Answer

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(1) (a) Given; Static sensitivity = K = 5 volts / input / volts

and, time constant = $\tau = 0.25$ second

also; The system can be modeled by first order differential equation

$$\Rightarrow \boxed{0.25 \frac{dy(t)}{dt} + y(t) = 5x(t)}$$

The above is the differential equation describing the system.

Where $y(t)$ is the output of the transducer and $x(t)$ is the input.

The frequency response for 1st order differential equation is given by;

$$\frac{K}{1 + j\tau\omega} = \frac{5}{1 + j(0.25)\omega}$$

Thus; the frequency response function is $\frac{5}{1 + j(0.25)\omega}$

(b) At very low frequency;
$\omega \approx 0$

Thus; output amplitude at low frequency = 5.

we need to find a frequency for which output amplitude = $5/2 = 2.5$

Let the required frequency be ω_0

So,

$$(2.5)^2 = \frac{K^2}{1 + (\tau\omega_0)^2} = \frac{5^2}{1 + 0.0625\,\omega_0^2}$$

$$\Rightarrow 1 + 0.0625\,\omega_0^2 = 4 \Rightarrow \omega_0 = \sqrt{3/0.0625}$$

and Thus, $\omega_0 = \sqrt{48}$ rad/s.

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Thus the frequency at which amplitude would be half $= \sqrt{48}$ rad/sec

(c) 1% amplitude distortion means that frequency response function's magnitude is either 0.99 or 1.01 times the magnitude at DC

It is a known fact that,
the magnitude of frequency response function decreases as ~~frequency~~ (ω) frequency increases. $\Rightarrow$ our new magnitude is 0.99 times of that at DC.

At DC; $\omega = 0 \Rightarrow$ magnitude of frequency response function $= 5$

Now, we need to find some frequency, ω_0 such that the magnitude of frequency response function is $5 \times 0.99 = 4.95$.

We need to solve for ω_0;

$$(4.95)^2 = \frac{5^2}{1 + 0.0625(\omega_0)^2}$$ [obtained from frequency response function]

$$\Rightarrow 24.5025 = \frac{5^2}{1 + 0.0625\omega_0^2}$$

Upon solving we get;

$$\omega_0 = 0.57 \text{ rad/s}$$

$\therefore$ The frequency range for which distortion would be less than 1% is 0 (DC) to 0.57 rad/s.

The phase shift at the upper frequency is given by;

phase $= \phi = -\tan^{-1} \tau\omega_1 = -\tan^{-1}(0.25 \times 0.57) = 0.1415$ rads

$= 8.110^\circ$

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