

LEMBAR CARA TUGAS 1

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- ① a. p = Proses Penyerbukan bunga dapat terjadi
 q = ada serangga yang membantu proses penyerbukan
 r = ada angin yang membantu proses penyerbukan

$$\therefore p \rightarrow (q \vee r)$$

- b. p = Budi mendapat nilai akhir dibawah 55
 q = Budi dapat lulus mata kuliah Matematika Diskrit I

$$\therefore \neg p \leftrightarrow q$$

- c. p = Dapat menaiki wahana Hysteria
 q = Pengunjung memiliki tinggi dibawah 195cm
 r = Pengunjung berusia lebih dari 12 tahun

$$\therefore (q \wedge r) \rightarrow p$$

② a.

p	q	r	$(p \wedge q)$	$\neg(p \wedge q)$	$\neg(p \wedge q) \vee r$	$p \wedge r$	$\neg(p \wedge q) \vee r \rightarrow p \wedge r$
F	F	F	F	T	T	F	F
F	F	T	F	T	T	F	F
F	T	F	F	T	T	F	F
F	T	T	F	T	T	F	F
T	F	F	F	T	T	F	F
T	F	T	F	T	T	T	T
T	T	F	T	F	F	F	T
T	T	T	T	F	T	T	T

$$\hookrightarrow (\neg(p \wedge q) \vee r) \rightarrow (p \wedge r)$$

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b. $(\neg q \vee (r \rightarrow s)) \rightarrow \neg s$

q	r	s	$\neg q$	$\neg s$	$r \rightarrow s$	$\neg q \vee (r \rightarrow s)$	$(\neg q \vee (r \rightarrow s)) \rightarrow \neg s$
F	F	F	T	T	T	T	T
F	F	T	T	F	T	T	F
F	T	F	T	T	F	F	T
F	T	T	T	F	T	T	F
T	F	F	F	T	T	T	T
T	F	T	F	F	T	T	F
T	T	F	F	T	F	T	T
T	T	T	F	F	T	T	F

3. a. kuning berkata : $m \rightarrow (\neg b \oplus \neg h) \equiv \neg m \vee \neg b \oplus \neg h$ ①
 Hijau berkata : $h \wedge b$ ②
 Biru berkata : $m \rightarrow (\neg h \vee \neg k) \equiv \neg m \vee \neg h \vee \neg k$ ③
 merah berkata : $k \wedge b$ ④

b.

k	h	b	m	①	②	③	④	
F	T	T	T	F	T	T	F	kuning pembunuh
T	F	T	T	F	T	T	T	hijau pembunuh
T	T	F	T	F	T	F	F	biru pembunuh
T	T	T	F	T	T	T	T	merah pembunuh

∴ hijau pembunuhnya karena di kemungkinan ini, hanya ada 1 pembunuh (pembunuh) kemungkinan lainnya mengakibatkan lebih dari 1 pembunuh atau tidak ada pembunuh sama sekali

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4. a. ① $\neg p$
 ② $\neg q \wedge r$
 ③ $(q \wedge \neg s) \leftrightarrow p$
 ④ $t \vee q$
 ⑤ $(\neg q \vee r) \rightarrow s$

- b. $\neg p \equiv T \rightarrow p \equiv F \dots$ ①
 $p \equiv F \rightarrow (q \wedge \neg s) \equiv F \dots$ ② $\rightarrow T$
 $(\neg q \wedge r) \equiv T \rightarrow q \equiv F \quad r \equiv T \dots$ ③
 $t \vee q \equiv F \rightarrow t \equiv T \dots$ ④
 $(\neg q \vee r) \rightarrow s \equiv T \rightarrow s \equiv T \dots$ ⑤

sistem konsisten dengan interpretasi
 $p, q, r, s, t \equiv F, F, T, T, T$

5. a. kedua pasangan formula tidak ekuivalen karena interpretasi
 $p \equiv q \equiv r \equiv F$ akan membuat

