CPSC 314
Computer Graphics

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Projective Transformation
Ch. 10.3, 11.2, 11.3

Preliminaries

- Today
  - View Frustum
  - Orthographic Projection
  - Projective Transformations (not the same as Projection)
  - Additional “fixed function” steps: Clipping, Normalized Device Coordinates

Motivation

- The simple projection from last class lost all depth information (like a real camera)
  - Mathematically, the Projection is “singular”, and has a zero column
- We want to retain some “depth like” information
- Depth demo https://threejs.org/docs/#api/materials/MeshDepthMaterial

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- SWITCH TO TABLET

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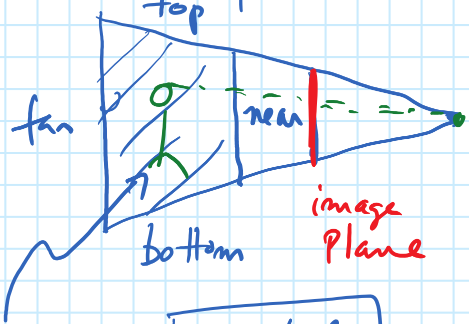
§ Orthographic Projection



View frustum

Central Projection

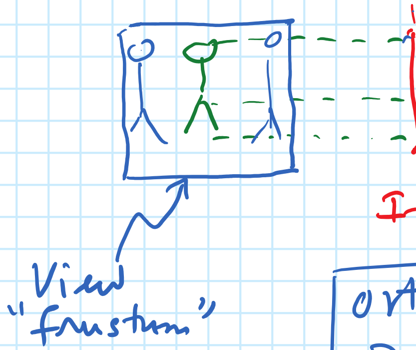
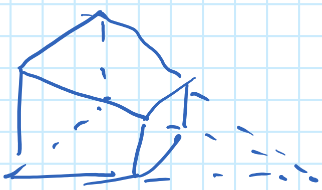
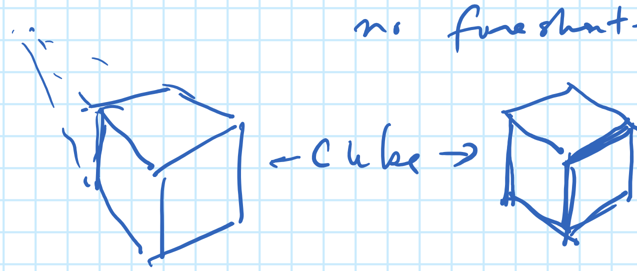


Image Plan

Orthographic Projection

no foreshortening





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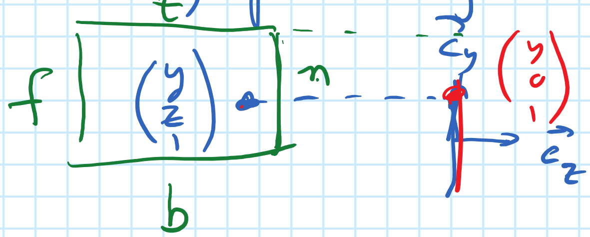
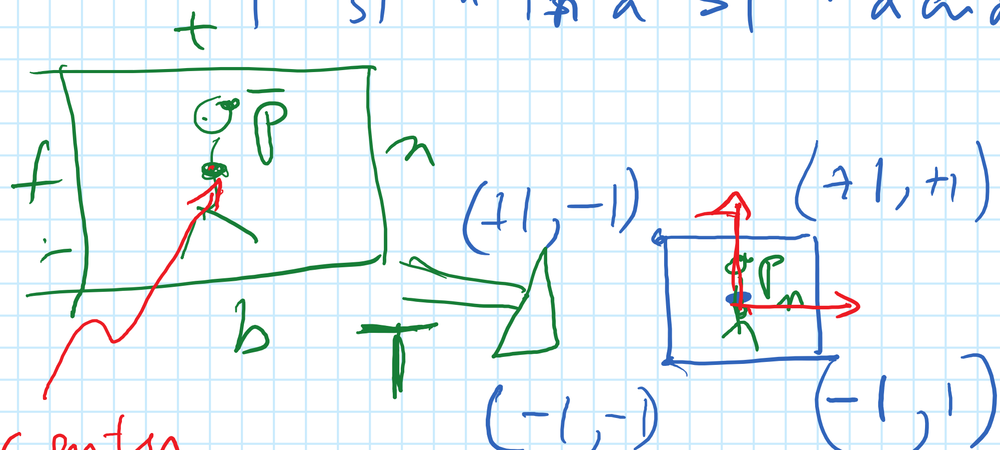
Orthographic Projection Matrix



$$\begin{pmatrix}y\\z\\1\end{pmatrix}\to\begin{pmatrix}y\\0\\1\end{pmatrix}=\underbrace{\left(\begin{array}{cc|c}1 & & \\ & 0 & \\ \hline & & 1\end{array}\right)}_{\text{define as } Q}\begin{pmatrix}y\\z\\1\end{pmatrix}$$

More general orthographic

We want to view things inside the frustum in a standard size "screen"



