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KALKULUS - C - PR 2

2 2 0 6 0 2 8 9 3 2

1. a.

$$\lim_{h\to 0} \frac{4(x+h)^4 + 4(x+h)^2 + 1 - (4x^4 + 4x^2 + 1)}{h}$$

$$= \frac{1 + 4(h^4 + 4h^3x + 6h^2x^2 + 4hx^3 + x^4) + 4(h^2 + 2hx + x^2) - (4x^4 + 4x^2 + 1)}{h}$$

$$= \frac{4h^4 + 16h^3x + 24h^2x^2 + 16hx^3 + 4x^2 + 4h^2 + 8hx + 4x^2 - 4x^4 - 4x^2 - 1 + 1}{h}$$

$$= \frac{4h^4 + 16h^3x + 24h^2x^2 + 16hx^3 + 8xh + 4h^2}{h}$$

$$= \frac{h(4h^3 + 16h^2x + 24hx^2 + 16x^3 + 8x + 4h)}{h}$$

$$= 0 + 0 + 0 + 16x^3 + 8x + 0 \quad \text{(substitusi)}$$

$$\therefore \frac{d}{dx} 4x^4 + 4x^2 + 1 = 16x^3 + 8x \quad \blacksquare$$

b.

$$\lim_{h\to 0} \frac{\frac{4}{(x+h)^2} - \frac{4}{x^2}}{h}$$

$$= \frac{4x^2 - 4(x+h)^2}{(x+h)^2 x^2 h} = \frac{4x^2 - 4(h^2 + 2hx + x^2)}{(x^2 + 2hx + h^2)x^2 h}$$

$$= \frac{4x^2 - 4x^2 - 8xh - 4h^2}{(x^4 + 2x^3h + x^2h)h} = \frac{h(-8x - 4h)}{(x^4 + 2x^3h + x^2h)h}$$

$$= \frac{-8x}{x^4} \quad \text{(substitusi)}$$

$$= -\frac{8}{x^3} \qquad \therefore \frac{d}{dx}\frac{4}{x^2} = -\frac{8}{x^3} \quad \blacksquare$$

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2\. a. Lemma: (i) $\tan x = \frac{\sin x}{\cos x}$

(ii) $\sin 2x = 2\sin x \cos x$

$\therefore \frac{d}{dx} \sin x \cos x \tan x$

$= \sin x \cos x \frac{\sin x}{\cos x} = \sin^2 x$

$= 2\sin x \, d\sin x$

$= 2\sin x \cos x$

$= \sin 2x$ ▨

b. Lemma: $\frac{d}{dx}\frac{u}{v} = \frac{u'v - v'u}{v^2}$, $\frac{d}{dx} uv = u'v + v'u$

(i) $\frac{d}{dx} \sin x \cdot x^2 = 2x\sin x + x^2 \cos x$

(ii) $\frac{d}{dx}(x^2+1) = 2x$

$\therefore \frac{d}{dx}\frac{(x^2 \sin x)}{(x^2+1)} = \frac{(x^2+1)(2x\sin x + x^2\cos x) + (-2x(x^2\sin x))}{(x^2+1)^2}$

$= \frac{(x^2+1)(2x\sin x) + (x^2+1)(x^2\cos x) - 2x(x^2\sin x)}{(x^2+1)^2}$

$= \frac{x^2 . 2x . \sin x + 2x\sin x + x^4\cos x + x^2\cos x - 2x . x^2 . \sin x}{(x^2+1)^2}$

$= \frac{x(2\sin x + (x^3+x)\cos x)}{(x^2+1)^2}$ ▨

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c. $\frac{d}{dx} \cos nx$

$= -\sin(nx)\, dnx$

$= -n \sin(nx)$ ▨

3\. a. $xy + 3y = 3x^2 - 7y^2$

$= x\,dy + y\,dx + 3dy = 6x\,dx + (-14y\,dy)$

$= x\,dy + 3dy + (14y\,dy) = 6x\,dx - y\,dx$

$= dy(x + 3 + 14y) = dx(6x - y)$

$\therefore \frac{dy}{dx} = \frac{6x - y}{x + 3 + 14y}$ ▨

b. $x^2 + y^2 = \sin xy$

$= 2x\,dx + 2y\,dy = \cos xy\,(x\,dy + y\,dx)$

$= x\cos xy\,dy + y\cos xy\,dx$

$= dy\,(2y - x\cos xy) = dx(y\cos xy - 2x)$

$\therefore \frac{dy}{dx} = \frac{y\cos xy - 2x}{2y - x\cos xy}$ ▨

4\. a. (i) $\frac{d}{dx} 2^{3x+2}$

misal $y = 2^{3x+2}$

$\rightarrow \ln y = (3x+2)\ln 2 \rightarrow \frac{dy}{y} = 3\ln 2\, dx$

$\therefore \frac{d}{dx} 2^{3x+2} = 2^{3x+2} . 3\ln 2$

(ii) $\frac{d}{dx} e^{-3x} = -3e^{3x}$

(iii) $\frac{d}{dx} \ln(x) = \frac{1}{x}$

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$$\therefore \frac{d}{dx} 2^{3x+2} + e^{-3x} + \ln x = 2^{3x+2} . 3\ln 2 - 3e^{-3x} + \frac{1}{x} \ \blacksquare$$

