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A computer based facial flaps simulator using projective dynamics

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ABSTRACT

Background and Objectives: Interactive surgical simulation using the finite element method to model human skin mechanics has been an elusive goal. Mass-spring networks, while fast, do not provide the required accuracy.
Methods: This paper presents an interactive, cognitive, facial flaps simulator based on a *projective dynamics* computational framework. Projective dynamics is able to generate rapid, stable results following changes to the facial soft tissues created by the surgeon, even in the face of sudden increases in skin resistance as its stretch limit is reached or collision between tissues occurs. Our prior work with the finite element method had been hampered by these considerations. Surgical tools are provided for; skin incision, undermining, deep tissue cutting, and excision. A spring-like “skin hook” is used for retraction. Spring-based sutures can be placed individually or automatically placed as a row between cardinal sutures.
Results: Examples of an Abbe/Estlander lip reconstruction, a paramedian forehead flap to the nose, a retroauricular flap reconstruction of the external ear, and a cervico-facial flap reconstruction of a cheek defect are presented.
Conclusions: Projective dynamics has significant advantages over mass-spring and finite element methods as the physics backbone for interactive soft tissue surgical simulation.

1. Introduction

For decades the plastic surgery community has been seeking a simulator to teach local flap closure of facial skin defects. These efforts have taken two paths; development of realistic physical models and computer-based simulation. Computer based simulators thus far have used mass spring networks and finite element software. Mass spring networks, while easy to implement and fast, are not able to generate realistic results for large deformations. The industry standard for solid simulation has been the finite element method. The first offline facial flap simulations were introduced by Pieper, Laub, and Rosen in 1995[1]. Since then, a number of offline finite element flap simulations have been reported [1–8]. Recently our group presented a novel finite element implementation of a flap simulator which allowed real time performance in a surgical simulation environment [9,10]. Unfortunately, the severely nonlinear stress-strain characteristics of human skin and the nonlinearities incurred by contact/collision processing made realistic flap simulation difficult to achieve at interactive frame rates.

In 2014 Bouaziz et al. [11] introduced *projective dynamics* as an alternative to the standard nonlinear solvers (based on the Newton-Raphson method) for finite element models in real time physics simulation. The principal difference is the standard Newton-Raphson solver combined with a solver for the linearized equations, such as Conjugate Gradients, are replaced by a weighted least squares solver. This numerical simulation scheme accommodates fast frame updates by leveraging precomputation after each surgical change to the model. Fast, consistent, and robust solutions are produced, even in the presence of severe nonlinear skin elasticity properties and collisions between tissues. These characteristics prompted its use in this application.

We present a cognitive facial flaps simulator that produces realistic results at interactive frame rates. It gives the surgeon the ability to make skin incisions, undermine flaps, do skin hook retraction, place sutures and make deep tissue cuts with minimal response time.

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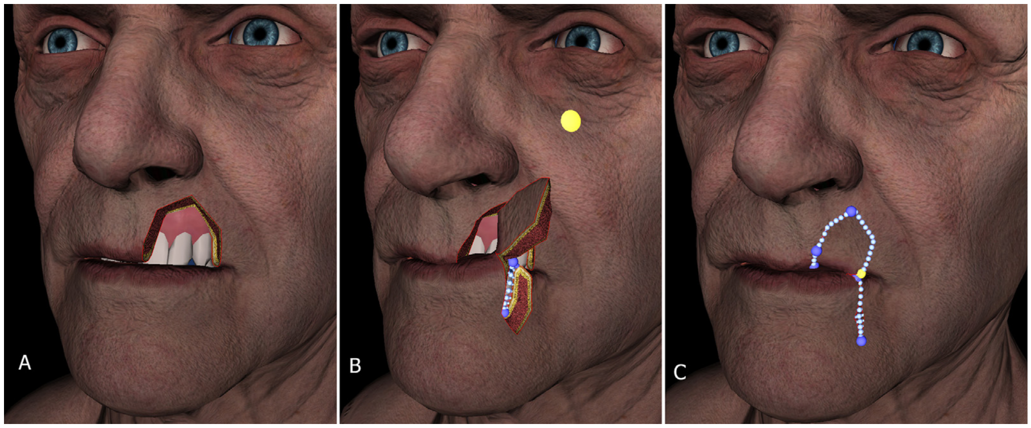

Fig. 1. The first stage of an Abbe/Estlander reconstruction of an upper lip defect is demonstrated. The ***deep cut*** tool is used to make full thickness incisions through the lip. A small bridge of tissue containing the inferior labial artery is preserved to maintain blood supply until the bridge can divided at a second stage.

