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Pertemuan 4 : Turunan

→ definisi derivative dan fungsi diferentiable

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

→ notasi $\frac{df(x)}{dx}$ atau $f'(x)$

→ newton needed this for physics

→ not all functions have derivatives

→ Leibniz Notation

- dx itu bekerja pada x
- $\lim_{\Delta x \to 0} \Delta x = dx$
- ex : $d x^2 = 2x\,dx$

2 syarat operator linear

- jika 2 fungsi dijumlahkan
  $d(f+g) = d(f) + d(g)$
- jika 2 fungsi dikalikan konstanta
  $d(kf) = k(df)$

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→ Teorema turunan

① $dx^n = n x^{n-1}\, dx$

→ $df(x)^n = n(f(x))^{n-1}\, df(x)$

② $d(fg) = g.df + f.dg$

③ $d\frac{f}{g} = \frac{g.df - f.dg}{g^2}$

④ $\frac{d f \circ g \circ h(x)}{dx} = \frac{df}{dg} \cdot \frac{dg}{dh} \cdot \frac{dh}{dx}$

→ jika differentiabe → kontinu

tapi tidak sebaliknya

⑤ $d \sin x = \cos x\, dx$

⑥ $d \cos x = \sin x\, dx$

⑦ $de^x = e^x dx$

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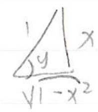
Pertemuan 5 : Turunan cont'd

→ Turunan dari inverse trancendental

d(arcsin x) :

∴ $y = \arcsin x \rightarrow dy = d\arcsin x$

$x = \sin y \rightarrow dx = d\sin y = \cos y\, dy$



$dy = \frac{dx}{\cos y} = \frac{1}{\sqrt{1-x^2}}\, dx$

bisa dipakai untuk menemukan inverse yang lain

→ $d\, f(x)^{g(x)}$

∴ $y = f^g \rightarrow \ln y = g.\ln f$

$d\ln y = dg.\ln f + d\ln f.g \rightarrow dy = y(dg \ln f + d\ln f.g)$

→ Teorema Rolle

· if f is differentiable in $[a,b]$ and $f(a) = f(b)$

then $\exists c,\ a < c < b$ such that $f'(c) = 0$

→ Teorema nilai tengah (Mean Value Theorem) (lagrange's)

jika f is differentiable in $[a,b]$ maka ada c sehingga $f'(c) = \frac{f(b)-f(a)}{b-a}$ (garis singgung di c sejajar dengan garis penghubung f(a) dan f(b))

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→ Implicit Differentiation
· kalo turunin y, tambah dy

→ fungsi parametrik
$x = f(t)$
$y = g(t)$ $\rightarrow \frac{dy}{dx} = \frac{dg(t)}{df(t)}$

→ koordinat kutub (polar)
$r = f(\theta)$
ex: $x = r\cos\theta \rightarrow dx = -r\sin\theta + \cos\frac{dr}{d\theta}$
$y = r\sin\theta \rightarrow dy = r\cos\theta + \sin\theta\frac{dr}{d\theta}$

→ variable x, y → persamaan rektanggulat

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Pertemuan 6 : Turunan contd...

→ Order n derivatives (tingkat tinggi)
· initial condition $d^0 f = f$ agar definisi bermakna
· $d^n f \rightarrow f$ diturunin n times

→ turunan parsial
· variabel lain jadi konstanta
· simbol $\frac{\partial f}{\partial x}$ atau f_x

→ Terapan Derivative
· jika $f'(x) > 0$ fungsi naik
· jika $f'(x) < 0$ fungsi turun
· jika $f'(x) = 0$ dan $f''(x) < 0$ fungsi titik maksimum
· jika $f'(x) = 0$ dan $f''(x) > 0$ titik minimum
· $f''(x) = 0$ undecideable
$\smile$ = cekung / concave $f''(x) > 0$
$\frown$ = cembung / convex $f''(x) < 0$

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→ Maks / Min lokal

- Maks / Min antara neighborsnya

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Pertemuan 7 : Integrals (indefinite)

→ $\int dx = x + C$

→ $\int dF(x) = F(x) + C$ ... ①

→ the first step to integrate is to find the form ①

→ $\int x^n \, dx = \frac{1}{n+1} x^{n+1} + C$

!! $x \neq -1$

→ $d \ln x$

↳ $y = \ln x \rightarrow x = e^y$

$= dx = y' e^y \rightarrow y' = e^{-y} \, dx = \frac{1}{x} dx$

so $\int \frac{1}{x} dx = \ln x$

ex ① $\int \underline{x} \cos \boxed{x^2} \, \underline{dx}$

since $x \, dx = \frac{1}{2} dx^2$

thus $\int x \cos x^2 = \frac{1}{2} \int \cos x^2 \, dx^2 = \frac{1}{2} \sin x^2 + C$

