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Evaluation of focus measures for the autofocus of line scan cameras

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ABSTRACT

Plenty of focus measures have been proposed and researched in the past few years. However, little attention is paid to their performance in the autofocus of line scan cameras. This paper investigates sixteen typical focus measures, and their performance is compared for the autofocus of line scan cameras. A comprehensive evaluation method is proposed to assess the performance of focus measures. Subjective evaluation is firstly utilized to exclude the focus measures of poor performance. Then, objective evaluation which includes seven criteria is utilized to rank the rest focus measures. Scale factors are introduced for the evaluation of focus measures in different search algorithms and focus styles. Experimental results show that Tenengrad provides the best performance in both global-based search and local-based search in the autofocus with auxiliary focus sheet. In the autofocus directly on the object when the object is variable, autocorrelation provides the best performance in global-based search while Tenengrad provides the best performance in local-based search. In the autofocus directly on the invariable object, the best focus measure is related to image content, and it should be selected in field test.

1. Introduction

Line scan cameras are widely used in many applications such as digital protection of cultural relics and real-time measurement systems [1]. Focus level is a key factor of the imaging system which determines the quality of images and the accuracy of measurement system. A well focused image contains more details and it is good for computer vision inspection. On the contrary, a defocused image is blurred and the measurement accuracy is difficult to ensure.

Focus measures are used to measure the focus level of an image. They have been used in many fields such as multi-focus image fusion [2], shape from focus [3], autofocus [4], etc. Some efforts have been made to test the performance of different focus measures. Huang and Jing [5] compared six spatial domain focus measures for multi-focus image fusion, and experimental results showed that sum of modified Laplacian provided better performance than others. Aslantas and Kurban [6] compared three groups of focus measures with frequency selective weighted median filter (FSWM) for the fusion of multi-focus noisy images, and the experimental results showed that FSWM provided better performance than others. Pertuz et al. [3] compared thirty-six focus measures for shape form focus, and experimental results showed that Laplacian based focus measures had the best overall performance at normal imaging conditions. Sun et al. [7] compared eighteen focus measures for the autofocus of microscopy, and experimental results showed that normalized variance provided the

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0030-4026/

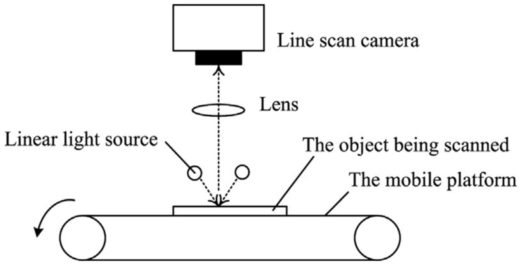


Fig. 1. The typical imaging system of line scan cameras.

best overall performance. Xu et al. [8] compared six focus measures in passive autofocus systems for low contrast images, and experimental results showed that Tenengrad and contrast measurement adaptive to noise yielded the best performance. Shih [9] compared seven focus measures in the autofocus on a variety of scenes, and experimental results showed that Tenengrad and spatial frequency yielded the best results in terms of accuracy and unimodality. Liu et al. [10] compared sixteen focus measures in the autofocus for automated microscopic evaluation of blood smear and pap smear, and experimental results demonstrated that variance provided the best overall performance. Lee et al. [11] compared reduced energy ratio with six other focus measures for the autofocus of digital camera, and concluded that the reduced energy ratio showed the best performance. However, the above comparisons are made for array cameras. In literatures, little attention is paid to the performance of focus measures in the autofocus of line scan cameras.

There are mainly two differences between the autofocus of line scan cameras and the autofocus of array cameras. Firstly, the imaging of line scan cameras is one-dimensional while that of array cameras is two-dimensional. Therefore, the focus measure functions for the autofocus of line scan cameras are one-dimensional while those for array cameras are two-dimensional. It can lead to the differences of performance of focus measures between line scan cameras and array cameras. For example, Tenengrad focus measure and Brenner focus measure are two focus measures of different performance for array cameras (see Eqs. (1) and (2)), but they are the same for line scan cameras (see Eq. (3)). The differences of performance of focus measures mean that the comparison of focus measures for array cameras in literatures cannot represent the performance of them in line scan cameras. It is necessary to test the performance of focus measures for the autofocus of line scan cameras. Secondly, the popular focus mode for the autofocus of line scan cameras is different from that of array cameras. There are three focus modes to obtain a focused image [12]: moving the sensor, moving the lens and moving the object. Moving the lens has been widely used in the autofocus of array cameras, but it is not the popular focus mode for line scan cameras. In the applications of line scan cameras such as digital protection of cultural relics and real-time measurement systems, usually more than one object needs to be captured. The imaging magnification is required to be a constant in favor of image stitching or continuous measurement. In the focus mode of moving the lens, the image distance is varying in the process of autofocus. It is difficult to make the imaging magnification consistent in the autofocus on different objects. In the focus mode of moving the object, the image distance is fixed throughout the process of autofocus. The object distance can be determined by Gaussian lens law, and it is a constant when the focal length is fixed. The imaging magnification can be set to a constant in this focus mode. Therefore, moving the object is the best focus mode for the autofocus of line scan cameras. In practice, moving the imaging system is usually adopted as the focus mode in the autofocus of line scan cameras to solve the problem when the object is immovable. There are some disadvantages in the focus mode of moving the imaging system. This focus mode can cause the increasing or decreasing of image content as moving the imaging system. Besides, the luminance on the image sensor is varying with the motion of the imaging system. The variation of image content and luminance will affect the performance of focus measures. It is necessary to assess the performance of different focus measures for the autofocus of line scan cameras.

The typical imaging system of line scan cameras is shown in Fig. 1. It is a close range imaging system and could capture high resolution images. The depth of field of the imaging system is short, which leads to the difficulty of the autofocus of line scan cameras. This paper discusses the evaluation of focus measures for the autofocus of line scan cameras in the system.

It is well known that focus measure and search algorithm are the two key factors which determine the autofocus result. A favorable autofocus result arises from the overall performance of the focus measure and the search algorithm. Since the evaluation of focus measures is aimed to obtain the best autofocus result, the evaluation of focus measures should be connected with the characteristics of search algorithms. Besides, the evaluation of focus measures should be connected with the autofocus styles of line scan cameras. There are two common styles in the autofocus of line scan cameras: autofocus with auxiliary focus sheet and autofocus directly on the object. The auxiliary focus sheet contains rich image content which are beneficial to focusing [13]. The imaging system can easily obtain sharp image in this focus style. However, the focus result is not accurate because of the thickness of focus sheet. It is often used in the digital protection of culture relics. Compared with the autofocus with auxiliary focus sheet, the autofocus directly on the object can obtain accurate focus result. It is usually used in the measurement system of line scan cameras. The defect of this focus style is that it is difficult to implement autofocus on the object with less image content. In the autofocus with auxiliary focus sheet, the focus scene is single and the performance of focus measures is stable. There is no need to consider the stability of focus measures in the evaluation of focus measures. In the autofocus directly on the object, the performance of a focus measure varies with image content. When the object is variable, it is necessary to consider the stability of focus measures in the evaluation of focus measures.

