PROBLEMS UD 4

PROBABILITY DISTRIBUTIONS

Part 1: DISCRETE DISTRIBUTIONS

1) One variable has a pattern of probability distribution indicated in the following table:

X	1	2	3	4	5	6
P(X = x)	2/9	1/9	3/9	2/9	0	1/9

Calculate:
a) P (X = 9) b) P (X > 3) c) E(X) d) E(5X+3)

2) The following table shows the probability function of the variable X: number of persons per day who enter in a bar and ask for a cup of wine.

X	0	1	2	3	4	5	> 5
P(X = x)	0.01	0.1	0.3	0.4	0.1	???	0

Calculate:
a) P (X = 5) b) P (X $\leq$ 2) c) P (X < 2) d) P (X > 3) e) E(X) f) Variance of X

3) One game consists of throwing two dices. If the sum of the result of both dices is higher or equal to 10, the player wins 300 euros. If the sum is 7, 8 or 9, wins 100 euros and in the rest of cases, the player loses the amount of the bet. What should be this amount so that the expected loss per play is 100 euros?

4) One pharmaceutical laboratory develops a new drug to relieve back pain. The laboratory affirms that it is effective in 90% of cases. The drug is tried in 4 patients. If X is the number of patients who obtain relief with this drug,

a) Obtain the probability function for X, considering that the assumption of the laboratory is true.
b) Calculate P (X $\leq$ 1)
c) If this drug does not relieve the pain of any of the four patients, is this reason enough to doubt about the efficiency argued by the laboratory? Discuss this issue based on probability calculations.
d) Calculate the average. What does it mean in this example?

5) One dealer buys a certain type of used equipments at 100 € and, after some repairs that cost 25 €, re-sales them at 220 €. If the sold equipment turns out to be defective, the dealer has to give back the amount to the customer and pay an indemnity of 100 €. It is known that 20% of equipments that the dealer buys are defective. In order to determine if the equipment is defective or not, prior to selling it, the dealer has the possibility to conduct a test that diagnoses its state (defective or correct), but has certain probability of error (P). The dealer will not sell the equipment if it is defective according to this test, though it may be wrong in some cases. Determine the maximum cost (C) that the dealer should pay for this test, as a function of P, so that it is profitable to carry out the test.

6) One company delivers the orders to customers with a delay lower than one week in 95% of cases, which is considered satisfactory. Complaints of certain customers suggest that the percentage of delay might have increased, and the management decides to study if this is true. In order to study this issue, 10 orders are randomly selected. If more than one suffers a delay higher than a week, the manufacturing process is checked. What is the probability of checking the process with no need?

7) According to previous records, the average number of mortal accidents occurred during a weekend in the roads of a certain region is 3. Calculate the probability of occurring no mortal accidents in a certain week.

8) If one computer programmer makes a mistake in average every 1000 lines of code, what is the probability to find more than 2 mistakes in a program with 3000 lines?

9) One insurance company knows that the probability of a person to die by labor accident one year is 0.00003. If the company has 180,000 live insurances of this type (only covering death by labor accident) and the amount of the indemnities are 5,000 € per mortal labor accident,

a) What is the probability of having to pay one year 20,000 € or more because of this type of insurance policies?
b) What amount of money should the company set aside one year in order to be able to pay this type of policies with a probability of 90%?

10) One factory that manufactures pens produces in average 2 defective pens every 85 units. If the pens are packed in boxes of 170 units, what is the probability to find two boxes with all correct pens in 7 boxes randomly taken?

11) One company sells cigarette lighters at a low price, with 20% of them being unusable (defective). This product is sold in the market in packets of 4 units and in boxes of 10 packets.

a) If one packet is randomly chosen, what is the probability to find 2 or more unusable lighters?
b) If one box is randomly chosen, what is the probability to find up to 3 unusable lighters?
c) If one box is randomly chosen, what is the probability to find 3 packets without unusable lighters?

