2 2 0 6 0 2 8 9 3 2

KALKULUS C - PR5 - ALDEN LUTHFI

1. $\sin^2 x = \dfrac{1-\cos 2x}{2}$, $\cos^2 x = \dfrac{\cos 2x + 1}{2}$

$$\int_0^{\frac{\pi}{2}} \sin^2 x\,(1-\sin^2 x)^2\,dx = \int_0^{\frac{\pi}{2}} \sin^2 x \cos^4 x\,dx$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1-\cos 2x}{2}\right)\left(\frac{\cos 2x + 1}{2}\right)^2 dx$$

$$= \frac{1}{8}\int_0^{\frac{\pi}{2}} (1-\cos 2x)(\cos 2x + 1)^2\,dx$$

$$= \frac{1}{8}\int_0^{\frac{\pi}{2}} (1-\cos^2 2x)(1+\cos 2x)\,dx$$

$$= \frac{1}{8}\int_0^{\frac{\pi}{2}} -\cos^3 2x - \cos^2 2x + \cos 2x + 1\,dx$$

$$= \frac{1}{8}\left(x \Big]_0^{\frac{\pi}{2}} + \frac{\sin 2x}{2}\Big]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \frac{\cos 4x + 1}{2}\,dx - \int_0^{\frac{\pi}{2}} \cos^3 2x\,dx \right)$$

$$= \frac{1}{8}\left(\frac{\pi}{2} - \frac{1}{2}\left(\int_0^{\frac{\pi}{2}} \cos 4x\,dx + \int_0^{\frac{\pi}{2}} dx \right) - \int_0^{\frac{\pi}{2}} \frac{\cos^2 2x}{2}\,d\sin 2x \right)$$

$$= \frac{1}{8}\left(\frac{\pi}{2} - \frac{1}{2}\left(\frac{\sin 4x}{4}\Big]_0^{\frac{\pi}{2}} + x\Big]_0^{\frac{\pi}{2}} \right) - \frac{1}{2}\int_0^{\frac{\pi}{2}} \cos^2 2x\,d\sin 2x \right)$$

$$= \frac{1}{16}\left(\pi - \frac{\pi}{2} - \left(\cos^2 2x \sin 2x - \int_0^{\frac{\pi}{2}} 2\cos 2x\,d\cos^2 2x\right)\Big]_0^{\frac{\pi}{2}} \right)$$

$$= \frac{1}{16}\left(\frac{\pi}{2} - \left(\cos^2 2x \sin 2x - \int_0^{\frac{\pi}{2}} -8\cos^2 2x \sin 2x\right)\Big]_0^{\frac{\pi}{2}} \right)$$

$$= \frac{1}{16}\left(\frac{\pi}{2} - \left(\cos^2 2x \sin 2x + 8\int_0^{\frac{\pi}{2}} -\frac{\cos^2 2x}{2}\,d\cos 2x\right)\Big]_0^{\frac{\pi}{2}} \right)$$

$$= \frac{1}{16}\left(\frac{\pi}{2} - \left(\cos^2 2x \sin 2x - 4\int_0^{\frac{\pi}{2}} \cos^2 2x\,d\cos 2x\right)\Big]_0^{\frac{\pi}{2}} \right)$$

$$= \frac{1}{16}\left(\frac{\pi}{2} - \left(\cos^2 2x \sin 2x - \frac{4}{3}\cos^3 2x\right)\Big]_0^{\frac{\pi}{2}} \right)$$

$$= \frac{\pi}{32} \quad \blacksquare$$

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(2.) Teorema dasar kalkulus pertama:
jika $f(t)$ kontinu di $[a,b]$ dan x adalah sebuah titik di (a,b) maka: $\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$

$$\therefore \int_0^{\frac{\pi}{2}} f(t)\,dt = \sin x + \int_0^x f(t)\cos^2 t\,dt$$

maka:

$$\frac{d}{dx}\int_0^{\frac{\pi}{2}} f(t)\,dt = \frac{d}{dx}\sin x + \frac{d}{dx}\int_0^x f(t)\cos^2 t\,dt$$

karena f kontinu di $[0,\infty)$ maka:

$$0 = \cos x + f(x)\cos^2 x$$

$$\therefore f(x) = -\sec x$$

$$\therefore \int_0^{\frac{\pi}{4}} f(x)\,dx = -\ln|\sec x + \tan x|\Big]_0^{\frac{\pi}{4}}$$

$$= -\ln|\sqrt{2}+1| - \ln|1|$$

$$= -\ln|\sqrt{2}+1| \blacksquare$$

(3.) $f(x)$ fungsi genap maka $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx = 2\int_{-a}^0 f(x)\,dx$

