Programming with C++

COMP2011: Function II — Recursion

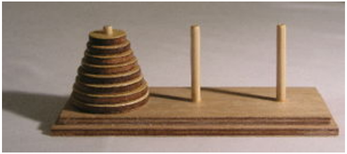
Cecia Chan
Cindy Li
Brian Mak

Department of Computer Science & Engineering
The Hong Kong University of Science and Technology
Hong Kong SAR, China

Part I

Think Recursively!
“God Help Those Who Help Themselves.”

Example: Tower of Hanoi Game



- It consists of 3 pegs, and a stack of discs of different sizes.
- It starts with all discs stacked up on one peg with smaller discs sitting on top of bigger discs.
- The goal is to move the entire stack of discs to another peg, making use of the remaining peg.
- Rules:
  - only one disc may be moved at a time
  - no disc may be placed on top of a smaller disc

Example: Factorial Function

Definition of the factorial function you learn in high school:

$$n! = n \times (n-1) \times (n-2) \times \cdots 2 \times 1$$

Recursive Definition of Factorial Function

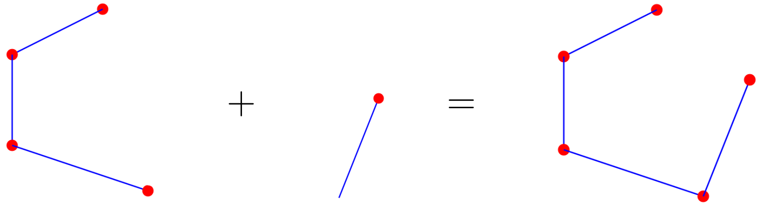
- $0! = 1$
- $n! = n \times (n-1)! \quad \text{if } n > 0$

Example: Natural Numbers

The formal definition of natural numbers in set theory:

- 0 is a natural number
- each natural number has a successor, which is also a natural number
- (definition of successor: successor of n is $n + 1$)

Example: Polyline



A polyline with n line segments is equivalent to adding another line segment to a polyline with $n - 1$ line segments.

Example: Fibonacci Numbers

$0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, \ldots$

Fibonacci (1202) investigated how fast rabbits could breed:

- A newly-born pair of rabbits, one male, one female, are put in a field.
- Rabbits mate at the age of one month so that at the end of its 2nd month, a female can produce another pair of rabbits.
- Suppose that our rabbits never die.
- Suppose the female always produces one new pair (one male, one female) every month from the 2nd month on.
- How many pairs will there be in one year?

Question : What is special with the above numbers?

Answer : Except for the first 2 numbers, each number is the sum of the last 2 numbers in the sequence.

Thinking Recursively

- In some problems, it may be natural to define the problem in terms of the problem itself!
- Recursion is useful for problems that can be represented by a simpler version of the same problem.
  1. To solve the Tower of Hanoi game that uses n discs, one first solves the tower of hanoi game that uses $n-1$ discs.
  2. To draw a polyline of n segments, first draw a polyline of $n-1$ segments, and then add the last line segment.
  3. To check if n is a natural number, it suffices to check if $n-1$ is a natural number instead.
  4. To find the value of $n!$, first find the value of $(n-1)!$ and then multiply the result with n.
  5. To find the nth Fibonacci number, first find the $(n-1)$th and $(n-2)$th Fibonacci numbers, and then sum them up.

Part II

Programming Recursion in C++

Implement Recursion by Recursive Function

- In programming, recursion means that a function calls itself!
- Although it looks strange in the beginning, solving a programming task by recursion renders the program
  - easier to write
  - easier to read (understand)
  - shorter (in codes).

Implement a Recursive Solution

1. Decompose the problem into sub-problems — which are smaller examples of the same problem — plus some additional work that "glues" the solutions of the sub-problems together.
2. The smallest sub-problem has a non-recursive solution.