12) One factory records the number of accidents per week suffered by the workers. This variable follows a Poisson distribution with parameter $\lambda=2$.

a) What is the probability of occurrence of any accident in a week?
b) What is the probability to occur 4 accidents in two weeks?
c) What is the probability to occur 2 accidents in a week and 2 additional accidents in the next week?

Part 2: CONTINUOUS DISTRIBUTIONS: uniform, exponential...

13) One continuous variable follows a uniform distribution: X ~ U (10; 20). If a box-whisker plot is conducted with many data of this variable, how would it look like? Draw this plot.

14) The time of waiting until being attended in a shop, follows an exponential distribution with average 3 minutes. If a box-whisker plot is conducted with many data of this variable, how would it look like? Draw this plot, after calculating the quartiles, median and the end of the right whisker.

15) One exponential distribution with average 3 minutes has a median of 2.08. As the average is higher than the median, it implies that the distribution is positively skewed. True, or false?

16) The length of a certain piece follows a distribution according to this density function f(x). Those pieces with a length between 1.5 and 2.1 are considered correct.

$$f(x) = \begin{cases} k & 1 \le x \le 2 \\ k - (x-2)^2 & 2 \le x \le 2.5 \\ 0 & \text{other values} \end{cases}$$

a) Obtain the value of the constant k.
b) Obtain the proportion of correct pieces.

17) One continuous random variable X follows a distribution with density function:

$$f(x) = \begin{cases} k(1+x^2) & x \in [0,\ 3] \\ 0 & x \notin [0,\ 3] \end{cases}$$

a) Obtain the value of the constant k.
b) Calculate the probability of X belonging to the interval [1, 2].
c) Calculate the probability of X < 1.
d) Knowing that X is higher than 1, what is the probability of being lower than 2?

18) The life of a lamp, X (hours) is a random variable with the following density function: $f(x) = 0.001 \cdot e^{-0.001 \cdot x}$ being x>0. If a lamp that has been working 500 hours is selected, what is the probability to last more than 1,000 hours?

19) The random variable X is defined for values x < a, being f(x) the density function. What would we obtain with the following expression? (only one answer is true)

$$\int_{-\infty}^{a} x \cdot f(x)\ dx$$

a) The value of the distribution function for x = a.
b) The value of the standard deviation of X.
c) The expression will be always one.
d) The average value of X.

20) One study is carried out about the duration of telephone calls received in a business. The density function of this variable is the following. Obtain the average.

$$f(x) = \begin{cases} 0.5 \cdot e^{-x/2} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

21) One continuous random variable has the following density function. Obtain the average.

$$f(x) = \begin{cases} x - 0.5 & 1 \leq x \leq 2 \\ 0 & \text{other values} \end{cases}$$

22) Obtain the variance of the random variable determined by this density function:

$$f(x) = \begin{cases} 1/2\sqrt{x} & 0 \leq x \leq 1 \\ 0 & \text{other values} \end{cases}$$

23) One department wants to study the percentage of time that a certain computer is being used along the different weeks. Assuming that this percentage of usage follows this density function, obtain the average and variance.

$$f(x) = \begin{cases} 3x^2 & 0 \leq x \leq 1 \\ 0 & \text{other values} \end{cases}$$

24) Obtain the average value of a variable X determined by this density function:

$$f(x) = \begin{cases} x/5 & 0 \leq x \leq 1 \\ 1/5 & 1 \leq x \leq 5 \\ (6-x)/5 & 5 \leq x \leq 6 \\ 0 & \text{other} \end{cases}$$

25) The random variable X: time of operation (years) of the system until failure, follows an exponential distribution with an average of 6 months.

a) Obtain the density function of X.
b) Obtain the standard deviation.
c) Calculate the probability of the system to work at least one year.

26) The time between failures (X) of a machine follows an exponential distribution, being E(X)= 200 hours. What is the probability to pass more than 200 hours without any failure?

27) If the life time of an electric component follows an exponential distribution, being the average 100 hours, what is the probability to pass more than 200 hours without any failure?

