Implementation of full and simplified likelihoods in CheckMATE

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December 15, 2023

Abstract

Multibin searches in Checkmate

1 Introduction

2 Technical implementation

In this section we introduce the methods of implementation the simplified and full likelihoods in CheckMATEand user switches for controlling their execution.

2.1 ATLAS

The functionality of combining signal regions for recasting in ATLAS searches can be implemented either using the full likelihood model [1] or following a simplified approach detailed in Ref. [2]. Table 1 lists the ATLAS analyses with likelihood functionality implemented in CheckMATE. The simplified likelihood method requires background rates and uncertainties that were already available in the implemented searches. The full likelihood requires an appropriate file in the JSON format and these file were released by ATLAS for 6 searches already implemented in CheckMATE. For the searches atlas_2004_14060 and atlas_2006_05880 the full model files are not available but using the published data one can still perform simplified model fitting in multibin signal regions.

The full likelihoods statistical models are encoded in the JSON files by the ATLAS collaboration. The information provided includes the number of background events for all signal and control regions and for each major background category separately. This results in a large number of nuisance parameters and the complexity of the procedure makes the hypothesis testing very CPU-expensive. Additionally, on the recasting side, in order to fully exploit the method one should also implement CRs, which was not a standard approach in CheckMATE. Currently, just one search atlas_2010_14293 has a full implementation of all CRs. In other searches it is assumed that the contribution of signal to CRs is negligible. This assumption is not obviously fulfilled in all imaginable new physics models.

On the technical side, after the usual evaluation of events within CheckMATE, a JSON patchset is created which encapsulates signal contributions to SRs (and CRs if applicable). The patchset is then combined with the background-only input from ATLAS. This is further evaluated using the package pyhf [3–5], which is a Python implementation of the HistFactory specification for binned statistical models [6, 7]. The signal strength μ is the parameter of interest. Depending on the user choices output can contain information about expected and observed upper limit on μ, with 2-σ bounds, along with observed and expected CL_s for $\mu = 1$. The default method of calculation is by using the asymptotic calculator, see [3].

By default the above calculation will be executed using the Spey program [8]. Spey is a cross-platform Python-based package that allows for a statistical inference of hypotheses using different likelihood prescriptions.[♮1] In our setup it gives a somewhat better control over the calculation than the above mentioned CheckMATE-pyhf interface, but nevertheless the calculation is still performed in pyhf framework. The main motivation, however, for using Spey was a possibility of combining different searches (also across experiments), which is planned in the next release of CheckMATE. In any case. a direct evaluation using pyhf and by-passing Spey also remains available.

Since the evaluation of full likelihoods is normally time consuming it is not practical for large scans of the parameter space. Therefore the alternative approach to likelihood evaluation relies on the concept of simplified likelihood [2]. In this case the background model is approximated with the total SM background rate obtained in the background-only fit in the full model. A single nuisance

[♮1] Installation of Spey is straightforward: pip install spey. Please refer to the Spey online documentation for more details [9].

Name	Description	#SR, N_{bin}	Full	Ref.
atlas_1908_03122	Search for bottom squarks in final states with Higgs bosons, b-jets and E_T^{miss}	2, 7	✓	[10]
atlas_1908_08215	Search for electroweak production of charginos and sleptons in final states with 2 leptons and E_T^{miss}	1, 52	✓	[11]
atlas_1911_06660	Search for direct stau production in events with two hadronic taus	1, 2	✓	[12]
atlas_1911_12606	Search for electroweak production of supersymmetric particles with compressed mass spectra	11, 78	✓	[13]
atlas_2004_14060	Search for stops in hadronic final states with E_T^{miss}	3, 14	✗	[14]
atlas_2006_05880	Search for top squarks in events with a Higgs or Z boson	3, 28	✗	[15]
atlas_2010_14293	Search for squarks and gluinos in final states with jets and E_T^{miss}	3, 60	✓	[16]
atlas_2101_01629	Search for squarks and gluinos in final states with one isolated lepton, jets, and E_T^{miss}	8, 32	✓	[17]
atlas_2106_01676	Search for chargino–neutralino production in final states with 3 leptons and E_T^{miss}	2, 72	✓	[18]
atlas_2111_08372	Search for associated production of a Z boson with an invisibly decaying Higgs boson or dark matter candidates	1, 22	✗	[19]
atlas_2202_07953	Search for invisible Higgs-boson decays in vector-boson fusion	1, 16	✗	[20]