2. Materials and methods

A personal computer required to run this software must have an Intel processor with an avx512 instruction set (Xeon Phi and above) with an NVIDIA graphics card (GeForce and above) and at least 8GB of RAM. Programming was done using the C++ and CUDA^{TM} languages. This software currently runs on Linux and Windows 10. A commercial model of an aging face (TurboSquid^{TM}) was altered with Blender software [12] to create a closed manifold surface, solid model of facial soft tissue anchored to underlying bone. Surgical tools were developed to allow skin incisions with undermining and deep incisions to alter the model. The skin subsurface model is precomputed offline, such that each surface triangle has an homologous, non-inverted subsurface triangle. This is done by perpendicular projection of a surface triangle down the desired skin depth to form a triangular prism composed of three tetrahedra. Border conditions for the model are set and all prisms assembled in three dimensions. Projective dynamics is used to solve the system to generate a subsurface. Any inverted tetrahedra at solution have their stiffness increased. The process is repeated iteratively until a non-inverting, triangular subsurface is produced. This subsurface then becomes part of the model. In this way parametric incision specifications made on the skin surface are duplicated on the subsurface to define a skin incision. A flood-fill undermine operation adjacent to an incision edge defined on the top side is simply duplicated in the subsurface to define a skin flap. A deep cut operator is implemented as a directed graph through a series of bilinear surfaces with user specified incision normal vectors at each point. This incision-undermine system produces an orientable, closed, manifold surface at each step in the surgical process for solids modeling.

The model is embedded in a tetrahedral mesh [13] which is presented to the physics engine after each surgical change to the model. The group at the University of Wisconsin developed custom projective dynamics software which allow half a million tetrahedra to be processed at interactive frame rates [14]. Forces are applied to the model using skin hooks and sutures implemented as springs, which are fully incorporated into the Projective Dynamics scheme. Tetrahedral elastic behavior was designed to closely match the distinctly nonlinear stress/strain characteristics measured from human skin [15–25].

A limited realtime collision capability is provided between the inner surface of the lips and the teeth as well as the undersurface of the eyelids with the globes. Collisions are also provided between the undersurface of flaps and the deep bed created by undermining. Technical details of this approach to collisions are provided elsewhere [14]. This program is rendered as a clickable web link available at: https://github.com/uwgraphics/SkinFlaps.

3. Results

Illustrations of the use of this simulator are given in the accompanying figures. An Abbe/Estlander flap reconstruction of an upper lip defect using the deep cut tool is presented in Fig. 1. A paramedian forehead flap reconstruction of a nasal tip skin defect using skin incisions and undermining is given in Fig. 2. A postauricular flap reconstruction of an auricle defect, using both incision methods, is shown in Fig. 3. Video, Supplemental Digital Content 1 demonstrates a cervico-facial rotation flap closure of a cheek defect as well as several features of the simulator. Initially the flap is too small to close the defect demonstrating the nonlinear elasticity characteristics of skin (i.e. “the flap won’t reach”). The high density, tetrahedral embedding of the model is demonstrated. The incision is then extended and more extensive undermining done to allow closure of the defect using the suture tool.

4. Discussion

Physical models of the skin have long been used to illustrate skin flap concepts to the student surgeon. These are usually porcine or cadaver skin or some type of silicone overlayed on a substrate [26–32]. Recently 3D printers have been used to generate a facial flaps simulation using this approach [33]. Physical models have the advantage that they are the most realistic and natural to the student surgeon. The disadvantage is that they can be expensive and usually are single use. Each simulation can also be time consuming to perform. For the plastic surgery student, the tactile experience of incision and suturing should already have been developed as a prerequisite to plastic surgery training. As a result, the psychomotor advantages of physical models are of less importance at this level of training. This is fortunate, as the refresh rate required for acceptable haptics (300 – 1000 Hz) [34] are orders of magnitude beyond the refresh rate of FEM or PD physics.

