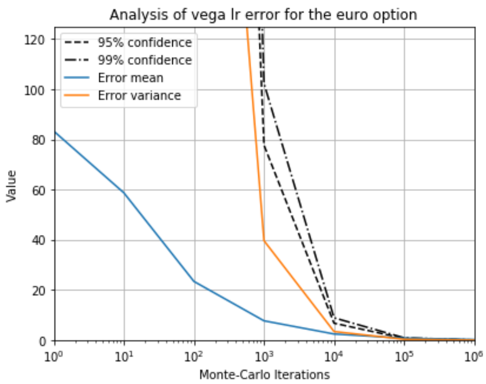
(c) Gamma with LR-LR method



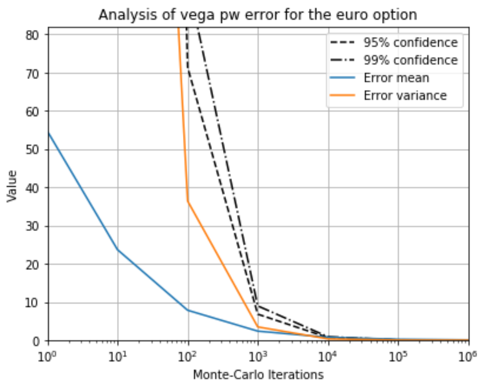
(d) Gamma with LR-PW method



(e) Gamma with PW-LR method



(f) Vega with LR method



(g) Vega with PW method

Figure 1: A graphical analysis of the errors of the Greeks depending on the number of simulations. We notice that there is a significant increase above 1000 simulations in all cases. This is very clear when looking at vega which tends to have a very high error variance below 1,000 simulations. In order to generate these graphs, we have done 1,000 simulations for each value, at every power of 10.

		Black -Scholes	Monte Carlo Simulation		
			1,000	**10,000**	**100,000**
Option Price		18.023	19.0162	17.6732	18.0014
Delta	**LR**	0.627409	0.662561	0.612362	0.627889
	PW		0.64485	0.623239	0.627453
Gamma	**PW LR**	0.0094605	0.0101213	0.00919597	0.00945029
	LR PW		0.00994418	0.00930474	0.00944593
	LR LR		0.00975465	0.00918059	0.0095502
Vega	**LR**	37.842	39.0186	36.7224	38.2008
	PW		40.4851	36.7839	37.8012

Table 2: Table with the result of our simulations for different Monte-Carlo iterations. We can clearly see the improvement between 1,000 and 10,000 simulations.

1,000,000 MC simulations	**Error Mean**	**Error Variance**	**Time (seconds)**
Option Price	na	na	5.0 e-6
Delta LR	4.1 e-3	9.7 e-6	1.4 e-3
Delta PW	1.8 e-3	1.8 e-6	9.9 e-4
Gamma PWLR	6.1 e-5	2.0 e-9	1.4 e-3
Gamma LRPW	3.5 e-5	7.4 e-10	1.4 e-3
Gamma LRLR	1.9 e-4	2.1 e-8	4.7 e-3
Vega LR	7.7 e-1	3.3 e-1	4.6 e-3
Vega PW	2.4 e-1	3.2 e-2	1.6 e-3

Table 3: Sample from our simlation dataset with 100,000 Monte-Carlo simlations. Note that the run time depends on the computer (here a 3.1 GHz Intel Core i5). We computed the absolute error and since the option price is computed using the closed form formula no error is represented.

but it is the slowest, so gamma PWLR seems to be a good compromise. Vega PW is both faster and more accurate than vega LR.

For the following section, PW methods will be infeasible to do for Barrier Option. Hence, we will use likelihood ratio method when calculating the Greeks.

Part II
Main Task

3 Barrier Option

3.1 Presentation

Let T denote option expiration time and $[0, T]$ lookback period. For $T_0 \leq t \leq T$ denote by

$$m_0^T = \underset{0 \leq t \leq T}{min} S_t, \quad M_0^t = \underset{0 \leq t \leq T}{max} S_t$$

For an up-and-out barrier call option $A_T = (S_T - K)^+ \mathbb{1}_{max0 \leq t \leq T \ S_t \leq B}$, where B is a barrier leve and $\mathbb{1}_S$ is an indicator function.

We simulate the path of S_t by dividing the maturity time T into 1,000 steps, and we simulate 5,000 such paths. We have an M_0^T for each path. However, this method is bordensome as we need to generate each step for each path. The simulation process is long.

3.2 Another Method using Rayleigh Distribution

Taking into consideration the arguments from above, we tried second method using Rayleigh Distribution to simulate the distribution of M_0^t directly. The maximum of a standard Brownian motion starting at the origin to be at b at time 1 over period [0, T] has the Rayleigh distribtuion [2, 3, 4]

$$F(x) = 1 - e^{-2x(x-b)}, \ x \geq b.$$

solving the equation $F(x) = u, u \in (0, 1)$ has roots

$$x = \frac{b}{2} \pm \frac{\sqrt{b^2 - 2log(1-u)}}{2}$$

Hence, at time T with S_T,

$$M^T = \frac{S_T + \sqrt{S_T^2 - 2TlogU}}{2}$$

And we simulate S_T by Black-Scholes formula

$$S_T = S_0 e^{(r - \frac{1}{2}\sigma^2)T + \sigma\sqrt{T}Z}$$

Therefore,

$$M^T = S_0 e^{\frac{1}{2}log(\frac{S_T}{S_0}) + \sqrt{log(\frac{S_T}{S_0})^2 - 2\sigma^2 TlogU}}$$

With this method, we only need to generate S_T for each path and one random variable for the uniform distribution to get the M^T. It reduces the amount of simulation by 500 times (as before, 1,000 S_t need to be generated for each path).

As we mentioned previously, it is impossible to find the partial direvative of the indicator function $\mathbb{1}_{M_0^T \leq B}$ with respect to S_0. Hence pathwise method is eliminated by us. For the likelihood ratio method, we find the differentiation of joint cumulative density function of S_T and the M_T to be

$$f_{uo}(x, m, T) = \frac{1}{\sqrt{T}} \left(\phi(\frac{x - \mu T}{\sqrt{T}}) - e^{2m\mu} \phi(\frac{x - 2m - \mu T}{\sqrt{T}}) \right)$$

	Closed Form	**Monte Carlo 100,000 Simulation**
Option Price	15.18	15.11
Delta LR	0.776	0.769
Gamma LRLR	2.58 e-3	2.89 e-3
Vega LR	15.55	16.02

Table 4: Theoretical results compared to simulation results with the same parameters as the European option and a barrier at 100.

100,000 MC simulations	**Error Mean**	**Error Variance**	**Time (seconds)**
Option Price	na	na	na
Delta LR	1.508 e-2	1.40 e-4	2.109
Gamma LRLR	6.186 e-4	2.131 e-7	2.118
Vega LR	2.405	3.435	2.113

(a) Error statistics and computation time for the Newton-Raphson method. We computed the absolute error and since the option price is computed using the closed form formula no error is represented.

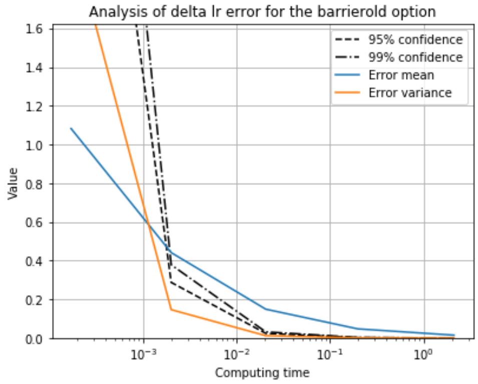
1,000,000 MC simulations	**Error Mean**	**Error Variance**	**Time (seconds)**
Option Price	na	na	..
Delta LR	1.036 e-2	3.029 e-5	9.284 e-2
Gamma LRLR	1.895 e-4	1.895 e-8	1.475 e-1
Vega LR	7.507 e-1	3.220 e-1	1.097 e-1

(b) Error statistics and computation time for the Rayleigh method. We computed the absolute error and since the option price is computed using the closed form formula no error is represented.