$(r \wedge (p \rightarrow q)) \vee (\neg q \rightarrow r) \equiv F$ sedangkan
 $(\neg r \rightarrow p) \leftrightarrow (p \leftrightarrow q) \wedge r \equiv T$

b. ekuivalen

$$\begin{aligned}
 &\equiv (p \rightarrow s) \vee (((r \wedge r) \vee (p \wedge \neg q)) \wedge (q \rightarrow r)) \dots ① \\
 &\equiv (\neg p \vee s) \vee (((r \wedge r) \vee (p \wedge \neg q)) \wedge (\neg q \vee r)) \quad (\text{definisi implikasi}) \\
 &\equiv (\neg p \vee s) \vee ((r \vee (p \wedge \neg q)) \wedge (\neg q \vee r)) \quad (\text{idempoten}) \\
 &\equiv (\neg p \vee s) \vee ((r \vee p) \wedge (r \vee \neg q) \wedge (\neg q \vee r)) \quad (\text{distributif}) \\
 &\equiv (\neg p \vee s) \vee ((r \vee p) \wedge (r \vee \neg q) \wedge (r \vee \neg q)) \quad (\text{komutatif}) \\
 &\equiv (\neg p \vee s) \vee ((r \vee p) \wedge (r \vee \neg q)) \quad (\text{idempoten}) \\
 &\equiv (\neg p \vee s) \vee (r \vee (p \wedge \neg q)) \quad (\text{distributif}) \\
 &\equiv \neg p \vee (p \wedge \neg q) \vee s \vee r \quad (\text{komutatif}) \\
 &\equiv ((\neg p \vee p) \wedge (\neg p \vee \neg q)) \vee s \vee r \quad (\text{distributif}) \\
 &\equiv (T \wedge (\neg p \vee \neg q)) \vee s \vee r \quad (\text{negasi}) \\
 &\equiv \neg p \vee \neg q \vee s \vee r \dots ②
 \end{aligned}$$

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c) kedua pasangan formula tidak ekuivalen karena interpretasi
 $p \equiv F, q \equiv T, r \equiv T$ akan membuat

$$(\neg p \leftrightarrow r) \wedge (q \rightarrow p) \wedge (r \rightarrow \neg p) \equiv F \text{ sedangkan } (\neg p \wedge r) \vee (p \wedge \neg r) \equiv T$$

6. a) Kontradiksi

$$\begin{aligned} &\equiv ((a \rightarrow b) \rightarrow c) \wedge a \wedge b \wedge \neg c \\ &\equiv ((a \vee b) \rightarrow c) \wedge a \wedge b \wedge \neg c && (\text{definisi implikasi}) \\ &\equiv ((\neg a \wedge \neg b) \vee c) \wedge a \wedge b \wedge \neg c && (\text{de morgan dan definisi implikasi}) \\ &\equiv (\neg a \vee c) \wedge \neg b \vee c \wedge a \wedge b \wedge \neg c && (\text{distributif}) \\ &\equiv (\neg a \vee c) \wedge a \wedge (\neg b \vee c) \wedge b \wedge \neg c && (\text{komutatif}) \\ &\equiv ((\neg a \wedge a) \vee (a \wedge c)) \wedge ((\neg b \wedge b) \vee (b \wedge c)) \wedge \neg c && (\text{distributif}) \\ &\equiv (F \vee (a \wedge c)) \wedge (F \vee (b \wedge c)) \wedge \neg c && (\text{negasi}) \\ &\equiv a \wedge c \wedge b \wedge (c \wedge \neg c) && (\text{identitas}) \\ &\equiv a \wedge b \wedge c \wedge F && (\text{negasi}) \\ &\equiv F && (\text{dominasi}) \end{aligned}$$

b) Tautologi

$$\begin{aligned} &\equiv (\neg((\neg a \vee b) \wedge a) \vee b) \leftrightarrow ((c \rightarrow b) \vee (b \rightarrow c)) \\ &\equiv (\neg((\neg a \wedge a) \vee (a \wedge b)) \vee b) \leftrightarrow ((c \rightarrow b) \vee (b \rightarrow c)) && (\text{distributif}) \\ &\equiv (\neg(F \vee (a \wedge b)) \vee b) \leftrightarrow ((c \rightarrow b) \vee (b \rightarrow c)) && (\text{negasi}) \\ &\equiv (\neg(a \wedge b) \vee b) \leftrightarrow ((c \rightarrow b) \vee (b \rightarrow c)) && (\text{identitas}) \\ &\equiv (\neg a \vee \neg b) \vee b \leftrightarrow ((c \rightarrow b) \vee (b \rightarrow c)) && (\text{de morgan}) \\ &\equiv \neg a \vee T \leftrightarrow ((c \rightarrow b) \vee (b \rightarrow c)) && (\text{identitas}) \\ &\equiv T \leftrightarrow ((c \rightarrow b) \vee (b \rightarrow c)) && (\text{negasi}) \\ &\equiv T \leftrightarrow (\neg c \vee b) \vee (\neg b \vee c) && (\text{definisi implikasi}) \\ &\equiv T \leftrightarrow (c \vee c) \vee (\neg b \vee b) && (\text{komutatif}) \\ &\equiv T \leftrightarrow T && (\text{negasi}) \\ &\equiv T \end{aligned}$$