Computer based simulations and are of two general types; psychomotor and cognitive. Psychomotor simulators have a rich history [28,35,36]. They attempt to reproduce the “look and feel” of performing the surgery. As such they usually employ 3D virtual reality display systems and haptic feedback to reproduce the tactile experience of the simulated procedure. Computer based psychomotor simulation can be expected to compete with physical model simulation for the foreseeable future.

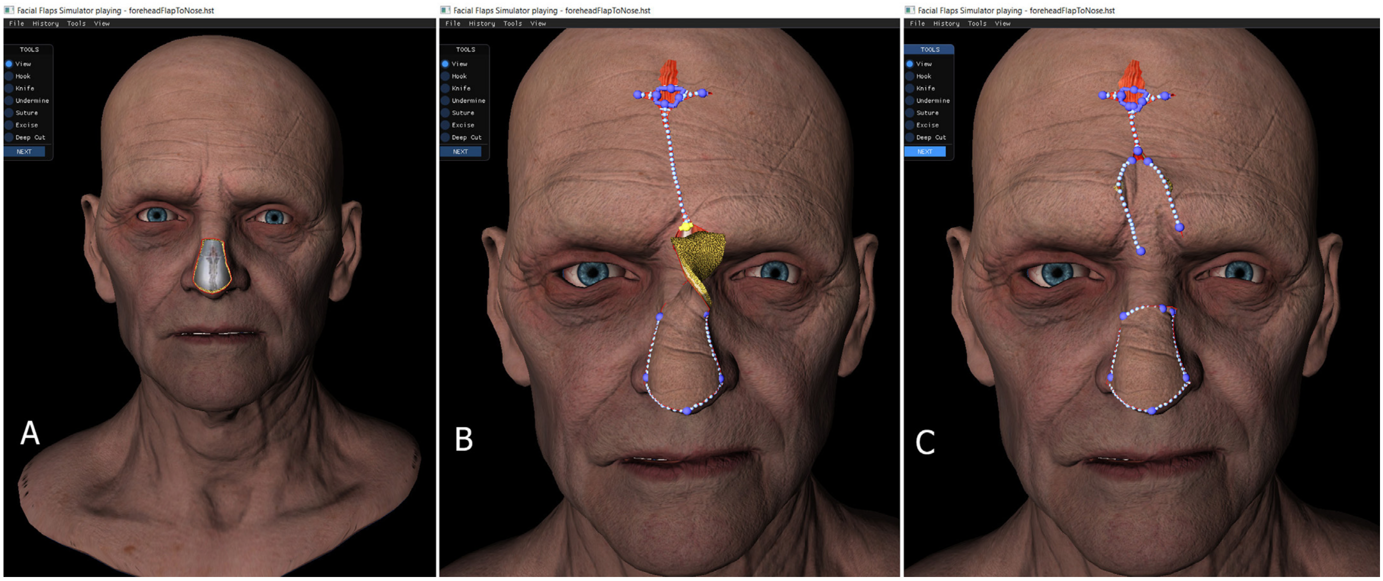

Fig. 2. A skin defect of the nasal tip is closed with a paramedian forehead flap using the ***skin incision*** and ***undermine*** tools. At the conclusion of the first stage B) the forehead is closed. Note the often experienced, excess tension at the top of the forehead wound making closure difficult. At the completion of the second stage C) the flap has been divided and the unused basal pedicle is returned to the lower forehead.