This paper investigates sixteen typical focus measures and analyzes their performance in the autofocus of line scan cameras. A comprehensive evaluation method is proposed to assess the performance of focus measures. In this method, subjective evaluation is firstly utilized to exclude the focus measures of poor performance. Then, objective evaluation which includes seven criteria is utilized to rank the rest focus measures. The best focus measures in different search algorithms and autofocus styles are selected for the autofocus of line scan cameras.

The paper is organized as follows. In Section 2, our comprehensive evaluation method is introduced. Then, focus measures are tested in the experiments for the autofocus of line scan cameras in Section 3. Finally, conclusions are drawn in Section 4.

2. Focus measures

Focus measures can be divided into six families [3]: gradient based focus measures, Laplacian based focus measures, wavelet based focus measures, statistics based focus measures, discrete cosine transform based focus measures, and miscellaneous focus measures. In this paper, sixteen typical focus measures are investigated and compared for the autofocus of line scan cameras.

2.1. Gradient based focus measures

Gradient based focus measures are based on the first derivative of the image. Two popular gradient based focus measures are investigated in this paper.

(1) Tenengrad (Ten) [14]. Tenengrad is a gradient based focus measure which has been widely used in autofocus. It can be expressed by [6]:

$$F_{Ten} = \sum_{i=1}^{M-2}\sum_{j=1}^{N-2}\left[s_x^2\ i,j + s_x^2\ i,j\right] \tag{1}$$

where $s_x(i,j)$ and $s_y(i,j)$ are respectively the convolution results of the image $f(i,j)$ with Sobel operators on the horizontal and vertical directions; M and N are respectively the image height and width.

Brenner [15] focus measure can be expressed by:

$$F_{Bre} = \sum_{i=0}^{M-3}\sum_{j=0}^{N-1}[f(i,j) - f(i+2,j)]^2 \tag{2}$$

It can be seen from Eqs. (1) and (2) that Tenengrad and Brenner are two different focus measures for array cameras. But they are the same focus measure for line scan cameras, which can be expressed by:

$$F = \sum_{i=0}^{M-3}[f(i) - f(i+2)]^2 \tag{3}$$

(2) Spatial frequency (SF) [16]. SF is a gradient based focus measure which is similar with Tenengrad. It uses the first difference of the image instead of applying Sobel operators [6]. The SF focus measure for line scan cameras can be expressed by:

$$F_{SF} = \sqrt{\frac{1}{M}\sum_{i=1}^{M-1}[f(i) - f(i-1)]^2} \tag{4}$$

2.2. Laplacian based focus measures

Laplacian based focus measures are based on Laplacian operators or the second derivative of the image. Two typical Laplacian based focus measures are investigated in this paper.

(3) Sum of Modified Laplacian (SML) [17]. SML is a popular focus measure which has been widely used in the fields of multi-focus image fusion, shape from focus, autofocus, etc. The SML focus measure for line scan cameras can be expressed by:

$$F_{SML} = \sum_{i=1}^{M-2}|2f\ i - f\ i-1 - f\ i+1\ | \tag{5}$$

(4) Energy of Laplacian (EOL) [18]. EOL is a Laplician based focus measure. The EOL focus measure for line scan cameras can be expressed by:

$$F_{EOL} = \sum_{i=1}^{M-2} [L\ i]^2 \tag{6}$$

where $L(i)$ is the convolution result of the image with Laplacian template.

2.3. Wavelet based focus measures

Wavelet based focus measures are based on the statistical properties of the discrete wavelet transform (DWT) of the image. Three common wavelet based focus measures are investigated in this paper.

(5) Sum of wavelet coefficients (SWC) [19]. SWC focus measure is defined as the sum of wavelet coefficients of the detail sub-bands in the first level DWT. The SWC focus measure for line scan cameras can be expressed by:

$$F_{SWC} = \sum |W_{H1}(x)| \tag{7}$$

where $W_{H1}(x)$ is the detail sub-bands in the first level DWT.

(6) Variance of wavelet coefficients (VWC) [19]. Instead of sum of wavelet coefficients, VWC uses the variance of wavelet coefficients to measure the focus level. It can be expressed by:

$$F_{VWC} = \sum [W_{H1}(x) - \mu_{H1}]^2 \tag{8}$$

where μ_{H1} is the mean values of $W_{H1}(x)$.

(7) Ratio of wavelet coefficients (RWC) [20]. RWC measures the focus level by the ratio between the high and low frequency coefficients. The RWC focus measure for line scan cameras can be expressed by:

$$F_{SWC} = \frac{\sum W_{Hk}(x)^2}{\sum W_{Lk}(x)^2} \tag{9}$$

where $W_{Hk}(x)$ is the level-k detail sub-band, and $W_{Lk}(x)$ is the level-k approximation sub-band.

2.4. Statistics based focus measures

Statistics based focus measures utilize the statistics data of an image to measure the focus level. Three common statistics based focus measures are investigated in this paper.

(8) Absolute central moment (ACM) [21]. ACM is a statistics based focus measure which was proposed by Shirvaikar and it can be expressed by:

$$F_{ACM} = \sum_{k=0}^{L-1} |k - \mu| p_k \tag{10}$$

where L is the grey level number of the image; μ is the mean grey intensity of the image; p_k is the normalized grey level histogram.

(9) Variance (Var) [22]. Variance is a statistics based focus measure which has been widely used in both autofocus and shape from focus [3]. The Var focus measure for line scan cameras can be expressed by:

$$F_{Var} = \frac{1}{M} \sum_{i=0}^{M-1} [f(i) - \mu]^2 \tag{11}$$

(10) Entropy (En) [23]. Entropy measures the information quality of an image. It can be used to measure the focus level of an image. Entropy can be expressed by:

$$F_{En} = -\sum_{k=0}^{L-1} p_k \log_2 p_k \tag{12}$$

2.5. Discrete cosine transform based focus measures

This family focus measures are based on the discrete cosine transform of the image. Discrete cosine transform energy ratio focus measure and Bayes spectral entropy focus measure are investigated in this paper.

(11) Discrete cosine transform energy ratio (DER) [24]. DER is a typical discrete cosine transform based focus measure. It can be expressed by:

$$F_{DER} = \frac{E_{AC}}{E_{DC}} \tag{13}$$

where E_{AC} and E_{DC} are respectively the sum of square of the AC coefficients and the square of the DC coefficient.