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c) Kontradiksi

$$\begin{aligned} &\equiv ((a \rightarrow b) \leftrightarrow (\neg a \wedge \neg c \wedge \neg b)) \wedge (a \wedge b \wedge \neg c) \\ &\equiv ((a \vee b) \leftrightarrow (\neg a \wedge \neg c \wedge \neg b)) \wedge (a \wedge b \wedge \neg c) && (\text{definisi implikasi}) \\ &\rightarrow ((a \vee b) \leftrightarrow (\neg a \wedge \neg c \wedge \neg b)) \\ &\equiv ((a \vee b) \wedge (\neg a \wedge \neg c \wedge \neg b)) \vee (\neg(a \vee b) \wedge \neg(\neg a \wedge \neg c \wedge \neg b)) && (\text{definisi bi implikasi}) \\ &\equiv (a \vee b) \wedge (\neg a \wedge \neg c \wedge \neg b) \vee (\neg a \wedge \neg b \wedge (a \vee c \vee b)) && (\text{de morgan}) \\ &\equiv (a \vee b) \wedge (\neg a \wedge \neg c \wedge \neg b) \vee (\neg a \wedge (a \vee c \vee b) \wedge \neg b) && (\text{komutatif}) \\ &\equiv (a \vee b) \wedge (\neg a \wedge \neg c \wedge \neg b) \vee ((\neg a \wedge a) \vee (\neg a \wedge c) \vee (\neg a \wedge b)) \wedge \neg b && (\text{distributif}) \\ &\equiv (a \vee b) \wedge (\neg a \wedge \neg c \wedge \neg b) \vee (F \vee (\neg a \wedge c) \vee (\neg a \wedge b)) \wedge \neg b && (\text{negasi}) \\ &\equiv (a \vee b) \wedge (\neg a \wedge \neg c \wedge \neg b) \vee (\neg a \wedge c) \vee (\neg a \wedge b) \wedge \neg b && (\text{identitas}) \\ &\equiv (a \vee b) \wedge (\neg a \wedge \neg c \wedge \neg b) \vee (\neg a \wedge (c \vee b)) \wedge \neg b && (\text{distributif}) \\ &\equiv (a \vee b) \wedge (\neg a \wedge \neg c \wedge \neg b) \vee (\neg a \wedge ((c \wedge \neg b) \vee (b \wedge \neg b))) && (\text{distributif}) \\ &\equiv (a \vee b) \wedge (\neg a \wedge \neg c \wedge \neg b) \vee (\neg a \wedge (c \wedge \neg b) \vee F) && (\text{negasi}) \\ &\equiv (a \vee b) \wedge (\neg a \wedge \neg c \wedge \neg b) \vee (\neg a \wedge c \wedge \neg b) && (\text{identitas}) \\ &\rightarrow ((a \vee b) \wedge (\neg a \wedge \neg b \wedge \neg c) \vee (\neg a \wedge c \wedge \neg b)) \wedge (a \wedge b \wedge \neg c) \\ &\equiv ((a \vee b) \wedge \neg a \wedge \neg b \wedge \neg c \wedge a \wedge b \wedge \neg c) \vee ((\neg a \wedge \neg b \wedge c) \wedge (a \wedge b \wedge \neg c)) && (\text{distributif}) \\ &\equiv ((a \vee b) \wedge \neg a \wedge \neg b \wedge \neg c \wedge a \wedge b \wedge \neg c) \vee (\neg a \wedge a \wedge \neg b \wedge b \wedge \neg c \wedge c) && (\text{komutatif}) \\ &\equiv (a \vee b) \wedge F \wedge F \wedge \neg c \wedge \neg c) \vee (F \wedge F \wedge F) && (\text{negasi}) \\ &\equiv F \vee F \\ &\equiv F \end{aligned}$$

7. a) TRUE

↳ ada nilai x yang memenuhi, misal $x=3$


b) FALSE

↳ tidak semua x memenuhi, misal $x=2$

c) FALSE

↳ tidak semua x memenuhi, misal $x=5$

ADENDUM


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6. a. formula tersebut adalah satisfiable

formula $\equiv T$ ketika $a \equiv T, b \equiv T$ dan $c \equiv F$
dan $\equiv F$ pada interpretasi selain itu