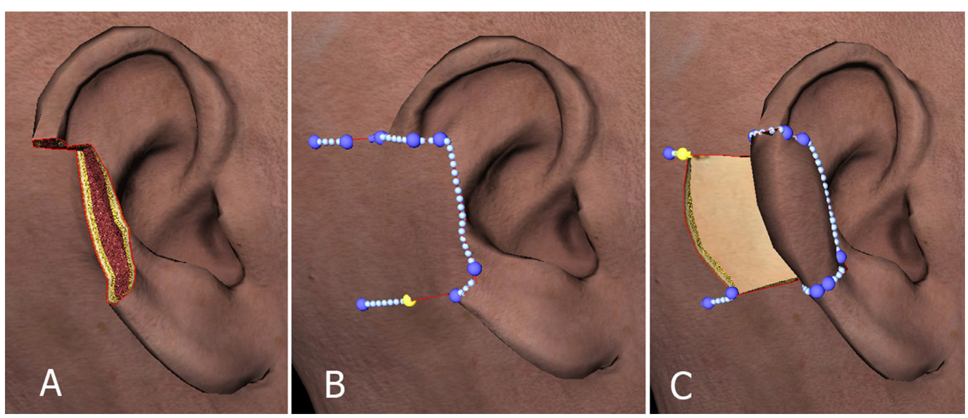

Fig. 3. A full thickness defect of the auricle is created with the ***deep cut*** tool (A). A post-auricular flap is elevated and sutured into the antero-lateral defect (B). At the second stage (C) the flap is divided and wrapped around itself and sutured posteriorly. The largely unseen postauricular defect can be skin grafted.

The simulator reported in this paper is of the cognitive type. The objective of a cognitive simulator is to explore what to do, rather than how it feels to do it. Skin incisions are specified by simply connecting points on the surface. Only deep tissue cuts require incision angulation information to be provided. A border line connecting two incision points is all that is required to undermine a skin flap. Sutures simply connect two incision points (or an automatically connected row between two cardinal sutures for speed). The only retraction tool provided is a spring-like “skin hook”.

The goal in a cognitive simulator is to provide as accurate a simulation outcome as possible. Reproducing realistic skin and soft tissue responses to surgical forces is critical. Skin has a remarkably nonlinear response to strain [15–25]. At low strain, skin is soft and pliable. At high strain the collagen network within the skin is pulled to its limit and the resistance to further stretching increases dramatically. In surgical parlance this is often expressed as “the flap won’t reach”. In the simulator reported in this paper, if this stretch limit is exceeded, sutures will simply not close the wound. Similarly, skin hook “springs” may be retracted to any point, but if the tissue stretch limit is exceeded, the hook spring simply stretches and the tissue does not follow.

Computational physics strategies attempting to reproduce realistic tissue response have a long history. The first applied, and simplest, approach uses a mass-spring network. Montgomery and Schendel reported a cleft lip simulator using this approach [37,38]. A flap simulation application of planar and facial flaps has recently been introduced running on an iPad or iPhone using a mass-spring model [39]. Based on prior 3D animation work [40–42], Cutting and Oliker developed a cleft lip simulator using a mass-spring approach [42]. This was found by the authors not to produce sufficiently realistic results. In a tetrahedral spring model if one of its points is compressed down near the center of the other three, the force exerted against the first point actually decreases. With further compression the tetrahedron will quickly move into a stable inversion of its original shape. This can in no way mimic normal tissue response.

The industry standard for modeling solid elastic behavior is the finite element (hereafter FE) method [43]. Pieper, Laub and Rosen were the first to present an offline facial flaps simulation using FE [44]. Robust modeling of tetrahedral elements with realistic stress/strain responses and inversion handling has been described by Irving, Teran and Fedkiw [45]. Offline FE simulation of complex flap designs and breast reduction has been reported [1–8, 46,47]. Due to the high computational requirements of the method, FE had not been used for interactive simulation. In 2016 advances in computational FE made possible interactive flap simulation using a linear elastic model of skin behavior [9,10]. Subsequent work trying to model the distinctly nonlinear elastic behavior of skin at high strain in an interactive simulator has proved difficult. Finite Element Methods yield a discretization of the governing laws of elasticity, typically resulting in a nonlinear system of equations (or in the case of quasistatic simulation, equivalently, a nonlinear energy minimization problem). Newton-Raphson methods are the commonplace solver for such problems, but are plagued by weak stability guarantees, often requiring heuristic and convergence-deteriorating line search safeguards. These hazards are even more pronounced in our surgical simulations that feature pronounced nonlinearities due to biphasic tissue elastic response and contact events.