Example: Factorial Function

```
int factorial(int n)     /* factorial.cpp */
{
    if (n < 0)           // Error checking
        return -1;
    else if (n == 0)     // Base case; ending case too!
        return 1;
    else                 // Recursive case
        return n * factorial(n-1);
}
```

Or, equivalently,

```
int factorial(int n)     /* factorial2.cpp */
{
    if (n < 0)           // Error checking
        return -1;
    if (n == 0)          // Base case; ending case too!
        return 1;
    return n * factorial(n-1); // Recursive case
}
```

How the Recursive Factorial Function Works?

factorial(3) :

$3 < 0$ false

$3 == 0$ false

3 * factorial(2)

factorial(2) :

$2 < 0$ false

$2 == 0$ false

2 * factorial(1)

factorial(1) :

$1 < 0$ false

$1 == 0$ false

1 * factorial(0)

factorial(0) :

$0 < 0$ false

$0 == 0$ true

return 1

return 1*1 = 1

return 2*1 = 2

return 3*2 = 6

Factorial Function: Recursive vs. Non-Recursive

```
int factorial(int n)     /* factorial.cpp */
{
    if (n < 0)           // Error checking
        return -1;
    else if (n == 0)     // Base case; ending case too!
        return 1;
    else                 // Recursive case
        return n * factorial(n-1);
}
```

```
int factorial(int n)     /* non-recursive-factorial.cpp */
{
    int result = 1;      // Default value for n = 0 or 1
    if (n < 0)           // Error checking
        return -1;

    for (int j = 2; j <= n; j++) // When n >= 2
            result *= j;
    return result;
}
```

Infinite Loop!

- When we work with loops, we have to be careful that the ending condition will be met eventually.
- Otherwise, we will get infinite loop!

```
for (int j = 1; j != 10; j += 2)
{
    // Ending condition never met!
    cout << j << endl;
    ...
}
```

Infinite Recursion!

- Just like infinite loops, we have to be careful that a recursion will eventually end up with a non-recursive base case.
- Otherwise, we will get infinite recursion!

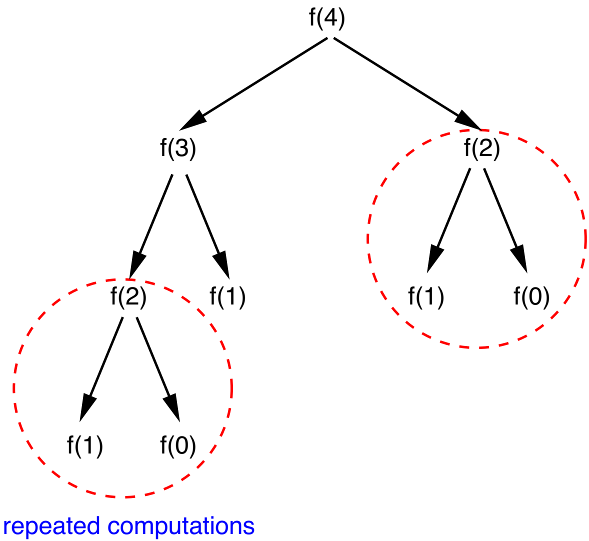
```
int factorial(int n)
{
    // Forget the base case, which is the ending case too!
    return n * factorial(n-1);
}
```

```
int factorial(int n)
{
    // Forget checking if n < 0
    if (n == 0)
        return 1;

    // Infinite recursion for negative n
    return n * factorial(n-1);
}
```

Example: Fibonacci Function as a Recursion

```
int fibonacci(int n)    /* File: fibonacci.cpp */
{
    if (n == 0)         // Base case #1
        return 0;

    if (n == 1)         // Base case #2
        return 1;

    return fibonacci(n-1) + fibonacci(n-2);
}
```