(a) $\int_{-5}^{5} f(x) + f(-x)\,dx = \int_{-5}^{5} 2f(x)\,dx = 4\int_0^5 f(x)\,dx$

$= -100 \blacksquare$

(b) karena $f(x) \leq 0$ maka $|f(x)| = -f(x)$

$$\int_{-5}^{5} |f(x)|\,dx = -\int_{-5}^{5} f(x)\,dx = -2\int_0^5 f(x)\,dx = 50 \blacksquare$$

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(4.) Teorema nilai rata-rata:
jika f kontinu di $[a,b]$ maka ada $c \in [a,b]$ sehingga

$$f(c) = \frac{1}{b-a}\int_a^b f(x)\,d(x)$$

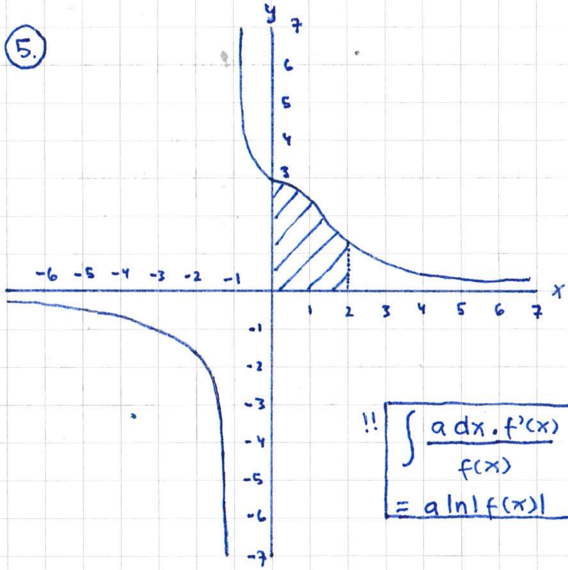
$$\therefore f(c) = \frac{1}{2}\int_0^2 \frac{x}{\sqrt{x^2+9}}\,dx \quad , \text{misal } t^2 = x^2+9, \; t\,dt = x\,dx$$

$$= \frac{1}{2}\int_3^{\sqrt{13}} dt = \frac{\sqrt{13}-3}{2} = \frac{4}{2\sqrt{13}+6}$$

$$\frac{c}{\sqrt{c^2+9}} = \frac{4}{2\sqrt{13}+6}$$

$$\Rightarrow (2\sqrt{13}+6)c = 4\sqrt{c^2+9}$$
$$\Rightarrow (2\sqrt{13}+6)^2 c^2 = 4^2(c^2+9)$$
$$\Rightarrow ((2\sqrt{13}+6)^2 - 16)c^2 = 144$$
$$\Rightarrow c = \sqrt{\frac{144}{(2\sqrt{13}+6)^2 - 16}}$$

(5.)



$$!! \quad \boxed{\int \frac{a\,dx \cdot f'(x)}{f(x)} = a\ln|f(x)|}$$

$$\frac{5x^2+2x+3}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$A = \frac{5x^2+2x+3}{x^2+1}\Big]_{-1} = 3$$

$B = 2$

$C = 0$

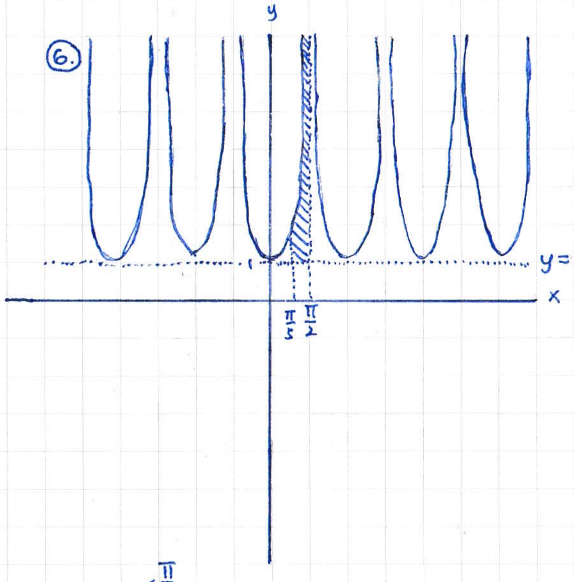
$$\therefore A = \int_0^2 \frac{5x^2+2x+3}{(x+1)(x^2+1)}\,dx$$

