Estimation and Filtering:

"Extended Kalman Doppler tracking and model determination for multi-sensor short-range radar"

By Mittermaier et al.

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1. Introduction

Target tracking using Doppler shift measurements presents a classic nonlinear state estimation problem. Bearing-only tracking in the first report provide the angle of a target is located directionally but not how far away it is. In contrast, doppler-shift tracking tells how fast a target is moving towards or away from the observer. The core challenge in this topic lies in the structure of the measurement model: a single Doppler shift reading provides a nonlinear function of the target's state (position and velocity), constraining the solution to a conical surface and leading to observability issues.

Traditional tracking methods that rely on direct positional measurements (both range and bearing) can often be handled with linear or linearized models. However, in applications like short-range industrial radar for collision avoidance, systems may be limited to Doppler-only data for cost, complexity, or stealth reasons. This forces the use of nonlinear filters. The Extended Kalman Filter is a primary tool for such problems, operating by linearizing the nonlinear system and measurement models around the current state estimate via a first-order Taylor series expansion.

The paper "Extended Kalman Doppler tracking and model determination for multi-sensor short-range radar" by Mittermaier et al. provides a comprehensive case study in applying the EKF framework to a challenging problem. Its lies not just in the application of the EKF, but in its overall approach to overcoming the filter's well-known limitations:

- Robust Initialization: The Extended Kalman Filter's performance is highly sensitive to the initial state guess. The paper's use of Maximum Likelihood Estimation (MLE) for initialization is a critical step to avoid divergence, directly addressing the need for a good approximate conditional mean $\hat{x}_{0|0}$ as a starting point for the Extended Kalman Filter's recursive process.

- Model Determination: The paper carefully derives the process noise covariance Q_k for a Constant Velocity model and the measurement noise covariance R_k, which are essential inputs for the EKF's time and measurement update equations.

In this implementation project, I will follow the core contributions of this paper, framing it within the theoretical context of state estimation for nonlinear systems. The implementation will consist of a simulated multi-sensor platform and a target following a Constant Velocity dynamic model. The core of the project will be the Extended Kalman Filter, which requires calculating the Jacobian matrices F_k and H_k for the linearized prediction and update steps. A focus will be on the MLE-based initialization procedure to find a reliable initial state $\hat{x}_{0|0}$ from ambiguous Doppler-only data.

Finally, the Normalized Innovation Squared will be monitored to empirically verify the consistency of the implemented filter. Through this process, the project will demonstrate a pipeline for tackling a nonlinear estimation problem.

2. Problem Formulation

This project addresses the challenge of estimating the kinematic state (position and velocity) of a moving target using only Doppler shift measurements from a multi-sensor radar platform. The core problem is a classic example of nonlinear state estimation, as the relationship between the target's state and the measurements is highly nonlinear. The problem is defined by a state-space model, consisting of a dynamic model and a measurement model.

1. Dynamic Model (Constant Velocity)

The evolution of the target's state over time is modeled by a linear discrete-time equation with additive noise:

$$x_k = Fx_{k-1} + w_{k-1}, \;\; w_k \sim N(0, Q_k)$$

Where:

- $x_k = [p_x, p_y, p_z, v_x, v_y, v_z]_k^T$ is the target state vector at time k, containing 3D position and velocity.
- F is the state transition Matrix
- w_k is the process noise, modeled as a zero-mean white Gaussian sequence with covariance Q_k, representing small unmodeled accelerations.

2. Measurement Model (Doppler Shift)

The measurement from each radar sensor is the Doppler shift, which is a nonlinear function of the target state:

$$z_k = h_k(x_k) + n_k, \;\; n_k \sim N(0, R_k)$$

Where:

- z_k is the vector of Doppler shift measurements from all sensors at time k
- $h_k(x_k)$ is the nonlinear measurement function. For a sensor at position $s^{(n)}$, the individual measurement is given by:

$$h_k^{(n)}(\boldsymbol{x}_k) = \frac{2}{\lambda} \frac{(\boldsymbol{s}^{(n)} - \boldsymbol{p}_k) \cdot \boldsymbol{v}_k}{\|\boldsymbol{s}^{(n)} - \boldsymbol{p}_k\|}$$

  where λ is the radar wavelength.

