ALDEN LUTHFI - PR 4 - KALKULUS C

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1. a. $\frac{1}{6}x^6 + \frac{3}{5}x^5 + \frac{1}{6}x^4 + \frac{7}{3}x^3 + 3x^2 + 6x + C$

b. $f'(x) = 8x^3 - 24x^2 + 2x + C$

$f(x) = 2x^4 - 8x^3 + x^2 + Cx + D$

$f(1) = -5 + C + D = -9 \rightarrow C + D = -4$

$f(-2) = 100 - 2C + D = -4 \rightarrow -2C + D = -104$

$C = \frac{100}{3}$, $D = -\frac{112}{3}$

$\therefore f(x) = 2x^4 - 8x^3 + x^2 + \frac{100}{3}x - \frac{112}{3}$

2. a. $\int \frac{x^2-1}{x-1}\,dx = \int x+1\,dx = \frac{x^2}{2} + x + C$

b. $\int x.\ln x\,dx$

$u = \ln x \rightarrow x = e^u$

$du = \frac{dx}{x}$

$= \int u.e^{2u}\,du = \frac{1}{2}\int u\,de^{2u} = \frac{1}{2}\left(u.e^{2u} - \int e^{2u}\,du\right)$

$= \frac{u.e^{2u}}{2} - \frac{e^{2u}}{4} = \frac{x^2\ln x}{2} - \frac{x^2}{4} = \frac{1}{4}(2x^2\ln x - x^2) + C$

$= \frac{x^2}{4}(2\ln x - 1) + C$

c. $\frac{1}{(x^2+1)(x-1)(x+1)} = -\frac{1}{2(x^2+1)} - \frac{1}{4(x+1)} + \frac{1}{4(x-1)}$

$\therefore \int \frac{dx}{4(x-1)} - \int \frac{dx}{4(x+1)} - \int \frac{dx}{2(x^2+1)}$

$= \frac{\ln|x-1|}{4} - \frac{\ln|x+1|}{4} - \frac{\arctan x}{2} + C$

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2 2 0 6 0 2 8 9 3 2

3\. a. $\int \sin x \csc x \, dx = \int dx = x + C$ $\blacksquare$

b. $\int \frac{\sec^2 x}{\tan x} \, dx = \int \frac{d\tan x}{\tan x} = \ln|\tan x| + C$ $\blacksquare$

c. $\int \frac{\sin\frac{1}{x}}{x^2} \, dx = -\int \sin\frac{1}{x} \, d\frac{1}{x} = \cos\frac{1}{x} + C$

d. $\int e^{2x}\sin 2x \, dx = \frac{e^{2x}\cos 2x}{2} - \int e^{2x}\cos 2x \, dx$

$= \frac{e^{2x}\cos 2x}{2} - \left(-\frac{e^{2x}\sin 2x}{2} + \int e^{2x}\sin 2x \, dx\right)$

$\therefore 2\int e^{2x}\sin 2x \, dx = \frac{e^{2x}(\cos 2x + \sin 2x)}{2}$

$\int e^{2x}\sin 2x \, dx = \frac{e^{2x}}{4}(\cos 2x + \sin 2x) + C$ $\blacksquare$

e. $\cos 2x = 2\cos^2 x - 1$

$\therefore \frac{1}{1+\cos 2x} = \frac{\sec^2 x}{2}$

$\int \frac{\sec^2 x}{2} \, dx = \frac{1}{2}\int d\tan x = \frac{\tan x}{2} + C$ $\blacksquare$

f. $\int \sec^3 x \, dx = \int \sec x \, d\tan x$

$= \sec x \tan x - \int \tan x \, d\sec x$

$= \sec x \tan x - \int \tan^2 x \sec x \, dx$

$= \sec x \tan x - \left(\int (\sec^2 x - 1)\sec x \, dx\right)$

$= \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$

$= \frac{1}{2}\left(\sec x \tan x + \ln|\sec x + \tan x|\right)$

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g. $\cos A \sin B = \frac{1}{2}\left(\sin(A+B) - \sin(A-B)\right)$

