Problema

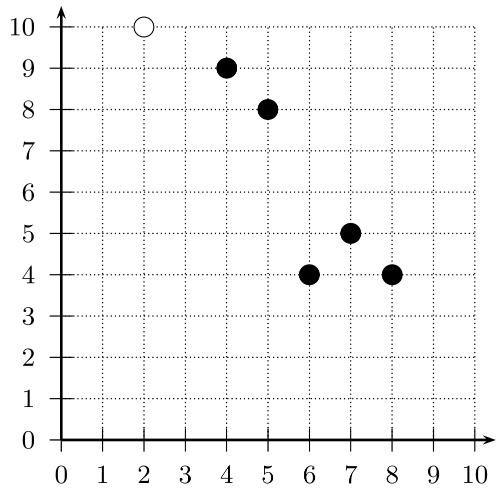
Se tienen los siguientes 6 vectores bidimensionales:

$$\boldsymbol{x}_1 = \begin{pmatrix} 2 \\ 10 \end{pmatrix}, \quad \boldsymbol{x}_2 = \begin{pmatrix} 8 \\ 4 \end{pmatrix}, \quad \boldsymbol{x}_3 = \begin{pmatrix} 5 \\ 8 \end{pmatrix}, \quad \boldsymbol{x}_4 = \begin{pmatrix} 7 \\ 5 \end{pmatrix} \quad \boldsymbol{x}_5 = \begin{pmatrix} 6 \\ 4 \end{pmatrix} \quad \boldsymbol{x}_6 = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$$

Partiendo de la partición

$$\Pi = \{X_1 = \{\boldsymbol{x}_1\}, X_2 = \{\boldsymbol{x}_2, \boldsymbol{x}_3, \boldsymbol{x}_4, \boldsymbol{x}_5, \boldsymbol{x}_6\}\}$$

Se desea agrupar estos vectores de manera no-supervisada en 2 clases aplicando el algoritmo *c-medias*.



Algoritmo C-medias

Algorithm C*-means* (versión "correcta" [Duda & Hart])

Input: X; C; $\Pi = \{X_1, \dots, X_C\}$;

Output: $\Pi^* = \{X_1, \dots, X_C\}$; $\boldsymbol{m}_1, \dots, \boldsymbol{m}_C$; J

for $c = 1$ **to** C **do** $\boldsymbol{m}_c = \frac{1}{n_c} \sum_{\boldsymbol{x} \in X_c} \boldsymbol{x}$ **endfor**

repeat

  $transfers$ = false

  forall $\boldsymbol{x} \in X$ (let $i : \boldsymbol{x} \in X_i$) **do**

    if $n_i > 1$ **then**

      $$j^* = \arg\min_{j \neq i} \frac{n_j}{n_j + 1} \|\boldsymbol{x} - \boldsymbol{m}_j\|^2$$

      $$\triangle J = \frac{n_{j^*}}{n_{j^*} + 1} \|\boldsymbol{x} - \boldsymbol{m}_{j^*}\|^2 - \frac{n_i}{n_i - 1} \|\boldsymbol{x} - \boldsymbol{m}_i\|^2$$

      if $\triangle J < 0$ **then**

        $transfers$ = true

        $$\boldsymbol{m}_i = \boldsymbol{m}_i - \frac{\boldsymbol{x} - \boldsymbol{m}_i}{n_i - 1} \qquad \boldsymbol{m}_{j^*} = \boldsymbol{m}_{j^*} + \frac{\boldsymbol{x} - \boldsymbol{m}_{j^*}}{n_{j^*} + 1}$$

