CPSC 314
Computer Graphics

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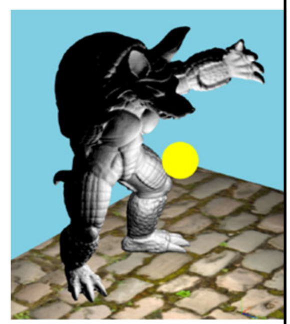
Geometry 1: Points, Vectors and Coordinates

Preliminaries

- Announcements
  - A1 is out! Please get started. Any questions so far?
  - Labs start this week. Zoom links posted on Canvas.
- Today: Theory (review Chapter 2 of text)
  - Points and Vectors
  - Basis and Coordinates (vec3). Orthonormal Basis.
  - Linear transformations and matrices (mat3 or mat3x3)

Assignment 1 notes

- Many of you have already made great progress! Congratulations!
- Note that there are many things in the provided code (e.g., “modelMatrix”), that you haven’t learned yet. Don’t panic! You don’t need to understand them deeply for this assignment. We’ll cover them later this week.
- For Part 1(c) the exact brightness is not important and can be adjusted by a scale factor.
  As long as you get shading that varies with the cosine of the angle between normal and light position, its fine for this assignment. Later we’ll look at diffuse shading more accurately.

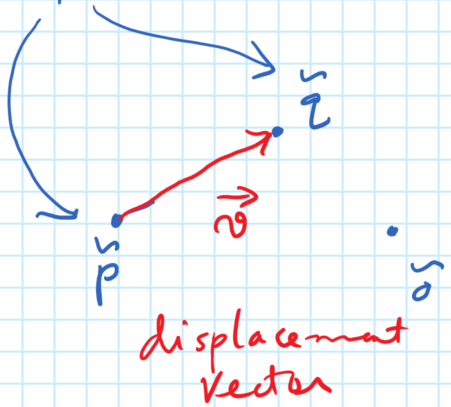


Today

- Learning objectives
  - Linear algebra review, with some new notation
  - **Points ≠ Vectors ≠ Arrays of numbers**
  - Basis and Coordinates
  - Operations: dot, norm, etc.(built into GLSL)
  - Coordinate transformations, matrices
- Switch to tablet

arrays $\neq$ vectors $\neq$ points

Point is a real location in space



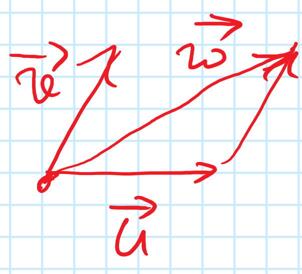
vector is an algebraic object. So you can

(1) Multiply with a scalar

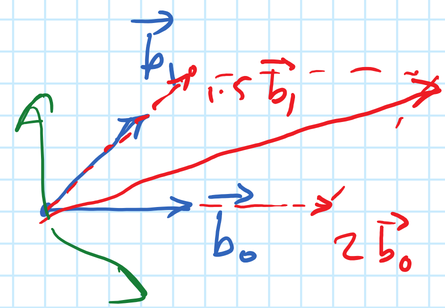
$$a\vec{v} \longrightarrow \text{a new vector}$$

(2) Add two vectors and get another vector

$$\vec{u} + \vec{v} = \vec{w} \quad \text{also a vector}$$



§ Basis



$\vec{v} = 2\,\vec{b}_0 + 1.5\,\vec{b}_1$

An independent set of vectors that can produce any vector in a given space by linear combination

The size of the basis set is the dimension of the space.

§ Coordinates of a vector in a basis $\vec{b}$

Can write any vector $\vec{v}$ as

$$\boxed{\vec{v} = v_0\,\vec{b}_0 + v_1\,\vec{b}_1}$$

vectors

Scalars = numbers

$$\underset{\text{Real}}{\vec{v}} \xrightarrow[\vec{b}]{\text{in basis}} \underset{\text{Coordinates}}{\begin{pmatrix} v_0 \\ v_1 \end{pmatrix}} = \bar{v}$$

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Notation:

$$\bar{v} = \begin{pmatrix} v_0 \\ v_1 \end{pmatrix} \qquad \underline{v} = \begin{pmatrix} v_0 & v_1 \end{pmatrix}$$

Column matrix

row matrix

With this, we can compactly write the basis

$$\underline{\vec{b}} = \begin{pmatrix} \vec{b}_0 & \vec{b}_1 \end{pmatrix}$$

Can write this

$$\vec{v} = v_0 \vec{b}_0 + v_1 \vec{b}_1$$

as

$$\vec{v} = \begin{pmatrix} \vec{b}_0 & \vec{b}_1 \end{pmatrix} \begin{pmatrix} v_0 \\ v_1 \end{pmatrix}$$

$$\boxed{\vec{v} = \underline{\vec{b}} \, \bar{v}}$$

Real vector

Basis

Coordinates of $\vec{v}$ in basis $\underline{\vec{b}}$

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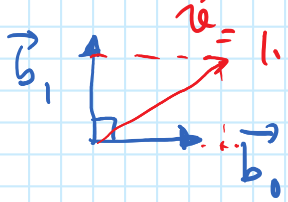
§ Notation

	Ours	Textbook
Points	$\tilde{p}$	$\tilde{p}$
vector	$\vec{v}$	$\vec{v}$
column matrix	$\bar{v}$	$\mathbf{v}$
row matrix	$\underline{v}$	$\mathbf{v}^T$
basis	$\underline{\vec{b}}$	$\vec{\mathbf{b}}^T$

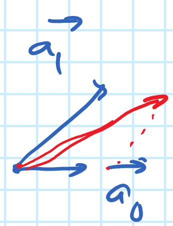
bold font

§ Orthonormal basis



$\vec{v} = 1.1\,\vec{b}_0 + 0.9\,\vec{b}_1$

Basis vectors are of unit length, and perpendicular to each other



$\vec{v} = 1.5\,\vec{a}_0 + 1.1\,\vec{a}_1$

Computing the component along any basis is easy.

$$\vec{v} \cdot \vec{b}_\gamma = 1.1\,\underbrace{\vec{b}_\gamma \cdot \vec{b}_1}_{1} + 0.9\,\underbrace{\vec{b}_0 \cdot \vec{b}_1}_{0}$$

For next class

- Review Chapter 3 and 4 of textbook.
- This is a key step: you'll solve the mystery of 4-vectors!