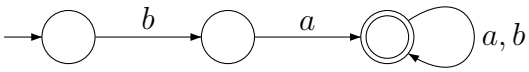
Substantiate the answers

Exercises

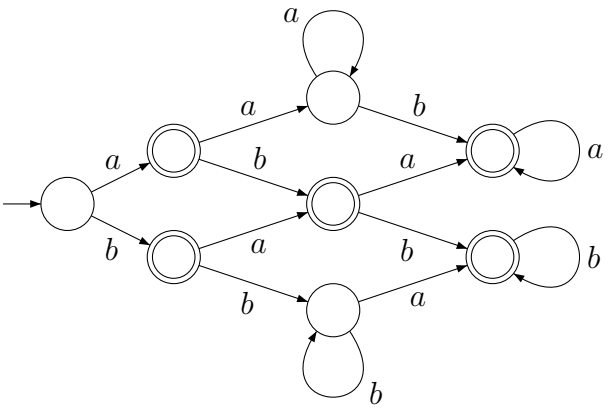
1. For each one of the following classes of languages, provide two languages in the class.
   (a) $\mathscr{L} = \{L \subseteq \Sigma^* \ : \ \Sigma^2 \subseteq L\}$
   (b) $\mathscr{L} = \{L \subseteq \Sigma^* \ : \ \forall n \geq 0, \ |L \cap \Sigma^n| \leq 1\}$
   (c) $\mathscr{L} = \{L \subseteq \Sigma^* \ : \ x \in L \Leftrightarrow x^r \in L\}$
   (d) $\mathscr{L} = \{L \subseteq \Sigma^* \ : \ L \cap (\Sigma\Sigma)^* = \emptyset\}$

2. Given $L = \{x \in \{a,b\}^* \ : \ ab \notin Suf(x)\}$ and the following automaton:



   obtain a DFA for the following languages:
   (a) $L \cap L(A)$
   (b) $L \cap L(A)^r$
   (c) $\overline{L} \cup L(A)$
   (d) $\overline{L} \cup \overline{L(A)^r}$
   (e) $L \cup \overline{L(A)}$

3. Given $L = \{axb \ : \ x \in \{a,b\}^*\}$, the following automaton:



   and the homomorphism:

$$\begin{cases} h(0) = ba \\ h(1) = ab \end{cases}$$

   obtain a DFA for the following languages:

(a) $(aab)^{-1}L$

(b) $(aba)^{-1}L(A)$

(c) $(aba)^{-1}\overline{L}$

(d) $h^{-1}(L(A))$

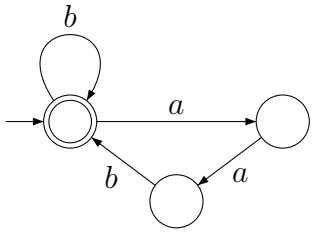
(e) $\overline{h^{-1}(L)}$

(f) $h^{-1}(L) \cap h^{-1}(\overline{L(A)})$

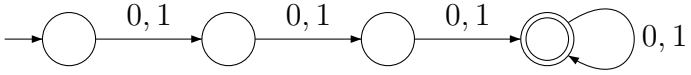
(g) $\overline{L(A)^r}$

4. Given the following automata:

A_1



A_2



and the homomorphism:

$$\begin{cases} h(0) = ba \\ h(1) = ab \end{cases}$$

obtain a DFA for the language $\overline{h^{-1}(L(A_1))} \cap L(A_2)$

5. Let consider the following classes of languages:

$$\mathscr{L}_p = \{L \subseteq \Sigma^* \ : \ \Sigma^2 \subseteq L\}$$
$$\mathscr{L}_t = \{L \subseteq \Sigma^* \ : \ \forall n \geq 0, \ |L \cap \Sigma^n| \leq 1\}$$
$$\mathscr{L}_r = \{L \subseteq \Sigma^* \ : \ x \in L \Leftrightarrow x^r \in L\}$$

(a) Is $\mathscr{L}_p$ closed under union operation?

(b) Is $\mathscr{L}_p$ closed under intersection?