Table 1: List of implemented ATLAS analyses which have likelihood-based signal regions (all searches at $\sqrt{s} = 13$ TeV and $\mathcal{L} = 139$ fb^{-1}).

parameter correlated over all bins and representing post-fit background uncertainty is constrained by unit normal distribution. The evaluation is also performed using the `pyhf` package.

2.2 CMS

The simplified likelihood framework was defined in Ref. [21]. This assumes correlation between background contributions that can be modelled using the multivariate Gaussian distribution:

$$\mathcal{L}_S(\mu, \boldsymbol{\theta}) = \prod_{i=1}^{N} \frac{(\mu \cdot s_i + b_i + \theta_i)^{n_i} e^{-(\mu \cdot s_i + b_i + \theta_i)}}{n_i!} \cdot \exp\left(-\frac{1}{2}\boldsymbol{\theta}^T \mathbf{V}^{-1} \boldsymbol{\theta}\right) \tag{1}$$

where the product runs over all bins and μ is the signal strength (and the Parameter of Interest), n_i the observed number of events, s_i an expected number of signal events, b_i an expected number of background events, θ_i a background nuisance parameter and $\mathbf{V}$ the covariance matrix. It is implemented using the covariance matrices provided by the CMS Collaboration which are included in the `CheckMATE` distribution in the JSON format. Evaluation of the above model is performed using the `Spey` package and the `default_pdf.correlated_background` PDF.

2.3 `CheckMATE` parameters

`CheckMATE` provides several switches and parameters to control details of statistical evaluation. These are summarized in Tab. 3. The switches are divided into two groups: one providing a control of what statistical tests are performed and the other to control different modes of calculation. By default no statistical evaluation is performed. For the sake of speed and stability one switch, `scan`, provides a quick and reliable way of obtaining `Allowed/Excluded` result but with limited

Name	Description	N_{bin}	Ref.
cms_1908_04722	Search for supersymmetry in final states with jets and E_T^{miss}	174	[25]
cms_1909_03460	Search for supersymmetry with M_{T2} variable in final states with jets and E_T^{miss}	282	[26]
cms_2107_13021	Search for new particles in events with energetic jets and large E_T^{miss}	66	[27]
cms_2205_09597	Search for production of charginos and neutralinos in final states containing hadronic decays of WW, WZ, or WH and E_T^{miss}	35	[27]

Table 2: List of implemented CMS analyses which have likelihood-based signal regions (all searches at $\sqrt{s} = 13$ TeV and $\mathcal{L} = 139$ fb^{-1}).

additional information. Generally the available statistics include CL_s tests and calculation of upper limits on signal strength, both of which can be obtained as observed and/or expected measures. By choosing a `select` switch users can control which statistics are calculated. If no explicit choice is made the *observed upper limit* will be calculated. Finally, the `detailed` switch can be used to request calculation of all available statistics, but it should be noted that its execution can be time consuming. The option `-so` can be used to request calculation of statistics for previous `CheckMATE` runs (it requires presence of the `evaluation/total_results.dat` file in the output directory.