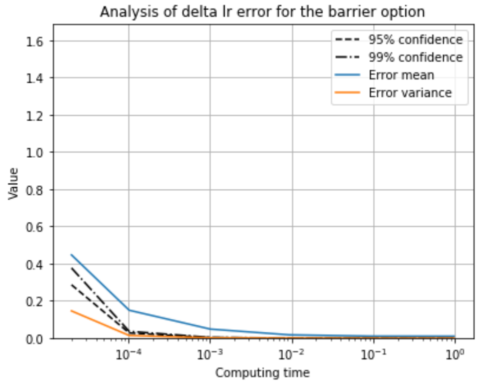
Table 5: Sample from our simulation dataset with the new fast method for barrier option simulation. It is clear that the results have significantly improved compared with the previous method. Note that the run time depends on the computer used (here a 3.4 GHz Intel Core i7). However, due to the extremely slow computation time for LR methods, we only computed up to 100,000 samples compared to 1,000,000 for the Rayleigh method.

where $x = \frac{1}{\sigma} ln \frac{S_T}{S_0}$, $\mu = \frac{1}{\sigma}(r - \frac{\sigma^2}{2})$ and $m = \frac{1}{\sigma} ln \frac{M_T}{S_0}$. Then we can find the Greeks accordingly, the details of the computation are in Appendix A.

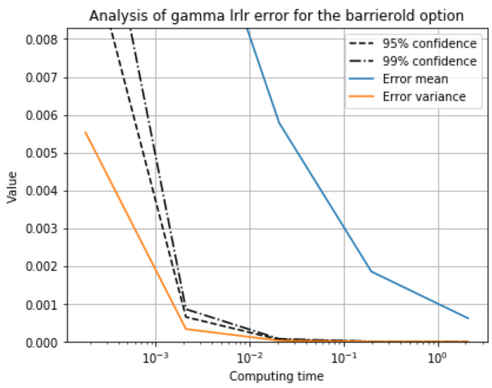
To analyze further the accuracy of our simulation, we plot the simulated option prices and Greeks versus their theoretical values as barrier increases (Figure 2).



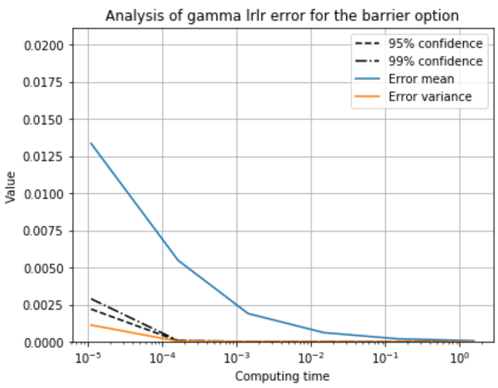
(a) Delta with LR method



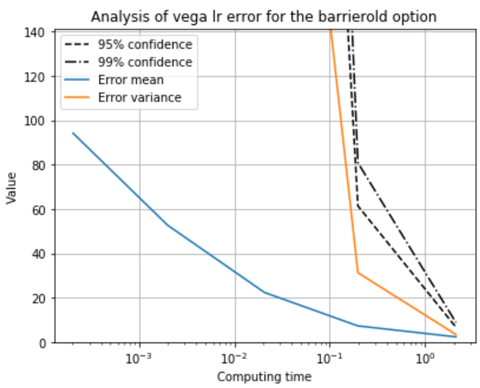
(b) Delta with Rayleigh method



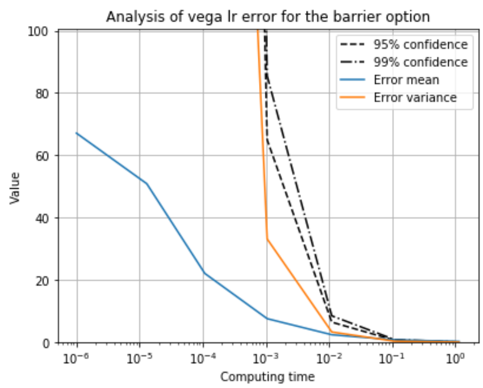
(c) Gamma with LRLR method



(d) Gamma with Rayleigh method



(e) Vega with LR method



(f) Vega with Rayleigh method

Figure 2: A graphical analysis of the error of the Greeks depending on the computation time. We notice that the method with Rayleigh distribution is between 100 and 1,000 times faster. In order to generate these graphs, we have done 1,000 simulations for each value. However, due to the extremely slow computation time for LR methods, we only computed up to 100,000 samples compared to 1,000,000 for the Rayleigh method.

4 Look-back Option

4.1 Presentation

Look-back call option with fixed strike price K has payoff $(M_0^T - K)^+$. The call option price at time t is

$$c(S_0, K, t) = e^{-r(T-t)}E[(max(M_0^t, M_t^T) - K)^+|\mathcal{F}_t]$$

The closed-form formula for fixed strike look-back call option at time 0 (see [1], [4]) is

$$c(S_0, K, 0) = C_{bs}(S_0, K) + \frac{S_0\sigma^2}{2r}\{\Phi[d_2(S_0, K)] - e^{-rT}\frac{S_0}{K}^{-\frac{2r}{\sigma^2}}\Phi[-d_1(K, S_0)]\}$$

The probability density function of the distribution of maximum S_t for standard Brownian motion for the period of (0, T) is

$$f_{(m)} = \frac{2}{\sqrt{T}}\phi\left(\frac{m - aT}{\sqrt{T}}\right) - 2ae^{2am}\Phi\left(\frac{-m - aT}{\sqrt{T}}\right)$$

and the cumulative density function is

$$F_{(M_T > m)} = \Phi\left(\frac{m - aT}{\sqrt{T}}\right) - e^{2am}\Phi\left(\frac{-m - aT}{\sqrt{T}}\right)$$

We used Newton-Raphson method to solve for the above cumulative density function $F = U$, the uniform distribution. Hence, for each random number we generate from [0,1], we receive one m by solving for F. To get M^T of stock price which follows geometric Browninan motion with starting price S_0, we let $M^T = S_0e^{\sigma m}$.

4.2 Another Method using Rayleigh Distribution

In a similar way as for the barrier option, we have also implemented a faster and more efficient method. The results can be compared on the plots.

100,000 MC Simulations		**Closed Form**	**Monte-Carlo 100,000 Simulation**
Option Price		37.76	37.715
Delta	**LR**	1.329	1.329
	PW		1.322
Gamma	**PW LR**	0.021	0.021
	LR PW		-0.0023
	LR LR		0.021
Vega	**LR**	98.68	100.56
	PW		111.02

Table 6: Theoretical results compared to simulation results with the same parameters as the European option. Note in this specific case we have a problem with Vega PW.

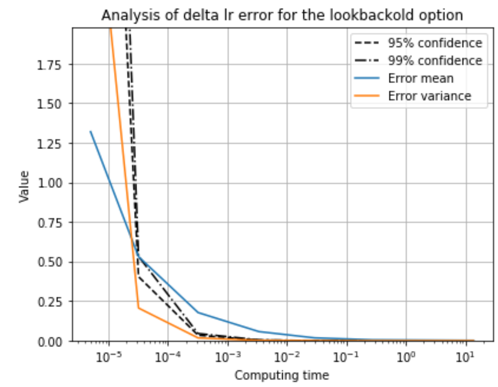
100,000 MC simulations	**Error Mean**	**Error Variance**	**Time (seconds)**
Delta LR	5.568 e-3	1.768 e-5	0.269
Gamma PWLR	8.383 e-5	3.638 e-9	0.261

(a) Error statistics and computation time for the Newton-Raphson method. We computed the absolute error and since the option price is computed using the closed form formula no error is represented.

