CS 170 Homework 1

Due **2025/2/10, at 10:00 pm (grace period until 11:59pm)**

1 Study Group

List the names and SIDs of the members in your study group. If you have no collaborators, you must explicitly write "none".

Solution: I worked on this homework with the following collaborators:

- none,which is only me,Sillycheese

2 Math Review

(a) Simplify the following expressions into a single logarithm. Your answer should be in the form $\log_a b$ or $\ln(b)$:

(i.) $\frac{\ln x}{\ln y}$

Solution: $log_y x$

(ii.) $\ln(x) + \ln(y)$

Solution: $\ln xy$

(iii.) $\ln(x) - \ln(y)$

Solution: $\ln \frac{x}{y}$

(iv.) $170 \ln(x)$

Solution: $\ln x^{170}$

(b) Give a simple proof for each of the following identities:

(i.) $x^{\log_{\frac{1}{x}} y} = \frac{1}{y}$

Solution: $x^{\log_{\frac{1}{x}} y} = ((x^{-1})^{-1})^{\log_{x^{-1}} y} = ((x^{-1})^{\log_{x^{-1}} y})^{-1}$

so,we can get y^{-1},and prove it

(ii.) $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

Solution: $\sum_{i=1}^{n} i = 1 + 2 + \cdots + n$

and begin is 1 ,end is n,so we can compute it with that formula,

which is the answer,so prove it.

(iii.) $\sum_{k=0}^{n} ar^k = \begin{cases} a(\frac{1-r^{n+1}}{1-r}), r \neq 1 \\ a(n+1), r = 1 \end{cases}$

Solution: if r=1:

formula=$a(1+1+\cdots+1)$,which has $n+1$ numbers

if $r \neq 1$:

formula=$a(1+r+r^1+\cdots+r^n)$

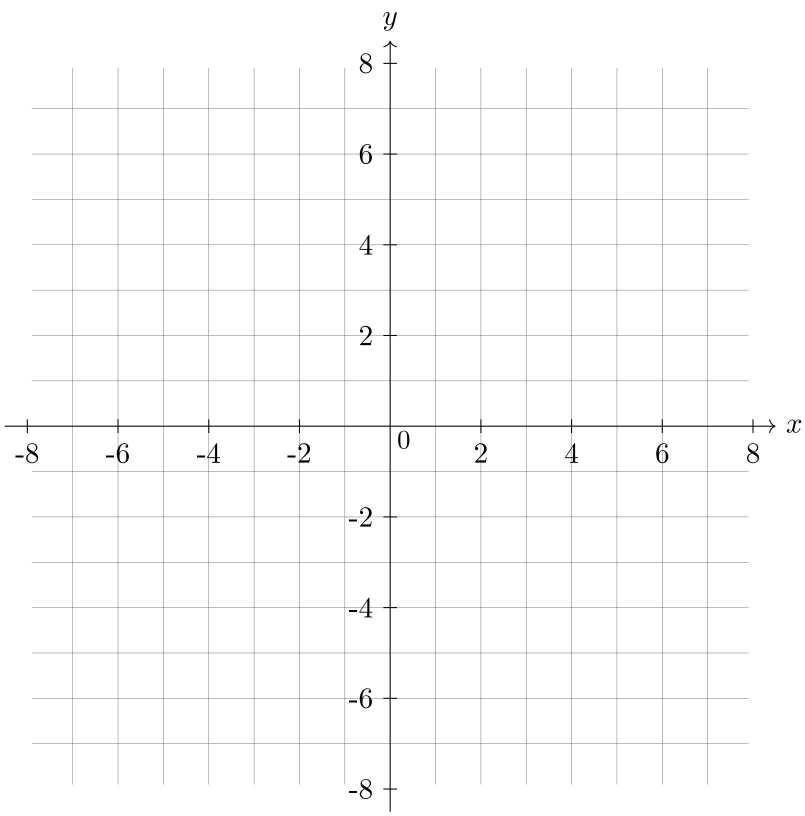
we can get q is r,and begin is1,end is$n$ so use that formula,we can get the answer. so prove it.

