Problema

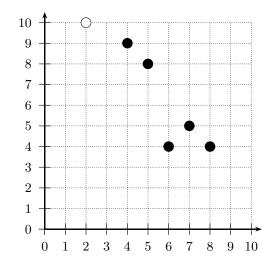
Se tienen los siguientes 6 vectores bidimensionales:

$$m{x}_1 = \begin{pmatrix} 2 \\ 10 \end{pmatrix}, \quad m{x}_2 = \begin{pmatrix} 8 \\ 4 \end{pmatrix}, \quad m{x}_3 = \begin{pmatrix} 5 \\ 8 \end{pmatrix}, \quad m{x}_4 = \begin{pmatrix} 7 \\ 5 \end{pmatrix} \quad m{x}_5 = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \quad m{x}_6 = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$$

Partiendo de la partición

$$\Pi = \{X_1 = \{x_1\}, X_2 = \{x_2, x_3, x_4, x_5, x_6\}\}$$

Se desea agrupar estos vectores de manera no-supervisada en 2 clases aplicando el algoritmo *c-medias*.



Algorithm C-means (versión "correcta" [Duda & Hart])

Input:
$$X; C; \Pi = \{X_1, \dots, X_C\};$$

Output:
$$\Pi^* = \{X_1, \dots, X_C\}; m_1, \dots, m_C; J$$

for
$$c=1$$
 to C do ${m m}_c=\frac{1}{n_c}\sum_{{m x}\in X_c}{m x}$ endfor repeat

transfers = false

forall
$$x \in X$$
 (let $i : x \in X_i$) do

if
$$n_i > 1$$
 then

$$j^* = \operatorname*{arg\,min}_{j \neq i} \frac{n_j}{n_j + 1} \left\| \boldsymbol{x} - \boldsymbol{m}_j \right\|^2$$

$$\triangle J = \frac{n_{j^*}}{n_{j^*} + 1} \| \boldsymbol{x} - \boldsymbol{m}_{j^*} \|^2 - \frac{n_i}{n_i - 1} \| \boldsymbol{x} - \boldsymbol{m}_i \|^2$$

if $\triangle J < 0$ then

transfers = true

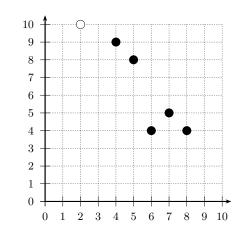
$$egin{aligned} oldsymbol{m}_i &= oldsymbol{m}_i - rac{oldsymbol{x} - oldsymbol{m}_i}{n_i - 1} & oldsymbol{m}_{j^*} &= oldsymbol{m}_{j^*} + rac{oldsymbol{x} - oldsymbol{m}_{j^*}}{n_{j^*} + 1} \ X_i &= X_i - \{oldsymbol{x}\} & X_{j^*} &= X_{j^*} + \{oldsymbol{x}\} \ J &= J + \triangle J \end{aligned}$$

endif

endif

endforall

П	$\{X_1 = \{x_1\}, X_2 = \{x_2, x_3 x_4, x_5, x_6\}\}$
m_1	
m_2	
J_1	
J_2	
J	
خ	Transferimos $oldsymbol{x}_2 = (8,4)^t$ de X_2 a X_1 ?
$\triangle J$	
خ	Transferimos $oldsymbol{x}_3=(5,8)^t$ de X_2 a X_1 ?
$\triangle J$	
خ	Transferimos $oldsymbol{x}_4=(7,5)^t$ de X_2 a X_1 ?
$\triangle J$	
خ	Transferimos $oldsymbol{x}_5 = (6,4)^t$ de X_2 a X_1 ?
$\triangle J$	
خ	Transferimos $oldsymbol{x}_6 = (4,9)^t$ de X_2 a X_1 ?
$\triangle J$	



Algorithm C-means (versión "correcta" [Duda & Hart])