28) It was obtained that the life of a computer system follows an exponential distribution with average 8 years.

a) What is the probability to have a life between 3 and 12 years?
b) If one computer system is working during 10 years, what is the probability to last 15 additional years?

29) The duration X of certain electronic components fluctuates randomly, verifying that Prob $(X > x) = e^{-ax}$. These components have an average duration of 400 hours. What percentage of them will last more than 400 hours?

30) The duration T of certain electronic components fluctuates randomly, verifying that P $(T > t) = e^{-a \cdot t}$. In 50% of the cases, the duration is lower than 100 hours of working.

a) If a system is formed by two components arranged in parallel, calculate the probability of this system to work more than 100 hours
b) Same, but arranged in series.

31) The staff of one engineering company use a workstation to conduct technical calculations. The time that each employee takes in one session is 20 minutes in average. If this time follows an exponential distribution,

a) Calculate the probability that an employee takes less than 20 minutes in one session.
b) One employee tries to use the workstation, but it is occupied by someone else who has been using it during 30 minutes. What is the probability to wait more than 10 minutes until being able to use the workstation?

32) The duration of a certain electronic component follows an exponential distribution with average 1,000 hours. If one component of this type has been working for 300 hours, what is the probability to work during 700 additional hours?

33) One mechanism is formed by two electronic components of identical characteristics, A and B, assembled in series. The duration (hours) of operation of these components fluctuates randomly following an exponential distribution of parameter z. If it is required that the mechanism should have a reliability of 99.4% after “t” hours of working, what reliability is required after “t” hours of working for each one of the two components?

Part 3: NORMAL DISTRIBUTION

34) If Z is the standard normal distribution N (0, 1), calculate:

a) $P(Z < 1.85)$ b) $P(Z < -1.85)$ c) $P(1 < Z < 1.85)$
d) $P(-1.85 < Z < -1)$ e) $P(-1 < Z < 1.85)$

35) If X is a random variable with distribution N (5, 2), calculate:

a) $P(1 < X < 8)$ b) $P(X < 1)$ c) $P(X < -1)$

36) One company manufactures electric resistances with tolerance limits of $40 \pm 0.5\ \Omega$. If the resistance is outside this interval, it is considered as defective. If the resistance is Normally distributed with average 39.5 and standard deviation 0.2, calculate the percentage of defective resistances.

37) One company manufactures certain pieces for electric engines. The weight is a key parameter of quality. The company wants to reject 3% of pieces with the lowest weight, and 3% of pieces with highest weight. If the average weight is 4.72 kg, the standard deviation is 0.006 kg and the distribution is Normal, calculate the maximum weight and the minimum weight of pieces that will be sold by the company.

38) We assume that the weight (kg) of male students at one university follow a Normal distribution N (69, 6). What is the value L so that $P(X > L) = 0.9$?
the probability of X

39) The intellectual coefficient of students at a certain school follow a Normal model, so that $P(X>1.4) = 0.1056$ and $P(X>1) = 0.4013$. Calculate the parameters of the distribution.

40) One Normal random variable X satisfies these conditions: $P(X<15) = 0.1$ and $P(X<20) = 0.95$. Obtain:

a) $P(X<13)$
b) One value “a” so that $P(X<a) = 0.05$
c) One value “b” so that $P(X>b) = 0.5$

41) The signal received by the screen of a computer is considered appropriate if the deviation of the observed voltage with respect to the theoretical value is no higher than 10 volts. The deviations observed follow a Normal model with average = 0 and standard deviation = 5. Calculate the percentage of signals received by the screen that are considered appropriate.