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① FALSE

↳ tidak semua x memenuhi, misal $x=2$

⑧ a. ada mahasiswa fasikom yang bertakamata dan pandai melukis

b. tidak ada mahasiswa fasikom yang bertakamata dan pandai melukis

c. ada mahasiswa fasikom yang tidak pandai melukis

d. ada mahasiswa fasikom yang tidak bertakamata dan tidak pandai melukis

e. Semua mahasiswa fasikom yang pandai melukis bertakamata

$$\begin{aligned} \neg \exists x (\neg P(x) \wedge Q(x)) &\equiv \forall x (P(x) \vee \neg Q(x)) \\ &\equiv \forall x (\neg P(x) \rightarrow \neg Q(x)) \\ &\equiv \forall x (Q(x) \rightarrow P(x)) \end{aligned}$$

⑨. a. misal domain $x \in \{x_1, x_2\}$

maka $\forall x (P(x) \leftrightarrow Q(x))$

$$\equiv (P(x_1) \leftrightarrow Q(x_1)) \wedge (P(x_2) \leftrightarrow Q(x_2)) \dots ①$$

maka $\forall x P(x) \leftrightarrow \forall x Q(x)$

$$\equiv (P(x_1) \wedge P(x_2)) \leftrightarrow (Q(x_1) \wedge Q(x_2)) \dots ②$$

∴ ada interpretasi $P(x_1) \equiv T, P(x_2) \equiv F, Q(x_1) \equiv F, Q(x_2) \equiv T$ sehingga ① $\equiv F$ namun ② $\equiv T$, sehingga kedua formula tidak ekuivalen

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$$\text{b. } \neg \forall x (\neg Q(x) \rightarrow P(x)) \dots ①$$

$$\equiv \exists x \neg (Q(x) \vee P(x)) \dots ② \quad (\text{definisi implikasi})$$

∴ kedua formula ekuivalen

c. misal domain $x \in \{x_1, x_2\}$

$$\forall x (P(x) \wedge Q(x))$$

$$\equiv (P(x_1) \wedge Q(x_1)) \wedge (P(x_2) \wedge Q(x_2)) \dots ①$$

$$\exists x P(x) \vee \exists x Q(x)$$

$$\equiv (P(x_1) \vee P(x_2)) \vee (Q(x_1) \vee Q(x_2)) \dots ②$$

interpretasi $P(x_1) \equiv F, P(x_2) \equiv T, Q(x_1) \equiv T, Q(x_2) \equiv T$ membuat ① $\equiv F$ namun ② $\equiv T$

∴ kedua formula ekuivalen

10. a. asumsi d = matematika diskrit 1

$$\equiv \neg \forall x (L(x, d) \rightarrow A(x, d))$$

$$\equiv \exists x (\neg L(x, d) \vee A(x, d)) \quad (\text{definisi implikasi})$$

$$\equiv \exists x (L(x, d) \wedge \neg A(x, d))$$

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b) asumsi

↳ d = Matematika distret 1

↳ S = Statistika & Probabilitas

↳ k = Kalkulus

$$\therefore \forall x (M(x, S) \rightarrow (L(x, d) \vee L(x, k)))$$

c)

$$\therefore \exists x \exists y \forall z (A(x, z) \wedge A(y, z) \wedge x \neq y \wedge \forall w (w \neq x \wedge w \neq y \rightarrow \neg A(w, z)))$$

d)

$$\therefore \exists x \forall y (L(x, y) \vee \exists z \exists w (A(x, z) \wedge A(x, w) \wedge w \neq z))$$

$$\begin{aligned} \text{ii) a) } & \neg \exists x < 0 \exists y > 0 (xy > 0) \quad x, y \in \mathbb{Z} \\ & \equiv \forall x < 0 \forall y > 0 (xy < 0) \quad x, y \in \mathbb{Z} \\ & \equiv \forall x \forall y ((x < 0 \wedge y > 0) \rightarrow xy < 0) \quad x, y \in \mathbb{Z} \end{aligned}$$

$$\begin{aligned} \text{b) } & \forall x \in \mathbb{Z} \forall y \in \mathbb{Z} (x + y \in \mathbb{Z}) \\ & \equiv \forall x \forall y (x \in \mathbb{Z} \wedge y \in \mathbb{Z} \rightarrow (x + y) \in \mathbb{Z}) \end{aligned}$$

$$\begin{aligned} \text{c) } & \forall x \in \mathbb{Z} (x^2 > 0) \\ & \equiv \forall x (x \in \mathbb{Z} \rightarrow x^2 > 0) \end{aligned}$$