(12) Bayes spectral entropy (BSE) [25]. BSE is a discrete cosine transform based focus measure which was proposed by Kristan et al. The BSE focus measure for line scan cameras can be expressed by:

$$F_{BSE} = 1 - \frac{\sum_{w \le t} |F_C(w)|^2}{\left[\sum_{w \le t} |F_C(w)|\right]^2} \tag{14}$$

where $F_C(w)$ is the discrete cosine transform of the image, and t is a predefined value for suppressing the noise.

2.6. Miscellaneous focus measures

This family focus measures utilize miscellaneous features of the image to measure the focus level. Typical features include eigenvalues, image curvature, autocorrelation, fast Fourier transform, etc.

(13) Eigenvalues (Eig) focus measure [26]. Eig is a focus measure which was proposed by Wee and Raramesran. It can be expressed by:

$$F_{Eig} = \text{trace}\left[\mathbf{\Lambda}\right] \tag{15}$$

where $\mathbf{\Lambda}$ is the diagonal eigenvalues matrix of the image and trace[·] is the sum of the diagonal elements.

(14) Image curvature (IC) [27]. IC is a focus measure which was proposed by Helmli and Scherer. It can be expressed by:

$$F_{IC} = |\boldsymbol{c_0}| + |\boldsymbol{c_1}| + |\boldsymbol{c_2}| + |\boldsymbol{c_3}| \tag{16}$$

where $\boldsymbol{c_0}$, $\boldsymbol{c_1}$, $\boldsymbol{c_2}$ and $\boldsymbol{c_3}$ are the vector of coefficients and they satisfy:

$$f(i,j) = c_0 i + c_1 j + c_2 i^2 + c_3 j^2 \tag{17}$$

The four vectors can be computed by:

$$\left.\begin{aligned} \boldsymbol{c_0} &= \boldsymbol{F} \times \mathrm{M}_1 \\ \boldsymbol{c_1} &= \boldsymbol{F} \times \boldsymbol{M_1^T} \\ \boldsymbol{c_2} &= \frac{3}{2}\boldsymbol{F} \times \boldsymbol{M_2} - \boldsymbol{F} \times \boldsymbol{M_2^T} \\ \boldsymbol{c_3} &= \frac{3}{2}\boldsymbol{F} \times \boldsymbol{M_2^T} - \boldsymbol{F} \times \boldsymbol{M_2} \end{aligned}\right\} \tag{18}$$

where:

$$\boldsymbol{M_1} = \frac{1}{6}\begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{M_2} = \frac{1}{5}\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \tag{19}$$

(15) Autocorrelation (AC) [28]. AC can be used to measure the focus level of an image. It can be expressed by:

$$F_{AC} = \sum_{i=0}^{M-3} f(i) f(i+1) - \sum_{i=0}^{M-3} f(i) f(i+2) \tag{20}$$

(16) Fast Fourier transform (FFT) [29]. FFT focus measure can be expressed by:

$$F_{FFT} = \frac{1}{L_0} \sum |[r(n)]^2 \varphi(n)| \tag{21}$$

where $r(n)$ is the modulus of FFT; $\varphi(n)$ is the phase of FFT; $L_0$is the length of input complex numbers.

3. Evaluation methodology

A comprehensive evaluation method is proposed to assess the performance of focus measures for the autofocus of line scan cameras. The method contains two steps: subjective evaluation and objective evaluation. Subjective evaluation is by means of observing the focus curve to find out whether the focus curve satisfies the focusing requirements. It is used to exclude the bad focus measures of poor performance. Objective evaluation utilizes quantitative analysis to assess the performance of focus measures. It is used to rank the good focus measures and select the best one for the autofocus of line scan cameras.

3.1. Subjective evaluation

An ideal focus measure should take the following observable features in the focus curve:

1) Monotonicity and unimodality.(单峰性和单调性) A focus measure is used to measure the focus level of an image. The values should increase as the focus level increases and decrease as the focus level decreases. It is required that the focus measure should have only one maximum which is located at the focused position.
2) Sharp focus peak. The sharper focus peak is beneficial for the search algorithm to locate the focused position accurately.
3) Low noise level. Noise can cause the misjudgment of focus measures. A good focus measure should have low noise level.
4) Independent of image content. A good focus measure should be independent of image content. That is, the focus measure should have good performance whether the image content is rich or poor.

As previously mentioned, the evaluation of focus measures should be connected with the characteristics of search algorithms. Search algorithms can be divided into two groups: global-based search and local-based search. Global-based search algorithms go through every possible position to search the maximum. The autofocus result is determined by the global property of focus measure values. Local flaw scarcely affects the autofocus result. Global-based search algorithms include global search algorithm, rule-based search algorithm, etc. Local-based search algorithms are based on the local property of focus measure values. They usually go through a portion of the possible positions. They are susceptible to local property of focus measure values, and local flaw will affect the autofocus result. Local-based search algorithms include Fibonacci search algorithm, hill-climbing search algorithm, etc. In global-based search algorithms, monotonicity and unimodality are not strictly required. The focus measure can be locally monotonous and multimodal. A good focus measure in global-based search is just required to have the global maximum and sharp peak in the focused position. In local-based search algorithms, monotonicity and unimodality are strictly required because local flaw in focus measure values will affect the autofocus result. The requirements 'low noise level' and 'independent of image content' are required in both global-based search and local-based search. Considering 'low noise level' and 'independent of image content' sometimes are not easy to observe in practice, we only use them as the auxiliary evaluation criterion in subjective evaluation. Further evaluation of noise level and independence of image content will be carried out in objective evaluation.

The basis of subjective evaluation is observing the focus curve to determine whether the focus measure satisfies the above requirements. The advantage of subjection evaluation is intuition and it is easy to exclude the focus measures of poor quality. The disadvantage is that it cannot accurately rank the good focus measures according to their performance. Therefore, we use subjective evaluation to exclude the bad focus measures which are not suitable for the autofocus of line scan cameras.

3.2. Objective evaluation

Seven criteria are used in objective evaluation to test the performance of focus measures. They are accuracy, noise level, real false range, number of real false maxima, real width, stability and execution time. Accuracy and execution time are the two criteria previously used in literatures [7,13,30]. Noise level, real false range, number of real false maxima and real width are the four modified criteria different from these in literatures [7,13,30]. Stability is the criterion proposed in this paper to measure the independence of focus measures on image content.