$\int \cos^2 3x \sin 2x \, dx = \int \cos 3x \left(\frac{1}{2}(\sin 5x - \sin x)\right) dx$

$= \frac{1}{2}\int \cos 3x \sin 5x \, dx - \frac{1}{2}\int \cos 3x \sin x \, dx$

$= \frac{1}{2}\left(\int \frac{1}{2}(\sin 8x + \sin 2x) \, dx - \int \frac{1}{2}(\sin 4x - \sin 2x) \, dx\right)$

$= \frac{1}{4}\left(-\frac{1}{8}\cos 8x - \frac{1}{2}\cos 2x + \frac{1}{4}\cos 4x - \frac{1}{2}\cos 2x\right)$

$= \frac{1}{4}\left(\frac{\cos 4x}{4} - \cos 2x - \frac{\cos 8x}{8}\right) + C$ $\blacksquare$

4\. a. $\int \frac{1}{x^2+1} \, dx$

$x = \tan u \rightarrow u = \arctan x$

$\tan^2 u + 1 = \sec^2 u$, $dx = \sec^2 u \, du$

$\therefore \int \frac{1}{\sec^2 u} \cdot \sec^2 u \, du = u + C$

$= \arctan x + C$

b. $\int x\sqrt{1-x^2} \, dx$

$x = \sin u \rightarrow dx = \cos u \, du$

$\sqrt{1-x^2} = \cos u$, $u = \arcsin x$

$\therefore \int \sin u \cos^2 u \, du = -\int \cos^2 u \, d\cos u$

$= -\frac{\cos^3 u}{3} = -\frac{\cos^3(\arcsin x)}{3} + C$ $\blacksquare$

$= -\frac{(1-x^2)^{\frac{3}{2}}}{3} + C$

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(a.)

(5.) $\int \sec^5 x \, dx = \int \sec^3 x \, d\tan x$

$= \sec^3 x \tan x - \int \tan x \, d\sec^3 x$

$= \sec^3 x \tan x - 3\int \tan^2 x \sec^3 x \, dx$

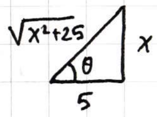
$= \sec^3 x \tan x + 3\int (1-\sec^2 x)\sec^3 x \, dx$

$= \sec^3 x \tan x + \left(3\int \sec^3 x \, dx - 3\int \sec^5 x \, dx\right)$

$4\int \sec^5 x \, dx = \frac{3}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + \sec^3 x \tan x + C$

$\int \sec^5 x \, dx = \frac{3}{8}(\sec x \tan x + \ln|\sec x + \tan x|) + \frac{\sec^3 x \tan x + C}{4}$

(b.) $d\sqrt{x^2+25} = \frac{x}{2\sqrt{x^2+25}}$



$\tan\theta = \frac{x}{5}$

$\cos\theta = \frac{5}{\sqrt{x^2+25}}$

$\int \frac{x^2}{\sqrt{x^2+25}} \, dx = \int x \, d\sqrt{x^2+25}$

$= x\sqrt{x^2+25} - \int \sqrt{x^2+25} \, dx$

$= x\sqrt{x^2+25} - 25\int \sec^3\theta \, d\theta$

$= x\sqrt{x^2+25} - 25\left(\frac{1}{2}(\sec\theta\tan\theta + \ln|\sec\theta + \tan\theta|)\right)$

$= x\sqrt{x^2+25} - \frac{25}{2}\left(\frac{x\sqrt{x^2+25}}{25} + \ln\left|\frac{\sqrt{x^2+25}}{5} + \frac{x}{5}\right|\right)$

$= \frac{1}{2}x\sqrt{x^2+25} - \frac{25}{2}\ln\left|\frac{\sqrt{x^2+25}}{5} + \frac{x}{5}\right| + C$

$= \frac{1}{2}\left(x\sqrt{x^2+25} - 25\ln|\sqrt{x^2+25} + x|\right) + C$

karena $\frac{25}{2} \cdot \ln\frac{1}{5}$ adalah konstanta