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1. $f(x) = \cos(x)\, e^{(\cos x^2)}$

$f'(x) = -\sin(x) e^{\cos x^2} + \cos(x^2)\, e^{\cos(x^2)} \cdot -\sin(x^2) \cdot 2x$

2. $\dfrac{\ln(y^2)}{x} + e^{xy} + \csc^2(x) = 1$

$\dfrac{\frac{x}{y^2} \cdot 2y\, dy - \ln(y^2)}{x^2} + e^{xy}(y + x\, dy) + (-2\csc(x) \cdot \csc(x) \cdot \cot(x)) = 0$

$\cancel{\dfrac{\frac{2x}{y} dy}{x^2} - \dfrac{\ln(y^2)}{x^2} + xy^2 e^{xy} + x^2 y e^{xy} dy - 2\csc^2(x)\cot(x))}$

$\cancel{\dfrac{2}{xy} dy}$

$\dfrac{\frac{2x}{y}\frac{dy}{dx} - \ln(y^2)}{x^2} - e^{xy}\left(y + x\dfrac{dy}{dx}\right) - 2\csc^2 x \cot^2 x = 0$

$\dfrac{2\,dy}{xy} - \dfrac{\ln(y^2)}{x^2} - ye^{xy} - xe^{xy}\dfrac{dy}{dx} - 2\csc^2 x \cot x = 0$

$\dfrac{2}{xy}\dfrac{dy}{dx} - xe^{xy}\dfrac{dy}{dx} = \dfrac{\ln(y^2)}{x^2} + ye^{xy} + 2\csc^2 x \cot x$

$\dfrac{dy}{dx} = \dfrac{\frac{\ln(y^2)}{x^2} + ye^{xy} + 2\csc^2 x \cot x}{\left(\frac{2}{xy} - xe^{xy}\right)}$

(5.)

3. $dt = \frac{1}{2} d(2t)$

$\int \frac{\sin^5(2t)}{\csc(2t)} dt = \frac{1}{2} \int \frac{\sin^3(2t)}{\csc(2t)} d(2t) \nearrow \csc = \frac{1}{\sin}$

$= \frac{1}{2} \int \sin^4(2t)\, d(2t) \rightarrow 2t = x$

$= \frac{1}{2} \int \left(\frac{1-\cos 2x}{2}\right)^2 dx$

$= \frac{1}{8} \int \cos^2 2x - 2\cos 2x + 1\, dx$

$= \frac{1}{8}\left(\left(\int \frac{1+\cos 4x}{2} dx\right) - \sin 2x + x\right) + C$

$= \frac{1}{8}\left(\frac{1}{2}x + \frac{1}{8}\sin 4x - \sin 2x + x\right) + C$

$= \frac{3}{16}x - \frac{\sin 2x}{8} + \frac{1}{64}\sin 4x = \frac{3}{8}t - \frac{\sin 4t}{8} + \frac{\sin 8t}{64} + C$

4. $\int \frac{1}{(x-2)(x+1)} dx = \int \frac{1}{3(x-2)} - \frac{1}{3(x+1)} dx$

$= \frac{1}{3} \int \frac{1}{x-2} - \frac{1}{x+1} dx = \frac{1}{3}\left(\ln|x-2| - \ln|x+1|\right) + C$

5. misal $x - 9 = t^2 \rightarrow dx = 2t\, dt$

$2x = 2(t^2+9)$

$\int \frac{2x}{\sqrt{x-9}} dx = 4\int (t^2+9)\, dt = 4\left(\frac{1}{3}t^3 + 9t\right)$

~~$\frac{4}{3}x^{\frac{3}{2}} + \frac{36}{3}\sqrt{x-9} + C$~~

~~$(4x - 36 - 108\sqrt{x-9})\frac{1}{3} + C$~~

$= \frac{4}{3}t^3 + 36t + C$

~~$\frac{4}{3}(x-9)^{3/2} + 36\sqrt{x-9} + C$~~

6. $\frac{4}{3}(x-9)^{\frac{3}{2}} + 36(x-9)^{\frac{1}{2}} + C$