Inefficiency of Recursive Fibonacci Function



Example: Non-Recursive Fibonacci Function

```
int fibonacci(int n)     /* non-recursive-fibonacci.cpp */
{
    int fn;                // keep track of f(n)
    int fn_1 = 1;          // keep track of f(n-1)
    int fn_2 = 0;          // keep track of f(n-2)

    if (n == 0) return 0; // Base case #1
    if (n == 1) return 1; // Base case #2

    for (int j = 2; j <= n; j++)
    {
        fn = fn_1 + fn_2; // f(n) = f(n-1) + f(n-2)

        // Prepare for the calculation of the next fibonacci number
        fn_2 = fn_1;       // f(n-2) = f(n-1)
        fn_1 = fn;         // f(n-1) = f(n)
    }

    return fn;
}
```

Example: Counting Zeros in an Integer

- Example: for the integer 120809, there are 2 zeros.
- Basic idea:
  - Break down the number into quotient and remainder.
  - Count the number of zeros in quotient and remainder.

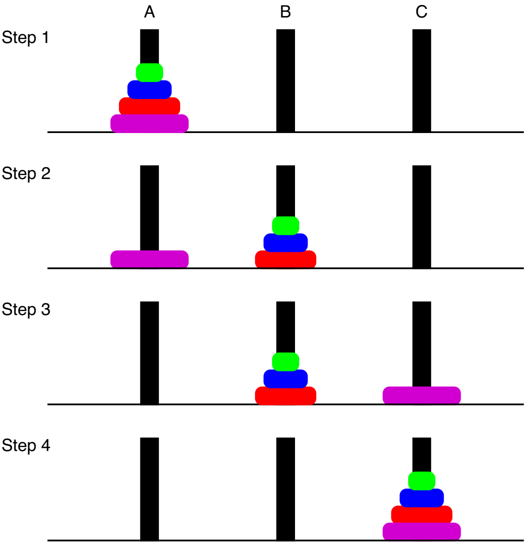
```
int num_zeros(int n)    /* File: num-zeros.cpp */
{
    if (n == 0)                 // Base case #1
        return 1;

    else if (n < 10 && n > 0) // Base case #2
        return 0;

    else
        return num_zeros(n/10) + num_zeros(n%10);
}
```

Example: Factoring

- Goal: find how many times factor m appears in the integer n.
- Example: if $n = 48$ and $m = 4$, since $48 = 4 \times 4 \times 3$, the answer is 2.
- Basic idea:
  - Divide n by m until the remainder is non-zero.
  - Increment the count by 1 for every successful division.

```
int num_factors(int n, int m) /* File: factor.cpp */
{
    if (n % m != 0)      // Base case
        return 0;
    else
        return 1 + num_factors(n/m, m);
}
```

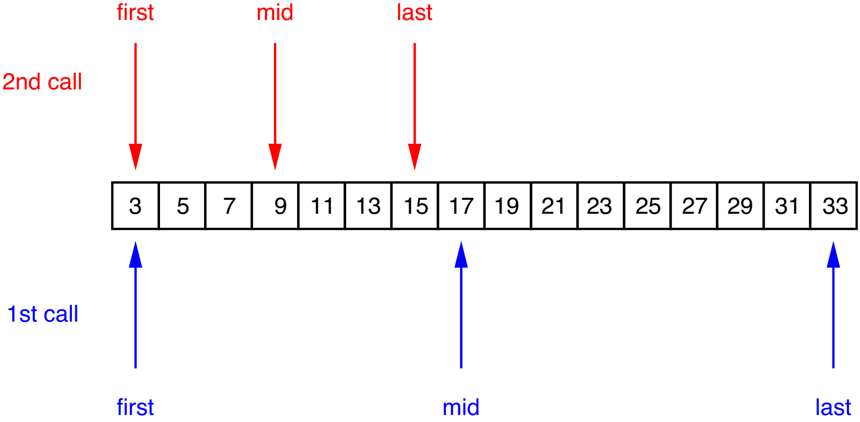
Recursive Solution of Tower of Hanoi



Example: Recursive Solution of Tower of Hanoi

```
#include <iostream>       /* File: toh.cpp */
using namespace std;

void tower_of_hanoi(int num_discs, char pegA, char pegB, char pegC)
{
    if (num_discs == 0) // Base case
        return;

    tower_of_hanoi(num_discs-1, pegA, pegC, pegB);

    cout << "move disc " << num_discs
         << " from peg " << pegA << " to peg " << pegC << endl;

    tower_of_hanoi(num_discs-1, pegB, pegA, pegC);
}
```