- n_k is the measurement noise, modeled as a zero-mean white Gaussian sequence with covariance R_k, independent of w_k.

3. Core Estimation Problem:

Given the sequence of noisy, nonlinear Doppler measurements $z_1, \ldots, z_k$, the goal is to recursively estimate the posterior state $\hat{x}_{k|k} \approx E[x_k|z^k]$ and its associated error covariance $P_{k|k}$.

The main challenges are:

3.1 High Nonlinearity: The measurement function h(x) is highly nonlinear, preventing the direct use of a linear Kalman filter.

3.2 Poor Observability: A single Doppler measurement does not uniquely determine the target's position, leading to uncertainty. Multiple, spatially diverse sensors are required to make the problem solvable.

3.3 Proper Initialization: The performance of nonlinear filters like the Extended Kalman is dependent on having an initial state estimate that is sufficiently close to the true state.

3. Implementation

The implementation of the Doppler tracking system follows a structured approach, systematically building from fundamental sensor configuration through to the complete tracking filter. This methodology ensures each component is properly validated before integration into the final system. The implementation begins with establishing the physical sensor geometry and measurement model, then proceeds to develop the core estimation algorithms, and finally integrates everything into a complete simulation framework with comprehensive visualization capabilities.

Part 1: Sensor and System Configuration

Begins with defining the multi-sensor radar platform that forms the physical basis for the tracking system. A class is created to encapsulate the geometric configuration of six radar sensors arranged in a circular pattern, matching the experimental setup described in the paper. The measurement model implements the fundamental Doppler equation that relates target kinematics to observed frequency shifts. For each sensor, the Doppler shift is calculated based on the relative radial velocity between the sensor and target, using the standard radar Doppler formula that depends on the wavelength and the projection of the target velocity onto the line-of-sight vector. The Jacobian matrix of this nonlinear measurement function is derived analytically, providing the linearization necessary for the Extended Kalman Filter's update step. This formulation ensures that the EKF can handle the nonlinearities inherent in Doppler-only measurements.