        $$X_i = X_i - \{\boldsymbol{x}\} \qquad X_{j^*} = X_{j^*} + \{\boldsymbol{x}\}$$

        $$J = J + \triangle J$$

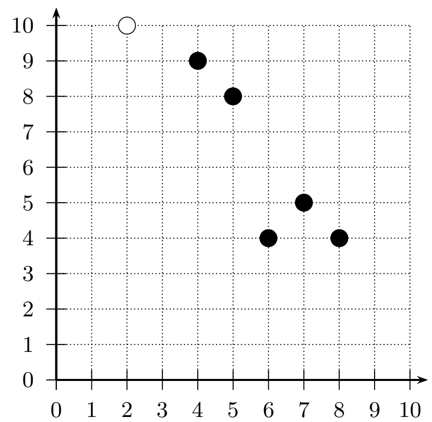
      endif

    endif

  endforall

until $\neg transfers$

Π	$\{X_1 = \{\boldsymbol{x}_1\}, X_2 = \{\boldsymbol{x}_2, \boldsymbol{x}_3\, \boldsymbol{x}_4, \boldsymbol{x}_5, \boldsymbol{x}_6\}\}$
m_1	
m_2	
J_1	
J_2	
J	
¿Transferimos $\boldsymbol{x}_2 = (8,4)^t$ de X_2 a X_1?	
$\triangle J$	
¿Transferimos $\boldsymbol{x}_3 = (5,8)^t$ de X_2 a X_1?	
$\triangle J$	
¿Transferimos $\boldsymbol{x}_4 = (7,5)^t$ de X_2 a X_1?	
$\triangle J$	
¿Transferimos $\boldsymbol{x}_5 = (6,4)^t$ de X_2 a X_1?	
$\triangle J$	
¿Transferimos $\boldsymbol{x}_6 = (4,9)^t$ de X_2 a X_1?	
$\triangle J$	



Algoritmo C-medias

Algorithm *C-means* (versión "correcta" [Duda & Hart])

Input: X; C; $\Pi = \{X_1, \ldots, X_C\}$;

Output: $\Pi^* = \{X_1, \ldots, X_C\}$; $\boldsymbol{m}_1, \ldots, \boldsymbol{m}_C$; J

for $c = 1$ **to** C **do** $\boldsymbol{m}_c = \frac{1}{n_c} \sum_{\boldsymbol{x} \in X_c} \boldsymbol{x}$ **endfor**

repeat

$transfers$ = false

forall $\boldsymbol{x} \in X$ (let $i : \boldsymbol{x} \in X_i$) **do**

if $n_i > 1$ **then**

$$j^* = \arg\min_{j \neq i} \frac{n_j}{n_j + 1} \|\boldsymbol{x} - \boldsymbol{m}_j\|^2$$

$$\triangle J = \frac{n_{j^*}}{n_{j^*} + 1} \|\boldsymbol{x} - \boldsymbol{m}_{j^*}\|^2 - \frac{n_i}{n_i - 1} \|\boldsymbol{x} - \boldsymbol{m}_i\|^2$$

if $\triangle J < 0$ **then**

$transfers$ = true

$$\boldsymbol{m}_i = \boldsymbol{m}_i - \frac{\boldsymbol{x} - \boldsymbol{m}_i}{n_i - 1} \qquad \boldsymbol{m}_{j^*} = \boldsymbol{m}_{j^*} + \frac{\boldsymbol{x} - \boldsymbol{m}_{j^*}}{n_{j^*} + 1}$$