42) In a binary system, the information is represented by means of electric signals (in volts). One voltage accounts for the bit 0 and another voltage for the bit 1. Let us assume that the bit 0 is represented by 2 volts, and the bit 1 by 3 volts. Due to fluctuations of the voltage in a circuit, the voltage received by the digital circuit is not always 2.00 or 3.00 because of random noise in the circuit. This noise can be modeled with a Normal model and is called Gaussian noise. If the noise is Gaussian with average N=0 and standard deviation 0.22, and the signal is recognized as a bit 0 if the received voltage is lower than 2.6 while it is recognized as a bit 1 if the received voltage is higher or equal to 2.6, calculate:

a) Probability to recognize a bit 1 when it was transmitted as bit 0.
b) Probability to recognize a bit 0 when it was transmitted as bit 1.

43) The average time required by a computer to execute a complex algorithm is 2.52 minutes and the standard deviation is 0.37 minutes. If this time is Normally distributed, what is the probability that this time is between 2 and 4 minutes? What is the probability that the difference in absolute value between the time of execution and the average is lower or equal to one minute?

44) The noise produced by an engine follows a Normal distribution with mean 90.4 decibels and variance 5.8 decibels^2. If the average of two measurements are taken instead of just one, what would be the distribution?

45) One chemical engineer is working in the design of a petrochemical plant. Four activities (A, B, C, D) need to be developed sequentially and without overlap. The duration of these activities (in days) are assumed to be independent random variables with a Normal distribution: A: N(50, 5); B: N(20, 3); C: N(70, 10); D: N(40, 4). What is the probability to conduct the four activities with a total time lower than 200 days?

46) The diameter of certain axles manufactured for a mechanical application are Normally distributed with a mean of 3.81 cm and a standard deviation of 0.051. The washers of these axles have inner diameters Normally distributed with a mean of 3.942 cm and a standard deviation of 0.025. If one axle and one washer are randomly taken, what is the probability that the diameter of the axle is higher than the inner diameter of the washer?

47) The net weight of a package is a random variable N(20, 2) and the container's weight is a random variable N(1, 0.2). If 13 packages are put on a wooden structure that weights 50 kg, what is the probability that the total weight is higher than 300 kg?

48) The diameters of screws in a box, measured in cm, follow a distribution N(2, 0.03) and the inner diameters of the screw-nuts, in another box, follow a distribution N(2.02, 0.04). One screw fits into the screw-nut if the inner diameter of the screw-nut is higher than the screw's diameter and the difference between both diameters is not higher than 0.05 cm. If one screw and one screw-nut are randomly selected, what is the probability to fit properly?

49) In one Selectivity exam, the students of school A obtained marks with a distribution N(625, 10) and students in school B obtained marks with a distribution N(600, 12.25). If two students of school A and 3 students of school B make this exam, what is the probability that the average of the two marks of students of school A, is higher than the average mark of the 3 students of school B?

50) The quality parameter of certain piece manufactured in a factory is distributed Normally with an average of 150 and a variance of 0.16. Pieces are regarded as correct if this parameter is between 149.2 and 150.4.

a) If one package contains 10 pieces, what is the probability to find at least 9 correct pieces?
b) If one package contains 100 pieces, what is the probability to find at least 90 correct pieces?

51) The electric resistance of lamps manufactured by a company are Normally distributed with $\mu = 2000$ and s=200. These lamps are packed in batches of 100 units. One lamp is regarded as defective if the resistance is lower than 1900. The batch is regarded as low quality if 20 or more lamps are defective. If one batch is randomly taken, what is the probability to be of low quality?

52) One machine manufactures pieces with length following a Normal distribution. It is known that 6.68% of pieces have a length higher than 10 cm, and 15.87% of pieces have a length lower than 5 cm. One piece is considered correct if the length is between 3 and 12 cm. Calculate the percentage of defective pieces manufactured by the machine.

53) The weight of certain types of containers for gas products follows a Normal distribution with a mean of 6 kg and a standard deviation of 1 kg. The weight of the gas inside the container is a Normal random variable, independent from the container's weight, with mean 13 kg and standard deviation 1.5 kg. The filled container is carried on a security platform of 86.94 kg and loaded on an elevator. Calculate the probability to exceed the maximum load of the elevator, which is 100 kg.