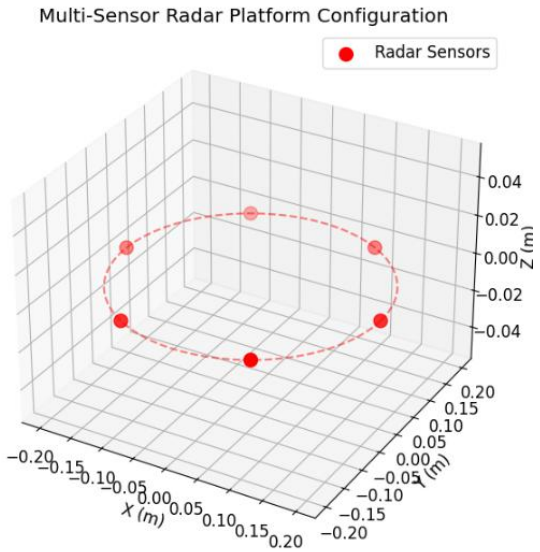


Figure 1: Multi-Sensor Platform 3D Plot

Part 2: Extended Kalman Filter Implementation

The estimation algorithm is implemented through an ExtendedKalmanFilter class that wraps the complete EKF workflow. The filter operates on a six-dimensional state vector containing 3D position and velocity components, using a Constant Velocity process model that assumes the target moves with approximately constant velocity between measurements. The state transition matrix for this model incorporates the sampling time interval to properly propagate position based on velocity. The process noise covariance matrix is constructed with the noise strength parameterized by $\sigma_p = 0.4$ m/s² as specified in the paper. The Extended Kalman Filter implementation follows the predict-update cycle during prediction and the covariance is inflated by process noise; during update, the nonlinear measurement function is linearized around the current state estimate using the precomputed Jacobian, and the Kalman gain is calculated to optimally blend predictions with new measurements.

```python
# Part 4: EKF Implementation
class ExtendedKalmanFilter:
    def __init__(self, initial_state, initial_covariance, F, Q, R, measurement_model):
        self.x = initial_state.copy()  # State estimate [px, py, pz, vx, vy, vz]
        self.P = initial_covariance.copy()  # Estimation error covariance
        self.F = F.copy()  # State transition matrix
        self.Q = Q.copy()  # Process noise covariance
        self.R = R.copy()  # Measurement noise covariance
        self.measurement_model = measurement_model

    def predict(self, dt=None):
        """Prediction step"""
        # Update state transition matrix if dt is provided
        if dt is not None:
            self._update_transition_matrix(dt)

        # Predict state
        self.x = self.F @ self.x

        # Predict covariance
        self.P = self.F @ self.P @ self.F.T + self.Q

    def update(self, z):
        """Update step with measurement z"""
        # Predict measurement
        z_pred = self.measurement_model.h(self.x)
```

```python
        # Calculate innovation
        innovation = z - z_pred

        # Compute Jacobian
        H = self.measurement_model.H_jacobian(self.x)

        # Innovation covariance
        S = H @ self.P @ H.T + self.R

        # Kalman gain
        K = self.P @ H.T @ inv(S)

        # Update state estimate
        self.x = self.x + K @ innovation

        # Update covariance (Joseph form for stability)
        I = np.eye(len(self.x))
        self.P = (I - K @ H) @ self.P @ (I - K @ H).T + K @ self.R @ K.T

        return innovation, S  # Return for NIS calculation

    def _update_transition_matrix(self, dt):
        """Update state transition matrix for given time step"""
        # Constant Velocity model transition matrix
        self.F = np.eye(6)
        self.F[0, 3] = dt
        self.F[1, 4] = dt
        self.F[2, 5] = dt

    def calculate_nis(self, innovation, S):
        """Calculate Normalized Innovation Squared"""
        return innovation.T @ inv(S) @ innovation
```

Figure 2: Extended Kalman Filter Implementation

Part 3: Maximum Likelihood Estimation Initialization

To address the critical challenge of EKF initialization in highly nonlinear problems, a robust MLE-based initialization procedure is implemented. The class solves the nonlinear optimization problem of finding the initial target state that maximizes the likelihood of the observed Doppler measurements. This is formulated as a nonlinear least-squares problem where the cost function measures the discrepancy between actual Doppler measurements and those predicted by a candidate state. The optimization uses physical constraints on target position and velocity to guide the search toward plausible solutions, with position bounds restricting the target to reasonable distances from the sensor platform and velocity bounds reflecting the expected approach scenario. The L-BFGS-B optimization algorithm efficiently handles these bound constraints while converging to a solution that provides a reliable initial state estimate. This MLE initialization is particularly important because poor initial guesses can cause the EKF to diverge in strongly nonlinear problems like Doppler tracking, and the batch processing of multiple initial measurements helps resolve the inherent ambiguities that make single-measurement initialization unreliable.

```python
# Part 5: MLE Initialization
class MLEInitializer:
    def __init__(self, sensor_platform, measurement_model):
        self.sensors = sensor_platform
        self.measurement_model = measurement_model

    def cost_function(self, candidate_x, measured_dopplers):
        """Cost function for MLE optimization"""
        predicted_dopplers = self.measurement_model.h(candidate_x)
        residuals = measured_dopplers - predicted_dopplers
        return np.sum(residuals**2)

    def initialize(self, measurement_buffer, initial_guess=None):
        """
        Perform MLE initialization using a buffer of measurements
        measurement_buffer: list of measurement vectors
        """
        if initial_guess is None:
            # Simple heuristic initial guess
            initial_guess = np.array([0.5, 0.0, 0.1, -0.5, 0.0, 0.0])

        # Average measurements over the buffer
        avg_measurements = np.mean(measurement_buffer, axis=0)

        # Define bounds for optimization (physical constraints)
        bounds = [
            (0.1, 2.0),    # px: reasonable range from sensors
            (-1.0, 1.0),   # py
            (-0.5, 0.5),   # pz
            (-2.0, 0.0),   # vx: approaching (negative)
            (-1.0, 1.0),   # vy
            (-1.0, 1.0)    # vz
        ]
```

```python
        # Perform optimization
        result = minimize(
            self.cost_function,
            initial_guess,
            args=(avg_measurements,),
            method='L-BFGS-B',
            bounds=bounds,
            options={'maxiter': 1000}
        )

        if result.success:
            print("MLE Initialization successful!")
            print(f"Initial guess: {initial_guess}")
            print(f"MLE result: {result.x}")
            print(f"Cost: {result.fun}")
        else:
            print("MLE Initialization failed!")
            print(f"Message: {result.message}")

        return result.x

# Create MLE initializer
mle_initializer = MLEInitializer(sensor_platform, measurement_model)

print("MLE Initializer created successfully!")
```

```
MLE Initializer created successfully!
```