(b.) Lemma : (i) $\ln x \rightarrow {}^{e}\log x$

(ii) ${}^{a}\log b + {}^{a}\log c = {}^{a}\log bc$

$$\therefore \ln \frac{1}{x^3} + \ln x^4 = \ln x$$

$$\frac{d}{dx} \ln \frac{1}{x^3} + \ln x^4 = \frac{1}{x} \ \blacksquare$$

(5.)

$$y = Ae^{px}$$

$$y' = p . Ae^{px} = \frac{dy}{dx}$$

$$y'' = p . p . Ae^{px} = p^2 Ae^{px} = \frac{d^2y}{dx^2}$$

lemma : $\frac{d}{dx} e^{f(x)} = f'(x) e^{f(x)}$

$$\therefore p^2 Ae^{px} - 2p . p Ae^{px} + p^2 . Ae^{px} = \frac{d^2y}{dx^2} - 2p . \frac{dy}{dx} + p^2 y$$

$$= Ae^{px} (p^2 - 2p^2 + p^2) \rightarrow Ae^{px} . 0 = 0 \ \blacksquare$$

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(6) Lemma : $\cos x = \sqrt{1 - \sin^2 x}$

(i) $\frac{d}{dx} 1 + 4^x$

$$\therefore y = 4^x$$

$$= \ln y = x \ln 4$$

$$\therefore \frac{d}{dx} 1 + 4^x = 4^x \ln 4 + 0 = 4^x . \ln 4$$

(ii) $\frac{d}{dx} 2^{x+1}$

$$\therefore y = 2^{x+1}$$

$$= \ln y = (x+1) \ln 2 = x\ln 2 + \ln 2$$

$$\therefore \frac{d}{dx} 2^{x+1} = 2^{x+1} . (\ln 2 + 0) = 2^{x+1} . \ln 2$$

(iii) $\frac{d}{dx} \sin^{-1} \left(\frac{2^{x+1}}{1+4^x} \right)$

$$y = \sin^{-1} \left(\frac{2^{x+1}}{1+4^x} \right) \rightarrow \sin y = \frac{2^{x+1}}{1+4^x} \rightarrow \cos y = \sqrt{1 - \frac{2^{2x+2}}{(1+4^x)^2}}$$

$$\therefore \cos y \ dy = \frac{(2^{x+1} \ln 2)(1+4^x) - (4^x \ln 4)(2^{x+1})}{(1+4^x)^2}$$

$$\therefore \frac{d}{dx} \sin^{-1} \left(\frac{2^{x+1}}{4^x+1} \right) = \frac{(2^{x+1} \ln 2)(1+4^x) - (4^x \ln 4)(2^{x+1})}{\sqrt{1 - \frac{2^{2x+2}}{(1+4^x)^2}} . (1+4^x)^2} = f'(x)$$

$f'(0) = $ undefined $\blacksquare$

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(2) (i) $\frac{d}{dx}(\tan x)^{\tan x}$

$\therefore \ y = (\tan x)^{\tan x}$

$= \ln y = \tan x \ln \tan x$

$\therefore \ \frac{dy}{y} = \sec^2 x \ln \tan x + \frac{\sec^2 x}{\tan x} \tan x$

$\therefore \ \frac{d}{dx}(\tan x)^{\tan x} = \tan x^{\tan x} . (\sec^2 x \ln \tan x + \sec^2 x)$

$\therefore \ \frac{d}{dx} \tan x^{\tan x^{\tan x}}$

$= y = \tan x^{\tan x^{\tan x}} \rightarrow \ln y = \tan x^{\tan x} \ln \tan x$

$\therefore \ \frac{dy}{y} = (\tan x^{\tan x}(\sec^2 x \ln \tan x + \sec^2 x))(\ln \tan x) + \left(\frac{\sec^2 x}{\tan x}\right)(\tan x^{\tan x})$

$= \tan x^{\tan x^{\tan x}}\left((\ln \tan x . \tan x^{\tan x} \sec^2 x (\ln \tan x + 1)) + \frac{\sec^2 x}{\tan x} . \tan x^{\tan x}\right)$

$\tan \frac{\pi}{4} = 1 \ , \ \sec^2 \frac{\pi}{4} = 2$

$= 1^{1^1}\left((\ln 1 . 1^1 . 2 . (\ln 1 + 1)) + \frac{2}{1} . 1^1\right)$

$= 1(0.1.2.1 + 2.1) = 2$ ■