Binary Search



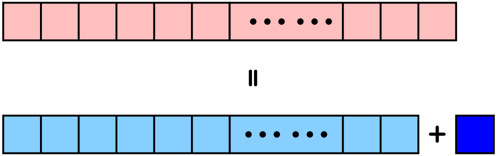
binary search for the value 9

Example: Recursive Solution of Binary Search

```
const int NOT_FOUND = -1;      /* File: bsearch.cpp */
int bsearch(const int data[ ], // sorted in ascending order
            int first,         // lower bound index
            int last,          // upper bound index
            int value)         // value to search
{
    if (last < first)          // Base case #1
        return NOT_FOUND;

    int mid = (first + last)/2;

    if (data[mid] == value)
        return mid;             // Base case #2

    else if (data[mid] > value) // Search the lower half
        return bsearch(data, first, mid-1, value);

    else                        // Search the upper half
        return bsearch(data, mid+1, last, value);
}
```

Example: Non-Recursive Solution of Binary Search

```
const int NOT_FOUND = -1;       /* File: non-recursive-bsearch.cpp */
int bsearch(const int data[ ], // sorted in ascending order
            int size,           // number of data in the array
            int value)          // value to search
{
    int first = 0;
    int last = size - 1;

    while (first <= last)
    {
        int mid = (first + last)/2;
        if (data[mid] == value)
            return mid;         // Value found!
        else if (data[mid] > value)
            last = mid - 1;     // Set up for searching the lower half
        else
            first = mid + 1;    // Set up for searching the upper half
    }

    return NOT_FOUND;
}
```



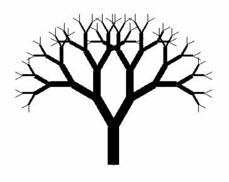
- Array is a recursive data structure in nature.
- For many problems, one may define a recursion on an array of size N which
  - will call *itself* with only $N - 1$ elements (either the top $N - 1$ or the last $N - 1$ elements),
  - with some extra codes to deal with the remaining element (last or first element).

Example: Sum Up Array Elements

```
#include <iostream> /* File: array-sum.cpp */
using namespace std;

// Summing up x[0] + x[1] + ... + x[num_elements-1]
int array_sum(const int x[], int num_elements)
{
    if (num_elements <= 0) return 0; // Base case
    return array_sum(x, num_elements-1) + x[num_elements-1];
}

int main()
{
    int a[] = { 1, 2, 3, 4, 5, 6 };
    int n;                 // #elements in an array to sum
    while (cin >> n)
        cout << array_sum(a, n) << endl;
    return 0;
}
```

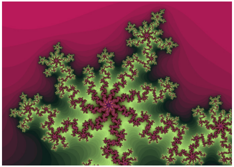
Question: What happens if you pass a value bigger than the size of the array size to *n*?

- Many natural phenomena are recursion: a smaller part of oneself is embedded in itself!



(a) tree

(b) infinite mirror images



(c) fractal

- As a result, it is usually easier for a programmer to write a solution using recursion $\Rightarrow$ greater productivity.

Disadvantages of Recursion

- The greater programming productivity is achieved at the expenses of the more computing resources. To run recursion, it usually requires
  - more memory
  - more computational time
- The reason is that whenever a function is called, the computer
  - has to memorize its current state, and passes control from the caller to the callee.
  - sets up a new data structure (you may think of it as a scratch paper for rough work) called activation record which contains information such as
    - where the caller stops
    - what actual parameters are passed to the callee
    - new local variables created by the callee function
    - the return value of the function at the end
  - removes the activation record of the callee when it finishes.
  - passes control back to the caller.