(c) Consider the following two inequalities:

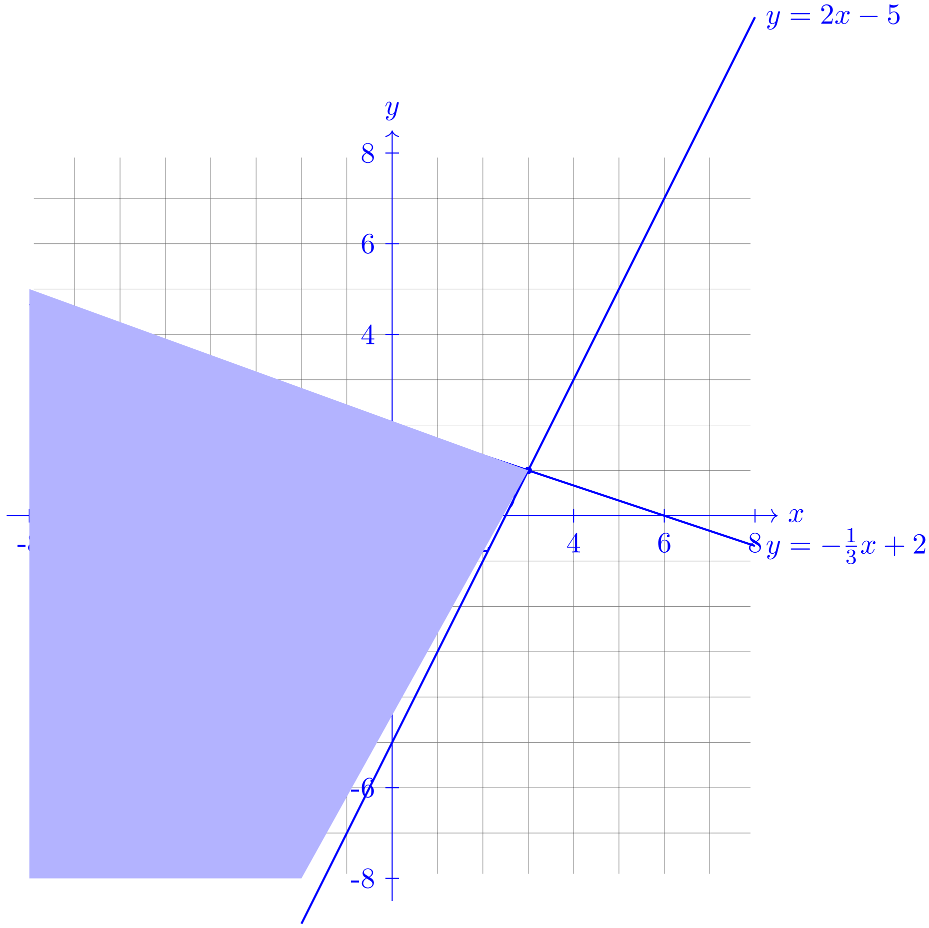
$$x + 3y \leqslant 6$$

$$y \geqslant 2x - 5$$

Graph out the set of all points (x, y) that satisfy both inequalities above. Clearly mark your lines and shaded region(s).



Solution:



3 Asymptotics Practice

(a) For each pair of functions f and g below, specify whether f = O(g), g = O(f), or both. No justification needed.

Solution:

i. both

ii. both

iii. $f = O(g)$

iv. $f = O(g)$

v. $f = O(g)$

vi. $f = O(g)$

(b) Consider the two functions f (n) = 1 + b + b2 + · · · + bn and g(n) = bn for an arbitrary constant b > 0. Similar to what you did in the previous part, specify whether f = O(g), g = O(f), or both. Justify your answer. Hint: the asymptotic relationship between f and g varies depending on the specific value of b. This content is protected and may not be shared, uploaded, or distributed.

Solution:

$g = O(f)$ for all b > 0, as $g(n)f(n)$ for all n.

only if $b > 1$, $f = O(g)$

4 Recurrence Relations

For each part, find the asymptotic order of growth of $T(n)$; that is, find a function g such that $T(n) = (g(n))$. Show your reasoning and do not directly apply the Master Theorem; doing so will yield 0 credit.

In all subparts, you may ignore any issues arising from whether a number is an integer.

1. $T(n) = 2T(n/3) + 5n$

   Solution: $\sum_{k=0}^{\log_3 n} (\frac{2}{3})^k \cdot 5n$

   then it is $\Theta(n)$

2. An algorithm A takes Θ(n log n) time to partition the input into 5 sub-problems of size n/5 each and then recursively runs itself on 3 of those subproblems. Describe the recurrence relation for the run-time T (n) of A and find its asymptotic order of growth.

   Solution: this is $\Theta(n \log n)$

3. $T(n) = T(3n/5) + T(4n/5)(We have T(1) = 1)$

   Solution: this is $\Theta(n^2)$

4. $T(n) = T(\sqrt{n}) + 1$

   Solution: this is $\Theta(loglogn)$