Simulation Methods for Finance

Barrier and Look-back Options

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March 11, 2018

Outline

Random Variable Generation - Methods

Random

Generate Uniform Distribution	**Convert U to SDN**
System built-in Rand	Central Limit Theory
Linear Congruential Generator	Box-Muller
	Marsagilia Polar

Random Variable Generation - Analysis

- Rand returns a random number in $(1, 2^{15} - 1)$, may show correlation between numbers.
- CLT requres a sufficient large number of uniform distributions, the computational time is long.
- Box-Muller involves calculation of Sine Cosine and log. Computational time can be long.

Random Variable Generation - Results

Generating 100 million random numbers

Method	Mean	Variance	Time
rand + CLT	1.77 e-5	0.999909	90.34
rand + Box-Muller	-6.80 e-5	1.000703	8.64
rand + Marsagilia Polar	1.70 e-5	1.000472	6.40
LCG + CLT	1.52 e-5	0.999973	236.3
LCG + Box-Muller	-1.27 e-4	0.999797	11.59
LCG + Marsagilia	7.36 e-5	0.999979	10.52
Random	3.58 e-4	1.000210	1.498

European Call Option - Task

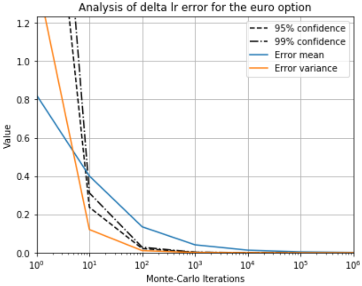
With the usual conventions, recall our model.

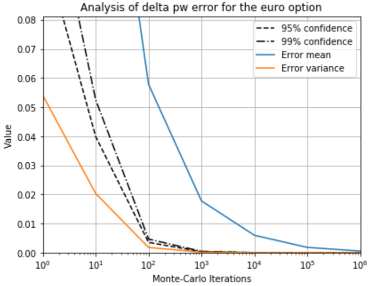
$$dS_t = rS_t dt + \sigma S_t dW_t, \ 0 \leq t \leq T$$

$$C_t = E[e^{-r(T-t)}(S_T - K)^+ | F_t]$$

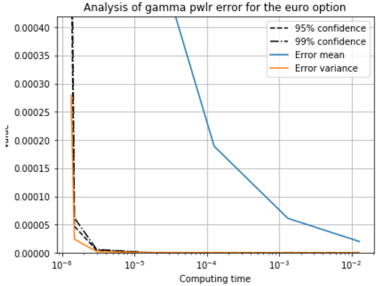
- Create a module to simulate the results for the European option.
- Compute the Greeks (delta, gamma, vega) using different methods.
- Analyse the error compared to the theoretical result.

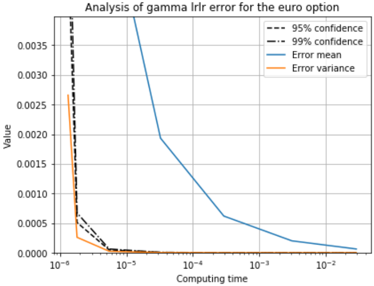
European Call Option - Analysis





European Call Option - Analysis





European Call Option - Results

- There is a significant accuracy gap beyond 1,000 simulations.
- We based our conclusions on the industry standard: 100,000 simulations.
- There is a trade-off computation time/accuracy.

	Error Mean	**Error Variance**	**Time**
Delta LR	Worst	Worst	Best
Delta PW	Best	Best	Worst
Gamma PWLR	Worst	Best	Best
Gamma LRPW	-	Worst	Best
Gamma LRLR	-	-	Worst
Vega LR	Worst	-	Worst
Vega PW	Best	-	Best

Barrier Option - Task

$$m_0^T = \min_{0 \leq t \leq T} S_t, \quad M_0^T = \max_{0 \leq t \leq T} S_t$$

For an up-and-out barrier call option, $B > S_0$
$A_T = (S_T - K)^+ 1_{M_0^T < B}$, where B is a barrier level and 1_S is an indicator function.

