LATIHAN

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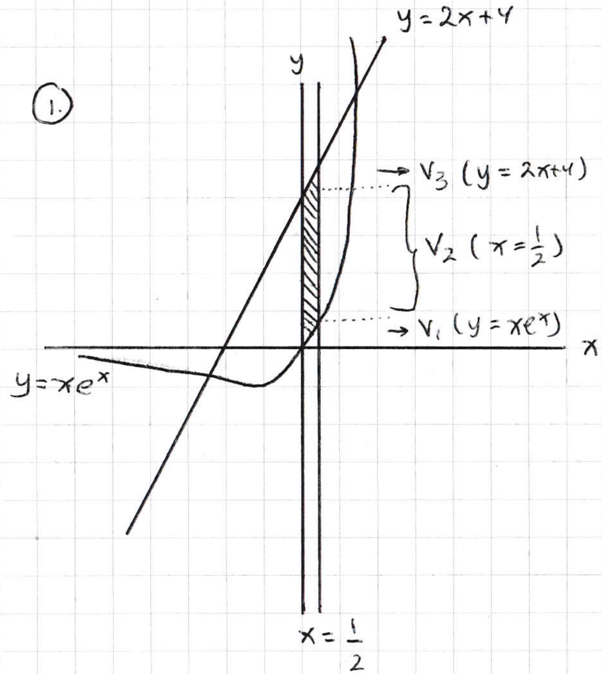
(1.)



(a.) $y = 2x + 4 \rightarrow x = \frac{y-4}{2}$

$y = xe^x \rightarrow dy = e^x(1+x)\,dx$

Disk / Washer Method :

$$V = \pi \int_a^b x^2 \, dy$$

$$V_1 = \pi \int_0^{\frac{\sqrt{e}}{2}} x^2 \, dy = \pi \int_0^{\frac{1}{2}} x^2 e^x (1+x) \, dx$$

$$V_2 = \pi \int_{\frac{\sqrt{e}}{2}}^{4} \frac{1}{4} \, dy$$

$$V_3 = \pi \int_4^5 \left(\frac{1}{2}\right)^2 - \left(\frac{y-4}{2}\right)^2 dy$$

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$$\therefore V = \pi\left(\int_0^{\frac{1}{2}} x^2 e^x \, dx + \int_0^{\frac{1}{2}} x^3 e^x \, dx + \int_{\frac{\sqrt{e}}{2}}^{4} \frac{1}{4} \, dy + \int_4^5 \frac{1}{4} - \frac{(y-4)^2}{4} \, dy \right)$$

$$= \pi\Big(x^2e^x - 2xe^x + 2e^x \Big]_0^{\frac{1}{2}} + x^3e^x - 3x^2e^x + 6xe^x - 6e^x \Big]_0^{\frac{1}{2}}$$

$$+ \frac{y}{4}\Big]_{\frac{\sqrt{e}}{2}}^{4} + \frac{1}{4}\Big(-\frac{y^3}{3}\Big]_4^5 + 4y^2\Big]_4^5 - 15y\Big]_4^5 \Big)\Big)$$

$$= \pi\Big(\big(\frac{\sqrt{e}}{4} - \sqrt{e} + 2\sqrt{e}\big) - 2 + \big(\frac{\sqrt{e}}{8} - \frac{3\sqrt{e}}{4} + 3\sqrt{e} - 6\sqrt{e}\big)$$

$$+ 6 + 1 - \frac{\sqrt{e}}{8} + \frac{1}{4}\Big(-\frac{125}{3} + \frac{64}{3} + 100 - 64 - (75 - 60) \Big)\Big)$$

$$= \pi\left(\frac{5\sqrt{e} - 8}{4} + \frac{48 - 29\sqrt{e}}{8} + 1 - \frac{\sqrt{e}}{8} - \frac{61}{12} + 25 - 16 - \frac{15}{4} \right)$$

$$= \pi\left(\frac{5\sqrt{e} - 23}{4} + \frac{48 - 30\sqrt{e}}{8} + 10 - \frac{61}{12} \right)$$

$$= \pi\left(\frac{-20\sqrt{e} + 2}{8} + \frac{59}{12} \right) = \pi\left(\frac{62 - 30\sqrt{e}}{12} \right) \blacksquare$$

b. Shell method

$$V = \int_a^b 2\pi x y \, dx$$

$$\therefore V = \int_0^{\frac{1}{2}} 2\pi x \, (2x + 4 - xe^x) \, dx$$

$$= 2\pi \int_0^{\frac{1}{2}} 2x^2 + 4x - x^2e^x \, dx$$

$$= 2\pi\left(\int_0^{\frac{1}{2}} 2x^2 \, dx + \int_0^{\frac{1}{2}} 4x \, dx - \int_0^{\frac{1}{2}} x^2e^x \, dx \right)$$

$$= 2\pi\left(\frac{2}{3}x^3\Big]_0^{\frac{1}{2}} + 2x^2\Big]_0^{\frac{1}{2}} - x^2e^x - 2xe^x + 2e^x\Big]_0^{\frac{1}{2}} \right)$$

$$= 2\pi\left(\frac{1}{12} + \frac{1}{2} - \Big(\frac{\sqrt{e}}{4} - \sqrt{e} + 2\sqrt{e} \Big) + 2 \right)$$

$$= 2\pi\left(\frac{31}{12} - \frac{5\sqrt{e}}{4} \right) = \pi\left(\frac{62 - 30\sqrt{e}}{12} \right) \blacksquare$$

2.

$$s = \int_a^b \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2} \, dt$$

$$dx = 1 - \cos t \, dt$$

$$dy = \sin t \, dt$$

$$\therefore s = \int_0^{3\pi} \sqrt{\sin^2 t + (1 - \cos t)^2} \, dt$$

$$= \int_0^{3\pi} \sqrt{2 - 2\cos t} \, dt = \int_0^{3\pi} \sqrt{2(1 - \cos t)} \, dt$$

$$= 2\int_0^{3\pi} \sin t \, dt = 2(-\cos t)\Big]_0^{3\pi} = 2 \blacksquare$$

ralat: harusnya t/2 jawabannya sama