Input:
$$X; C; \Pi = \{X_1, \dots, X_C\};$$

Output:
$$\Pi^* = \{X_1, \dots, X_C\}; m_1, \dots, m_C; J$$

for
$$c=1$$
 to C do ${m m}_c=rac{1}{n_c}\sum_{{m x}\in X_c}{m x}$ endfor repeat

transfers = false

forall
$$x \in X$$
 (let $i : x \in X_i$) do

if
$$n_i > 1$$
 then

$$j^* = \operatorname*{arg\,min}_{j \neq i} \frac{n_j}{n_j + 1} \left\| \boldsymbol{x} - \boldsymbol{m}_j \right\|^2$$

$$\Delta J = \frac{n_{j^*}}{n_{i^*} + 1} \| \boldsymbol{x} - \boldsymbol{m}_{j^*} \|^2 - \frac{n_i}{n_i - 1} \| \boldsymbol{x} - \boldsymbol{m}_i \|^2$$

if $\triangle J < 0$ then

transfers = true

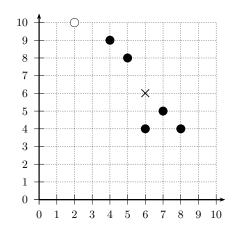
$$egin{aligned} oldsymbol{m}_i &= oldsymbol{m}_i - rac{oldsymbol{x} - oldsymbol{m}_i}{n_i - 1} & oldsymbol{m}_{j^*} &= oldsymbol{m}_{j^*} + rac{oldsymbol{x} - oldsymbol{m}_{j^*}}{n_{j^*} + 1} \ X_i &= X_i - \{oldsymbol{x}\} & X_{j^*} &= X_{j^*} + \{oldsymbol{x}\} \ J &= J + \triangle J \end{aligned}$$

endif

endif

endforall

Π	$\{X_1 = \{x_1\}, X_2 = \{x_2, x_3 x_4, x_5, x_6\}\}$
m_1	$(2,10)^t$
m_2	$(6,6)^{t}$
J_1	0
J_2	32
J	32
خ	,Transferimos $oldsymbol{x}_2 = (8,4)^t$ de X_2 a X_1 ?
$\triangle J$	26
¿Transferimos $oldsymbol{x}_3=(5,8)^t$ de X_2 a X_1 ?	
$\triangle J$	0.25
خ	,Transferimos $oldsymbol{x}_4 = (7,5)^t$ de X_2 a X_1 ?
$\triangle J$	47.5
خ	,Transferimos $oldsymbol{x}_5 = (6,4)^t$ de X_2 a X_1 ?
$\triangle J$	21
خ	Transferimos $oldsymbol{x}_6=(4,9)^t$ de X_2 a X_1 ?
$\triangle J$	-13.75

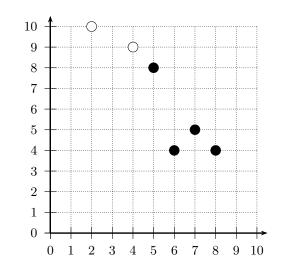


Algorithm *C-means* (versión "correcta" [Duda & Hart])

```
Input: X; C; \Pi = \{X_1, \dots, X_C\};
Output: \Pi^* = \{X_1, \dots, X_C\}; m_1, \dots, m_C; J
for c=1 to C do m{m}_c=rac{1}{n_c}\sum_{m{x}\in X_c}m{x} endfor
repeat
    transfers = false
    forall x \in X (let i : x \in X_i) do
         if n_i > 1 then
            j^* = \operatorname*{arg\,min}_{j \neq i} \frac{n_j}{n_j + 1} \|\boldsymbol{x} - \boldsymbol{m}_j\|^2
            \Delta J = \frac{n_{j^*}}{n_{i^*} + 1} \| \boldsymbol{x} - \boldsymbol{m}_{j^*} \|^2 - \frac{n_i}{n_i - 1} \| \boldsymbol{x} - \boldsymbol{m}_i \|^2
             if \triangle J < 0 then
                 transfers = true
                egin{aligned} m{m}_i &= m{m}_i - rac{m{x} - m{m}_i}{n_i - 1} & m{m}_{j^*} &= m{m}_{j^*} + rac{m{x} - m{m}_{j^*}}{n_{j^*} + 1} \ X_i &= X_i - \{m{x}\} & X_{j^*} &= X_{j^*} + \{m{x}\} \end{aligned}
                 J = J + \triangle J
             endif
         endif
```