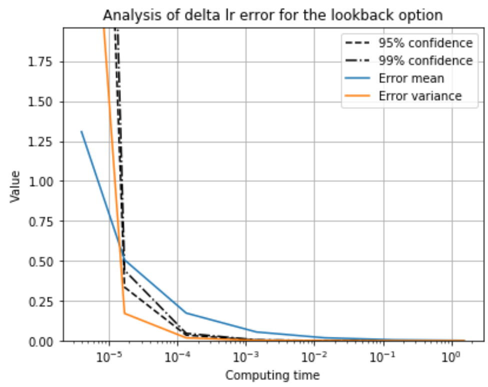
1,000,000 MC simulations	**Error Mean**	**Error Variance**	**Time (seconds)**
Price			
Delta LR	5.44 e-3	1.67 e-5	0.145
Delta PW	9.34 e-4	5.05 e-7	0.068
Gamma LRLR	2.80 e-4	4.39 e-8	0.153
Gamma PWLR	2.91 e-4	4.93 e-8	0.153
Vega LR	1.189	0.779	0.157
Vega PW	7.503	0.050	0.061

(b) Error statistics and computation time for the Rayleigh method. We computed the absolute error and since the option price is computed using the closed form formula no error is represented.

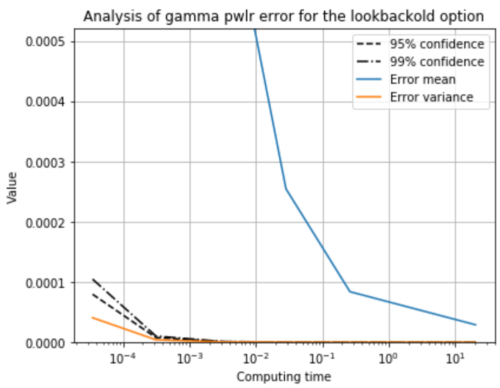
Table 7: Sample from our simulation dataset with the new fast method for barrier option simulation. It is clear that the results have significantly improved compared to the previous method. Note that the run time depends on the computer used (here a 3.4 GHz Intel Core i7).



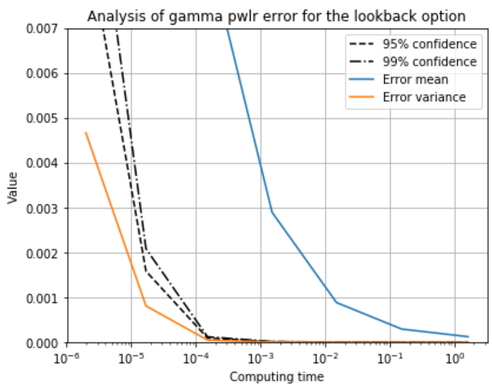
(a) Delta with Newton-Raphson method



(b) Delta with Rayleigh method



(c) Gamma with Newton-Raphson method



(d) Gamma with Rayleigh method

Figure 3: A graphical analysis of the error of the Greeks depending on the computation time. We notice that the method with Rayleigh distribution is between 10 and 100 times faster. In order to generate these graphs, we have done 1,000 simulations for each value.

References

[1] Harry Zheng (Pr.), *Simulation Methods for Finance*, Imperial College London, London, 2018.

[2] Diogo Monteiro da Costa Soares Justino, *Hedging of Barrier Options*, Instituto Universitário de Lisboa, Lisbon, 2010.

[3] Paul Glasserman (Pr.), *Monte Carlo Methods in Financial Engineering*, Springer Sciencea, New York, 2013.

[4] Peter G. Zhang (Pr.), *Exotic Options, A Guide to Second Generation Options*, World Scientific Publishing, Hong Kong, 1998 (second edition).

Appendix

A Sample of closed-formed formula of Barrier Options

$$UOC(S_0,K,B) = \mathbb{1}_{B>K}\Big(C_{bs}(S_0,K) - C_{bs}(S_0,B) - (B-K)e^{-rT}\Phi[d_1(S_0,B)] \\ - \frac{B}{S_0}^{\frac{2v^2}{\sigma^2}}\Big[C_{bs}(\frac{B^2}{S_0},K) - C_{bs}(\frac{B^2}{S_0},B) - (B_0-K)e^{-rT}\Phi[d_1(S_0,B)]\Big]\tag{1}$$

Where C_{bs} and d_1 are as stated in the Black-Scholes formula (1) and (2), and $\upsilon = r - \frac{\sigma^2}{2}$.

closed form for DOC price is

$$C_{DO}(S_0,K,B) = C_{bs}(S_0,K) - (\frac{S_0}{B})^{-2\frac{\upsilon}{\sigma^2}}C_{bs}(\frac{B^2}{S_0},K)$$

where $\upsilon = r - \frac{\sigma^2}{2}$.

$$\begin{aligned}\delta_D OC =&\delta_{BS}(S_0,K) - \delta_{BS}(S_0,B) - \frac{B-K}{\sigma S_0\sqrt{T}}e^{-rT}\Phi(d_2(S_0,B)) \\ &+ \left(\frac{2\upsilon}{\sigma^2 S_0}\left(\frac{B}{S_0}\right)^{2\upsilon/\sigma^2}\right) \\ &\times\left(C_{BS}\left(\frac{B^2}{S_0},K\right) - C_{BS}\left(\frac{B^2}{S_0},B\right) - (B-K)e^{-rT}\Phi(d_2(B,S_0))\right) \\ &- \left(\frac{B}{S_0}\right)^{2\upsilon/\sigma^2}(\left(\frac{-B}{S_0}\right)^2\delta_{BS}\left(\frac{B^2}{S_0},K\right) + \left(\frac{B}{S_0}\right)^2\delta_{BS}\left(\frac{B^2}{S_0},B\right) \\ &+ \left(\frac{B-K}{\sigma S_0\sqrt{T}}\right)e^{-rT}\Phi(d_2(B,S_0)));\end{aligned}$$

$$\begin{aligned}\delta_{DOC} =&\Phi\left(\frac{\log\frac{S_0}{K} + (r+\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\right) - \left(\frac{B}{S_0}\right)^{r/\sigma^2-1} \\ &\times\left(-\frac{B}{S_0}^2\Phi\left(\frac{log\frac{B^2}{S_0K}+\upsilon T}{\sigma\sqrt{T}}+\sigma\sqrt{T}\right) - \frac{2\upsilon C_{BS}(B^2/S_0,K)}{(S_0\sigma^2)}\right)\end{aligned}$$

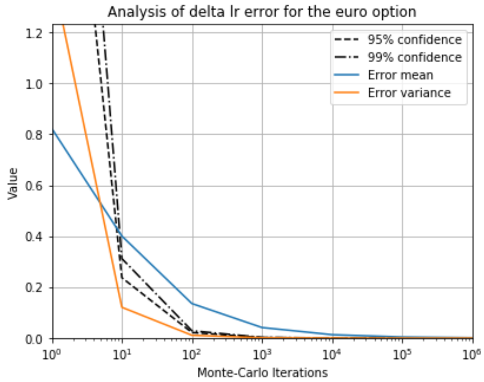
$$\gamma_{DOC} =\frac{\phi(d_2)}{S_0\sigma\sqrt{T}} - \left(\frac{B}{S_0}\right)^{2\upsilon/\sigma^2}\left(\frac{4\upsilon^2+2\upsilon\sigma^2}{S_0^2\sigma^4}C_{BS}(B^2/S_0,K) + \gamma_{bs} - \frac{4\upsilon\delta_{bs}}{S_0\sigma^2}\right)$$

$$\nu_{DOC} = S_0\phi\left(\frac{log(\frac{S_0}{K}) + (r+\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\right)\sqrt{T} - \left(\frac{B}{S_0}\right)^{\frac{2\upsilon}{\sigma^2}}\left(\nu_{bs} - \frac{4rC_{bs}(\frac{B^2}{S_0},K)log(\frac{B}{S_0})}{\sigma^3}\right)$$

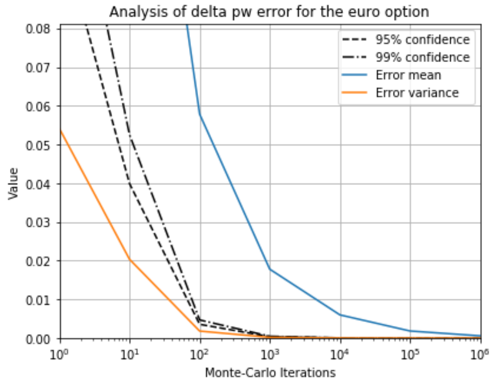
B Code

All our code is available on our GitHub repository:
`github.com/tjespel/barrier-and-look-back-options`.