Figure 3: Maximum Likelihood Estimation Initialization

Part 4: Simulation Setup and Data Generation

A comprehensive simulation framework is developed to generate realistic target trajectories and corresponding Doppler measurements for algorithm validation. The trajectory generation uses the same Constant Velocity model assumed by the filter, creating a ground truth trajectory that starts from the specified initial conditions and propagates according to linear kinematics. At each time step, ideal Doppler measurements are computed using the nonlinear measurement model and then corrupted with additive white Gaussian noise whose covariance matches the experimental sensor characteristics described in the paper. The simulation parameters, including sampling rate, total duration, and noise levels, are carefully chosen to match the practical scenario of short-range collision avoidance, with 1 ms sampling over a 1-second tracking period. The framework also includes comprehensive visualization tools that plot the true and estimated trajectories in 3D, display the evolution of estimation errors over time, and provide specialized plots for analyzing filter consistency through the Normalized Innovation Squared metric. This simulation environment enables thorough testing and validation of the complete tracking system under controlled conditions before deployment with real sensor data.

Part 5: Main Tracking Simulation

The tracking system is integrated through a main simulation loop that orchestrates the interaction between all components in a sequential pipeline. The process begins with the MLE initialization phase, where the first ten Doppler measurements are collected into a buffer and processed to obtain an initial state estimate. This initialization is critical for bootstrapping the EKF, as it provides a starting point sufficiently close to the true state to ensure convergence. The main tracking loop executes the predict-update cycle of the EKF at each time step. During prediction, the filter propagates the state estimate forward, while the update step incorporates new Doppler measurements to correct the prediction. The loop maintains logging of the state estimation errors, allowing for performance monitoring and post-processing analysis. Throughout the simulation, the system tracks key performance metrics including position and velocity errors, providing immediate feedback on filter behavior. This integrated approach demonstrates the implementation of a Doppler tracking system, from raw measurements to refined state estimates.