$$X_i = X_i - \{\boldsymbol{x}\} \qquad X_{j^*} = X_{j^*} + \{\boldsymbol{x}\}$$

$$J = J + \triangle J$$

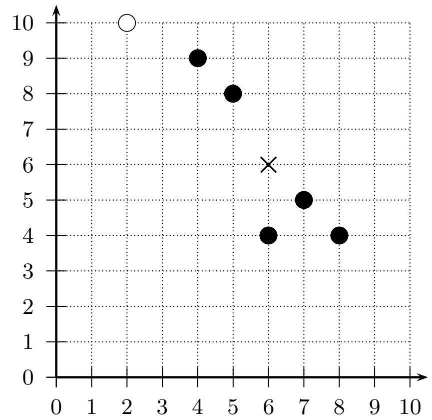
endif

endif

endforall

until $\neg transfers$

Π	$\{X_1 = \{\boldsymbol{x}_1\}, X_2 = \{\boldsymbol{x}_2, \boldsymbol{x}_3\, \boldsymbol{x}_4, \boldsymbol{x}_5, \boldsymbol{x}_6\}\}$
m_1	$(2, 10)^t$
m_2	$(6, 6)^t$
J_1	0
J_2	32
J	32
¿Transferimos $\boldsymbol{x}_2 = (8, 4)^t$ de X_2 a X_1?	
$\triangle J$	26
¿Transferimos $\boldsymbol{x}_3 = (5, 8)^t$ de X_2 a X_1?	
$\triangle J$	0.25
¿Transferimos $\boldsymbol{x}_4 = (7, 5)^t$ de X_2 a X_1?	
$\triangle J$	47.5
¿Transferimos $\boldsymbol{x}_5 = (6, 4)^t$ de X_2 a X_1?	
$\triangle J$	21
¿Transferimos $\boldsymbol{x}_6 = (4, 9)^t$ de X_2 a X_1?	
$\triangle J$	-13.75



Algoritmo C-medias

Algorithm C*-means* (versión "correcta" [Duda & Hart])

Input: $X; C; \Pi = \{X_1, \ldots, X_C\};$

Output: $\Pi^* = \{X_1, \ldots, X_C\}; \boldsymbol{m}_1, \ldots, \boldsymbol{m}_C; J$

for $c = 1$ **to** C **do** $\boldsymbol{m}_c = \frac{1}{n_c} \sum_{\boldsymbol{x} \in X_c} \boldsymbol{x}$ **endfor**

repeat

  $transfers =$ false

  forall $\boldsymbol{x} \in X$ (let $i : \boldsymbol{x} \in X_i$) **do**

    if $n_i > 1$ **then**

$$j^* = \arg\min_{j \neq i} \frac{n_j}{n_j + 1} \|\boldsymbol{x} - \boldsymbol{m}_j\|^2$$

$$\triangle J = \frac{n_{j^*}}{n_{j^*} + 1} \|\boldsymbol{x} - \boldsymbol{m}_{j^*}\|^2 - \frac{n_i}{n_i - 1} \|\boldsymbol{x} - \boldsymbol{m}_i\|^2$$

      if $\triangle J < 0$ **then**

        $transfers =$ true

$$\boldsymbol{m}_i = \boldsymbol{m}_i - \frac{\boldsymbol{x} - \boldsymbol{m}_i}{n_i - 1} \qquad \boldsymbol{m}_{j^*} = \boldsymbol{m}_{j^*} + \frac{\boldsymbol{x} - \boldsymbol{m}_{j^*}}{n_{j^*} + 1}$$

$$X_i = X_i - \{\boldsymbol{x}\} \qquad X_{j^*} = X_{j^*} + \{\boldsymbol{x}\}$$

$$J = J + \triangle J$$

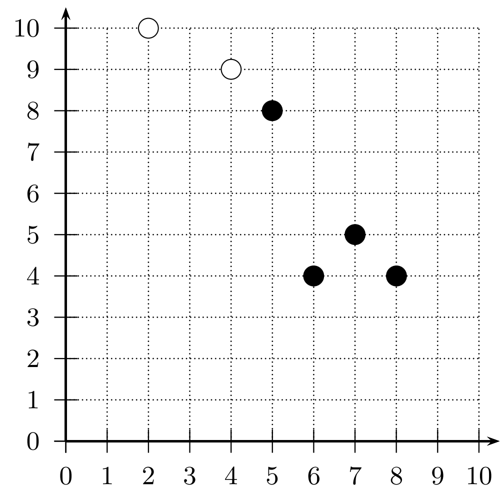
      endif

    endif

  endforall

until $\neg transfers$

Π	$\{X_1 = \{\boldsymbol{x}_1, \boldsymbol{x}_6\}, X_2 = \{\boldsymbol{x}_2, \boldsymbol{x}_3\, \boldsymbol{x}_4, \boldsymbol{x}_5\}\}$
m_1	
m_2	
J	
¿Transferimos $\boldsymbol{x}_1 = (2, 10)^t$ de X_1 a X_2?	
$\triangle J$	
¿Transferimos $\boldsymbol{x}_2 = (8, 4)^t$ de X_2 a X_1?	
$\triangle J$	
¿Transferimos $\boldsymbol{x}_3 = (5, 8)^t$ de X_2 a X_1?	
$\triangle J$	



