The answers to the exercises must be reasoned

Exercises

Exercise 1

Given the string $x = 0011$, obtain

(a) $Pref(x)$

(b) $Suf(x)$

(c) $Seg(x)$

Exercise 2

Given $L = \{x \in \{a,b\}^* \ : \ |x|_a \leq 2\}$

(a) Enumerate, in canonical order, the first ten strings of L

(b) Give a description of the language $(aba)^{-1}L$

Exercise 3

Given $L = \{x \in \{a,b\}^* \ : \ |x|_a \bmod 2 = 0\}$

(a) Enumerate, in canonical order, the first ten strings of L

(b) Give a description of the languages $(abb)^{-1}L$ and $(bbaba)^{-1}L$

(c) Describe the languages $Pref(L)$ and $Suf(L)$

Exercise 4

Given the languages $L_1 = \{0^n1^n \ : \ n \geq 0\}$ and $L_2 = \{0^n1^m \ : \ n, m \geq 0\}$

(a) Enumerate, in canonical order, the first ten strings of L_1

(b) Enumerate, in canonical order, the first ten strings of L_2

(c) Describe the language $L_1 \cup L_2$

Exercise 5

Given $L_1 = \{xaa \ : \ x \in \{a,b\}^*\}$ and $L_2 = \{xaay \ : \ x, y \in \{a,b\}^*\}$, describe the following languages:

(a) $L_1 \cap L_2$

(b) $L_1 \cup L_2$

(c) $(aba)^{-1}L_1$

Exercise 6

Given $L_1 = \{xaby \ : \ x, y \in \{a,b\}^*\}$ and $L_2 = \{xbay \ : \ x, y \in \{a,b\}^*\}$, describe the language $L_1 \cap \overline{L_2}$

Exercise 7

Describe the language over $\{a,b\}$ whose words begin with a and do not contain the segment ba.

Exercise 8

Given the language $L = \{a, abb\}^*\{b, baa\}^*$, enumerate, in canonical order, the first ten strings of the language.

Exercise 9

Given the language $L = \{x \in \{a, b\}^* \ : \ x = x^r\}$, describe the language L^r.

Exercise 10

Taking into account the following languages:

$$L_1 = \{axb \ : \ x \in \{a, b\}^*\}$$
$$L_2 = \{x \in \{a, b\}^* \ : \ (|x|_a = 1) \ \vee \ (|x|_b = 1)\}$$

and the homomorphism:

$$\begin{cases} h(0) = ba \\ h(1) = ab \end{cases}$$

describe the following languages:

(a) L_1^2

(b) L_1^*

(c) $(aab)^{-1}L_1$

(d) $(aba)^{-1}L_1$

(e) $\overline{L_1}$

(f) $h^{-1}(L_1)$

(g) $L_1 \cap L_2$

(h) $h^{-1}(L_2)$

Exercise 11

Decide on the validity of the following statement:

$$Suf(L) = (Pref(L^r))^r$$

Exercise 12

Decide on the validity of the following statement:

Let $L \subset \Sigma^*$ be a language such that there exists a number n such that, for any $x \in \Sigma^*$, if $|x| > n$, then it is fulfilled that $x \in L$. The language $\overline{L}$ is finite.

Exercise 13

Describe the language L^* taking into account that $L = L^2$.

Exercise 14

Let L_1 and L_2 be such that $L_1^* = L_2^*$. It is fulfilled that $L_1 = L_2$?

Exercise 15

Let u, v two strings in Σ^* and let $L_1 = \{ux \ : \ x \in \Sigma^*\}$ and $L_2 = \{uvx \ : \ x \in \Sigma^*\}$ be two languages. Is there any relationship between L_1 and L_2?

Exercise 16

Given the language $L = \{a\}^+\{b\}^+$

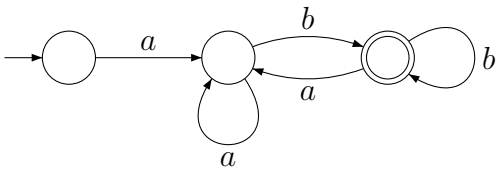
(a) Obtain a DFA that accepts L

(b) Describe the language $\overline{L}$

(c) Obtain a DFA for the language $\overline{L}$

Exercise 17

Given $L = \{xab \ : \ x \in \{a, b\}^*\}$ and the following automaton:



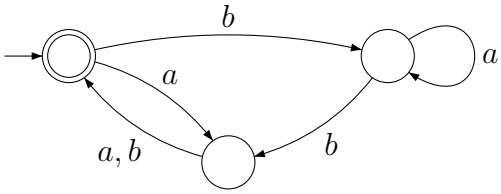
Decide the validity of the following statements:

(a) $L \subseteq L(A)$

(b) $L(A) \subseteq L$

Exercise 18

Given $L = \{x \in \{a, b\}^* \ : \ |x|_b \equiv 1 \pmod 2\}$ and the following automaton:



Decide the validity of the following statements:

(a) $L \subseteq L(A)$

(b) $L(A) \subseteq L$

Exercise 19

Decide the validity of the following statement:

Given a finite automaton $A = (Q, \Sigma, \delta, q_0, F)$, it is hold that $\lambda \in L(A)$ if and only if $q_0 \in F$

Exercise 20

Decide the validity of the following statement:

Let A_1 y A_2 be two accesible and completely specified DFA. If $L(A_1) \subseteq L(A_2)$, then, the number of states of A_1 is smaller or equal than the number of states of A_2

Exercise 21

Obtain a DFA for the following languages:

(a) $L = \{xaa \ : \ x \in \{a,b\}^*\}$ (strings that have sufix aa).

(b) $L = \{xabay \ : \ x,y \in \{a,b\}^*\}$ (strings that contain the segment aba).

(c) $L = \{a, abb\}^*\{b, baa\}^*$

(d) Language over the alphabet $\Sigma = \{a,b\}$ starting with a and ending with b.

(e) $L = \{(ab)^n \ : \ n > 0\}$

(f) $L = \{x \in \{a,b\}^* \ : \ aa \in Seg(x) \wedge a \in Suf(x)\}$

(g) $L = \{x \in \{a,b\}^* \ : \ |x|_a > 0 \wedge |x|_b > 0\}$

(h) $L = \{x \in \{a,b\}^* \ : \ |x|_a = 2\}$

(i) $L = \{x \in \{a,b\}^* \ : \ aa \notin Seg(x) \wedge bb \notin Seg(x)\}$

Exercise 22

Given $L = \{xa \ : \ x \in \{a,b\}^*\}$

(a) Obtain a DFA that accepts L

(b) Describe the language L^2

(c) Obtain a DFA for L^2

Exercise 23

Given $L = \{x \in \{a,b\}^* \ : \ x = a^n, \ n > 0\}$, obtain a DFA for $\overline{L}$

Exercise 24

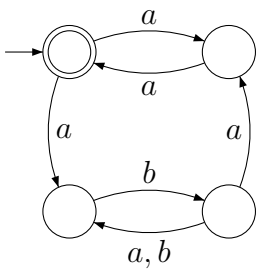
Decide the validity of the following statement:

Let A be an accesible DFA with n states. If there exists $x \in L$ such that $|x| > n$, then it is fulfilled that L is infinite.

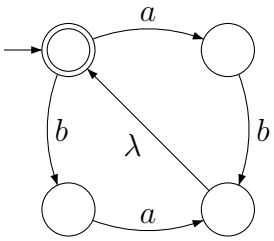
Exercise 25

Compute a equivalent DFA for the following automata:

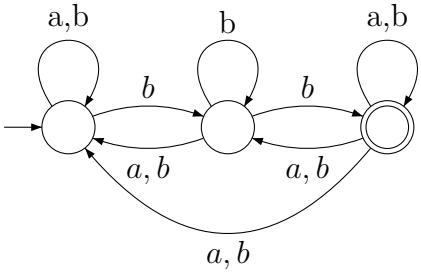
(a)



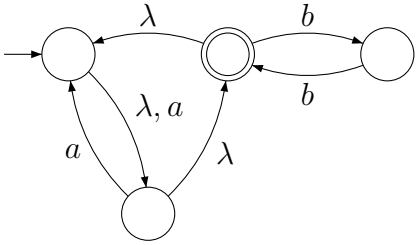
(b)



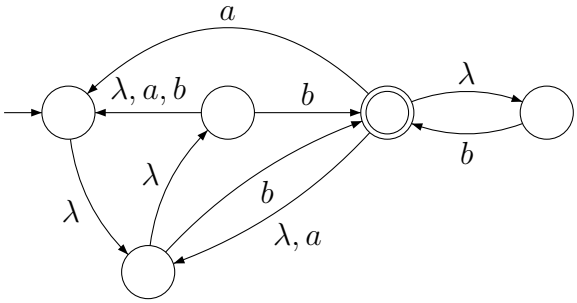
(c)



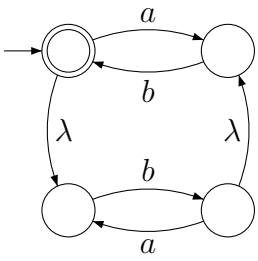
(d)



(e)



(f)



Exercise 26

Let L be a language for which there exist $u_1, u_2, \ldots, u_{1001} \in \Sigma^*$ such that, for any pair of strings u_i, u_j, if $i \neq j$ then there exists $z \in \Sigma^*$ that fulfills that $u_i z \in L \Leftrightarrow u_j z \notin L$.

(a) Is L a regular language?

(b) Is L a non-regular language?