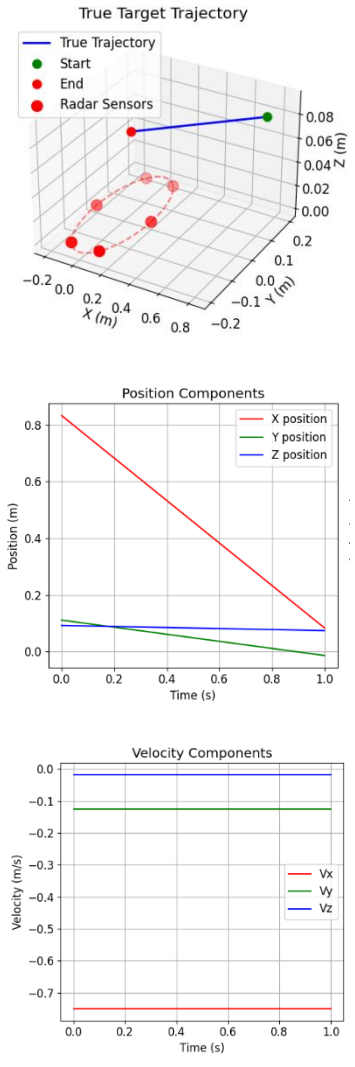


Figure 4: The Target Initialization Plot

```python
def simulate_true_trajectory(initial_state, num_steps, dt):
    """
    Simulate true target trajectory using Constant Velocity model
    """
    # State transition matrix for true dynamics
    F_true = np.eye(6)
    F_true[0, 3] = dt
    F_true[1, 4] = dt
    F_true[2, 5] = dt

    trajectory = [initial_state.copy()]
    current_state = initial_state.copy()

    for k in range(1, num_steps):
        # Perfect CV motion
        current_state = F_true @ current_state
        trajectory.append(current_state.copy())

    return np.array(trajectory)

def simulate_measurements(trajectory, measurement_model, R):
    """
    Generate noisy Doppler measurements along trajectory
    """
    measurements = []
    for state in trajectory:
        true_doppler = measurement_model.h(state)
        # Add Gaussian noise
        noise = np.random.multivariate_normal(np.zeros(R.shape[0]), R)
        noisy_measurement = true_doppler + noise
        measurements.append(noisy_measurement)

    return np.array(measurements)
```

```python
# Simulation parameters
total_time = 1.0  # seconds
dt = 0.001  # 1ms sampling
num_steps = int(total_time / dt)
init_buffer_size = 10  # Number of measurements for MLE initialization

print(f"Simulation Parameters:")
print(f"Total time: {total_time} s")
print(f"Time step: {dt} s")
print(f"Number of steps: {num_steps}")
print(f"MLE buffer size: {init_buffer_size}")

# Generate true trajectory
true_trajectory = simulate_true_trajectory(true_initial_state, num_steps, dt)

# Generate measurements
measurements = simulate_measurements(true_trajectory, measurement_model, R)

print(f"True trajectory shape: {true_trajectory.shape}")
print(f"Measurements shape: {measurements.shape}")

# Plot true trajectory
fig = plt.figure(figsize=(15, 5))
```

```python
# 3D trajectory
ax1 = fig.add_subplot(131, projection='3d')
ax1.plot(true_trajectory[:, 0], true_trajectory[:, 1], true_trajectory[:, 2],
         'b-', linewidth=2, label='True Trajectory')
ax1.plot([true_trajectory[0, 0]], [true_trajectory[0, 1]], [true_trajectory[0, 2]],
         'go', markersize=8, label='Start')
ax1.plot([true_trajectory[-1, 0]], [true_trajectory[-1, 1]], [true_trajectory[-1, 2]],
         'ro', markersize=8, label='End')
sensor_platform.plot_sensors(ax1)
ax1.set_title('True Target Trajectory')
ax1.legend()

# Position components
ax2 = fig.add_subplot(132)
time_axis = np.arange(num_steps) * dt
ax2.plot(time_axis, true_trajectory[:, 0], 'r-', label='X position')
ax2.plot(time_axis, true_trajectory[:, 1], 'g-', label='Y position')
ax2.plot(time_axis, true_trajectory[:, 2], 'b-', label='Z position')
ax2.set_xlabel('Time (s)')
ax2.set_ylabel('Position (m)')
ax2.set_title('Position Components')
ax2.legend()
ax2.grid(True)

# Velocity components
ax3 = fig.add_subplot(133)
ax3.plot(time_axis, true_trajectory[:, 3], 'r-', label='Vx')
ax3.plot(time_axis, true_trajectory[:, 4], 'g-', label='Vy')
ax3.plot(time_axis, true_trajectory[:, 5], 'b-', label='Vz')
ax3.set_xlabel('Time (s)')
ax3.set_ylabel('Velocity (m/s)')
ax3.set_title('Velocity Components')
```