Normalized
Device
Coordinates

center of frustum

$$p=\begin{pmatrix}\frac{t+b}{2}\\ \frac{n+f}{2}\\ 1\end{pmatrix}$$

$$p_n = T\,p$$

$$\begin{pmatrix}0\\0\\1\end{pmatrix}=\left[\begin{array}{cc|c}1 & & -\left(\frac{t+b}{2}\right)\\ & 1 & -\left(\frac{f+n}{2}\right)\\ \hline & & 1\end{array}\right]\begin{pmatrix}\frac{t+b}{2}\\ \vdots\\ 1\end{pmatrix}$$

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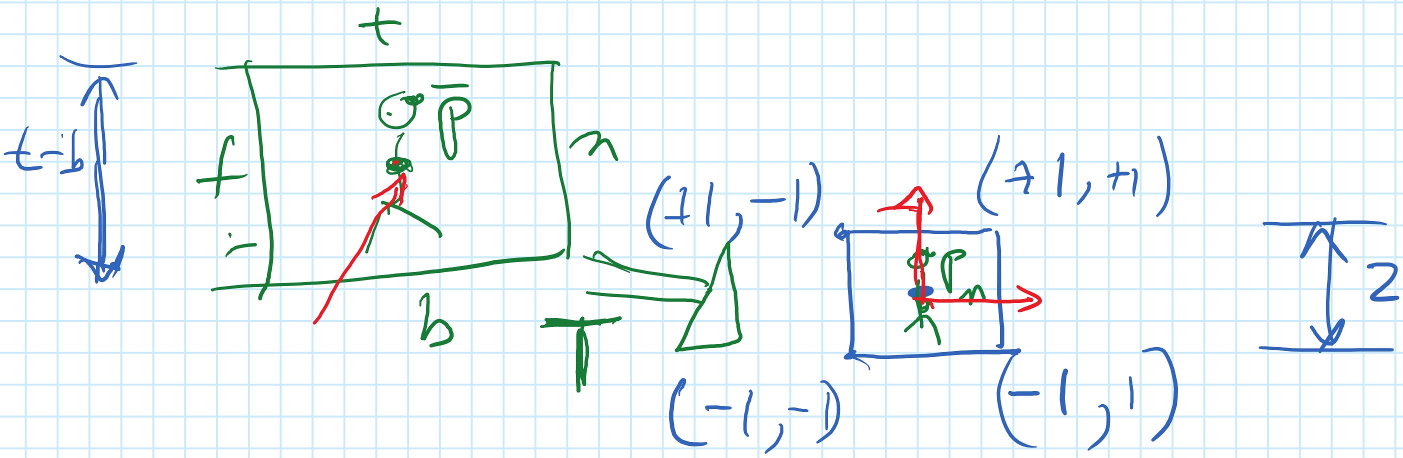

Next, scale the box to make its size (as. $t-b$) $\rightsquigarrow$ 2

$$S = \begin{bmatrix} \frac{2}{t-b} & & \\ & \frac{2}{n-f} & \\ & & 1 \end{bmatrix}$$

So the full transform is

$$P_n = S\,T\,p$$

$$= \begin{bmatrix} \frac{2}{t-b} & & -\frac{t+b}{t-b} \\ & \frac{2}{n-f} & -\frac{n+f}{n-f} \\ & & 1 \end{bmatrix}$$

Correction after class

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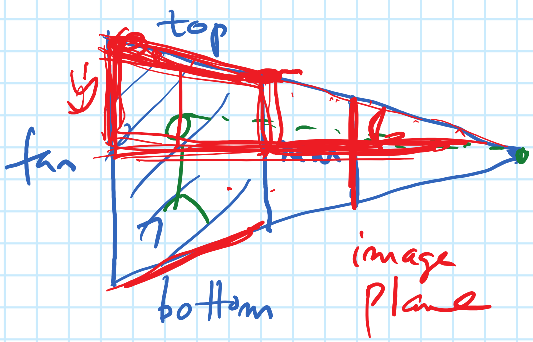


Previously

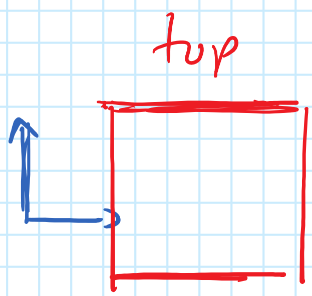
$$P = \left(\begin{array}{cc|c} 1 & & 0 \\ & 1 & 0 \\ \hline & -1 & 0 \end{array}\right)$$

Now

$$P_u = \left(\begin{array}{cc|c} 1 & & \\ & 0 & 1 \\ \hline & -1 & \end{array}\right)$$

"unhinging"

$$P_u \qquad \begin{pmatrix} y \\ z \\ 1 \end{pmatrix} \longrightarrow \begin{pmatrix} y \\ 1 \\ -z \end{pmatrix} \xrightarrow[\text{divide by "w"}]{\text{Perspective}} \begin{pmatrix} -y/z \\ -1/z \\ 1 \end{pmatrix}$$



A box!