② $\int x \sqrt{1 - x^2} = -\frac{1}{2} \int (1 - x^2)^{\frac{1}{2}} \, d(1 - x^2)$

$= -\frac{1}{2} \left(\frac{2}{3} (1 - x^2)^{\frac{3}{2}} \right) = -\frac{1}{3} (1 - x^2)^{\frac{3}{2}}$

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→ integration operator is linear

→ since $d uv = u'v + v'u$

thus $\int duv = \int u'v + \int v'u$

$\int u'v = uv - \int v'u$

→ General Trigonometric Integrals

- $\sec^2 x = 1 + \tan^2 x$ ...①

$\int \sec^m x \tan^n x$ or $\int \csc^m x \cot^n x$

if m is even :

use ①

if m odd and n odd

use $d \sec x = \sec x \tan x$

if m odd and n even

use $\int \sec^{m-n} x (\sec^n \tan^n)$

then solve integration by parts

sec and csc , tan and cot is interchangeable

!! LIPET for U sub

log, Invers trig, Polynom, Exponent (e), Trig

→ for sinAx cosBx or sinAx sinBx or cos Ax cos Bx

$\sin Ax \cos Bx = \frac{1}{2}(\sin(A+B)x + \sin(A-B)x)$

$\sin Ax \sin Bx = \frac{1}{2}(\cos(A-B)x - \cos(A+B)x)$

$\cos Ax \cos Bx = \frac{1}{2}(\cos(A+B)x + \cos(A-B)x)$

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Pertemuan 8 ; Integrals (contd ...)

→ useful trig identities

$\cos^2 x = \frac{\cos 2x + 1}{2}$, $\sin^2 x = \frac{1 - \cos 2x}{2}$

→ Substitusi ~~trigonometri~~

↳ $\sqrt{a^2 - x^2}$ → $a \sin\theta$

↳ $\sqrt{x^2 - a^2}$ → $a \sec\theta$

↳ $\sqrt{x^2 + a^2}$ /atau/ $\frac{1}{x^2+a^2}$ → $a \tan\theta$

↳ kalo $\sqrt{ax^2 + bx + c}$ harus diconvert dulu

ex $\sqrt{3 - 2x - x^2} = \sqrt{5 - (x+1)^2}$

$a = \sqrt{5}$, $\square^2 = (x+1)^2$

→ Integral of Rational functions

↳ if $\frac{P(x)}{q(x)}$ and roots of $q(x)$ are $(x - x_1) \dots (x - x_k)$

$\frac{A_1}{(x - x_1)} + \dots + \frac{A_k}{(x - x_k)}$ from $\frac{P(x)}{q(x)}$

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Pertemuan 10 : definite integral

→ Sifat definite integral

① $\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x) \pm \int_a^b g(x)$

② $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$ for any c

③ sama seperti indefinite integral

→ 1st fundamental theorem of Calculus

· $F(x) = \int_a^x f(t)\,dt$

$\frac{d}{dx}F(x) = f(x)$

→ 2nd fundamental theorem of Calculus

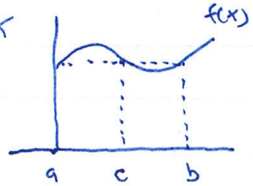
$\int_a^b f(x)\,dx = F(a) - F(b)$

→ Jika f continous di almost everywhere on $[a,b]$ then f is integrable di $[a,b]$ (titik dmn $f(x)$ itu discontinous is finite)

→ mean value theorem for integrals

· there is $c \in [a,b]$ such that $\int_a^b f(x)\,dx = f(c)(b-a)$

· ex



$\int_a^b f(x)\,dx = f(c)(b-a)$ ↳ luas rectangle

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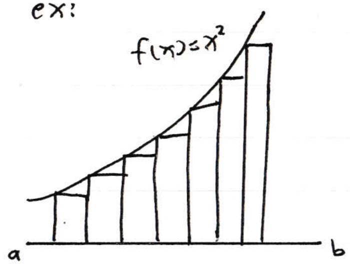
Pertemuan 9: Riemann integrals