1) Accuracy. Accuracy is one of the most important criteria to assess the performance of focus measures. It is defined as the distance between the best focus position and the maximum of the focus measure value. The best focus position can be determined by a proficient operator. The smaller the criterion, the better the focus measure.
2) Noise level. Noise level is a criterion to measure the noise level of focus measures. The number of the maxima which are caused by noise can reflect the noise level of focus measures. It is equal to the number of total maxima minus the number of real maxima. The number of real maxima is the number of the maxima in the focus measure data after median filter. The size of median filter can be obtained by an iterative algorithm. The size increases from 3 until the number of the maxima does not decrease, and the final size is the best for median filter. The smaller the noise level, the better the focus measure.
3) Real false range. Real false range is a criterion to measure the monotonicity of focus measures. The valid range is defined as the distance between the two local minima next to the global maximum [7]. The false range is equal to the total range minus the valid range. 'Real' here means the result without the effect of noise. The smaller the real false range, the better the focus measure.
4) Number of real false maxima. Number of real false maxima is a criterion to measure the unimodality of focus measures. Real false maxima here means the false maxima which are not caused by noise. It is equal to the number of real maxima minus 1. The smaller the number of real false maxima, the better the focus measure.

5) Real width. Real width is a criterion to measure the sharpness of focus peak. Usually, the width of a focus curve at 50% maximum of the peak is computed as the width of the focus curve [7,13]. 'Real' here also means the result without the effect of noise. The smaller the real width, the better the focus measure.

The above five criteria are the basis criteria for the evaluation of focus measures. They describe the performance of focus measures under a specific image content. The values of them are varying with image content. There are another two crucial criteria for the evaluation of focus measures. They are stability and execution time, which are independent of image content.

- Stability. Stability is proposed in this paper to measure the independence of focus measures on image content. It is computed by the above five criteria under different image content. The algorithm is as follows:

(1) Compute the accuracy, noise level, real false range, number of real false maxima, and real width of focus measures under different image content. The smaller the differences in each criterion, the more stable the focus measure.
(2) Normalize each criterion by divided by the maximum under different image content. The range of the normalized criteria is [0, 1]. It can be seen from this step that the more closer to 1 the normalized criteria, the more stable the focus measure.
(3) Use 1 minus each normalized criterion, and sum up the results as the stability. We can see that the stability defined in this step is a value greater or equal to zero. The smaller stability means the more stable focus measure.

- Execution time. Execution time is a criterion proposed by Santos et al. [13] to measure the efficiency of focus measures. In the autofocus of line scan cameras, the focus mode 'moving the imaging system' leads to a long stroke for searching. The long stroke can be as long as several hundred millimeters. Continuous motion is needed to improve the efficiency of autofocus in some cases such as using global search algorithm in the autofocus of line scan cameras. Execution time is a significant criterion which determines the system response time and the accuracy of autofocus. The shorter the execution time, the faster the system response, the better the focus accuracy, and the better the focus measure.

To assess the overall performance of focus measures, an overall score is computed as follows:

1) Compute the values of the seven criteria.
2) Normalize the criteria to equally weight each criterion.
3) The normalized criteria are multiplied by scale factors for different search algorithms and autofocus styles. The range of scale factors is [0, 1]. The larger scale factor means the more significance of the criterion.
4) Compute the Euclidean distance of a focus measure to the ideal focus measure as the overall score using the multiplication results in Step 3. The criteria of the ideal focus measure are all zero. The smaller overall score means the better focus measure.

There are mainly three differences between our method and the methods in references [7,13]. The first difference is that the criterion 'stability' is proposed in our method to measure the independence of focus measures on image content. Stability is a significant criterion to assess the performance of focus measures especially when the focus scene is variable. The second difference is the calculation of noise level. It is computed by the number of false maxima which are caused by noise rather than the sum of squares of the second derivatives. The sum of squares of the second derivatives is determined by both noise level and the shape of focus curve. It cannot accurately represent the noise level of focus measures. The third difference is the calculation of 'real' criteria. For example, we use number of real false maxima rather than number of false maxima to measure the unimodality of focus measures. Number of real false maxima is more proper because it excludes the false number which is caused by noise. The fourth difference is that scale factors are introduced in our method to assess focus measures for different search algorithms and autofocus styles.

Note that the determination of scale factors is related to search algorithms. In global-based search, continuous motion is needed to improve the efficiency of autofocus. The autofocus result is determined by accuracy, noise level and execution time. The effects of the real false range, number of real false number and real width are minor and can be ignored. In local-based search, the motion of imaging system is intermittent. The interrupted time is longer than the execution time of focus measures. The effect of execution time can be ignored. The autofocus result is determined by accuracy, noise level, real false range, number of real false maxima and real width. For simplicity, we set the scale factors of the criteria which determine the autofocus result to 1, and set the scale factors of the criteria which can be ignored to 0. The scale factors of accuracy, noise level, real false range, number of real false maxima, real width and execution time we set for global-based search are [1, 1, 0, 0, 0, 1]. The scale factors we set for local-based search are [1, 1, 1, 1, 1, 0]. The scale factors of the criteria which determine the autofocus result are all set to 1 because all of them can determine the autofocus result to be correct or incorrect. It is a good choice for the fresh autofocus operator. An expert autofocus operator can set the scale factors to different values according to his/her experience.

In the autofocus with auxiliary focus sheet, the focus scene is a single focus sheet. Stability is an unnecessary criterion in the evaluation of focus measures. The scale factor of stability we set for this focus style is 0. In the autofocus directly on the object when the object is invariable, stability is also unnecessary and the scale factor we set is 0. In the autofocus directly

Fig. 2. The sinusoidal grating stripe of the focus sheet in experiments.

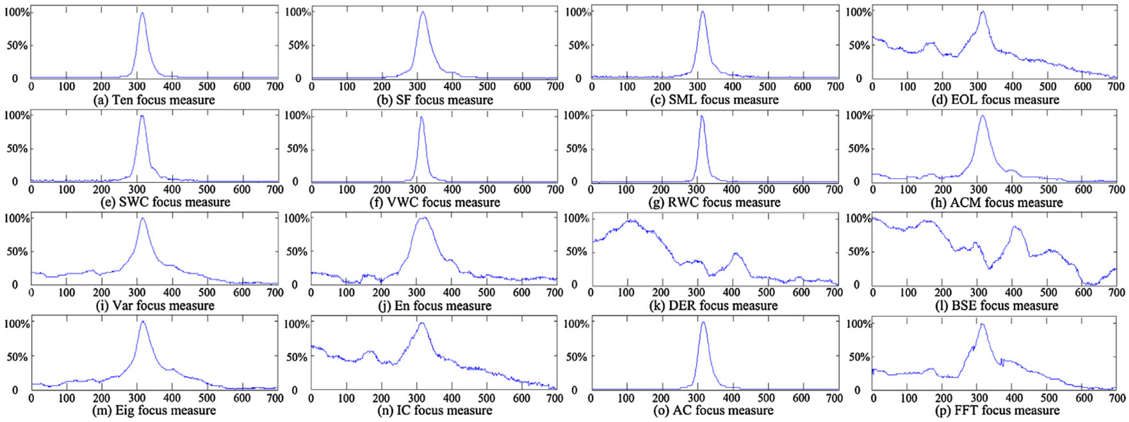


Fig. 3. The focus curves of the focus measures in the experiment of auxiliary focus sheet.

on the object when the object is variable, stability is a significant criterion to assess the performance of focus measures. The scale factor of stability we set for this focus style is 1.

