CS 170 Homework 2

Due **2025/2/23, at 10:00 am (grace period until 11:59pm)**

1 Study Group

List the names and SIDs of the members in your study group. If you have no collaborators, you must explicitly write “none”.

Solution: I worked on this homework with the following collaborators:

- none,which is only me,Sillycheese

2 Median of Medians

(a) (2 points) Let us see an example of QuickSelect in action. Suppose you always pick the first element as the pivot. Compute QuickSelect(A, 6) for the following array:

Solution: [78,13,97,45,48,26,85,100,78] k=4

[13,45,48,26] k=4

[45,48,26] k=3

[48] k=1

(b) (2 points) Consider the array

shuffled into some arbitrary order. What is the worst-case runtime of QuickSelect(A, n/2) in terms of n? Construct a sequence of ‘bad’ pivot choices that achieves this worst-case runtime.

Solution:

$$\frac{n(n+1)}{2} = O(n^2)$$

(c) (3 points) Let p be the pivot chosen by DeterministicSelect on A. Show that at least 3n/10 elements in A are less than or equal to p, and that at least 3n/10 elements are greater than or equal p.

Solution: firstly,at least half of medians array is less than p.so it is $n/10$. then go back to every median’s subarray and add them.It is $\frac{2n+n}{10}$(also need to add itself). so at least $\frac{3n}{10}$ elements in A are Less than or equal to p.

Same to the greater issue.

(d) In this problem, we will show that the worst-case runtime of DeterministicSelect(A, k) using the ‘Median of Medians’ strategy is O(n).

i. Find a recurrence relation for the time complexity of the algorithm, T (n).

Solution:

$$T(n) <= T(n/5) + T(7n/10) + O(n)$$

$T(n/5)$ is to find the medians in each subarray.

$T(7n/10)$ is the at most size of partition size,which is used to recursive call to the DC

$O(n)$ is just build array and partition time.

ii. Use the recurrence relation to show that, for some sufficently large c 0, the inequality T (n) c · n always holds.

Solution:

$$T(n) \leq c(n/5) + c(7n/10) + O(n) \leq c$$