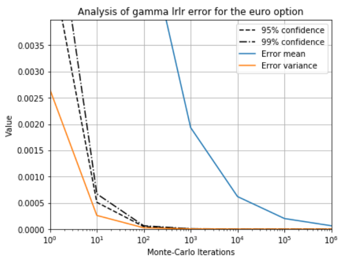
C Figures



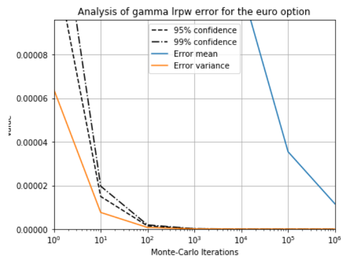
(a) Delta with LR method



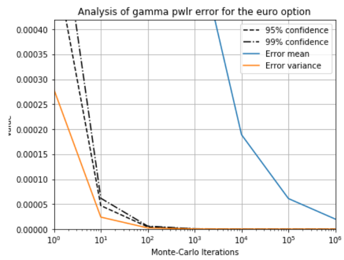
(b) Delta with PW method



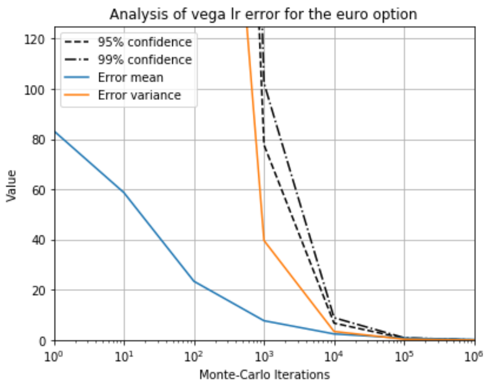
(c) Gamma with LR-LR method



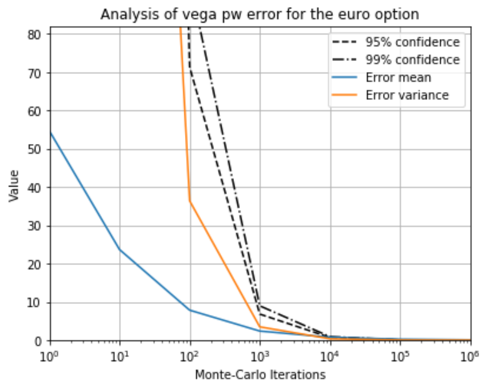
(d) Gamma with LR-PW method



(e) Gamma with PW-LR method

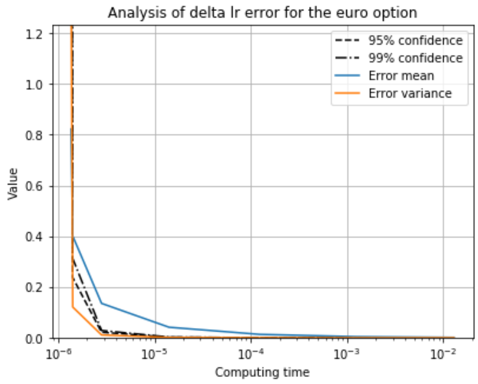


(f) Vega with LR method

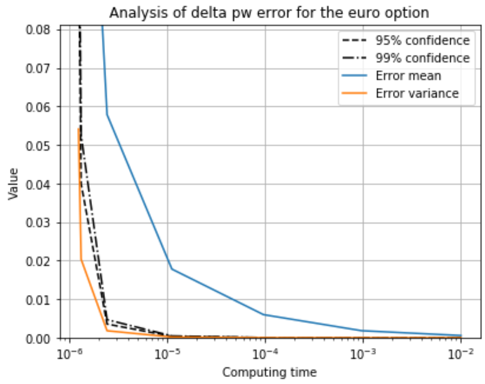


(g) Vega with PW method

Figure 4: **European Call Option**. A graphical analysis of the error of the Greeks depending on the number of simulations. In order to generate these graphs, we have done 1,000 simulations for each value.



(a) Delta with LR method



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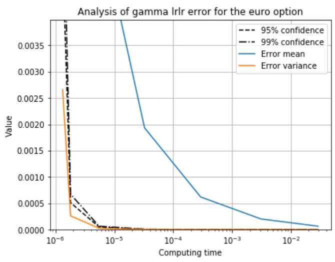
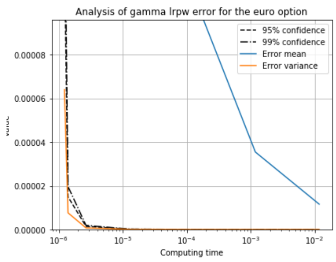
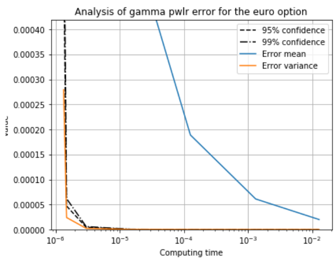