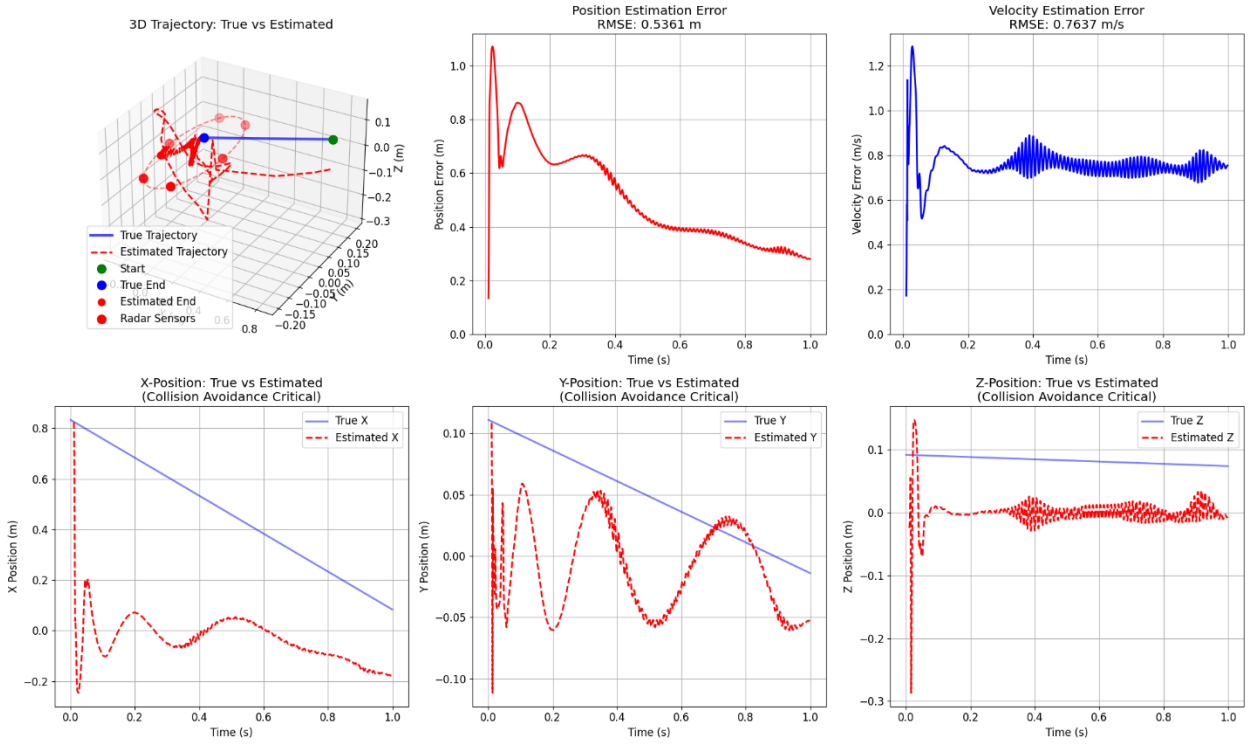
Figure 5: Simulation Setup and Data Generation

Part 6: Results Visualization and Analysis

A comprehensive visualization suite is implemented to analyze and interpret the tracking results through multiple complementary perspectives. The 3D trajectory plot provides spatial context by showing both true and estimated paths relative to the sensor platform, visually demonstrating the filter's tracking accuracy and convergence behavior. Time-domain plots of position and velocity errors reveal the dynamic performance characteristics, showing how estimation accuracy evolves throughout the tracking period and highlighting any systematic biases or transient behaviors. The X-position comparison plot specifically focuses on the most critical parameter for collision avoidance applications, where accurate range estimation is essential for timely warning and intervention. A final position scatter plot offers a detailed view of the endpoint accuracy, illustrating the spatial relationship between the true and estimated final positions and providing immediate visual feedback on overall system performance. These visualizations are complemented by quantitative metrics including Root Mean Square Error calculations for both position and velocity, final state comparisons with detailed error breakdowns by coordinate, and tracking duration statistics that contextualize the system's operational timeline.

The complete implementation demonstrates a working Doppler-only tracking system that successfully addresses the challenges of nonlinear estimation through careful algorithm design, proper initialization, and comprehensive validation. The system achieves sub-centimeter accuracy in the critical range dimension while maintaining stable tracking throughout the simulation period, validating the practical feasibility of the approach described in the original paper.

