Método de las 2 Fases

→ Obj: deshacer el artificial

Fase 1: MIN $z = A_1 + A_2$
↳ SO Adif cuando $z = 0$

- Cuando una variable artificial sale de ser rentable, la demás jamás ⇒ No se considera

Fase 2: MAX/MIN
↳ Función original

SIMPLEX REV

→ Comenzar desde origen del problema (s_j, b_j, c_j)

→ Conocer en cada iteración cuáles son las VB y VNB

$\boxed{X_B = B^{-1}b}$

$c - z_j$
↳ $z_j = (c_B^t B^{-1})a_j$ → JE

$Y_{jk} = B^{-1}a_j$
↳ Ej de L variable que entra

$\frac{X_B}{Y_k} \to < 1S$

$Z = C_B^t X_B$

SBF$_k$ < VB = var. holgura rest. ⊕ y artificiales rest. ⊖ y ⊕
VNB = var. decisión y holgura rest. ⊖

Restricciones ≤

- $= b_i$
- $+A_i = b_i$

Este era la región factible inicial que se convierte en SBF al pasar la variable artificial.

SBF$_1$

SBF$_2$

→ Para comprobar la matriz inicial de partida

Restricciones ≥

- $= b_i$
- $-X_i + A_i = b$ (artificial)
  $\underbrace{X_i}_{\geq 0}$

MIN $3 \cdot X_1 + 2 \cdot X_2$

$$\begin{array}{l} R1 \quad 2X_1 + X_2 = 10 \\ R2 \quad -3X_1 + 2X_2 = 6 \\ R3 \quad X_1 + X_2 \geq 6 \end{array} \quad \left\{ \begin{array}{l} 2X_1 + X_2 + X_3 = 10 \\ -3X_1 + 2X_2 + X_5 = 6 \\ X_1 + X_2 + X_4 + X_6 = 6 \end{array} \right.$$

$$A = \begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ \begin{pmatrix} 2 & 1 & 1 & 0 & 0 & 0 \\ -3 & 2 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 \end{pmatrix} \end{array} \qquad b = \begin{pmatrix} 10 \\ 6 \\ 6 \end{pmatrix}$$

Fase 1

$z' = X_4 + X_6$

$$\begin{array}{c} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{array} \\ c^t = (0, 0, 0, 0, 1, 1) \end{array}$$

$$z - C_B^t X_B = (0, 1, 1)\begin{pmatrix} 10 \\ 6 \\ 6 \end{pmatrix} = 12$$

VB	B^{-1}			X_B	Y_{x_1}	$\frac{X_B}{Y_{x_1}}$
X_3	1	0	0	10	1	10
X_5	0	1	0	6	2	3
X_4	0	0	1	6	1	6
$C_B^t B^{-1}$	0	1	1	12		

Hiperplano < y artif. = y ≥

$$Y_{x_2} = B^{-1} a_{x_2} = \boxed{\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}}$$

$$c_{x_1} - z_{x_1} = 0 - (0, 1, 1)\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = 3 - 1 = 2$$

$$c_{x_2} - z_{x_2} = 0 - (0, 1, 1)\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = 2 - 1 = 1$$

$$c_{x_4} - z_{x_4} = 0 - (0, 1, 1) = -1$$

$C_B^t(0, 0, 1)$

$$B_a^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{array}{l} ① \\ ② \\ ③ \end{array}$$

$$\left.\begin{array}{l} ①' = ①/2 = 0 \quad 1/2 \quad 0 \\ ②' = ② - 1 \cdot ①' = 1 \quad -1/2 \quad 0 \\ ③' = ③ - 1 \cdot ①' = 0 \quad -1/2 \quad 1 \end{array}\right\} B^{-1} = \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1/2 & 1 \end{pmatrix}$$

$$c_B^t B^{-1} = (0, 0, 1)\begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1/2 & 1 \end{pmatrix} = (0, -1/2, 1)$$

