Random Walk with absorbing barriers:

Consider a sequence of random variables $X_0, X_1, \ldots, X_n, \ldots$ with values in the set $\mathcal{X} = \{0, 1, \ldots, k\}$ defined recursively as follows.

- $X_0 := x_0$ (fixed, deterministic $x_0 \in \mathcal{X}$).
- If $X_n = x$ and $0 < x < k$ then

$$X_{n+1} = \begin{cases} x + 1 \text{ with probability } p; \\ x - 1 \text{ with probability } 1 - p. \end{cases}$$

- If $X_n = x$ and $x \in \{0, k\}$ then $X_{n+1} = x$ with probability 1.

Write a code for simulating $X_0, X_1, \ldots, X_n, \ldots$. Choose, for example, $k = 20$, $p = 0.4$.

Compute empirically

$\psi_0 = \mathbb{P}(X_n = 0 \text{ for some } n | X_0 = x_0)$ and $\psi_k = \mathbb{P}(X_n = k \text{ for some } n | X_0 = x_0)$

as functions of x_0.

Random Walk with elastic barriers:

Consider a sequence of random variables $X_0, X_1, \ldots, X_n, \ldots$ with values in the set $\mathcal{X} = \{0, 1, \ldots, k\}$ defined recursively as follows.

- $X_0 := x_0$ (fixed, deterministic $x_0 \in \mathcal{X}$).
- If $X_n = x$ then

$$X_{n+1} = \begin{cases} \min(x + 1, k) \text{ with probability } p; \\ \max(x - 1, 0) \text{ with probability } 1 - p. \end{cases}$$

Choose, for example, $k = 20$, $p = 0.4$. Compute empirically

$\pi(x) = \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}(X_i = x)$

for $x \in \mathcal{X}$.

Remark. Both above questions have simple analytic answers. You may try to find them out theoretically.