The disadvantage of objective evaluation is 'not intuitive' and complex computation. But it can quantitatively and accurately rank the good focus measures according to their performance.

4. Experiments and discussions

Experiments are conducted with a line scan camera with 7300 pixels and a lens with the focal length of 120 mm. The f-number adopted in experiments is 5.6. Dozens of experiments have been conducted with different imaging resolutions. Limited by the space, only a typical group of experiments under the imaging resolution of 1273 pixels per inch are present here. The execution time is based on the computer with configuration of 2.0 GHz CPU, 2G DDR and Visual C++ 2005 programming environment.

4.1. Experiments on the autofocus with auxiliary focus sheet

In the autofocus of line scan cameras, sinusoidal grating stripe is the popular pattern on the focus sheet. The response of an imaging system to the sine wave input is a wave reduced in amplitude but unmodified in shape [31]. High frequency information will be well preserved in the image, which is beneficial to the autofocus. The sinusoidal grating stripe used in experiments is shown in Fig. 2. The focus curves of the sixteen focus measures on the scene of the pattern are shown in Fig. 3.

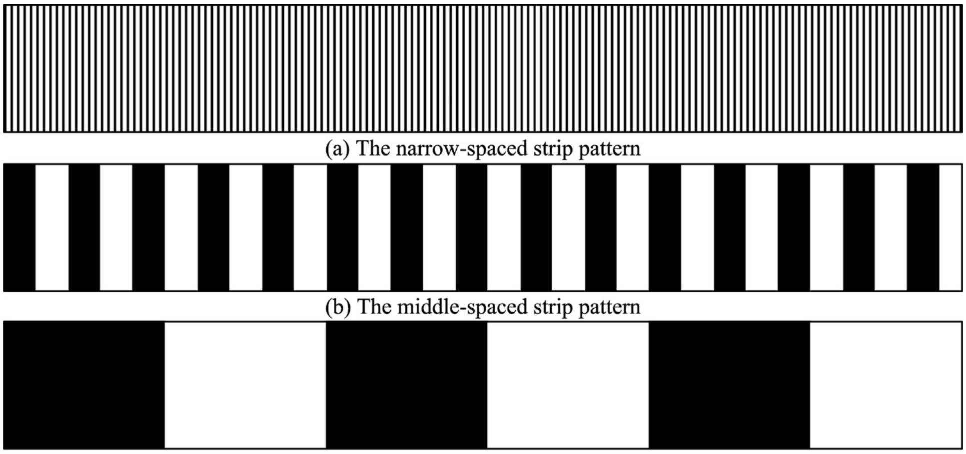
4.1.1. Subjective evaluation

In global-based search, a good focus measure is required to have the global maximum and sharp peak in the focused position. It can be seen from Fig. 3 that Ten, SF, SML, EOL, SWC, VWC, RWC, ACM, Var, Eig, IC, AC and FFT satisfy the requirement while En, DER and BSE do not. The peak of En is flat. DER and BSE do not have global maxima in the focused position. Thus, the good focus measures in global-based search for the autofocus of line scan cameras with auxiliary focus sheet are Ten, SF, SML, EOL, SWC, VWC, RWC, ACM, Var, Eig, IC, AC and FFT.

In local-based search, a good focus measure is required to be monotonous and unimodal. Besides, a good focus measure is also required to have the global maximum and sharp peak in the focused position. We can see from Fig. 3 that Ten, SF, SML, SWC, VWC, RWC and AC satisfy the requirement while EOL, ACM, Var, En, DER, BSE, Eig, IC and FFT do not. EOL, ACM, Var, Eig, IC and FFT are multimodal. The peak of En is flat. DER and BSE do not have global maxima in the focused positions. Thus, the good focus measures in local-based search for the autofocus of line scan cameras with auxiliary focus sheet are Ten, SF, SML, SWC, VWC, RWC and AC.

Table 1
The objective evaluation results in the experiment of auxiliary focus sheet.

Focus measure	Accuracy (step)	Real false range (step)	Number of real false maxima	Real width (step)	Noise level	Stability	Execution time (ms)	Overall score 1	Overall score 2
Ten	0(1)	449(1)	20(8)	33(5)	143(1)		5049.8(6)	0.7531(1)	1.5133(1)
SF	0(1)	469(3)	24(10)	42(7)	150(2)		2992.4(4)	0.7868(2)	1.7180(2)
SML	0(1)	519(6)	28(11)	32(3)	163(4)		6098.2(8)	0.8591(3)	1.7342(3)
EOL	0(1)	543(10)	10(5)	77(12)	186(11)		5151.2(7)	0.9774(5)	
SWC	1(6)	532(7)	34(13)	32(3)	168(7)		8547.1(10)	1.0211(10)	2.1042(7)
VWC	1(6)	544(11)	33(12)	23(1)	165(6)		7896.4(9)	1.0062(8)	2.0251(6)
RWC	1(6)	544(11)	20(8)	23(1)	178(10)		18888.3(12)	1.1006(11)	1.9094(5)
ACM	1(6)	547(13)	7(4)	51(8)	163(4)		2349.5(3)	0.9898(6)	
Var	1(6)	500(5)	1(1)	55(9)	169(8)		935.9(1)	1.0164(9)	
Eig	2(12)	496(4)	2(2)	58(10)	186(11)		9089.6(11)	1.4035(12)	
IC	0(1)	542(9)	10(5)	103(13)	191(12)		3215.9(5)	1.0013(7)	
AC	1(6)	451(2)	10(5)	33(5)	154(3)		1054.0(2)	0.9489(4)	1.7719(4)
FFT	2(12)	540(8)	4(3)	73(11)	173(9)		62009.7(13)	1.6794(13)	



(a) The narrow-spaced strip pattern

(b) The middle-spaced strip pattern

(c) The wide-spaced strip pattern

Fig. 4. The simulated patterns used in experiments.