endforall

Π	$\{X_1 = \{x_1, x_6\}, X_2 = \{x_2, x_3 x_4, x_5\}\}$
m_1	
m_2	
J	
زTransferimos $oldsymbol{x}_1=(2,10)^t$ de X_1 a X_2 ?	
$\triangle J$	
خ	,Transferimos $oldsymbol{x}_2 = (8,4)^t$ de X_2 a X_1 ?
$\triangle J$	
خ	,Transferimos $oldsymbol{x}_3 = \left(5,8 ight)^t$ de X_2 a X_1 ?
$\triangle J$	

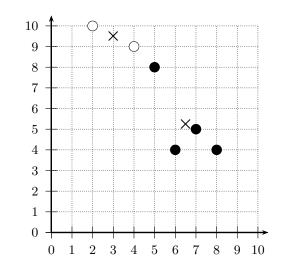


Algorithm C-means (versión "correcta" [Duda & Hart])

```
Input: X; C; \Pi = \{X_1, \dots, X_C\};
Output: \Pi^* = \{X_1, \dots, X_C\}; m_1, \dots, m_C; J
for c=1 to C do m{m}_c=rac{1}{n_c}\sum_{m{x}\in X_c} m{x} endfor
repeat
    transfers = false
    forall x \in X (let i : x \in X_i) do
        if n_i > 1 then
            j^* = \operatorname*{arg\,min}_{j \neq i} \frac{n_j}{n_j + 1} \|\boldsymbol{x} - \boldsymbol{m}_j\|^2
            \Delta J = \frac{n_{j^*}}{n_{i^*} + 1} \| \boldsymbol{x} - \boldsymbol{m}_{j^*} \|^2 - \frac{n_i}{n_i - 1} \| \boldsymbol{x} - \boldsymbol{m}_i \|^2
             if \triangle J < 0 then
                 transfers = true
                egin{aligned} m{m}_i &= m{m}_i - rac{m{x} - m{m}_i}{n_i - 1} & m{m}_{j^*} &= m{m}_{j^*} + rac{m{x} - m{m}_{j^*}}{n_{j^*} + 1} \ X_i &= X_i - \{m{x}\} & X_{j^*} &= X_{j^*} + \{m{x}\} \end{aligned}
                 J = J + \triangle J
            endif
         endif
```

endforall

Π	$\{X_1 = \{x_1, x_6\}, X_2 = \{x_2, x_3 x_4, x_5\}\}$	
m_1	$(3,9,5)^t$	
m_2	$(6,5,5,25)^t$	
J	$18,\!25$	
ز Transferimos $oldsymbol{x}_1 = (2,10)^t$ de X_1 a X_2 ?		
$\triangle J$	31.75	
¿Transferimos $oldsymbol{x}_2 = (8,4)^t$ de X_2 a X_1 ?		
$\triangle J$	31.75	
زTransferimos $oldsymbol{x}_3=(5,8)^t$ de X_2 a X_1 ?		
$\triangle J$	-9.58	