Algoritmo C-medias

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Input: X; C; $\Pi = \{X_1, \ldots, X_C\}$;

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  $transfers =$ false

  forall $\boldsymbol{x} \in X$ (let $i : \boldsymbol{x} \in X_i$) **do**

    if $n_i > 1$ **then**

$$j^* = \arg\min_{j \neq i} \frac{n_j}{n_j + 1} \|\boldsymbol{x} - \boldsymbol{m}_j\|^2$$

$$\triangle J = \frac{n_{j^*}}{n_{j^*} + 1} \|\boldsymbol{x} - \boldsymbol{m}_{j^*}\|^2 - \frac{n_i}{n_i - 1} \|\boldsymbol{x} - \boldsymbol{m}_i\|^2$$

      if $\triangle J < 0$ **then**

        $transfers =$ true

$$\boldsymbol{m}_i = \boldsymbol{m}_i - \frac{\boldsymbol{x} - \boldsymbol{m}_i}{n_i - 1} \qquad \boldsymbol{m}_{j^*} = \boldsymbol{m}_{j^*} + \frac{\boldsymbol{x} - \boldsymbol{m}_{j^*}}{n_{j^*} + 1}$$

$$X_i = X_i - \{\boldsymbol{x}\} \qquad X_{j^*} = X_{j^*} + \{\boldsymbol{x}\}$$

$$J = J + \triangle J$$

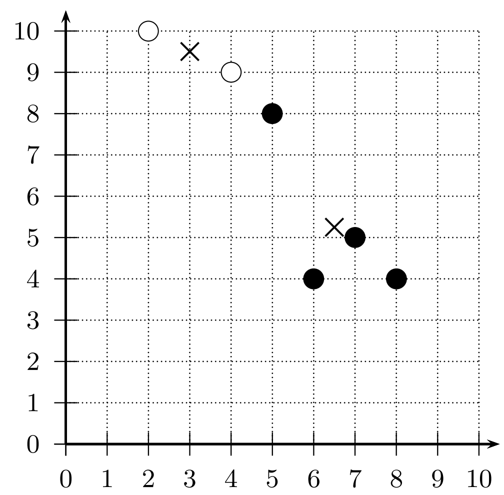
      endif

    endif

  endforall

until $\neg transfers$

Π	$\{X_1 = \{\boldsymbol{x}_1, \boldsymbol{x}_6\}, X_2 = \{\boldsymbol{x}_2, \boldsymbol{x}_3\, \boldsymbol{x}_4, \boldsymbol{x}_5\}\}$
m_1	$(3, 9{,}5)^t$
m_2	$(6{,}5, 5{,}25)^t$
J	$18{,}25$
¿Transferimos $\boldsymbol{x}_1 = (2, 10)^t$ de X_1 a X_2?	
$\triangle J$	31.75
¿Transferimos $\boldsymbol{x}_2 = (8, 4)^t$ de X_2 a X_1?	
$\triangle J$	31.75
¿Transferimos $\boldsymbol{x}_3 = (5, 8)^t$ de X_2 a X_1?	
$\triangle J$	-9.58



Algoritmo C-medias

Algorithm C*-means* (versión "correcta" [Duda & Hart])

Input: $X; C; \Pi = \{X_1, \ldots, X_C\};$

Output: $\Pi^* = \{X_1, \ldots, X_C\}; \boldsymbol{m}_1, \ldots, \boldsymbol{m}_C; J$

for $c = 1$ **to** C **do** $\boldsymbol{m}_c = \frac{1}{n_c} \sum_{\boldsymbol{x} \in X_c} \boldsymbol{x}$ **endfor**

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if $\triangle J < 0$ **then**