4.1.2. *Objective evaluation*

Objective evaluation is used to rank the good focus measures according to their performance. The objective evaluation results are shown in Table 1. The values which cannot be computed (for example, the stability values) or do not need to be computed (for example, some overall score 2 values) are marked with oblique lines in the table. The execution time in Table 1 is the time of each focus measure executed 10 thousand times. The values in round brackets are the ranking of focus measures according to the criteria. The overall score computed with the scale factors of global-based search is denoted by overall score 1. The overall score computed with the scale factors of local-based search is denoted by overall score 2.

It can be seen from Table 1 that Ten is the best focus measure in both global-based search and local-based search. Therefore, Ten is recommended for the autofocus of line scan cameras with auxiliary focus sheet, whether in global-based search or in local-based search.

4.2. *Experiments on the autofocus directly on the object*

It is well known that the quality of autofocus is closely related to image content [13]. Rich image content is good for the autofocus of cameras while poor image content is difficult to focus. Considering the line scan camera can only obtain a line image at a time, we adopt vertical stripe patterns as the experimental objects. The density of stripe in patterns reflects the amount of image content. To test the performance of focus measures, three types of stripe patterns including narrow-spaced stripes, middle-spaced stripes and wide-spaced stripes are used in the experiments. As shown in Fig. 4, these patterns consist of black and white stripes. The widths of each stripe in the three patterns are respectively 1 mm, 10 mm and 50 mm. The three patterns represent the image content from rich to poor. The focus curves of the sixteen focus measures on the scene of the three patterns are respectively shown in Figs. 5–7 .

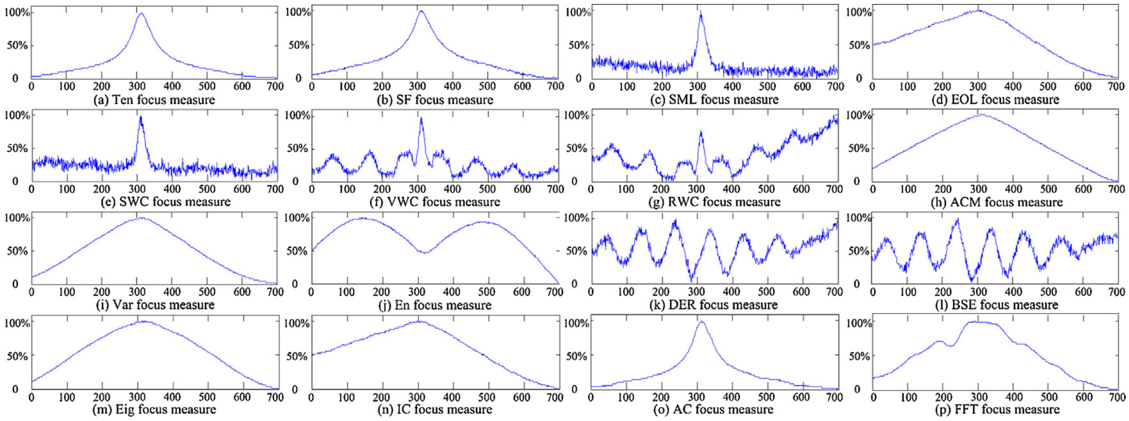


Fig. 5. The focus curves in the experiment of narrow-spaced strip pattern.

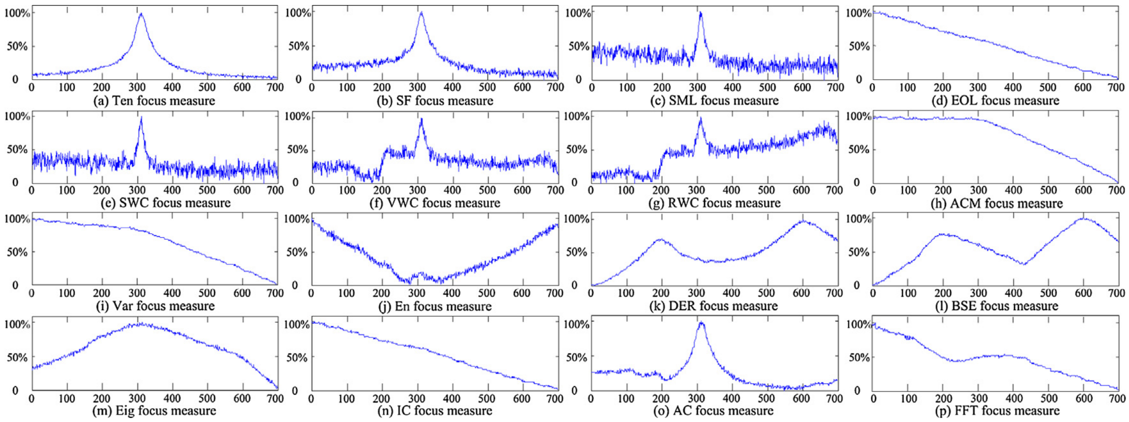


Fig. 6. The focus curves in the experiment of middle-spaced strip pattern.

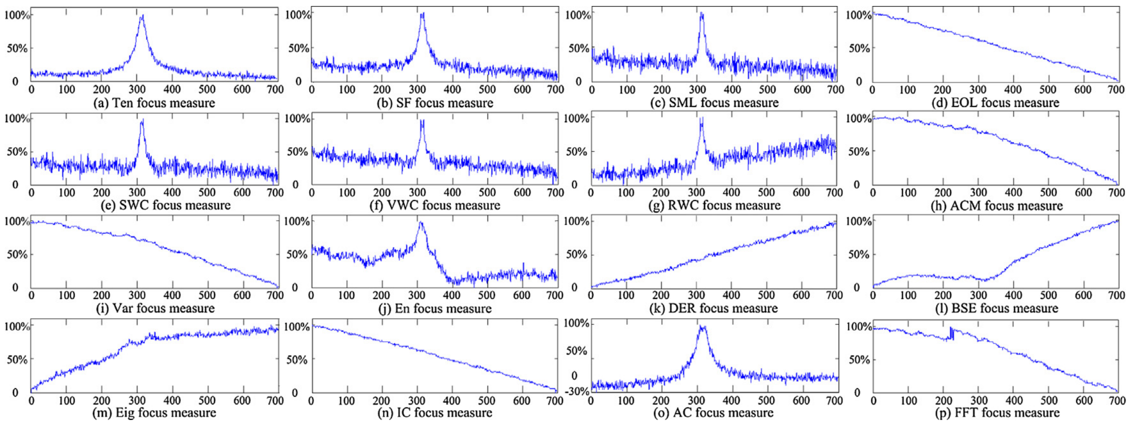


Fig. 7. The focus curves in the experiment of wide-spaced strip pattern.