$$\Rightarrow \int_0^2 \frac{3}{x+1} + \frac{2x}{x^2+1}\,dx$$

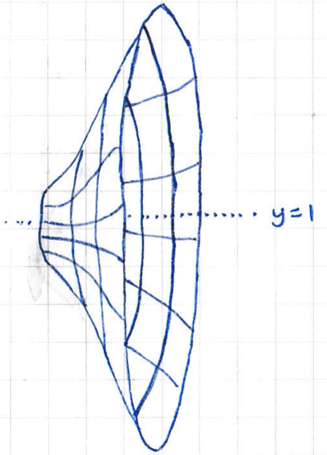
$$\Rightarrow 3\ln|x+1| + \ln|x^2+1|\Big]_0^2$$

$$\Rightarrow 3\ln 3 + \ln 5 \blacksquare$$

(6.)



$\Longrightarrow$



$$V = \pi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (\sec x - 1)^2 \, dx = \pi \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 1 + \sec^{12} x - 2\sec^6 x \, dx$$

$$= \pi \left(x \Big]_{\frac{\pi}{3}}^{\frac{\pi}{2}} + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (1+\tan^2 x)^5 d(\tan x) - 2\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} (1+\tan^2 x)^2 \, d(\tan x) \right)$$

$$= \pi \left(\frac{\pi}{6} + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \tan^{10} x \, d(\tan x) + 5 \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \tan^8 x \, d(\tan x) + 10 \int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \tan^6 x \, d(\tan x) \right.$$

$$+ 10 \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \tan^4 x \, d(\tan x) + 5 \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \tan^2 x \, d(\tan x) + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} d(\tan x)$$

$$\left. -2 \left(\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \tan^4 x \, d(\tan x) + 2 \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \tan^2 x \, d(\tan x) + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} d(\tan x) \right) \right)$$

$$= \pi \left(\frac{\pi}{6} + \frac{\tan^{11} x}{11} \Big]_{\frac{\pi}{3}}^{\frac{\pi}{2}} + \frac{5\tan^9 x}{9} \Big]_{\frac{\pi}{3}}^{\frac{\pi}{2}} + \frac{10\tan^7 x}{7} \Big]_{\frac{\pi}{3}}^{\frac{\pi}{2}} + \frac{8\tan^5 x}{5} \Big]_{\frac{\pi}{3}}^{\frac{\pi}{2}} + \frac{\tan^3 x}{3} \Big]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \right.$$

$$\left. - \tan x \Big]_{\frac{\pi}{3}}^{\frac{\pi}{2}} \right)$$

$$= \pi \left(\frac{\pi}{6} + \text{undefined} \right) = \text{undefined}$$

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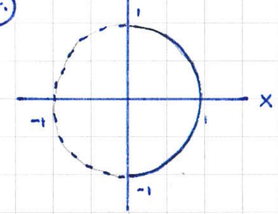
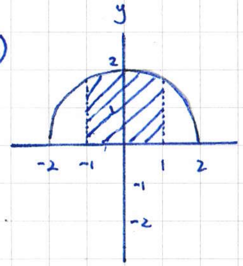
(7.)



$$S = \int_0^{\pi} \sqrt{(\cos t)^2 + (-\sin t)^2} \, dt$$

$$= \int_0^{\pi} dt = t \Big]_0^{\pi} = \pi$$

(8.)



$$dS = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx$$

$$= \sqrt{1 + \frac{x^2}{4 - x^2}} \, dx$$

$$A = 2\pi \int_a^b y \, ds$$

$$= 2\pi \int_{-1}^{1} \sqrt{4 - x^2} \sqrt{\frac{4}{4 - x^2}} \, dx = 4\pi \int_{-1}^{1} dx$$

$$= 4\pi \left(x \Big]_{-1}^{1} \right) = 8\pi$$