Exercises

Exercise 1

Let consider the following languages over $\{0,1\}$:

$$\begin{aligned}
L_1 &= \{0x \ : \ x \in \{0,1\}^*\} \\
L_2 &= \{x1 \ : \ x \in \{0,1\}^*\} \\
L_3 &= \{0x1 \ : \ x \in \{0,1\}^*\} \\
L_4 &= \{x \in \{0,1\}^* \ : \ |x|_0 = 2\} \\
L_5 &= \{x \in \{0,1\}^* \ : \ |x|_0 \ mod\ 2 = 0\} \\
L_6 &= \{x \in \{0,1\}^* \ : \ 001 \in Suf(x)\} \\
L_7 &= \{x \in \{0,1\}^* \ : \ 001 \in Seg(x)\}
\end{aligned}$$

(a) Taking into account the equivalence relation of L_1, decide whether the following string pairs are equivalent or not: $(001, 10)$, $(000, 0)$, $(11101001, 10)$, $(\lambda, 001)$, $(\lambda, 1001)$

(b) Taking into account the equivalence relation of L_2, describe each class of the equivalence relation R_{L_2} and which is the first string in canonical order of each class.

(c) Taking into account the equivalence relation of L_2, decide whether the following string pairs are equivalent or not: $(001, 10)$, $(000, 0)$, $(11101001, 10)$, $(\lambda, 001)$, $(\lambda, 10010)$

(d) Taking into account the equivalence relation of L_3, provide examples of three pairs of equivalent strings, and three pairs of non-equivalent strings.

(e) Taking into account the equivalence relation of L_3, provide examples of three pairs of equivalent strings, and three pairs of non-equivalent strings.

(f) Taking into account the equivalence relation of L_5, describe each class of the equivalence relation R_{L_5} and which is the first string in canonical order of each class.

(g) Taking into account the equivalence relation of L_6, describe each class of the equivalence relation R_{L_6} and which is the first string in canonical order of each class.

(h) Taking into account the equivalence relation of L_6, provide examples of three pairs of equivalent strings, and three pairs of non-equivalent strings.

(i) Provide three examples of strings in each of the following languages: $(11)^{-1}L_7$, $(001)^{-1}L_7$, $(1001)^{-1}L_7$, $(1100)^{-1}L_7$

Exercise 2

Decide whether the following languages are regular or not.

(a) $L = \{x \in \{0,1\}^* \ : \ \ x = x^r\}$

(b) $L = \{x \in \{0,1\}^* \ : \ |x|_0 = |x|_1\}$

(c) $L = \{x \in \{0,1,2\}^* \ : \ 2|x|_0 = |x|\}$

(d) $L = \{x \in \{0,1,2\}^* \ : \ |x|_0 = |x|_1 \times |x|_2\}$

(e) $L = \{x \in \{0,1,2\}^* \ : \ |x|_2 = |x|_0 + |x|_1\}$

(f) $L = \{xx \ : \ x \in \{0,1\}^*\}$

(g) Let L be the language over $\{0,1\}^*$ that contains the words such that the longest segment of symbols 0 has the same length than the longest segments of symbols 1.

(h) $L = \{a^p b^q c^r d^s \ : \ p = r \vee q = s\}$

(i) Let L be the language over $\{0,1\}^*$ that contains the words such that the longest segment of 0 symbols has odd length.

(j) Let L be the language of the words such that the number of 0 symbols in even and odd positions is the same.