4.2.1. Subjective evaluation

In global-based search, a good focus measure is required to have the global maximum and sharp peak in the focused position. It can be seen from the focus curves in the experiment of narrow-spaced stripe pattern (Fig. 5) that eleven focus measures including Ten, SF, SML, EOL, SWC, VWC, ACM, Var, Eig, IC and AC satisfy the requirement while the other five focus measures including RWC, En, DER, BSE and FFT do not. RWC, En, DER and BSE do not have global maxima in the focused positions. The peak of FFT is flat. Thus, the good focus measures in global-based search for the autofocus on the scene of narrow-spaced stripe pattern are Ten, SF, SML, EOL, SWC, VWC, ACM, Var, Eig, IC and AC. It can be seen from the focus curves

Table 2
The objective evaluation results in the experiment of narrow-spaced strip pattern.

Focus measure	Accuracy (step)	Real false range (step)	Number of real false maxima	Real width (step)	Noise level	Stability	Execution time (ms)	Overall score 1	Overall score 2
Ten	0(1)	0(1)	0(1)	91(4)	141(5)		5049.8(6)	0.9178(4)	0.7539(2)
SF	2(5)	0(1)	0(1)	111(6)	192(9)		2992.4(4)	1.0635(6)	1.0365(6)
SML	3(8)	612(9)	41(11)	27(2)	185(7)		6098.2(8)	1.2014(8)	
EOL	11(11)	0(1)	0(1)	470(10)	193(10)		5151.2(7)	1.5235(11)	1.7099(8)
SWC	2(5)	618(10)	39(10)	27(2)	185(7)		8547.1(10)	1.3550(10)	
VWC	2(5)	645(11)	12(9)	21(1)	107(4)		7896.4(9)	1.0465(5)	
ACM	1(3)	0(1)	0(1)	395(9)	28(1)		2349.5(3)	0.3100(2)	0.8257(4)
Var	1(3)	0(1)	0(1)	340(7)	43(2)		935.9(1)	0.2617(1)	0.7358(1)
Eig	5(9)	0(1)	0(1)	394(8)	69(3)		9089.6(11)	1.1552(7)	0.9918(5)
IC	9(10)	0(1)	0(1)	489(11)	193(10)		3215.9(5)	1.3396(9)	1.6338(7)
AC	0(1)	0(1)	0(1)	94(5)	153(6)		1054.0(2)	0.8012(3)	0.8157(3)

in the experiment of middle-spaced stripe pattern (Fig. 6) that eight focus measures including Ten, SF, SML, SWC, VWC, RWC, Eig and AC satisfy the requirement while the other eight focus measures including EOL, ACM, Var, En, DER, BSE, IC and FFT do not have global maxima in the focused positions. Thus, the good focus measures in global-based search for the autofocus on the scene of middle-spaced stripe pattern are Ten, SF, SML, SWC, VWC, RWC, Eig and AC. It can be seen from the focus curves in the experiment of wide-spaced stripe pattern (Fig. 7) that eight focus measures including Ten, SF, SML, SWC, VWC, RWC, En and AC satisfy the requirement while the other eight focus measures including EOL, ACM, Var, DER, BSE, Eig, IC and FFT do not have global maxima in the focused positions. Thus, the good focus measures in global-based search for the autofocus on the scene of wide-spaced stripe pattern are Ten, SF, SML, SWC, VWC, RWC, En and AC. In summary, the good focus measures in global-based search for the autofocus on the scene of the three patterns are Ten, SF, SML, SWC, VWC and AC.

In local-based search, a good focus measure is required to be monotonous and unimodal. Besides, a good focus measure is also required to have the global maximum and sharp peak in the focused position. It can be seen from the focus curves in the experiment of narrow-spaced stripe pattern (Fig. 5) that Ten, SF, EOL, ACM, Var, Eig, IC and AC satisfy the requirement while SML, SWC, VWC, RWC, En, DER, BSE and FFT do not. SML and SWC are not monotonous. VWC, RWC, En, DER, BSE and FFT are multimodal. Thus, the good focus measures in local-based search for the autofocus on the scene of narrow-spaced stripe pattern are Ten, SF, SML, EOL, SWC, ACM, Var, Eig, IC and AC. It can be seen from the focus curves in the experiment of middle-spaced stripe pattern (Fig. 6) that Ten, SF and Eig satisfy the requirement while SML, EOL, SWC, VWC, RWC, ACM, Var, En, DER, BSE, IC, AC and FFT do not. SML and SWC are not monotonous. VWC, RWC and AC are multimodal. EOL, ACM, Var, En, DER, BSE, IC and FFT do not have global maxima in the focused positions. Thus, the good focus measures in local-based search for the autofocus on the scene of middle-spaced stripe pattern are Ten, SF and Eig. It can be seen from the focus curves in the experiment of wide-spaced stripe pattern (Fig. 7) that Ten, SF and AC satisfy the requirement while SML, EOL, SWC, VWC, RWC, ACM, Var, En, DER, BSE, Eig, IC and FFT do not. SML, SWC, VWC, RWC and En are not monotonous. EOL, ACM, Var, DER, BSE, Eig, IC and FFT do not have global maxima in the focused positions. Thus, the good focus measures in local-based search for the autofocus on the scene of wide-spaced stripe pattern are Ten, SF and AC. In summary, the good focus measures in local-based search for the autofocus on the scene of the three patterns are Ten and SF.

4.2.2. *Objective evaluation*

Objective evaluation is used to rank the good focus measures according to their performance. Denote the overall score computed with the scale factors of global-based search overall score 1. Denote the overall score computed with the scale factors of local-based search overall score 2. The objective evaluation results in the experiments of narrow-spaced stripe pattern, middle-spaced stripe pattern and wide-spaced stripe pattern are respectively shown in Tables 2–4 . The values which cannot be computed (for example, the stability values) or do not need to be computed (for example, some overall score 2 values) are marked with oblique lines in the three tables. The execution time in tables is the time of each focus measure executed 10 thousand times. The values in round brackets are the ranking of focus measures according to the criteria.

It can be seen from Tables 2–4 that the performance of a focus measure varies with the image content. For example, the accuracy of Ten declines as the image content changes from narrow-spaced stripe pattern to wide-spaced stripe pattern; the number of real false maxima of AC increases as the image content changes from narrow-spaced stripe pattern to wide-spaced stripe pattern; etc. The performance of a focus measure relative to others also varies with the image content. Ten provides the best performance in accuracy, real false range and number of real false maxima in the narrow-spaced stripe pattern (see Table 2). As the image content changes, it provides the best performance only in accuracy and real false range in the middle-spaced stripe pattern (see Table 3), and provides the best performance only in real false range in the wide-spaced stripe pattern (see Table 4). The performance of focus measures varies as the image content changes.

Table 3
The objective evaluation results in the experiment of middle-spaced strip pattern.