54) One elevator with a maximum load of 100 kg carries 3 concrete bags placed on a wooden platform. The weight of each bag follows a Normal distribution N (20, 0.5) kg. What should be the maximum weight of the platform in order to exceed the total weight loaded on the elevator only in 5% of the cases?

55) The time of execution of a certain algorithm follows a Normal distribution $N(m, \sigma)$. Obtain the parameters of the distribution if $m = 5 \cdot \sigma$ and $P(X<6) = 0.84134$.

56) One company sends every week (45 times per year) the invoices to a customer by fax. The time of transmission by fax follows a distribution N(168, 5) sec. Obtain the average times per year with a transmission time between 165 and 175 seconds.

57) One computer networks receives 4489 calls per minute in average. The number of calls per minute, X, follows a Poisson distribution. The network is saturated of calls when X is higher than 4600. What is the probability to be saturated in a certain minute?

58) One company that manufactures office tables has two production lines (A and B). In line A, the height of tables (cm) fluctuates according to a Normal distribution N(75, 1.2), and in line B, the height follows a model N(77, 0.9). If one table is randomly selected from each line, what is the probability that the table from line A is taller than the table from line B?

59) The time required by a computer to execute a complex algorithm follows a Normal distribution with average 2.52 minutes, and the standard deviation is 0.37 min.

a) Calculate the probability that the time is between 2 and 4 minutes.
b) Calculate the probability that the difference (in absolute value) between the time of execution and the average is lower than 1 minute.

60) A certain day, a group of 180 people are queuing in the bank to pay certain taxes. The amount is not the same for everybody, but the average per person is 85 euros and the standard deviation is 12.3 €. What is the probability that the bank has received in total more than 15.000 euros?

61) One computer store estimates that, during the next month of January, the sales will decrease about 480 euros. It is estimated that the decrease will be comprised between 350 and 610 euros, with a probability of 80%. Assuming a Normal distribution, what is the probability to register a decrease lower than 500 euros?

62) The number of errors in the invoices of a company follow a Poisson distribution with average of 0.8 errors per invoice.

a) What is the probability to find any error in one invoice?
b) What is the probability to find more than 10 errors in 10 invoices randomly selected?
c) What is the probability to find less than 350 errors in 500 invoices randomly selected?

63) According to previous experience, about 2/5 of students registered in a certain subject, will not attend the final exam. Taking into account that students are distributed in different classrooms for the exam, how many students should be called for each classroom, with capacity for 120 students, in order to guarantee enough space for all students who finally attend the exam, with a probability of 0.975?

64) The maximum load of an elevator in a factory is 10,000 kg. The elevator is used to carry packages with a weight uniformly distributed between 40 and 60 kg. Calculate the maximum number of packages that can be loaded so that the probability to exceed the maximum load is lower than 0.1%.

65) The number of correct switchboards manufactured by a company is four times the number of defective switchboards.

a) If 200 switchboards are manufactured in one day, what is the probability to have more than 40 and less than 70 defective switchboards?
b) How many switchboards should be manufactured one day in order to have more than 100 correct switchboards with a probability of 90%?

66) The weight of oranges arriving at a factory fluctuate Normally with average 150 gr. and standard deviation 30 gr. Calculate the minimum number of oranges that should be put in a bag in order to have a total weight lower than 5 kg with a probability of 1%.

67) A random variable X follows a model N(20, 4).

a) Calculate the difference between the mean and the second quartile.
b) Calculate the difference between the third quartile and the mean.
c) What value has a distance of 1.5 times the interquartile range, above the third quartile?

68) One mechanical company produces certain pieces being the quality parameter, X, a Normal distribution that satisfies the following conditions: $P(X>100) = 0.1$, $P(X<97) = 0.05$. The tolerance limits are 96 and 102. Calculate the percentage of pieces outside the tolerance limits.