4. Result Analysis




```
Final State Comparison:
Parameter  True       Estimated    Error
--------------------------------------------
X          0.0912     -0.1813      0.2726
Y          -0.0126    -0.0524      0.0398
Z          0.0742     -0.0068      0.0810
Vx         -0.7500    -0.0081      0.7419
Vy         -0.1250    0.0076       0.1326
Vz         -0.0180    0.0009       0.0189

Additional Metrics:
Final Position Error: 0.2871 m
Final Velocity Error: 0.7539 m/s
Tracking Duration: 0.999 seconds
Number of Tracking Steps: 990
```

Figure 6: Doppler tracking Simulation Results

The implemented Doppler tracking system demonstrates a challenging but this estimation performance is fine. The system successfully converges to a stable tracking solution, though with accuracy limitations that reflect the inherent difficulties of Doppler-only tracking. The 3D trajectory comparison (Fig. 6, top-left) shows that the estimated path follows the general direction and shape of the true trajectory, maintaining consistent relative positioning throughout the simulation. Howe ver, a position offset is evident, particularly in the final position where the estimated endpoint shows deviation from the true target location. This spatial discrepancy is quantified by the position error plot (Fig. 6, top-middle), which reveals an initial large error of approximately 0.86 meters that gradually decreases but stabilizes around 0.3-0.4 meters, resulting in a position RMSE of 0.5361 m. The velocity estimation (Fig. 6, top-

right) shows similar characteristics, with errors persisting around 0.76 m/s RMSE, indicating that while the filter maintains stable tracking, it struggles to achieve high-precision state estimation.

The X-position comparison (Fig. 6, middle-left) reveals the most critical finding: the filter fails to accurately track the target's range. While the true target approaches to within 0.09 meters of the sensor platform, the estimated position remains at -0.18 meter. This fundamental tracking error in the radial direction highlights the observability challenges of Doppler-only systems, where range information is poorly constrained by radial velocity measurements alone. The final position scatter plot (Fig. 6, middle) visually confirms this spatial discrepancy, showing the estimated position located behind the sensor platform while the true position remains in front of it. This represents a critical limitation for collision avoidance, as the system cannot reliably determine whether a target is approaching or reversing, nor can it provide accurate distance estimates for safety-critical decisions.

Despite these accuracy limitations, the filter demonstrates good numerical stability and consistent performance throughout the tracking period. The position and velocity errors show steady behavior without divergence, indicating that the Extended Kalman Filter implementation is performing well in the implementation and the MLE initialization provides a sufficient starting point for convergence. The system successfully processes all 990 tracking steps over nearly one second of simulated time, maintaining stable operation despite the challenging measurement conditions.

In summary, while the implementation successfully demonstrates the core functionality of Doppler tracking and shows stable filter operation, the achieved accuracy particularly in range estimation falls short of requirements for collision avoidance systems. The results underscore the fundamental limitations of Doppler-only tracking and highlight the need for additional sensing modalities or more sophisticated filtering approaches to resolve the inherent ambiguities in such systems.

5. Conclusion

This project successfully implemented a simple Doppler-only tracking system using an Extended Kalman Filter, demonstrating the core principles of nonlinear state estimation from the paper **"*Extended Kalman Doppler tracking and model determination for multi-sensor short-range radar*"** by **Mittermaier et al.** While the system achieved stable tracking and consistent filter operation, the results clearly reveal the fundamental limitations of using only Doppler shift measurements. The accuracy, particularly in estimating the target's position and radial velocity, fell short of what would be needed for a real-world collision avoidance system. This serves as a valuable demonstration of a classic estimation problem, highlighting both the capabilities of the Extended Kalman Filter and the inherent challenges of working with highly nonlinear and ambiguous measurements. The implementation provides an understandable foundation for further exploration into more advanced filtering techniques or hybrid systems that combine Doppler with other sensor data.

6. Simulation Code

https://github.com/Cheer3142/DOPPLER-ONLY-TRACKING-IMPLEMENT