Focus measure	Accuracy (step)	Real false range (step)	Number of real false maxima	Real width (step)	Noise level	Stability	Execution time (ms)	Overall score 1	Overall score 2
Ten	1(1)	399(1)	22(4)	63(5)	190(4)		5049.8(3)	0.9965(1)	1.3133(1)
SF	2(3)	560(3)	33(5)	57(4)	180(2)		2992.4(2)	1.0228(3)	1.6458(2)
SML	3(7)	619(4)	41(8)	29(2)	186(3)		6098.2(4)	1.2206(6)	
SWC	2(3)	625(5)	39(6)	26(1)	191(5)		8547.1(6)	1.1501(5)	
VWC	2(3)	631(6)	40(7)	43(3)	178(1)		7896.4(5)	1.0857(4)	
RWC	2(3)	632(7)	18(2)	64(6)	192(6)		18888.3(8)	1.4585(7)	
Eig	4(8)	682(8)	8(1)	492(8)	205(8)		9089.6(7)	1.4938(8)	2.0146(3)
AC	1(1)	479(2)	18(2)	66(7)	199(7)		1054.0(1)	1.0040(2)	

Table 4
The objective evaluation results in the experiment of wide-spaced strip pattern.

Focus measure	Accuracy (step)	Real false range (step)	Number of real false maxima	Real width (step)	Noise level	Stability	Execution time (ms)	Overall score 1	Overall score 2
Ten	2(2)	527(1)	40(6)	40(6)	184(3)		5049.8(3)	1.0592(2)	2.0160(3)
SF	2(2)	591(4)	32(3)	36(5)	189(6)		2992.4(2)	1.0604(3)	1.9713(1)
SML	3(7)	632(8)	34(4)	21(1)	187(5)		6098.2(4)	1.1764(6)	
SWC	2(2)	629(7)	43(8)	21(1)	173(1)		8547.1(6)	1.0732(4)	
VWC	2(2)	617(5)	40(6)	26(3)	178(2)		7896.4(5)	1.0807(5)	
RWC	2(2)	627(6)	17(1)	26(3)	186(4)		18888.3(8)	1.4387(7)	
En	5(8)	578(3)	23(2)	119(8)	193(7)		9089.6(7)	1.4870(8)	
AC	1(1)	575(2)	34(4)	65(7)	195(8)		1054.0(1)	1.0213(1)	1.9797(2)

Table 5
The objective evaluation results based on the three experiments.

Focus measure	Accuracy (step)	Real false range (step)	Number of real false maxima	Real width (step)	Noise level	Stability	Execution time (ms)	Overall score 1	Overall score 2	Overall score 3
Ten	1.00(2)	308.67(1)	20.67(2)	64.67(4)	171.67(2)	5.3505(6)	5049.8(3)	1.5195(4)	2.1149(1)	1.7904(2)
SF	2.00(3)	394.33(3)	22.33(3)	68.00(5)	186.33(6)	3.1881(4)	2992.4(2)	1.3864(2)	2.3141(2)	1.8217(4)
SML	3.00(6)	621.00(4)	38.67(5)	25.67(2)	186.00(5)	0.5838(1)	6098.2(4)	1.5866(6)		2.0034(6)
SWC	2.00(3)	624.00(5)	40.33(6)	24.67(1)	183.00(4)	0.5948(2)	8547.1(6)	1.5561(5)		1.8728(5)
VWC	2.00(3)	631.00(6)	30.67(4)	29.33(3)	154.33(1)	2.0710(3)	7896.4(5)	1.4608(3)		1.7354(1)
AC	0.67(1)	351.33(2)	17.33(1)	75.00(6)	182.33(3)	4.4952(5)	1054.0(1)	1.3147(1)		1.7911(3)

From Table 2, we can see that Var is the best focus measure in both global-based search and local-based search in the narrow-spaced stripe pattern. From Table 3, we can see that Ten is the best focus measure in both global-based search and local-based search in the middle-spaced stripe pattern. From Table 4, we can see that AC is the best focus measure in global-based search while SF is the best focus measure in local-based search in the wide-spaced stripe pattern. It can be seen from the above results that the best focus measure for the autofocus directly on the object is related to the image content. It means, when the object is invariable, which occurs in the monofunctional measurement system, the best focus measure should be selected in field test according to the texture of the object.

A comprehensive objective evaluation results based on the three experiments are shown in Table 5. The values of accuracy, noise level, real false range, number of real false maxima and real width are the mean of the data in Table 2–4. The overall score computed with the scale factors of global-based search is denoted by overall score 1. The overall score computed with the scale factors of local-based search is denoted by overall score 2. The evaluation results reflect the overall performance of focus measures in the image content from rich to poor.

It can be seen from Table 5 that AC is the best focus measure in global-based search and Ten is the best focus measure in local-based search. Therefore, AC and Ten are respectively recommended in global-based search and local-based search for the autofocus directly on the variable object, which occurs in the multifunction measurement system of line scan cameras.

To verify the necessity of subjective evaluation, the evaluation of focus measures only by objective evaluation is carried out. Overall score 3 in Table 5 is the evaluation results of Ten, SF, SML, SWC, VWC and AC in local based search without subjective evaluation. It can be seen from overall score 3 that VWC (the first place) is better than Ten (the second place) and SF (the fourth place). In fact, VWC is not monotonous in any one of the three experiments, and therefore it is not a good focus measure for local-based search. The evaluation of focus measures only by objective evaluation produces false result. The fact illustrates the evaluation results only by objective evaluation cannot exactly assess the performance of focus measures. It is necessary to exclude the bad focus measures by subjective evaluation before objective evaluation.

5. Conclusions

This paper proposes a comprehensive evaluation method of focus measures for the autofocus of line scan cameras. In this method, subjective evaluation is firstly utilized to exclude the bad focus measures. Then, objective evaluation is utilized to rank the good focus measures according to their performance in seven criteria. Stability is proposed as a criterion in this paper to measure the independence of focus measures on image content. Scale factors are introduced to rank focus measures for different search algorithms and focus styles. Experimental results show the evaluation of focus measures only by objective evaluation cannot exactly assess the performance of focus measures. Subjective evaluation is a necessary step before ranking the focus measures by objective evaluation.

In this paper, two common autofocus styles of line scan cameras are tested in the experiments. One is the autofocus with auxiliary focus sheet. The other is the autofocus directly on the object. In the autofocus with auxiliary focus sheet, the popular sinusoidal grating pattern is tested in experiments. Experimental results show that Ten is the best focus measure in both global-based search and local-based search. In the autofocus directly on the object, the performance of focus measures varies with image content. In spite of this, we assess the overall performance of focus measures with the image content from rich to poor. Experimental results show that AC provides the best overall performance in global-based search and Ten provides the best overall performance in local-based search. When the object is invariable, the best focus measure should be selected in field test.

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