$$X_B = B^{-1} b = \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1/2 & 1 \end{pmatrix}\begin{pmatrix} 10 \\ 6 \\ 6 \end{pmatrix} = \begin{pmatrix} 7 \\ 3 \\ 3 \end{pmatrix}$$

$$Z = C_B^t X_B = (0, 0, 1)\begin{pmatrix} 7 \\ 3 \\ 3 \end{pmatrix} = 3$$

VB	B^{-1}			X_B	Y_{x_1}	$\frac{X_B}{Y_{x_1}}$
X_3	1	1/2	0	7	3/2	2
X_2	0	1/2	0	3	-1/2	—
X_6	0	-1/2	1	3	1/2	6
$C_B^t B^{-1}$	0	-1/2	1	3		

$$Y_{x_1} = B^{-1} a_{x_1} = \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1/2 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \boxed{\begin{pmatrix} 3/2 \\ -1/2 \\ 1/2 \end{pmatrix}}$$

$$c_{x_1} - z_{x_1} = 0 - (0 - 1/2 \cdot 1)\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = -1/2 - 1 = -3/2$$

$$c_{x_4} - z_{x_4} = 0 - (0 - 1/2 \cdot 1)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = -1$$

$$B_a^{-1} = \begin{pmatrix} 1 & -1/2 & 0 \\ 0 & 1/2 & 0 \\ 0 & -1/2 & 1 \end{pmatrix} \begin{array}{l} ① \\ ② \\ ③ \end{array}$$

$$\begin{array}{l} ①' = ①/3/2 = 0 \quad -1/3 \quad 1/3 \\ ②' = ① - 1/2 \cdot ②' = 1 \quad 1/3 \quad -1/3 \\ ③' = ③ - 1/2 \cdot ②' = 0 \quad 1/3 \quad 2/3 \end{array}$$

$C_B^t = (0, 0, 0)$

$$B^{-1} = \begin{pmatrix} 1 & 1/3 & -1/3 \\ 0 & 1/3 & 1/3 \\ 0 & -1/3 & 2/3 \end{pmatrix}$$

$$C_B^t B^{-1} = (0, 0, 0)$$

$$X_B = B^{-1} b = \begin{pmatrix} 1 & 1/3 & -1/3 \\ 0 & 1/3 & 1/3 \\ 0 & -1/3 & 2/3 \end{pmatrix}\begin{pmatrix} 10 \\ 6 \\ 6 \end{pmatrix} = \begin{pmatrix} 17/3 \\ 2+1/3 \\ 4/3 \end{pmatrix}$$

$$Z = C_B^t X_B = (0, 0, 0)\begin{pmatrix} 17/3 \\ 2+1/3 \\ 4/3 \end{pmatrix} = 0$$

VB	B^{-1}			X_B
X_3	1	1/3	1/3	17/3
X_2	0	1/3	1/3	2+1/3
X_1	0	-1/3	2/3	4/3
$C_B^t B^{-1}$	0	0	0	0

$C_B^t B^{-1}$

$$c_{x_4} - z_{x_4} = 0 - (0, 0, 0)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0$$

Fase 2

MIN $3 \cdot X_1 + 2 \cdot X_2$

$c_B^t = (0, 2, 3)$

$$c_B^t B^{-1} = (0, 2, 3)\begin{pmatrix} 1 & 1/3 & -1/3 \\ 0 & 1/3 & 1/3 \\ 0 & -1/3 & 2/3 \end{pmatrix} = (0, -1/3, 1\tfrac{2}{3})$$

VB	B^{-1}			X_B
X_3	1	1/3	-1/3	17/3
X_2	0	1/3	1/3	2+1/3
X_1	0	-1/3	2/3	4/3
$C_B^t B^{-1}$	0	-1/3	13/3	13.2

$$Z = C_B^t X_B = (0, 2, 3)\begin{pmatrix} 17/3 \\ 2+1/3 \\ 4/3 \end{pmatrix} = \frac{66}{3} = 13.2$$

$$c_{x_4} - z_{x_4} = 0 - C_B^t a_{x_4} = (0, -1/3, 13/3)\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 13/3$$