CPSC 314
Computer Graphics

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Other useful bits about Transformations
rotations, normal, Three.js scene graph

Preliminaries

- Today: wrapping up transformations
  - Appetizer: what you can do with vertex shaders
  - Mathematics of Rotation (Textbook Chapter 2.5)
  - How to transform a surface normal
  - Scene graph in Three.js

What you can do with vertex shaders

- Tyler posted some useful links on https://piazza.com/class/ky4qwpnvrqa5pq?cid=103
  eg. https://www.shadertoy.com/view/Ms2SD1
- Revisit this early example (from SIGGRAPH 2002)

DyRT: Dynamic Response Textures
for Real Time Deformation Simulation with Graphics Hardware

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- Switch to tablet

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§ Rotations

The special property is "isometry"

$$\begin{pmatrix} u \\ 0 \end{pmatrix} \overset{\text{def}}{=} \begin{pmatrix} Rv \\ 0 \end{pmatrix} = \left(\begin{array}{c|c} R & \begin{matrix} \hat{r} \\ \hat{x} \\ \hat{\cancel{z}} \end{matrix} \\ \hline & 1 \end{array}\right) \begin{pmatrix} v \\ 0 \end{pmatrix}$$

ignore, doesn't affect vector

Focus on the 3x3 "linear" part

$$u = Rv$$

want $\|u\| = \|v\|$ where $\|u\| = \sqrt{\underbrace{u_x^2 + u_y^2 + u_z^2}}$

$= dot(u, u)$

$= u^T u$

$$\Rightarrow \underbrace{u^T}\,\underbrace{u} = v^T v$$

$$v^T \underbrace{R^T R} v = v^T v$$

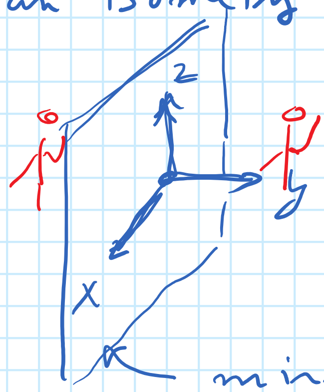
$$\Rightarrow \boxed{R^T R = I}$$

So if $R = \left[\begin{array}{c|c|c} \bar{r}_x & \bar{r}_y & \bar{r}_z \end{array}\right]$

Then $R^T R = \left[\begin{array}{c|c|c} \bar{r}_x^T \bar{r}_x & \bar{r}_{\cancel{x}}^T \bar{r}_y & \cdots \\ \hline & & \\ \hline & & \end{array}\right] = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix}$

§ Reflections

Also an isometry, but handedness changes



$$T = \begin{bmatrix} 1 & & \\ & -1 & \\ & & 1 \end{bmatrix}$$

mirror in x-z plane

$$T^T T = I$$

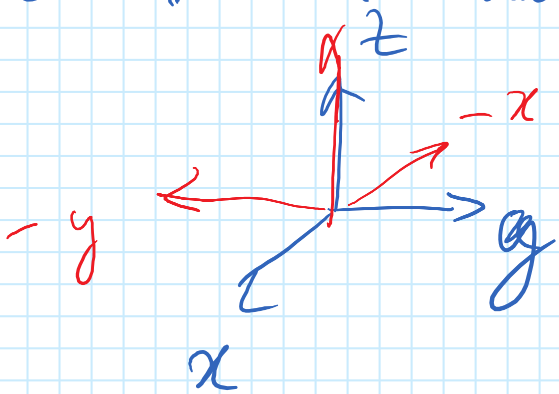
In general, T R

can detect using the

determinant,

$$\det(T) \begin{cases} = +1 & \text{Rotation} \\ = -1 & \text{Reflection} \end{cases}$$

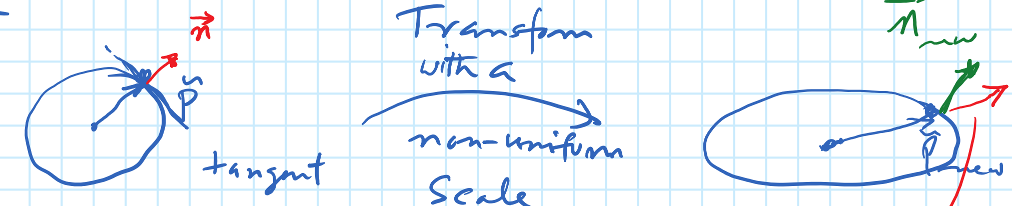
Check: what about



$$\begin{bmatrix} -1 & & \\ & -1 & \\ & & 1 \end{bmatrix}$$

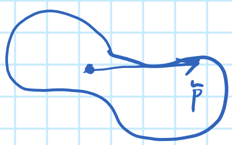
§ Normals aren't "normal"!

Example



$$T = \left[\begin{array}{cc|c} 3 & & \\ & 1 & \\ \hline & & 1 \end{array}\right]$$

$$\left[\begin{array}{c|c} A & \\ \hline & \end{array}\right] \bar n$$

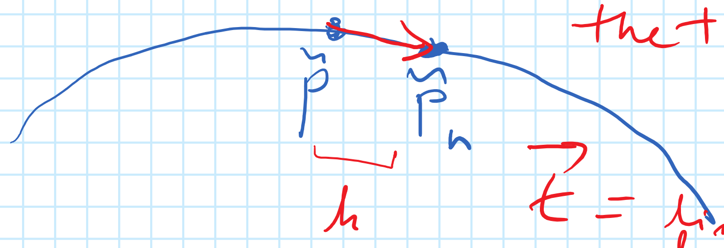


$$\bar p_{new} = \left[\begin{array}{c|c} A & \\ \hline & 1 \end{array}\right] \bar p$$

$$\bar n_{new} = \cancel{\left[\begin{array}{c|c} A? & \\ \hline & \end{array}\right]} \bar n$$

Good news: the tangent vector $\vec t$ transform like points do!!

$\vec t$ in the limit this is the tangent



$$\vec t = \lim_{h \to 0} \frac{\bar p_h - \bar p}{h}$$

So tangent $\simeq$ difference of two points

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Define $\vec{n}$ as $\vec{n} \cdot \vec{t} = \bar{n}^T \bar{t} = 0$

So want $n_{new}^T \underbrace{t_{new}}_{} = 0 = \underline{n^T t}$

$\underbrace{n_{new}^T T}_{n^T} t$

So $\boxed{n^T = n_{new}^T T}$

$n^T T^{-1} = n_{new}^T$

↓ transpose

$$\boxed{n_{new} = \left(T^{-1}\right)^T n}$$

if T is the modelView matrix

$$\begin{bmatrix} A & \\ \hline & 1 \end{bmatrix}^{-T} = \begin{bmatrix} A^{-T} & 0 \\ \hline 0 & 1 \end{bmatrix}$$

called normalMatrix in Three.js