→ Sometimes functions are hard to integrat

ex:



$$A(R) = \int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} (f(x)\cdot \Delta x)$$

$$\Delta x = \frac{b-a}{n}$$

$$x = a + k\cdot\Delta x = a + k\frac{b-a}{n} = \frac{an}{n} + \frac{b-a}{n}$$

$$= \frac{a(n-k)+bk}{n}$$

$$f(x) = \frac{a^2(n-k)^2 + 2abk(n-k) + b^2k^2}{n^2}$$

$$A(R) = \lim_{n\to\infty}\left(\sum_{k=1}^{n} \frac{(b-a)}{n}\cdot\frac{(a^2(n-k)^2 + 2abk(n-k) + b^2k^2)}{n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{b-a}{n^3}\left(\sum_{k=1}^{n} a^2(n-k)^2 + 2abk(n-k) + b^2k^2\right)\right)$$

$$= \lim_{n\to\infty}\left(\frac{b-a}{n^3}\left(\sum_{k=1}^{n} a^2n^2 - \sum_{k=1}^{n} 2a^2nk + \sum_{k=1}^{n} a^2k^2 + \sum_{k=1}^{n} 2abnk - \sum_{k=1}^{n} 2abk^2 + \sum_{k=1}^{n} b^2k^2\right)\right)$$

$$= \lim_{n\to\infty} \frac{b-a}{n^3}\left(a^2n^3 - 2a^2n\sum_{k=1}^{n} k + a^2\sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k\cdot 2abn - \sum_{k=1}^{n} k^2\cdot 2ab + b^2\sum_{k=1}^{n} k^2\right)$$

$$= \lim_{n\to\infty} \frac{b-a}{n^3}\left(a^2n^3 + (2abn - 2a^2n)\sum_{k=1}^{n} k + (a^2+b^2-2ab)\sum_{k=1}^{n} k^2\right)$$

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→ Integral tak wajar

1. salah satu batas tak hingga

$$\int_{-\infty}^{b} f(x)\,dx = \lim_{a \to -\infty} \int_a^b f(x)\,dx$$

or $$\int_b^{\infty} f(x)\,dx = \lim_{a \to \infty} \int_b^a f(x)\,dx$$

jika limit di ruas kanan ada maka integral tersebut konvergen

2. kedua batas tak hingga

jika $\int_{-\infty}^{0} f(x)\,dx$ konvergen dan $\int_0^{\infty} f(x)\,dx$ konvergen

maka $\int_{\infty}^{\infty}$ juga konvergen

3. mempunyai nilai tak hingga pada suatu batas

jika f kontinu di $[a,b)$ dan $\lim_{x \to b^-} |f(x)| = \infty$

$$\int_a^b f(x)\,d(x) = \lim_{\alpha \to b^-} \int_a^{\alpha} f(x)\,dx$$

↓

konvergen jika di ruas kanan limitnya ada

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→ Aplikasi Integral

- luas grafik
- volume benda putar
- luas permukaan benda putar
- panjang kurva

→ Parametric

misal ada $y(t)$ dan $x(t)$

then $L = \int_a^b y\,dx = \int_a^b y(t)\,\frac{dx}{dt}\,dt$

→ volume of $\int_a^b f(x)\,dx$ rotated on x

. $\pi \int_a^b f^2(x)\,dx = V$

→ benda diputar seputar sumbu-x $(f(x) - g(x))$

. $\pi \int_a^b f^2(x) - g^2(x)\,dx = V$

→ diputar sekitar $y = c$

. $V = \pi \int_a^b (f(x) - c)^2\,dx$

→ Panjang selimut

. $S = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\,dy$

→ luas permukaan

. $L = 2\pi \int_a^b f(x) \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$

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→ bentuk tak terdefinisi

1. $\frac{0}{0}$
2. $\frac{\infty}{\infty}$
3. $0 \cdot \infty$
4. $\infty - \infty$
5. 0^0
6. ∞^0
7. 1^∞

→ L'hopital untuk bentuk $\frac{0}{0}$

Jika $\lim_{x \to u} f(x) = \lim_{x \to u} g(x) = 0$ maka jika $\lim_{x \to u} \frac{f'(x)}{g'(x)}$ ada

maka $\lim_{x \to u} \frac{f(x)}{g(x)} = \lim_{x \to u} \frac{f'(x)}{g'(x)}$

→ Cauchy's Mean Value Theorem

jika $f(x)$ dan $g(x)$ diferensiable di (a,b) dan kontinu di $[a,b]$ maka ada c di (a,b) sehingga

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$$