Institut National des Sciences
Appliquées et de Technologies
Département de Mathématiques
et Informatiques

Exercises sheet: Two continuous random variables

Exercise 1: Let $f(x,y) = cxy$ for $0 < x < y < 1$ be a joint probability distribution function.

1. Determine the value of c.
2. Compute $\mathbb{P}\{X < \frac{1}{2}, Y < 1\}$.
3. Determine the marginal probability distributions of X and Y.
4. Compute $\mathbb{E}(X)$ and $\mathbb{E}(Y)$.
5. Determine the regression function $\varphi(x) = \mathbb{E}(Y|X = x)$ for all $x \in \mathbb{R}_+$ and deduce the conditional expectation random variable $\mathbb{E}(Y|X)$. Comment.
6. What about the strongness and/or significance of the relationship between X and Y?

Exercise 2: Let $f(x,y) = ce^{-2x-3y}$ for $0 < y < x$ be a joint probability distribution function.

1. Determine the value of c.
2. Compute $\mathbb{P}\{X < 1, Y < 2\}$ and $\mathbb{P}\{0 < y < \frac{x}{2}\}$.
3. Determine the marginal probability distributions of X and Y.
4. Compute $\mathbb{E}(X)$ and $\mathbb{E}(Y)$.
5. Determine the regression function $\varphi(x) = \mathbb{E}(Y|X = x)$ for all $x \in \mathbb{R}_+$ and deduce the conditional expectation random variable $\mathbb{E}(Y|X)$. Comment.
6. What about the strongness and/or significance of the relationship between X and Y ?

Exercise 3: The conditional probability density of Y given $X = x$ is $f_{Y|X=x}(y) = xe^{-xy}$ for $0 < y$ and the marginal probability distribution of X is the uniform distribution $\mathcal{U}([0,10])$.

1. Compute the probability $\mathbb{P}\{Y < 2|X = 2\}$.
2. Determine the function $\varphi(x) = \mathbb{E}(Y|X = x)$ for all $x \in [0,10]$ and deduce the conditional expectation random variable $\mathbb{E}(Y|X)$. Comment.
3. Determine the marginal distribution of Y, f_Y.
4. What about the strongness of the relationship between X and Y ?

Exercise 4: Consider the unit disc

$$D = \{(x, y)|x^2 + y^2 \leq 1\}.$$

Suppose that we choose a point (X, Y) uniformly at random in D. That is, the joint PDF of X and Y is given by

$$f_{XY}(x, y) = \begin{cases} c & (x, y) \in D \\ 0 & \text{otherwise} \end{cases}$$

i. Find the constant c.

ii. Find the marginal PDFs f_X and f_Y.

iii. Find the conditional PDF of $(X|Y = y)$, for $-1 \leq y \leq 1$.

iv. Are X and Y independent? If not determine the regression function of X given Y and gauge it.