(c) Is reverse a closed operation in $\mathscr{L}_p$?

(d) Is the class $\mathscr{L}_t$ closed with respect to the union operation?

(e) Is intersection a closed operation in the class $\mathscr{L}_t$?

(f) Is complementary a closed operation in the class $\mathscr{L}_t$?

(g) Is reverse a closed operation in the class $\mathscr{L}_r$?

(h) Is product (concatenation) a closed operation in the class $\mathscr{L}_r$?

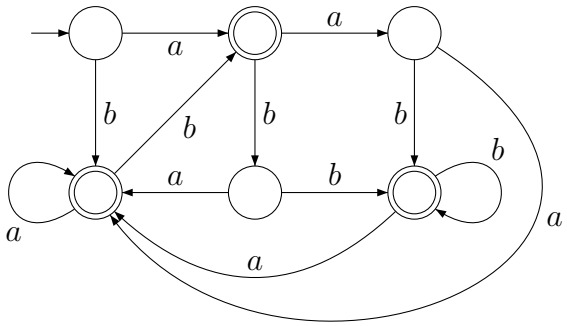
6. Let n be a positive integer. We define the class of languages $\mathscr{L}_n$ as follows:

$$\mathscr{L}_n = \{\Sigma^* - D \ : \ card(D) > n\}$$

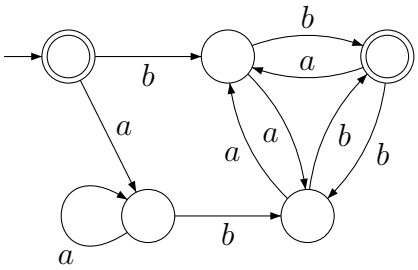
Is $\mathscr{L}_r$ closed with respect to the boolean operations (union, intersection and complement)?

7. Obtain the minimum DFA equivalent to the following automata.

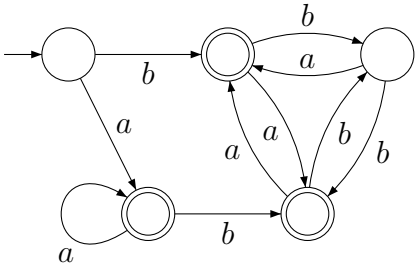
(a)



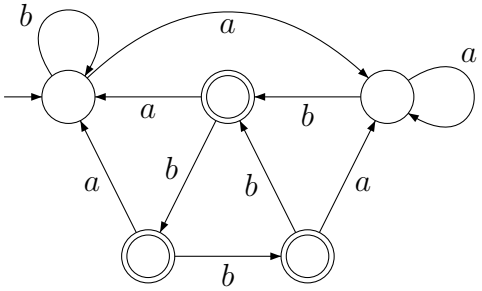
(b)



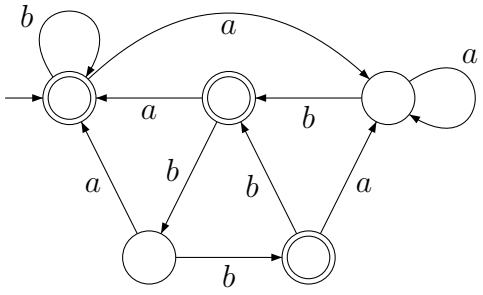
(c)



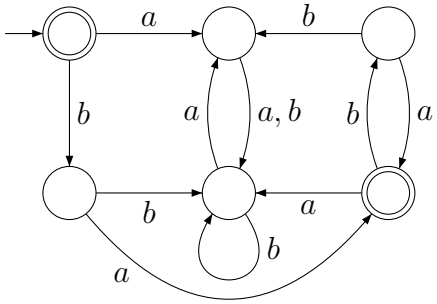
(d)



(e)

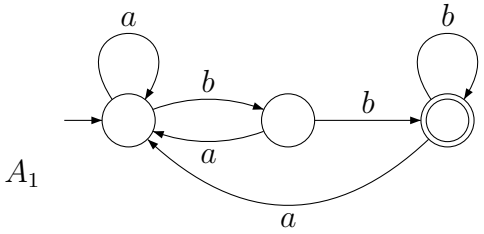


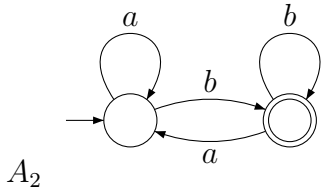
(f)



8. Decide whether the following statement is true or not.
   Let A be a DFA such that all its states are final. Then, $L(A) = \Sigma^*$.