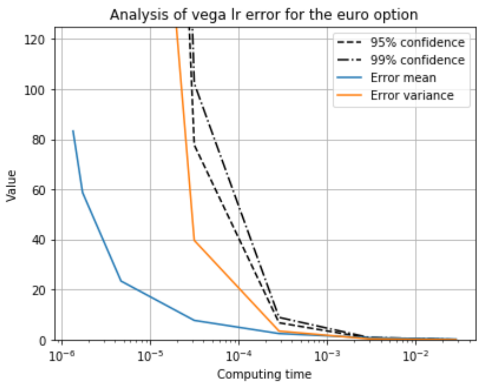
(c) Gamma with LR-LR method



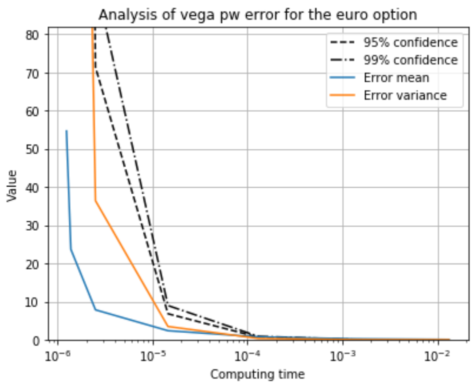
(d) Gamma with LR-PW method



(e) Gamma with PW-LR method

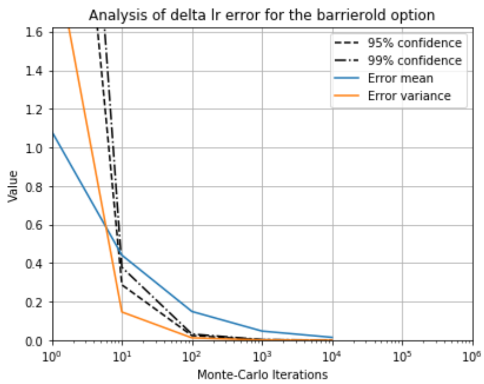


(f) Vega with LR method

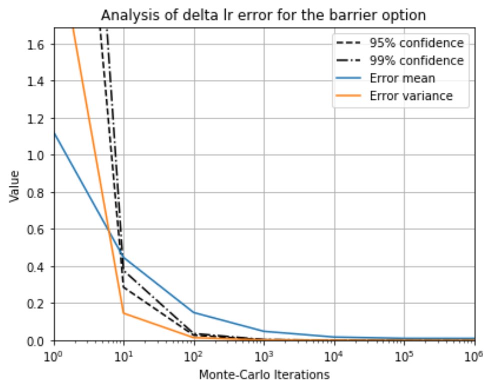


(g) Vega with PW method

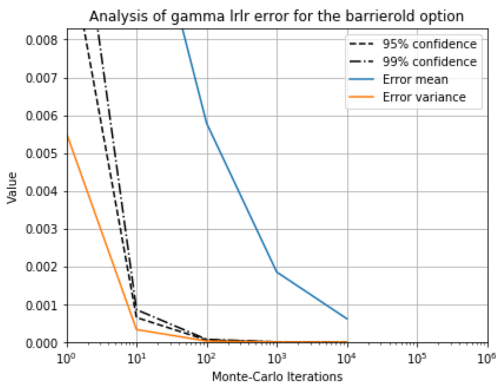
Figure 5: **European Call Option - Time**. A graphical analysis of the error of the Greeks depending on the number of simulations. In order to generate these graphs, we have done 1,000 simulations for each value.



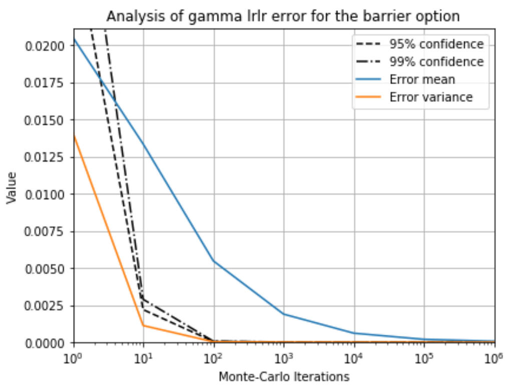
(a) Delta with LR method



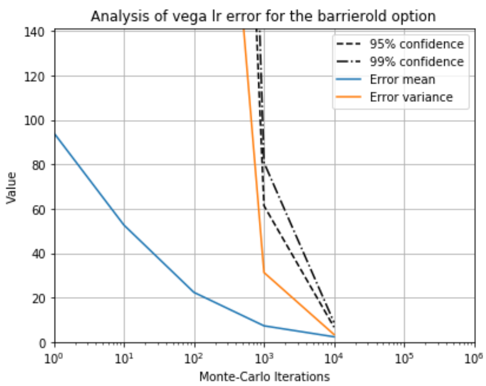
(b) Delta with LR Rayleigh method



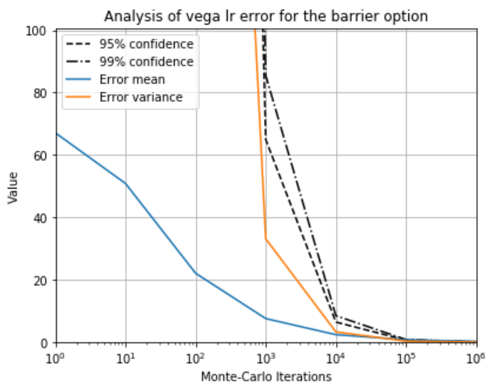
(c) Gamma with LR-LR method



(d) Gamma with LR-LR Rayleigh method

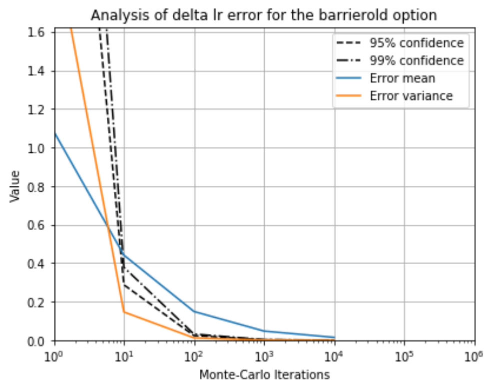


(e) Vega with LR method

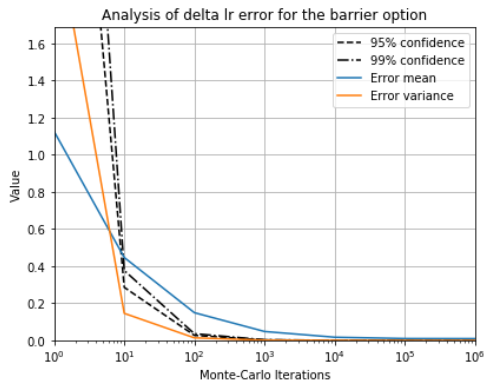


(f) Vega with LR Rayleigh method

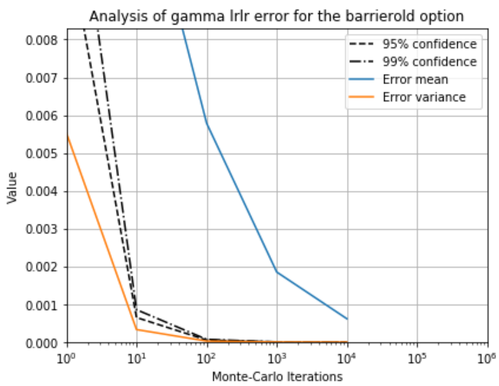
Figure 6: **Barrier Option**. A graphical analysis of the error of the Greeks depending on the number of simulations. In order to generate these graphs, we have done 1,000 simlations for each value, at every power of 10.