Barrier Option - Two Methods

1. Generate whole paths of S_t with 1,000 stops each path.
Time used for ***simulation : ***seconds/minutes/hours

Barrier Option - Two Methods

2. Use Rayleigh distribution generate the M_T directly.
The maximum of a standard Brownian motion starting at the origin to be at b at time 1 over period [0, T] has the Rayleigh distribtuion

$$F(x) = 1 - e^{-2x(x-b)},\ x \geq b.$$

Hense, at time T with S_T,

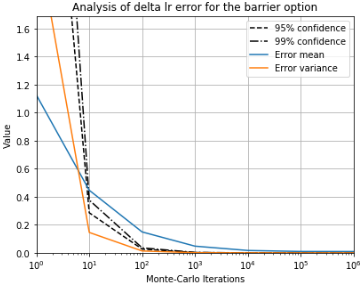
$$M^T = \frac{S_T + \sqrt{S_T^2 - 2TlogU}}{2}$$

Method 2 is 500 times faster!

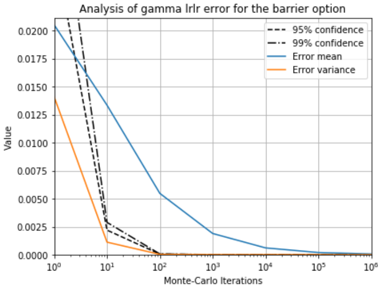
Barrier Option - Analysis

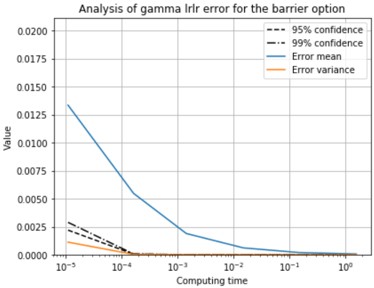
Simulate Greeks using Likelihood ratio method as Pathwise method is not feasible for barrier option.

Barrier Option - Analysis with a New Method



Barrier Option - Analysis with a New Method





Barrier Option - Results

	Error Mean	Error Variance	Time (s)
Price	na	na	..
Delta LR	1.04 e-2	3.03 e-5	9.28 e-2
Gamma LRLR	1.89 e-4	1.89 e-8	1.48 e-1
Vega LR	7.51 e-1	3.22 e-1	1.09 e-1

Table: Sample from our simulation dataset with the new fast method for barrier option simulation. It is clear that the results have significantly improved compared with the previous method. note that the run time depends on the computer used (here a 3.4 GHz Intel Core i7).

Look-back Option - Task

Lookback call option with fixed strik price K has payoff $(M_0^T - K)^+$. The call option price at time t is

$$c(S_0, K, t) = e^{-r(T-t)} E[(max(M_0^t, M_t^T) - K)^+ | \mathcal{F}_t]$$

Look-back Option - Method

The cdf of the distribution of Maximum S_t for standard Brownian motion for the period of (0, T) is

$$F = \Phi(\frac{m - aT}{\sqrt{T}}) - e^{2am}\Phi(\frac{-m - aT}{\sqrt{T}})$$

We used Newton-Raphson method to solve for the above cdf F = U. Hense, for each random number we generate from [0,1], we receive one m in by solving for F. And to get M^T of stock price follows geometric Browninan motion with starting price S_0, we let $M^T = S_0 e^{\sigma m}$.

Look-back Option - Analysis

Look-back Option - Results

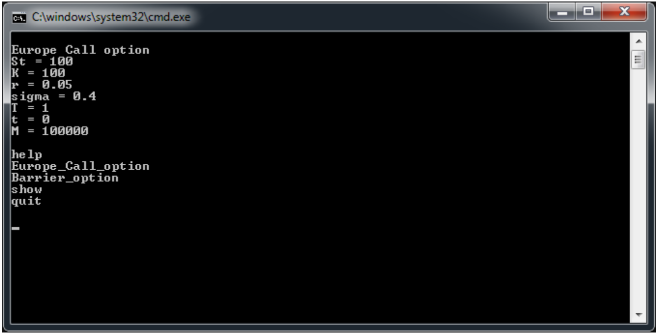
Using our Code - C++ Package

- We have created a package for C++ users.
- Uses industry standards with dynamic library files.
- Package is very intuitive.

```
#include "BarrierLookBackOptions.h"


european_option call(S_t, K, r, s, T, t, iterations);

double price = call.price();
double gamma = call.gamma("lrlr");
double vega = call.vega("lr");

```

Using our Code - "Code Free" terminal

- We have created an intuitive interface.
- This is suitable for users who do not code: you just have to type the command and there is a help mode.
- It supports european call option, barrier out call option and the look-back call option price and greeks.



Conclusion

- We have created a module with the core task.
- The module also can also compute elements related to the barrier option and the look-back option.
- Our package is user-friendly and has a ”code-free” interface.

Thank you!

github.com/tjespel/barrier-and-look-back-options