$transfers =$ true

$$\boldsymbol{m}_i = \boldsymbol{m}_i - \frac{\boldsymbol{x} - \boldsymbol{m}_i}{n_i - 1} \qquad \boldsymbol{m}_{j^*} = \boldsymbol{m}_{j^*} + \frac{\boldsymbol{x} - \boldsymbol{m}_{j^*}}{n_{j^*} + 1}$$

$$X_i = X_i - \{\boldsymbol{x}\} \qquad X_{j^*} = X_{j^*} + \{\boldsymbol{x}\}$$

$$J = J + \triangle J$$

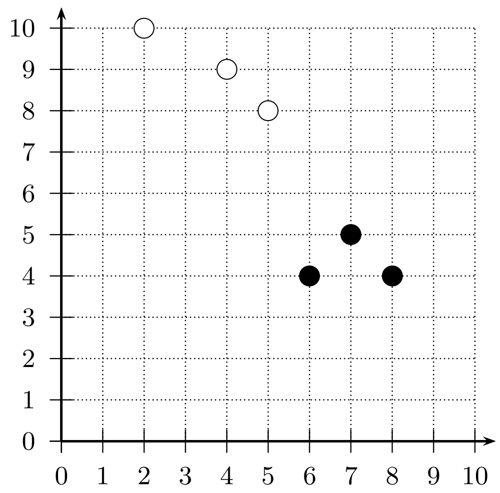
endif

endif

endforall

until $\neg transfers$

Π	$\{X_1 = \{\boldsymbol{x}_1, \boldsymbol{x}_3, \boldsymbol{x}_6\}, X_2 = \{\boldsymbol{x}_2, \boldsymbol{x}_4, \boldsymbol{x}_5\}\}$
m_1	
m_2	
J	
¿Transferimos $\boldsymbol{x}_4 = (7,5)^t$ de X_2 a X_1?	
$\triangle J$	
¿Transferimos $\boldsymbol{x}_5 = (6,4)^t$ de X_2 a X_1?	
$\triangle J$	
¿Transferimos $\boldsymbol{x}_6 = (4,9)^t$ de X_1 a X_2?	
$\triangle J$	



Algoritmo C-medias

Algorithm C*-means* (versión “correcta” [Duda & Hart])

Input: $X; C; \Pi = \{X_1, \ldots, X_C\};$

Output: $\Pi^* = \{X_1, \ldots, X_C\}; \boldsymbol{m}_1, \ldots, \boldsymbol{m}_C; J$

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repeat

  $transfers =$ false

  forall $\boldsymbol{x} \in X$ (let $i : \boldsymbol{x} \in X_i$) **do**

    if $n_i > 1$ **then**

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      if $\triangle J < 0$ **then**

        $transfers =$ true

$$\boldsymbol{m}_i = \boldsymbol{m}_i - \frac{\boldsymbol{x} - \boldsymbol{m}_i}{n_i - 1} \qquad \boldsymbol{m}_{j^*} = \boldsymbol{m}_{j^*} + \frac{\boldsymbol{x} - \boldsymbol{m}_{j^*}}{n_{j^*} + 1}$$

$$X_i = X_i - \{\boldsymbol{x}\} \qquad X_{j^*} = X_{j^*} + \{\boldsymbol{x}\}$$

$$J = J + \triangle J$$

      endif

    endif

  endforall

until $\neg transfers$

Π	$\{X_1 = \{\boldsymbol{x}_1, \boldsymbol{x}_3, \boldsymbol{x}_6\}, X_2 = \{\boldsymbol{x}_2, \boldsymbol{x}_4, \boldsymbol{x}_5\}\}$
m_1	$(3{,}67, 9)^t$
m_2	$(7, 4{,}33)^t$
J	$8{,}67$
¿Transferimos $\boldsymbol{x}_4 = (7, 5)^t$ de X_2 a X_1?	
$\triangle J$	20.32
¿Transferimos $\boldsymbol{x}_5 = (6, 4)^t$ de X_2 a X_1?	
$\triangle J$	21.16
¿Transferimos $\boldsymbol{x}_6 = (4, 9)^t$ de X_1 a X_2?	
$\triangle J$	22.94