In 2014 Bouaziz et al. [11] introduced the ***projective dynamics*** (hereafter PD) approach to real time physics simulation. PD shares many features in common with Newton-type methods for FE, including the underlying discretization. The underlying discretization used in PD is shared with the standard Finite Element Method, with the caveat that the material laws that can be supported within the PD paradigm are limited to corotated elasticity [48] and its close variants The principal difference is replacing the conjugate gradient solver with a weighted least squares solver, for which the Hessian matrix can be assembled and factorized ahead of time. The net effect is akin to replacing the “true” Newton iteration with a Modified Newton loop, that can afford to use a constant approximation to the Hessian while safeguarding rigorous stability and convergence guarantees. This allows precomputation of much of the solution only at times of change in the surgical topology (i.e. new flaps, deep cuts, hooks or sutures). All intermediate changes can be done at interactive frame rates. Most important is that the solutions are robust and do not require a line search. For this reason, the markedly nonlinear strain response of skin is modeled in a stable manner. The implementation in this paper allows the facial soft tissue to be embedded in over a half million tetrahedra in an interactive environment on a personal computer [14]. One limitation of PD is its natural affinity to the corotated elasticity constitutive model, and a few embellishments that remain compatible with this paradigm; thankfully, strain-limiting in the form of a biphasic elastic response to tissue expansion remains compatible with this paradigm and makes the aggregate material model quite adequate for the cognitive surgical tasks at hand.

The only difficulty with PD has been collision response. Since tissue collisions are evanescent events, they become involved in the aforementioned precomputation step, as their robust handling within the PD paradigm requires the matrix that is nominally constructed as precomputation to sustain some (local) updates near the collision site. By limiting the collision set to the inner lips with the teeth, the eyelids with the globes, and the underside of flaps with the undermined bed, a novel approach to collision handling [14] has allowed limited collisions to be modeled at interactive frame rates.

The mechanical properties of skin are patient, age, site specific, anisotropic, and visco-elasto-plastic. These properties are of varying importance surgically. The hysteresis in the stress strain curve of skin is of little surgical consequence. The mild to moderate anisotropy in facial skin is somewhat important surgically. The most important parameter to the surgeon is the strain at which the skin goes from stretching easily to stretching no further. This is at the foot of region 3 of the stress-strain curve [16]. This parameter has been shown to vary significantly with age, anatomic site, and between individuals, both experimentally [15,16,21,23,25] and over the surgical author’s career experience. In a single surgical procedure limited to a few hours the plastic nature of skin can be largely discounted, although a very small amount of “tissue creep” can be observed. Over a much longer time frame, measured in weeks, the plastic nature of skin can be quite pronounced, as dramatically demonstrated by tissue expanders [49,50].

While this paper focuses on the use of PD in teaching facial flap concepts, it is only at the beginning of its application to soft tissue surgery. Deeper open surgery problems with multiple different tissues and complex anatomic models become tractable (e.g. hand, breast, extremity, face, and thorax). One obvious extension is modeling patient specific facial skin defects and allowing the surgeon to develop a customized operative strategy for his/her patient. To become truly accurate, patient and site-specific data should be collected and input into the simulator [16]. Initial simulator output would benefit from offline, more rigorous, nonlinear FEM evaluation [49] in producing a definitive, patient specific, surgical design.

5. Conclusions

We present a computer-based, cognitive facial flap simulator using projective dynamics as a computational foundation. The advantages of projective dynamics over mass-spring networks and the traditional finite element method in modeling the nonlinear elastic properties of skin at interactive frame rates are demonstrated and discussed. A limited collision response is also demonstrated using this method. Several clinical examples are presented.

Authors roles

Qisi Wang, BS, Yutian Tao, BS and Eftychios D. Sifakis, PhD designed and programmed the projective dynamics based, surgical physics library that underlies the simulator presented in this paper.

Court B. Cutting, MD designed and programmed the surgical aspects of the software including surgical tools, graphics and model preparation.

Declaration of Competing Interest

The authors have no financial or personal relationships with people or organizations that could inappropriately influence (bias) this work.

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.cmpb.2022.106730.

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