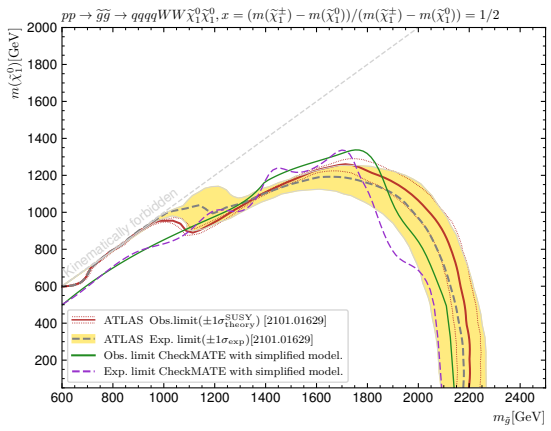
The second group of parameters is used to choose a method of calculation of requested statistics for the ATLAS searches (it does not affect the calculation for the CMS searches as described in the previous Section). For the default method, `simple`, calculation is performed using simplified likelihood and the `CheckMATE-pyhf` interface. The `full` switch chooses a calculation using the full likelihood and the `CheckMATE-Spey` interface. Finally, the `fullpyhf` switch requests calculation sing the full likelihood and `CheckMATE-pyhf` interface (this is somewhat less flexible regarding the output compared to the previous options). In any case, users should remember that the full likelihood calculation can be time consuming if many searches and signal regions are requested.

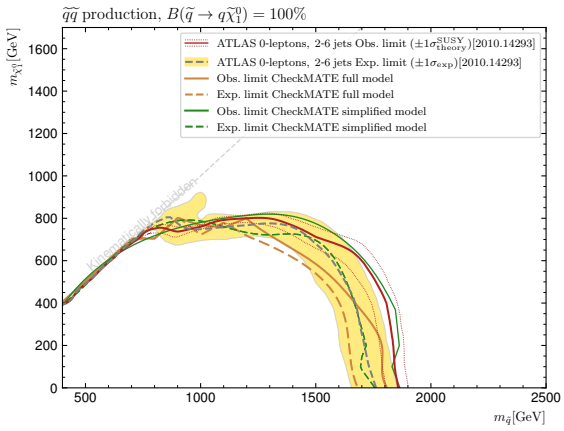
The results of the calculation for each of the multibin signal regions and all requested analyses are stored in the `multibin_limits/results.dat` file. In order to follow the progress of calculation the observed limits are also displayed on screen for each of the signal regions. To calculation sometimes results in bogus numbers and so as an additional precaution the CL_s results smaller than 10^{-6} are ignored. The final evaluation is decided using the upper limit on the signal strength μ (if available):

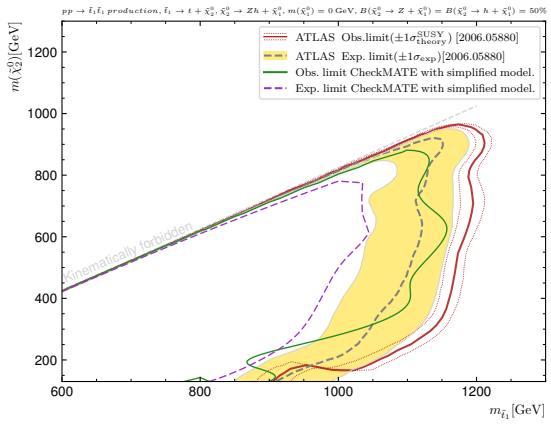
$$\mu < 0 \implies \texttt{Exclued}$$
$$\mu \geq 0 \implies \texttt{Allowed}.$$

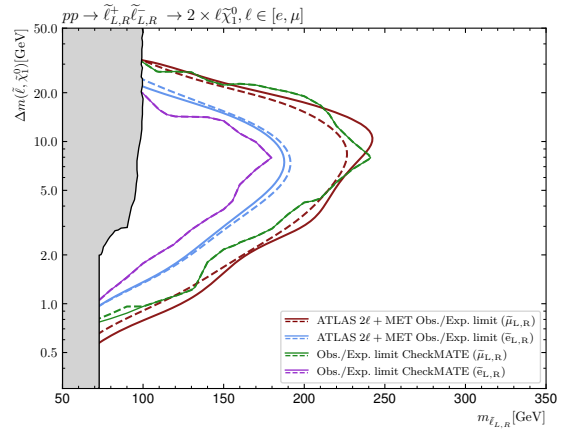
If the results for upper limit are not available the decision is made using the observed CL_s statistics at the 95% confidence level:

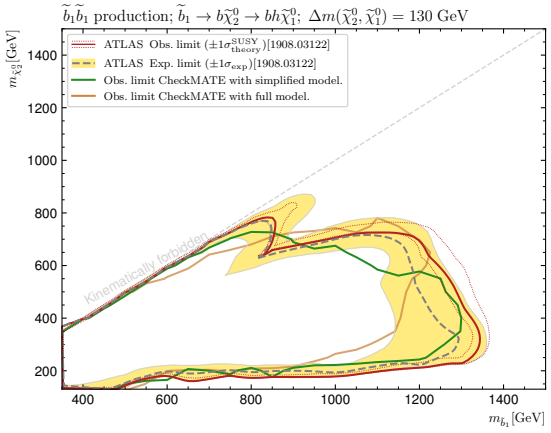
$$CL_s < 0.05 \implies \texttt{Exclued}$$
$$CL_s \geq 0.05 \implies \texttt{Allowed}.$$

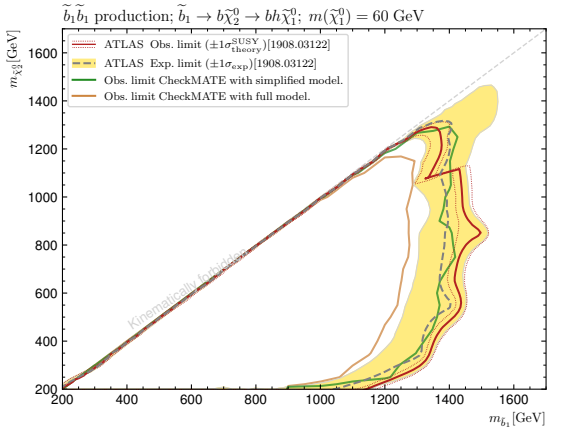

$pp \to \widetilde{g}\widetilde{g} \to qqqqWW\widetilde{\chi}_1^0\widetilde{\chi}_1^0, x = (m(\widetilde{\chi}_1^\pm) - m(\widetilde{\chi}_1^0))/(m(\widetilde{\chi}_1^\pm) - m(\widetilde{\chi}_1^0)) = 1/2$
$m(\widetilde{\chi}_1^0)$[GeV]
$m_{\widetilde{g}}$[GeV]
Kinematically forbidden
ATLAS Obs.limit($\pm 1\sigma^{\rm SUSY}_{\rm theory}$) [2101.01629]
ATLAS Exp. limit($\pm 1\sigma_{\rm exp}$)[2101.01629]
Obs. limit CheckMATE with simplified model.
Exp. limit CheckMATE with simplified model.

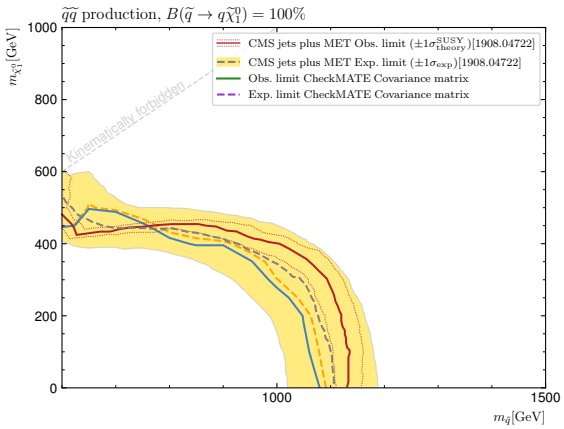

$\widetilde{q}\widetilde{q}$ production, $B(\widetilde{q} \to q\widetilde{\chi}_1^0) = 100\%$
$m_{\widetilde{\chi}_1^0}$[GeV]
$m_{\widetilde{q}}$[GeV]
Kinematically forbidden
ATLAS 0-leptons, 2-6 jets Obs. limit ($\pm 1\sigma^{\rm SUSY}_{\rm theory}$)[2010.14293]
ATLAS 0-leptons, 2-6 jets Exp. limit ($\pm 1\sigma_{\rm exp}$)[2010.14293]
Obs. limit CheckMATE full model
Exp. limit CheckMATE full model
Obs. limit CheckMATE simplified model
Exp. limit CheckMATE simplified model