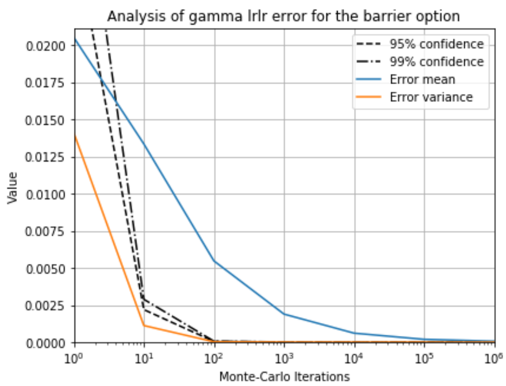
(a) Delta with LR



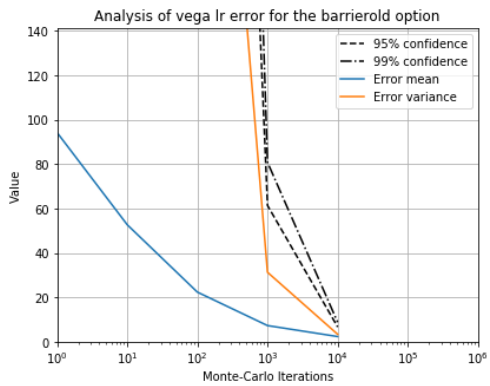
(b) Delta with LR Rayleigh



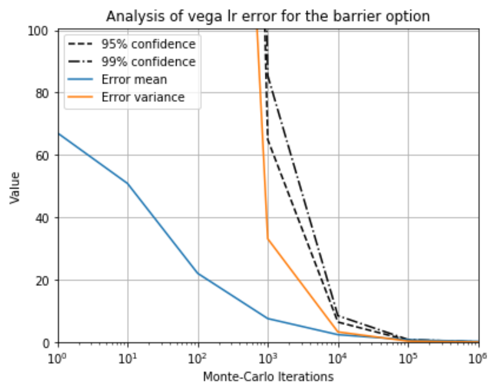
(c) Gamma with LR-LR method



(d) Gamma with LR-LR Rayleigh method

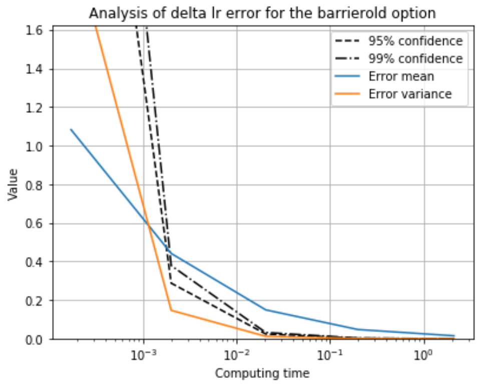


(e) Vega with LR method

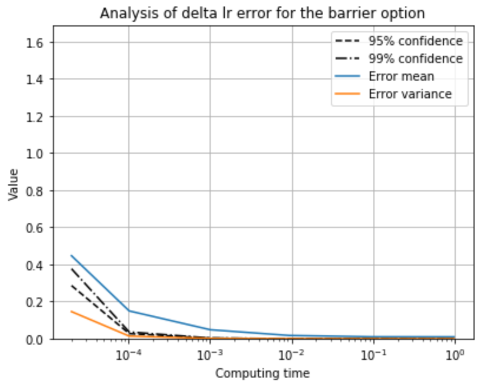


(f) Vega with LR Rayleigh method

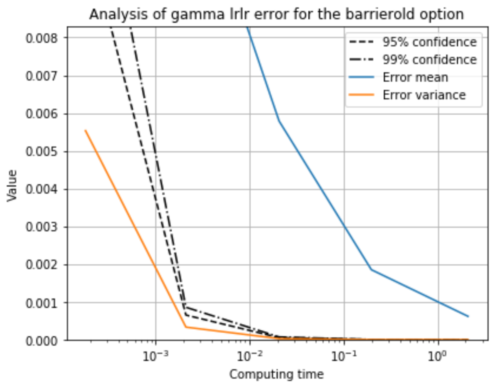
Figure 7: **Look-back Option**. A graphical analysis of the error of the Greeks depending on the number of simulations. In order to generate these graphs, we have done 1,000 simlations for each value, at every power of 10.



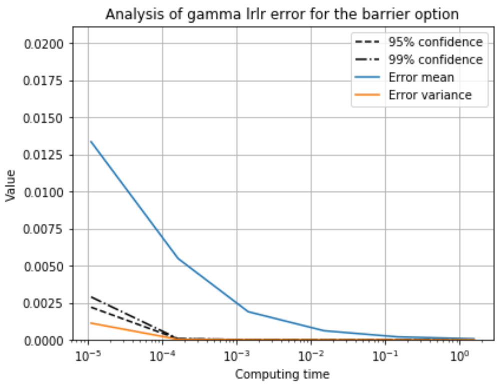
(a) Delta with LR



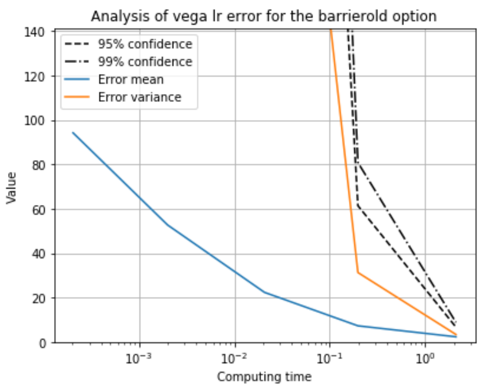
(b) Delta with LR Rayleigh



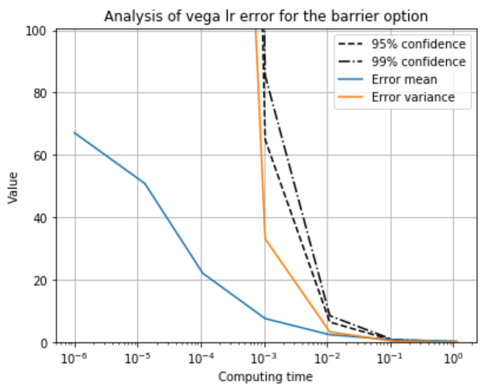
(c) Gamma with LR-LR method



(d) Gamma with LR-LR Rayleigh method

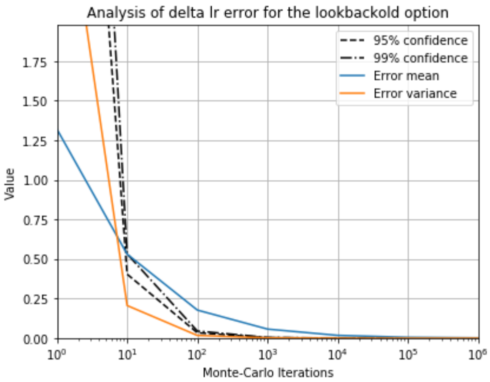


(e) Vega with LR method

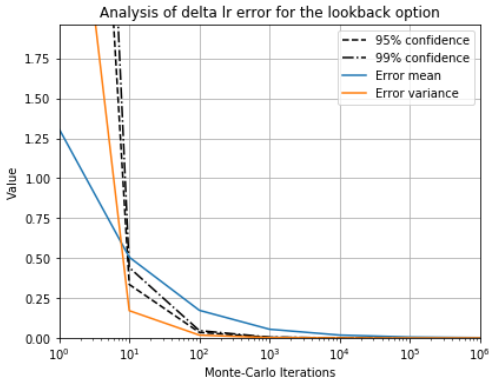


(f) Vega with LR Rayleigh method

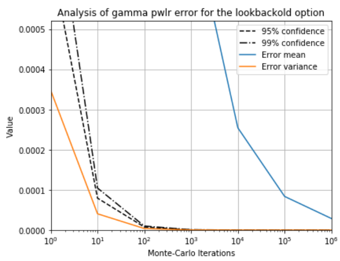
Figure 8: **Look-back Option - Time**. A graphical analysis of the error of the Greeks depending on the number of simulations. In order to generate these graphs, we have done 1,000 simlations for each value, at every power of 10.