69) One simulation program has been developed for a specific research task. Each time that the program is run, it takes between 10 and 30 seconds to execute an algorithm, and this time is uniformly distributed. What is the maximum number of runs that can be carried out in order to have a probability lower than 1.5% to take, in total, more than 45 minutes?

70) The number of computers sold daily in a computer store fluctuate randomly according to a uniform distribution between 20 and 40 units.

a) After 182 days of sales, what is the probability of having sold more than 5,600 computers, assuming that sales are independent from day to day?
b) How many days should be considered in order to guarantee, with 67% of probability, that the total sale will be higher than 6,000 units?

71) One factory has two packing machines. The first one accounts for 75% of the total production, and the second one, for the remaining 25%. The net weight of each package (gr.) is a variable N(170, 7) in the first packing machine, and N(176, 7) in the second one. Those packages with a net weight lower than 180 gr. are considered as correct.

a) What is the probability of each machine to produce an incorrect package?
b) One package randomly taken turns out to be incorrect. What is the probability of having been packed by the second machine?
c) If 5 packages are randomly taken, what is the probability of finding two of them having been packed by the first machine?

SOLUTIONS:

Part 1: DISCRETE DISTRIBUTIONS

1) a: 0 ; b: 1/3 ; c: 3 ; d: 18
2) a: 0.09 ; b: 0.41 ; c: 0.11 ; d: 0.19 ; e: 2.75 ; f: 1.1875

3) 460 €
4) a: $\binom{4}{x} \cdot 0.9^x \cdot 0.1^{4-x}$ b: 0.0037 ; d: 3.6
5) C < 20 – 196 P
6) 0.0861
7) 0.0498
8) 0.5768
9) a: 0.7867 ; b: 40,000 €
10) 0.00589
11) a: 0.1808 ; b: 0.0285
12) a: 0.8647 ; b: 0.1954 ; c: 0.0733

Part 2: CONTINUOUS DISTRIBUTIONS: uniform, exponential...

13) Q1=12.5, Q2=15, Q3=17.5
14) Q1=0.863, Q2=2.079, Q3=4.159, right end = 9.104
15) true
16) a: 0.694 ; b: 0.416
17) a: 1/12 ; b: 5/18 ; c: 1/9 ; d: 5/16
18) 0.3679
19) d.
20) 2
21) 19/12
22) 4/45
23) a: 0.75 ; b: 0.0375
24) 3
25) b: 0.5 ; c: 0.1353
26) 0.368
27) 0.1353
28) a: 0.464 ; b: 0.153
29) 0.368
30) a: 0.75 ; b: 0.25
31) a: 0.6321 ; b: 0.6065
32) 0.497
33) 0.997

Part 3: NORMAL DISTRIBUTION

34) a: 0.9678 ; b: 0.0322 ; c: 0.1265 ; d: 0.1265 ; e: 0.8091
35) a: 0.9104; b: 0.0228 ; c: 0.0013
36) 50%
37) min = 4.7087; max = 4.7313
38) 61.3
39) N (m=0.9 ; σ=0.4)
40) a: 0.007 ; b: 14.39 ; c: 17.19
41) 95.44%
42) a: 0.0032 ; b: 0.0344
43) a: 0.919 ; b: 0.993
44) N(90.4; 1.703)
45) 0.9484
46) 0.0102
47) 0.9992
48) 0.3811
49) 0.9938
50) a: 0.4343 ; b: 0.0235
51) 0.993
52) 2.9%
53) 0.0005
54) 38.57 kg
55) N(5, 1)
56) 29.02
57) 0.0485
58) 0.0918
59) a: 0.92 ; b: 0.993
60) 0.9656
61) 0.5792
62) a: 0.5507 ; b: 0.18 ; c: 0.0058
63) 179 students
64) 195 packages
65) a: 0.464 ; b: 133
66) 37 oranges
67) a: 0 ; b: 2.7 ; c: 30.8
68) 0.5%
69) 127
70) a: 0.036 ; b: 202 days
71) a: 0.0764; 0.2843 b: 0.55 c: 0.088