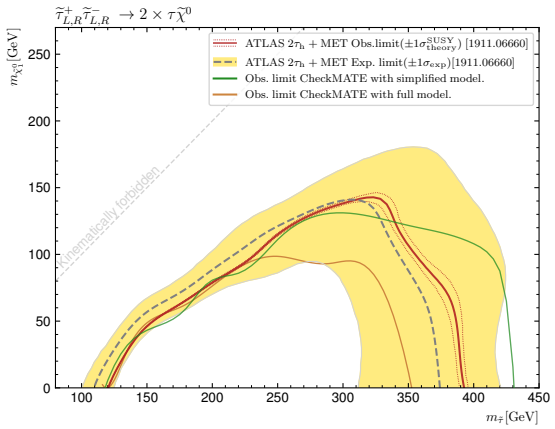

$pp \to \tilde{t}_1\tilde{t}_1$ production, $\tilde{t}_1 \to t + \tilde{\chi}_2^0, \tilde{\chi}_2^0 \to Zh + \tilde{\chi}_1^0, m(\tilde{\chi}_1^0) = 0$ GeV, $B(\tilde{\chi}_2^0 \to Z + \tilde{\chi}_1^0) = B(\tilde{\chi}_2^0 \to h + \tilde{\chi}_1^0) = 50\%$
$m(\tilde{\chi}_2^0)$[GeV]
$m_{\tilde{t}_1}$[GeV]
Kinematically forbidden
ATLAS Obs.limit($\pm 1\sigma^{\rm SUSY}_{\rm theory}$) [2006.05880]
ATLAS Exp. limit($\pm 1\sigma_{\rm exp}$)[2006.05880]
Obs. limit CheckMATE with simplified model.
Exp. limit CheckMATE with simplified model.

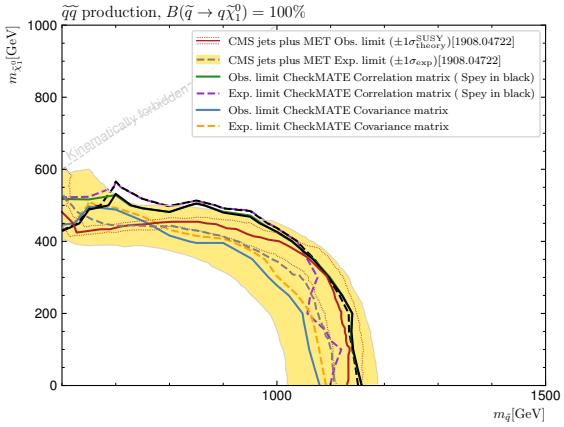

$pp \to \widetilde{\ell}^+_{L,R}\widetilde{\ell}^-_{L,R} \to 2 \times \ell\widetilde{\chi}_1^0, \ell \in [e, \mu]$
$\Delta m(\widetilde{\ell}, \widetilde{\chi}_1^0)$[GeV]
$m_{\widetilde{\ell}_{L,R}}$[GeV]
ATLAS 2ℓ + MET Obs./Exp. limit ($\widetilde{\mu}_{\rm L,R}$)
ATLAS 2ℓ + MET Obs./Exp. limit ($\widetilde{e}_{\rm L,R}$)
Obs./Exp. limit CheckMATE ($\widetilde{\mu}_{\rm L,R}$)
Obs./Exp. limit CheckMATE ($\widetilde{e}_{\rm L,R}$)

