Objectives

- Work with pairs and lists

Activities

1. Define a SCHEME function to take a number, and return a pair with the number and its square.

2. Define a SCHEME function `square-l` which will take any list of numbers as input, and returns a list with each of those numbers squared.

3. Define a SCHEME function `range` that takes a pair (x, y) of natural numbers as input and returns a list of all integers between them, inclusive.
   For example if $p = (0, 10)$ then

   ```
   (range p)  →  '(0 1 2 3 4 5 6 7 8 9 10)
   ```

4. **Scalar-Vector Multiplication** Multiplying a vector by a scalar produces a vector where each component is the corresponding value in the original vector multiplied by the scalar quantity. For example:

$$a(x_1 \quad x_2 \quad x_3) = (ax_1 \quad ax_2 \quad ax_3)$$

   Define a SCHEME function, named `sv-mult`, which takes a list and a value as parameters and performs scalar-vector multiplication on them.

5. **Vector Addition** Adding two vectors produces a vector where each component is the sum of the corresponding values in the original vectors. For example:

$$(x_1 \quad x_2 \quad x_3) + (y_1 \quad y_2 \quad y_3) = (x_1 + y_1 \quad x_2 + y_2 \quad x_3 + y_3)$$

   Define a SCHEME function, named `v-add`, which takes two lists and performs vector addition on them. *Note: **vector subtraction** is structured in the same way.*

6. The **dot product** of two lists of numbers $(x1\ x2\ x3)$ and $(y1\ y2\ y3)$ is

$$x1 * y1 + x2 * y2 + x3 * y3$$

   Define a recursive SCHEME function `(dot x y)` that takes two lists of numbers as its inputs and returns the dot product of those two lists. Do not use the in-built `map` function. You can assume the two lists have the same length.

7. The **cross product** of two sets represented by lists of numbers $X = (x1 \ x2 \ x3)$ and $Y = (y1 \ y2 \ y3)$, denoted $X \times Y$, is the set (list) of all possible pairs of numbers where the first number in each pair is a member of the first set (list) and the second number in each pair is a member of the second set (list). In our example, $X \times Y$ is $((x1 \ . \ y1)(x1 \ . \ y2)(x1 \ . \ y3)(x2 \ . \ y1)(x2 \ . \ y2)(x2 \ . \ y3)(x3 \ . \ y1)(x3 \ . \ y2)(x3 \ . \ y3))$. Define a SCHEME function `(cross x y)` which takes two lists as parameters and returns the list of pairs representing the cross product of the items in the lists `x` and `y`.