EMA 605 Project Progress Report

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For project option 2, FEM with quadratic triangle elements will be used to determine the stress concentration factor for a hole in an infinite plate under uniform tension.

Problem Description

As shown in **Figure 1**[1], a thin rectangular plate with a hole of radius a is under a uniaxial tension represented by σ_∞.

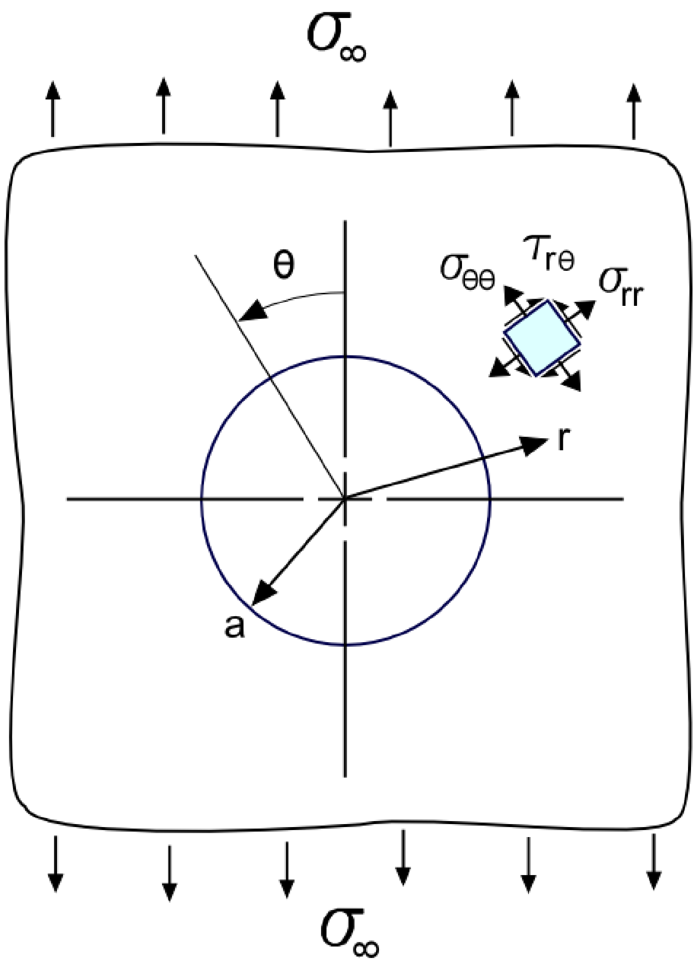

Figure 1. Rectangular plate with a circular hole under uniaxial tension

From the Kirsch equations[2], the analytical solution of the stress in the problem in polar coordinate system is

$$\sigma_{rr} = \frac{\sigma_\infty}{2}\left(1-\left(\frac{a}{r}\right)^2\right) + \frac{\sigma_\infty}{2}\left(1-4\left(\frac{a}{r}\right)^2+3\left(\frac{a}{r}\right)^4\right)\cos 2\theta$$

$$\sigma_{\theta\theta} = \frac{\sigma_\infty}{2}\left(1+\left(\frac{a}{r}\right)^2\right) - \frac{\sigma_\infty}{2}\left(1+3\left(\frac{a}{r}\right)^4\right)\cos 2\theta$$

$$\tau_{r\theta} = -\frac{\sigma_\infty}{2}\left(1+2\left(\frac{a}{r}\right)^2-3\left(\frac{a}{r}\right)^4\right)\sin 2\theta$$

Where, as indicated in **Figure 1.** r is the radial coordinate and $\theta = 0$ aligns with the loading direction.

Element Description

The element to be utilized to solve this problem is quadratic triangles[3] (LST).

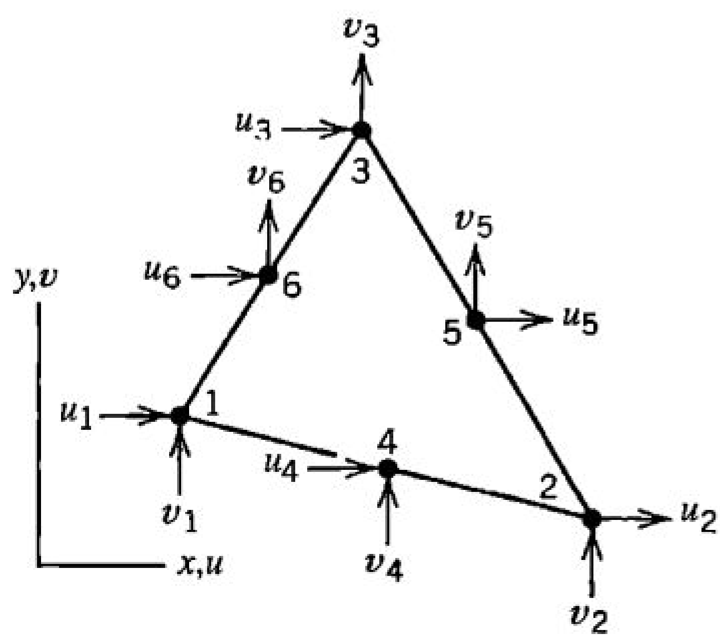

Figure 2. Nodes and DOF's of a quadratic triangle.

The displacement field on an element can thus be represented by

$u = a_1 + a_2x + a_3y + a_4x^2 + a_5xy + a_6y^2$

$v = a_7 + a_8x + a_9y + a_{10}x^2 + a_{11}xy + a_{12}y^2$

And the element strains can be computed as

$\varepsilon_x = a_2 + 2a_4x + a_5y$
$\varepsilon_y = a_9 + 2a_{12}y + a_{11}x$
$\gamma_{xy} = (a_3 + a_8) + (a_5 + 2a_{10})x + (2a_6 + a_{11})y$

Reference

1. Stress concentration at holes: http://www.fracturemechanics.org/hole.html
2. Kirsch equations: https://en.wikipedia.org/wiki/Kirsch_equations
3. Quadratic Triangle: CMPW e4, §3.5