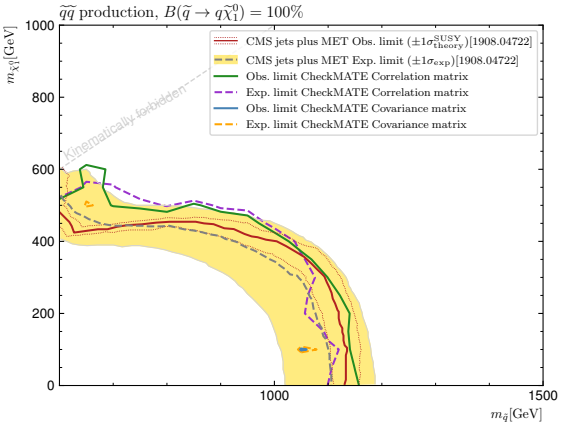

$\widetilde{b}_1\widetilde{b}_1$ production; $\widetilde{b}_1 \to b\widetilde{\chi}_2^0 \to bh\widetilde{\chi}_1^0$; $\Delta m(\widetilde{\chi}_2^0, \widetilde{\chi}_1^0) = 130$ GeV
$m_{\widetilde{\chi}_2^0}$[GeV]
$m_{\widetilde{b}_1}$[GeV]
Kinematically forbidden
ATLAS Obs. limit ($\pm 1\sigma^{\rm SUSY}_{\rm theory}$)[1908.03122]
ATLAS Exp. limit ($\pm 1\sigma_{\rm exp}$)[1908.03122]
Obs. limit CheckMATE with simplified model.
Obs. limit CheckMATE with full model.

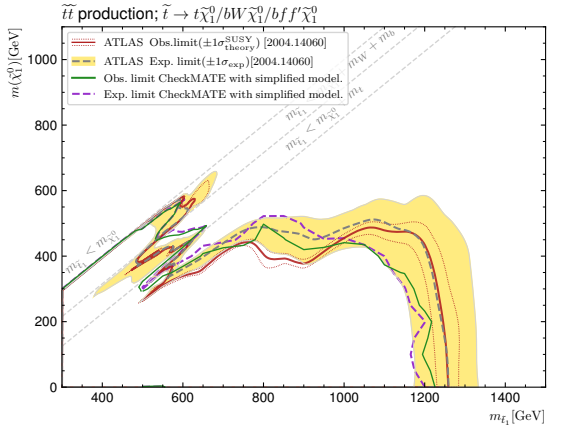

$\widetilde{b}_1\widetilde{b}_1$ production; $\widetilde{b}_1 \to b\widetilde{\chi}_2^0 \to bh\widetilde{\chi}_1^0$; $m(\widetilde{\chi}_1^0) = 60$ GeV
$m_{\widetilde{\chi}_2^0}$[GeV]
$m_{\widetilde{b}_1}$[GeV]
Kinematically forbidden
ATLAS Obs. limit ($\pm 1\sigma^{\rm SUSY}_{\rm theory}$)[1908.03122]
ATLAS Exp. limit ($\pm 1\sigma_{\rm exp}$)[1908.03122]
Obs. limit CheckMATE with simplified model.
Obs. limit CheckMATE with full model.

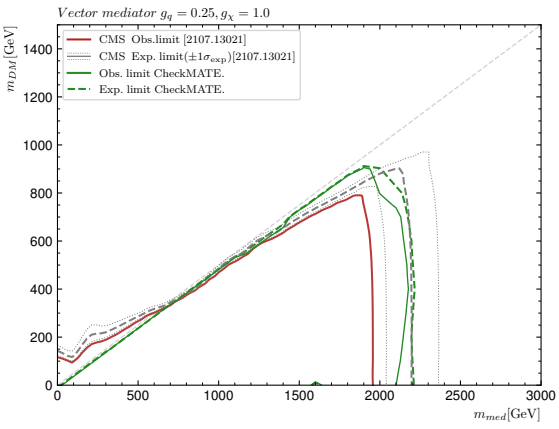

$\widetilde{q}\widetilde{q}$ production, $B(\widetilde{q} \to q\widetilde{\chi}_1^0) = 100\%$
CMS jets plus MET Obs. limit ($\pm 1\sigma^{\rm SUSY}_{\rm theory}$)[1908.04722]
CMS jets plus MET Exp. limit ($\pm 1\sigma_{\rm exp}$)[1908.04722]
Obs. limit CheckMATE Covariance matrix
Exp. limit CheckMATE Covariance matrix
Kinematically forbidden
$m_{\widetilde{\chi}_1^0}$[GeV]
$m_{\widetilde{q}}$[GeV]


