

## CS 170 Homework 3

Due 2025/3/1, at 10:00 pm (grace period until 11:59pm)

### 1 Study Group

List the names and SIDs of the members in your study group. If you have no collaborators, you must explicitly write “none”.

**Solution:** I worked on this homework with the following collaborators:

- none, which is only me, Sillycheese

### 2 Depth First Search

- (a) (4 points) In each of the following cases, PreVisit and PostVisit have been defined for you. After execution, the array  $A[v]$  will hold a value for each vertex  $v$ . Describe in words what  $A[v]$  represents.

- i. Describe in words what  $A[v]$  represents.

**Solution:** this is the longest path from root of subtrees to leaf.

- ii. Describe in words what  $A[v]$  represents.

**Solution:** this is maximum degree.

- (b) (6 points) In each of the following cases, write down pseudocode for PreVisit and PostVisit routines to perform the computation needed.

- i. For each vertex  $v$ , compute the maximum weight of an edge along the path from root  $r$  to vertex  $v$  and store it in array  $A[v]$ .

**Solution:**

```
procedure PreVisit(u, v)
   $A[v] \leftarrow \max(A[u], w(u, v))$ 
```

```
procedure PostVisit(u, v)
  return
```

- ii. For each vertex  $v$ , compute the maximum weight of any edge in the subtree rooted at vertex  $v$  and store it in array  $A[v]$ .

**Solution:**

```
procedure PreVisit(u, v)
  return
```

```

procedure PostVisit(u, v)
  A[u]  $\leftarrow$  max(A[u], A[v], w(u, v))

```

- iii. For each vertex  $v$ , compute the maximum pre-order number of any of its children and store it in array  $A[v]$ . If  $v$  has no children, then  $A[v]$  should be 0.

**Solution:**

```

procedure PreVisit(u, v)
  t  $\leftarrow$  t+1
  A[u]  $\leftarrow$  t

```

```

procedure PostVisit(u, v)
  t  $\leftarrow$  t+1

```

### 3 Biconnected Components

- (a) Suppose that  $|V| \geq 2$ . Can you always find a vertex  $v \in V$  that is not critical? What about an edge that is not critical?

**Solution:** DFS, and find that leaf

if all  $V$ s are critical

- (b) Give a linear time algorithm to find all the critical edges of  $G$ .

**Solution:** PreVisit, and maintain a low value. if  $\text{pre}(v) < \text{low}(n)$ , then  $u-v$  is critical.

- (c) Modify your algorithm above to find all the critical vertices of  $G$ .

**Solution:** DFS

### 4 Topological Sort Proofs

**Solution:** SKIP! all is because of my Ph.D Exam coming:(

### 5 Distant Descendants

- (a) Write an  $O(|V|)$  algorithm that computes the total size of the subtree (number of descendants plus 1 for the vertex itself) of each vertex  $v$  in an array  $s[v]$ . Give a brief justification that your algorithm is correct and runs in  $O(|V|)$  time. Do not just cite an algorithm from class; reproduce anything you use in your solution.

**Solution:** To find the size of subtree. we need to DFS( $v$ ), and maintain  $s[u] = s[u] + s[v]$  recursively.

- (b) Write an  $O(|V|)$  algorithm that computes the  $K$ -th level ancestor of each vertex  $v$  (null if the depth of  $v$  is less than  $K$ ) in an array  $a[v]$ . Give a brief justification that your algorithm is correct and runs in  $O(|V|)$  time. Make sure your algorithm runs in  $O(|V|)$  time and not  $O(K|V|)$  time.

**Solution:** say a array called a to note the members of ancestors.

if  $\text{len}(a) > K$ , then return  $a[\text{len}(a)-1-k]$

else return null

then recursively loop.

for  $(u,v)$  in  $G$

$a.\text{push\_back}(v)$

DFS( $v$ )

$a.\text{pop\_back}(v)$

clean the path for the next search.

remember to set a visited array to optimize it!

- (c) Write an  $O(|V|)$  algorithm to compute  $d[v]$  for each vertex  $v$  using  $s[v]$  and  $a[v]$ . Give a brief justification that your algorithm is correct and runs in  $O(|V|)$  time.

**Solution:** if  $a[v]$  exists

$d[a[v]] = s[v] + d[a[v]]$

loop and loop