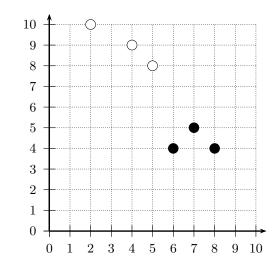


Algorithm *C-means* (versión "correcta" [Duda & Hart])

```
Input: X; C; \Pi = \{X_1, \dots, X_C\};
Output: \Pi^* = \{X_1, \dots, X_C\}; m_1, \dots, m_C; J
for c=1 to C do m{m}_c=rac{1}{n_c}\sum_{m{x}\in X_c}m{x} endfor
repeat
    transfers = false
    forall x \in X (let i : x \in X_i) do
        if n_i > 1 then
            j^* = \operatorname*{arg\,min}_{j \neq i} \frac{n_j}{n_j + 1} \|\boldsymbol{x} - \boldsymbol{m}_j\|^2
            \Delta J = \frac{n_{j^*}}{n_{i^*} + 1} \| \boldsymbol{x} - \boldsymbol{m}_{j^*} \|^2 - \frac{n_i}{n_i - 1} \| \boldsymbol{x} - \boldsymbol{m}_i \|^2
             if \triangle J < 0 then
                 transfers = true
                egin{aligned} m{m}_i &= m{m}_i - rac{m{x} - m{m}_i}{n_i - 1} & m{m}_{j^*} &= m{m}_{j^*} + rac{m{x} - m{m}_{j^*}}{n_{j^*} + 1} \ X_i &= X_i - \{m{x}\} & X_{j^*} &= X_{j^*} + \{m{x}\} \end{aligned}
                 J = J + \triangle J
            endif
         endif
```

endforall

Π	$\{X_1 = \{x_1, x_3, x_6\}, X_2 = \{x_2, x_4, x_5\}\}$
m_1	
m_2	
J	
Č	Transferimos $oldsymbol{x}_4=(7,5)^t$ de X_2 a X_1 ?
$\triangle J$	
Č	Transferimos $oldsymbol{x}_5 = (6,4)^t$ de X_2 a X_1 ?
$\triangle J$	
	Transferimos $oldsymbol{x}_6 = (4,9)^t$ de X_1 a X_2 ?
$\triangle J$	



Algorithm C-means (versión "correcta" [Duda & Hart])

```
Input: X; C; \Pi = \{X_1, \dots, X_C\};
Output: \Pi^* = \{X_1, \dots, X_C\}; m_1, \dots, m_C; J
for c=1 to C do m{m}_c=rac{1}{n_c}\sum_{m{x}\in X_c} m{x} endfor
repeat
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    forall x \in X (let i : x \in X_i) do
         if n_i > 1 then
            j^* = \operatorname*{arg\,min}_{j \neq i} \frac{n_j}{n_j + 1} \|\boldsymbol{x} - \boldsymbol{m}_j\|^2
            \Delta J = \frac{n_{j^*}}{n_{i^*} + 1} \| \boldsymbol{x} - \boldsymbol{m}_{j^*} \|^2 - \frac{n_i}{n_i - 1} \| \boldsymbol{x} - \boldsymbol{m}_i \|^2
             if \triangle J < 0 then
                 transfers = true
                egin{aligned} m{m}_i &= m{m}_i - rac{m{x} - m{m}_i}{n_i - 1} & m{m}_{j^*} &= m{m}_{j^*} + rac{m{x} - m{m}_{j^*}}{n_{j^*} + 1} \ X_i &= X_i - \{m{x}\} & X_{j^*} &= X_{j^*} + \{m{x}\} \end{aligned}
                 J = J + \triangle J
             endif
         endif
```

endforall

Π	$\{X_1 = \{x_1, x_3, x_6\}, X_2 = \{x_2, x_4, x_5\}\}$
m_1	$(3,67,9)^t$
m_2	$(7,4,33)^t$
J	8,67
¿Transferimos $oldsymbol{x}_4=(7,5)^t$ de X_2 a X_1 ?	
$\triangle J$	20.32
	Transferimos $oldsymbol{x}_5=(6,4)^t$ de X_2 a X_1 ?
$\triangle J$	21.16
زTransferimos $oldsymbol{x}_6=(4,9)^t$ de X_1 a X_2 ?	
$\triangle J$	22.94