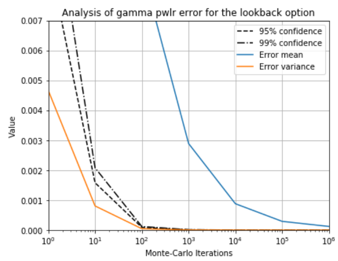
(a) Delta with LR method



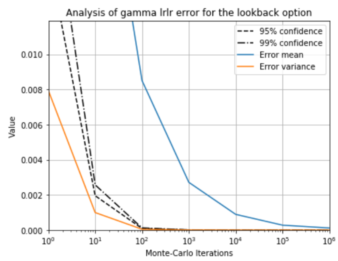
(b) Delta with LR Rayleigh



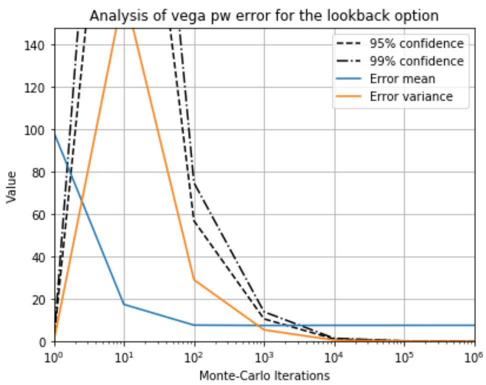
(c) Gamma with PW-LR method



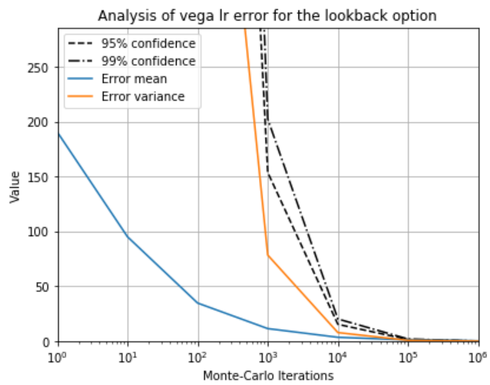
(d) Gamma with PW-LR method



(e) Gamma with LR-LR Rayleigh method

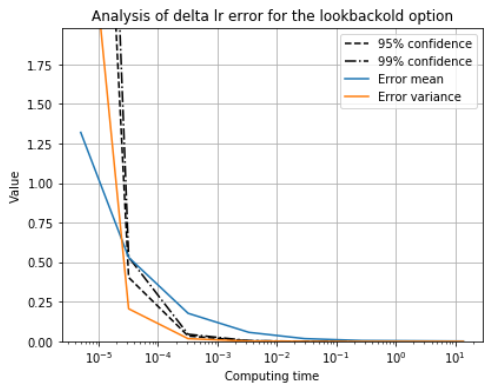


(f) Vega with PW Rayleigh method

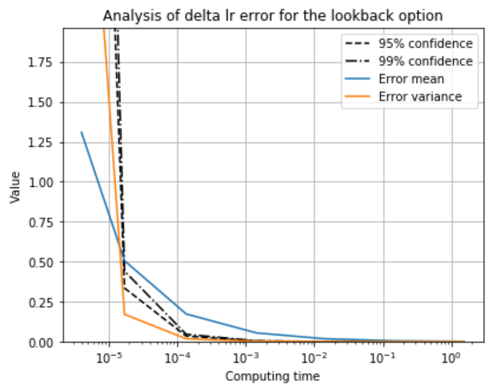


(g) Vega with LR Rayleigh method

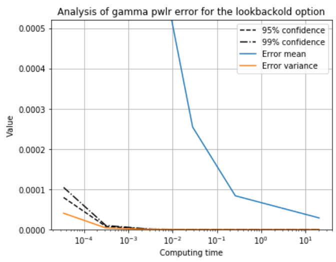
Figure 9: **LookBack Option**. A graphical analysis of the error of the Greeks depending on the number of simulations. In order to generate these graphs, we have done 1,000 simlations for each value.



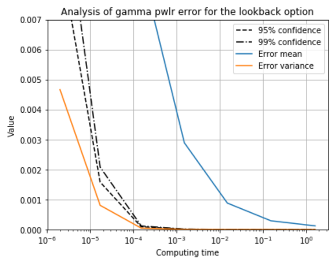
(a) Delta with LR method



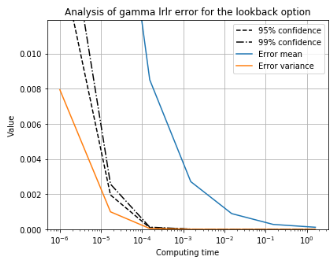
(b) Delta with LR Rayleigh



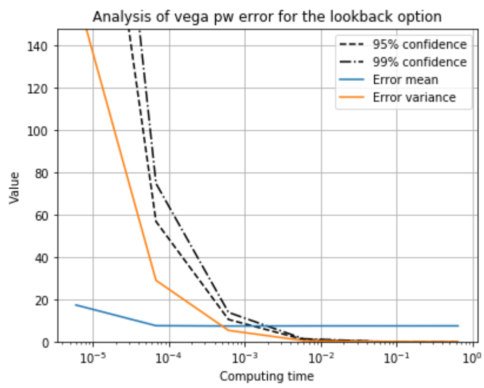
(c) Gamma with PW-LR method



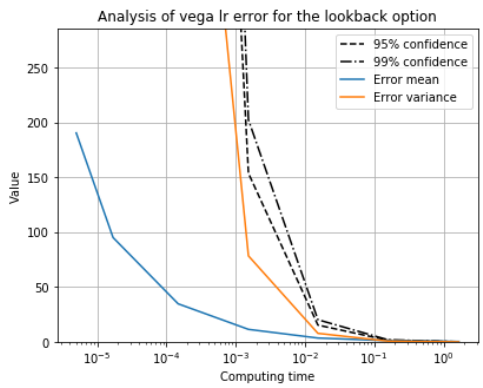
(d) Gamma with PW-LR method



(e) Gamma with LR-LR Rayleigh method



(f) Vega with PW Rayleigh method



(g) Vega with LR Rayleigh method

Figure 10: **LookBack Option-Time**. A graphical analysis of the error of the Greeks depending on the number of simulations. In order to generate these graphs, we have done 1,000 simlations for each value.

D App User Guide

App User Guide

This user guide is written for the users of the console interface of our program.

Table of Contents

Features

- Compute the European call option, the barrier option and the look-back price
- Compute the European call option, the barrier option and the look-back option greeks:
  - delta
  - gamma
  - vega
- Provide statistical informations on the random variables.

Installation

This app only works on Windows NT systems. We recommend using Windows 7 or any more recent Windows distribution.

Make sure you have the following files.

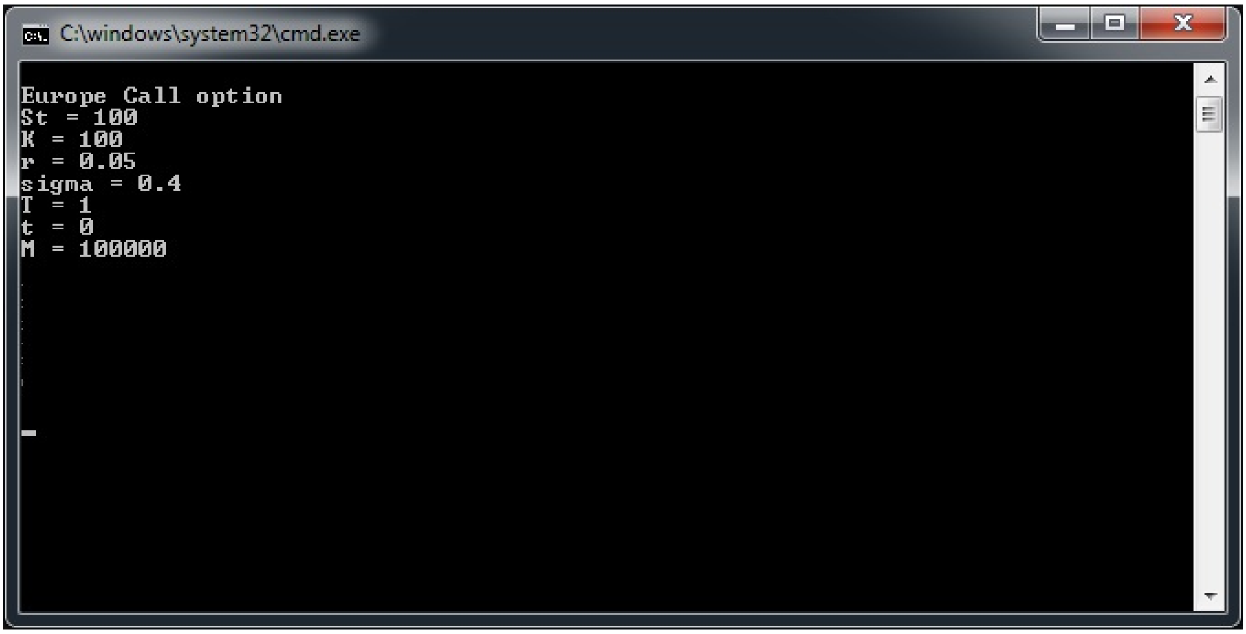
```
BarrierLookBackOptions.exe
```

Usage

Main Principles

To open the app, double-click on the `BarrierLookBackOptions.exe` file. A black window should appear.

You can type your instructions instructions directly in the window.



Working with the European Call option

The European call option is the default product. However, if you have to switch back to this product, enter the following command.

```
Europe_Call_option
```

The parameters are displayed on start-up. Here are the equivalences in plain language.