$\widetilde{\tau}^+_{L,R}\widetilde{\tau}^-_{L,R} \to 2 \times \tau\widetilde{\chi}^0$
ATLAS $2\tau_{\rm h}$ + MET Obs.limit($\pm 1\sigma^{\rm SUSY}_{\rm theory}$) [1911.06660]
ATLAS $2\tau_{\rm h}$ + MET Exp. limit($\pm 1\sigma_{\rm exp}$)[1911.06660]
Obs. limit CheckMATE with simplified model.
Obs. limit CheckMATE with full model.
Kinematically forbidden
$m_{\widetilde{\chi}_1^0}$[GeV]
$m_{\widetilde{\tau}}$[GeV]


$\widetilde{q}\widetilde{q}$ production, $B(\widetilde{q} \to q\widetilde{\chi}_1^0) = 100\%$
CMS jets plus MET Obs. limit ($\pm 1\sigma^{\rm SUSY}_{\rm theory}$)[1908.04722]
CMS jets plus MET Exp. limit ($\pm 1\sigma_{\rm exp}$)[1908.04722]
Obs. limit CheckMATE Correlation matrix (Spey in black)
Exp. limit CheckMATE Correlation matrix (Spey in black)
Obs. limit CheckMATE Covariance matrix
Exp. limit CheckMATE Covariance matrix
Kinematically forbidden
$m_{\widetilde{\chi}_1^0}$[GeV]
$m_{\widetilde{q}}$[GeV]


$\widetilde{q}\widetilde{q}$ production, $B(\widetilde{q} \to q\widetilde{\chi}_1^0) = 100\%$
CMS jets plus MET Obs. limit ($\pm 1\sigma^{\rm SUSY}_{\rm theory}$)[1908.04722]
CMS jets plus MET Exp. limit ($\pm 1\sigma_{\rm exp}$)[1908.04722]
Obs. limit CheckMATE Correlation matrix
Exp. limit CheckMATE Correlation matrix
Obs. limit CheckMATE Covariance matrix
Exp. limit CheckMATE Covariance matrix
Kinematically forbidden
$m_{\widetilde{\chi}_1^0}$[GeV]
$m_{\widetilde{q}}$[GeV]


$\widetilde{t}\widetilde{t}$ production; $\widetilde{t} \to t\widetilde{\chi}_1^0/bW\widetilde{\chi}_1^0/bff'\widetilde{\chi}_1^0$
ATLAS Obs.limit($\pm 1\sigma^{\rm SUSY}_{\rm theory}$) [2004.14060]
ATLAS Exp. limit($\pm 1\sigma_{\rm exp}$)[2004.14060]
Obs. limit CheckMATE with simplified model.
Exp. limit CheckMATE with simplified model.
$m(\widetilde{\chi}_1^0)$[GeV]
$m_{\widetilde{t}_1}$[GeV]


$Vector\ mediator\ g_q = 0.25, g_\chi = 1.0$
CMS Obs.limit [2107.13021]
CMS Exp. limit($\pm 1\sigma_{\rm exp}$)[2107.13021]
Obs. limit CheckMATE.
Exp. limit CheckMATE.
m_{DM}[GeV]
m_{med}[GeV]

Parameter card		Terminal	X	Description and available choices
`Multibin:`	`X`	`-mb X`	`none`	No signal region combination is performed (default).
			`select`	Calculates user selected statistics.
			`scan`	Calculates observed CL_s; fast and reliable for quick assessment of exclusion.
			`detailed`	Calculates observed and expected upper limits and CL_s.
`Expected:`	`False`	`-exp`		Selects calculation of expected limits.
`CLs:`	`False`	`-cls`		Selects calculation of CL_s.
`Uplim:`	`False`	`-uplim`		Selects calculation of upper limits.
`Statonly:`	`False`	`-so`		Calculates statistical combinations without event-level analysis provided the analysis and evaluation steps were already completed.
`Model:`	`X`	`-mod X`	`simple`	The simplified likelihood model for ATLAS searches (default).
			`full`	The Spey interface to the full likelihood model for ATLAS searches.
			`fullpyhf`	The full likelihood model for ATLAS searches with `pyhf` interface.

Table 3: Summary of options related to multibin signal regions.

3 Validation

4 Conclusions and outlook

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