9. Given the following automata:





   Decide algorithmically whether $L(A_1) \subseteq L(A_2)$.

10. Provide a regular expression for each one of the following languages:
    (a) $L = \{x \in \{a,b\}^* \ : \ |x| \text{ mód } 2 = 0\}$
    (b) $L = \{x \in \{a,b\}^* \ : \ ab \notin Seg(x)\}$
    (c) $L = \{x \in \{a,b\}^* \ : \ ab \notin Seg(x) \wedge ba \in Seg(x)\}$

11. Enumerate the first ten words in canonical order of the following languages:
    (a) $bb^*a^* + a^*$
    (b) $(bab + a)^*$
    (c) $a(b + ba)^* + b(a + ab)^*$

12. Obtain the position automaton from each one of the following regular expressions.

(a) $(ab^*a)^*$

(b) $a(b + ba)^*$

(c) $(ba)^* + (aa + bb)^*$

(d) $(a + b(ca)^*)^*$

(e) $(a + bb)^*a(b + aa)^*$

(f) $b(bb)^*a^*(cc)^* + \lambda$

(g) $ab^*a + b(aa)^*b$

13. Obtain the follow automaton from each one of the following regular expressions.

(a) $(ab^*a)^*$

(b) $b(a + ab)^*$

(c) $(ab)^* + (bb + aa)^*$

(d) $(a + b(ca)^*)^*$

(e) $(a + bb)^*a(b + aa)^*$

(f) $b(bb)^*a^*(cc)^* + \lambda$

(g) $ab^*a + b(aa)^*b$

(h) $(bb^*a + (aa)^*)^*$

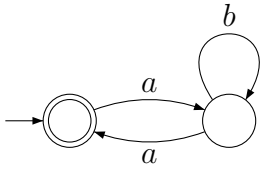
14. Given the expression $\alpha = a^*ba^* + b^*ab^*$ and the homomorphism:

$$\begin{cases} h(0) = ba \\ h(1) = ab \end{cases}$$

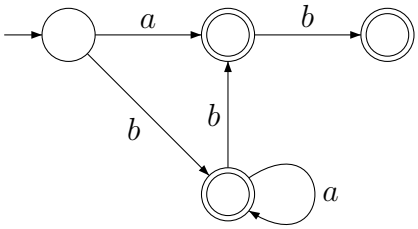
obtain the minimum DFA of the language $h^{-1}(L(\alpha))$

15. Obtain a regular expression to represent the languages accepted by each one of the following automata:

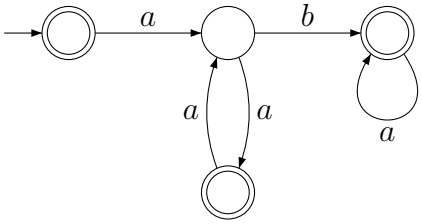
(a)



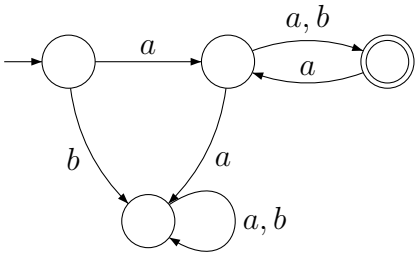
(b)



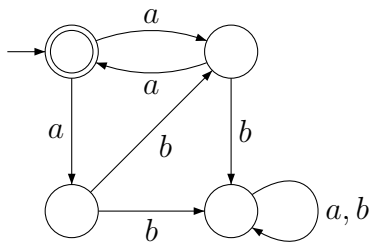
(c)



(d)



(e)



(f)