Abbreviation	Plain language	Description
`St`	Initial stock value	The value of the stock at the initial time of the simulation.
`K`	Strike	The value above which the call will allow one to make profits.
`r`	Interest rate	The rate the bank will pay one for leaving money in the bank account.
`sigma`	Volatility	Volatility is one of the main measures for the simulation of the evolution of the stock price in the Black-Scholes model. It is usually givent in percentage. In this porgram, type `0.04` for 4%.
`T`	Final simulation time / Maturity	The time at which the simulation stop. It is also the time at which the owner of the option will choose either to take or reject the contract.
`t`	Initial simulation time	The time at which `St` is recorded.
`M`	Number of Monte-Carlo simulations	The number of iterations in the Monte-Carlo method.

Changing parameters

To change the parameters, just type the abbreviation equated to the new value. If you want to change multiple parameters, you can simply type a coma between them.

Here is an example.

```
St=120,K=80
```

This command will simulatneously change `St` the initial stock value to `120` units and `K` the value of the strike to `80` units. The values must be separated by `,`. You should not time fractions. As an example type `0.3333` instead of 1/3.

You can change as many parameters as you want. You can display at any time the current parameters using the following command.

```
show
```

Price

In order to get the price, please type the following command.

```
price
```

Additional details such as the methods used, the error and the computation time are provided.

Greeks

You can ask for the following Greeks : delta, gamma, vega.

```
delta
```

```
gamma
```

```
vega
```

Additional details such as the methods used, the error and the computation time are provided.

Working with the Barrier option

To use the barrier option, enter the following command.

```
Barrier_option
```

The parameters are displayed on start-up. Here are the equivalences in plain language.

Abbreviation	Plain language	Description
`St`	Initial stock value	The value of the stock at the initial time of the simulation.
`K`	Strike	The value above which the call will allow one to make profits.
`r`	Interest rate	The rate the bank will pay one for leaving money in the bank account.
`sigma`	Volatility	Volatility is one of the main measures for the simulation of the evolution of the stock price in the Black-Scholes model. It is usually givent in percentage. In this porgram, type `0.04` for 4%.
`T`	Final simulation time / Maturity	The time at which the simulation stop. It is also the time at which the owner of the option will choose either to take or reject the contract.
`t`	Initial simulation time	The time at which `St` is recorded.
`B`	Barrier	The value of the barrier. It will automatically be recognised as a down-and-out call option or an upper-and-out call option.
`M`	Number of Monte-Carlo simulations	The number of iterations in the Monte-Carlo method.

Changing parameters

To change the parameters, just type the abbreviation equated to the new value. If you want to change multiple parameters, you can simply type a coma between them.

Here is an example.

```
St=120,K=80
```

This command will simulatneously change `St` the initial stock value to `120` units and `K` the value of the strike to `80` units.

You can change as many parameters as you want. You can display at any time the current parameters using the following command.

```
show
```

Price

In order to get the price, please type the following command.

```
price
```

Additional details such as the methods used, the error and the computation time are provided.

Greeks

You can ask for the following Greeks : delta, gamma, vega.

```
delta
```

```
gamma
```

```
vega
```

Additional details such as the methods used, the error and the computation time are provided.

Working with the Look-back option

To use the look-back option, enter the following command.

```
Lookback_option
```

The parameters of the Look-back option displayed when switching to this product. Here are the equivalences in plain language.

Abbreviation	Plain language	Description
`St`	Initial stock value	The value of the stock at the initial time of the simulation.
`K`	Strike	The value above which the call will allow one to make profits.
`r`	Interest rate	The rate the bank will pay one for leaving money in the bank account.
`sigma`	Volatility	Volatility is one of the main measures for the simulation of the evolution of the stock price in the Black-Scholes model. It is usually givent in percentage. In this porgram, type `0.04` for 4%.
`T`	Final simulation time / Maturity	The time at which the simulation stop. It is also the time at which the owner of the option will choose either to take or reject the contract.
`t`	Initial simulation time	The time at which `St` is recorded.
`M`	Number of Monte-Carlo simulations	The number of iterations in the Monte-Carlo method.

Changing parameters

To change the parameters, just type the abbreviation equated to the new value. If you want to change multiple parameters, you can simply type a coma between them.

Here is an example.

```
St=120,K=80
```

This command will simulatneously change `St` the initial stock value to `120` units and `K` the value of the strike to `80` units.

You can change as many parameters as you want. You can display at any time the current parameters using the following command.

```
show
```

Price

In order to get the price, please type the following command.

```
price
```

Additional details such as the methods used, the error and the computation time are provided.

Greeks

You can ask for the look-back option delta, gamma, vega.

```
delta
```

```
gamma
```

```
vega
```

Additional details such as the methods used, the error and the computation time are provided.

Switching Random Generation

The generation method of the random variables is a key challenge, and the performance can vary depending on the computer used. The use has the oppotunity to choose between two (uniform) random number generation methods.

To select the linear congruential method, type the following command. This one is used by default.

```
linear_congruential
```

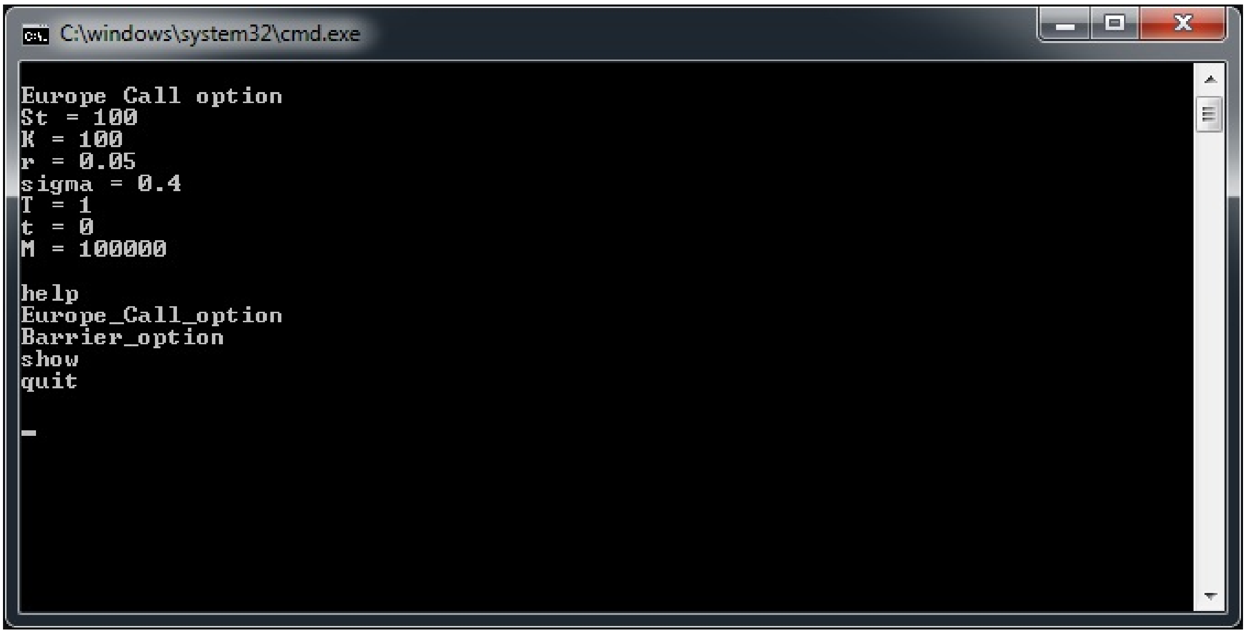
Some computers are more efficient using one of the system built-in functions, such as `mt19937`. To switch to this method, type the following command.

```
mt19937
```

Troubleshooting

At any time, you can type the following command in the terminal. A list of the instructions available will appear.

```
help
```



You can display at any time the current parameters using the following command.

```
show
```

To switch between different products, please type directly the name of the product. As an example, to switch to the barrier option, simply type the following instruction.

```
Barrier_ption
```

Closing the application

You can either type the following command,

```
quit
```

or simply close the window.