Kartesische Darstellung

$z = x + iy$

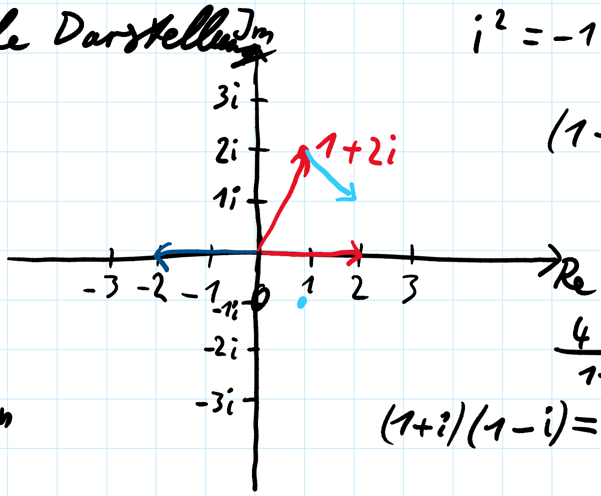
$\bar{z} = x - iy$

$|z| = \sqrt{x^2+y^2}$

$Re\{z\} = x$

$Im\{z\} = y$

$z^n = (x+iy)^n$



$i^2 = -1 \Leftrightarrow i = \sqrt{-1}$

$(1+2i) + (1-1i) = 2 + 1i$

$\frac{1}{i} \cdot \frac{i}{i} = \frac{i}{i^2} = -i$

$\frac{4+4i}{1+i} \cdot \frac{1-i}{1-i} = \frac{(4+4i)(1-i)}{2}$

$(1+i)(1-i) = 1 - i^2 = 1+1 = 2$

$\sqrt{-5} = \sqrt{5} \cdot \sqrt{-1} = i \cdot \sqrt{5}$

1. Verständnis kartesischer Darstellung

Forme die folgenden Ausdrücke sinnvoll um, indem du $z = x + iy$ als Darstellungsform nutzt.

1. $z + \bar{z} = (x+iy) + (x-iy) = 2x$
2. $z - \bar{z} = (x+iy) - (x-iy) = 2iy$
3. $z\bar{z} = (x+iy)(x-iy) = x^2 - (iy)^2 = x^2 + y^2 = |z|^2$
4. $iz = i(x+iy) = -y + ix$

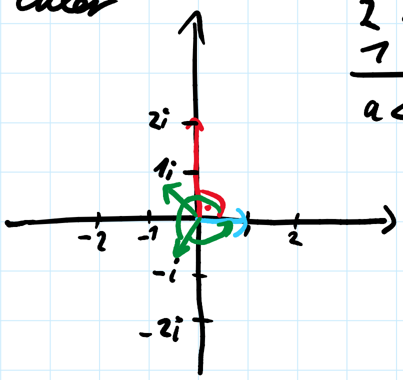
Polar / Euler

$|z| \cdot e^{i\varphi}$

$z = r \cdot e^{i\varphi}$

$w = R \cdot e^{i\psi}$

$\bar{z} = r\, e^{-i\varphi}$



$2 \angle 90° \left(\frac{\pi}{2}\right) = 2i$

$1 \angle 2\pi = 1 \angle 0$

$a \angle \pi$

$z \cdot w = r \cdot e^{i\varphi} \cdot R \cdot e^{i\psi} = rR \cdot e^{i(\varphi+\psi)}$

$z^n = (r \cdot e^{i\varphi})^n = r^n \cdot e^{in\varphi}$

$\frac{z}{w} = \frac{r e^{i\varphi}}{R e^{i\psi}} = \frac{r}{R} \cdot e^{i\varphi} \cdot e^{-i\psi} = \frac{r}{R} \cdot e^{i(\varphi-\psi)}$

$i^3 = i^2 \cdot i = -i$

$z^3 = e^{3i\varphi} = e^{0i} = e^{2\pi i}$

$3\varphi = 2\pi \quad |:3$

$\varphi = \frac{2\pi}{3} = 120°$

$\sqrt{1} = x \Leftrightarrow x_1 = 1$

$x_2 = -1$

$\sqrt[3]{1} = x$

$\Leftrightarrow x^3 = 1 \Rightarrow x_1 = 1$

$\Rightarrow x_2 = 1 \angle \frac{2}{3}\pi$

$x_3 = 1 \angle \frac{4}{3}\pi$

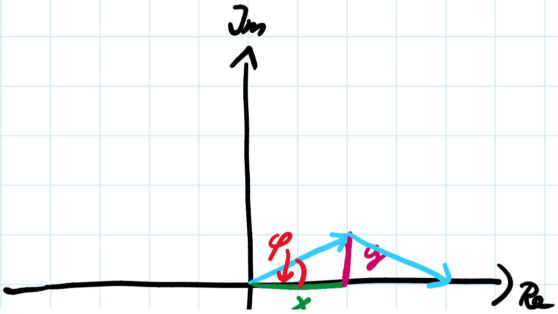
$\sqrt{1} = x$

$\Leftrightarrow x^2 = 1 \Rightarrow x_1 = 1$

$x_2 = -1$

$x_3 = -(-1)$

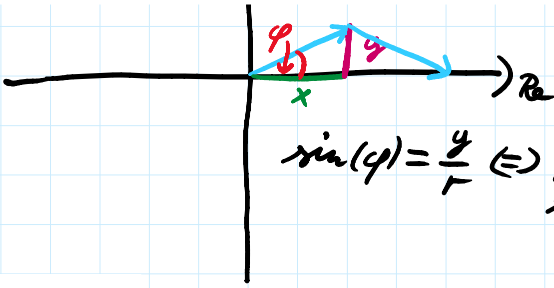
$x_4 = -(-(-1))$



$2i\pi$ — $1\,0\,(\;)$

$x_4 = -(-(-1))$

$$\cancel{e^{2i\pi} = 1 \quad | \ln(\dots)}$$
$$\cancel{2i\pi = \ln(1) = 0}$$



$\sin(\varphi) = \frac{y}{r} \Leftrightarrow y = r\sin(\varphi)$

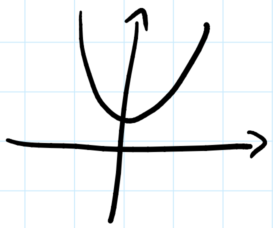
$x = r\cos(\varphi)$

3. Verständnis polarer Darstellung

Forme die folgenden Ausdrücke sinnvoll um. $z \in \mathbb{C}$

$i = e^{\frac{\pi}{2}i}$

1. $iz = e^{\frac{\pi}{2}i} \cdot r e^{i\varphi} = r e^{i(\varphi + \frac{\pi}{2})}$
2. $r(e^{i\varphi} + \frac{1}{e^{i\varphi}}) = r(e^{i\varphi} + e^{-i\varphi}) = z + \bar{z} = 2x$
3. $re^{i(\varphi + \pi)} = -z$
4. $z^n, n \in \mathbb{N} = r^n \cdot e^{in\varphi}$
5. $e^z = e^{x+iy} = e^x \cdot e^{iy}$
6. $z\bar{z} = r \cdot e^{i\varphi} \cdot r \cdot e^{-i\varphi} = r^2 \cdot e^{i\varphi - i\varphi} = r^2 = x^2 + y^2$



$$x^2 + 1 = 0$$
$$\Leftrightarrow \quad x^2 = -1$$
$$x = \pm\sqrt{-1}$$
$$x = \pm i$$