Problemes Tema 2 (bloc 1)

7) Si A, B i C són subconjunts qualsevol d'un conjunt E, proveu

a) $A^c \setminus B^c = B \setminus A$

$A^c \setminus B^c \overset{\text{def} \setminus}{=} A^c \cap (B^c)^c \overset{\text{doble comp}}{=} A^c \cap B \overset{\text{com}}{=} B \cap A^c =$

$\overset{\text{def} \setminus}{=} B \setminus A$

b) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

$(A \setminus B) \cap (A \setminus C) \overset{\text{def} \setminus}{=} (A \cap B^c) \cap (A \cap C^c) \underset{\text{Commut}}{\overset{\text{Associat.}}{=}} (A \cap A) \cap (B^c \cap C^c)$

$\overset{\text{idempot.}}{=} A \cap (B \cup C)^c \overset{\text{def} \setminus}{=} A \setminus (B \cup C)$

c) $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$

$(A \setminus C) \cup (B \setminus C) \overset{\text{def} \setminus}{=} (A \cap C^c) \cup (B \cap C^c) \overset{\text{dist.}}{=}$

$= (A \cup B) \cap C^c \overset{\text{def} \setminus}{=} (A \cup B) \setminus C$

d) $(A \cap B) \setminus C = (A \setminus C) \cap (B \setminus C)$

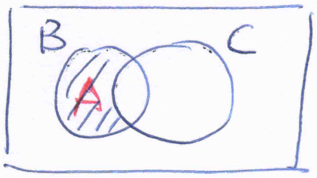
$(A \setminus C) \cap (B \setminus C) \overset{\text{def} \setminus}{=} (A \cap C^c) \cap (B \cap C^c) \underset{\text{commut}}{\overset{\text{associat}}{=}} (A \cap B) \cap (C^c \cap C^c)$

$\overset{\text{idemp}}{=} (A \cap B) \cap C^c \overset{\text{def} \setminus}{=} (A \cap B) \setminus C$

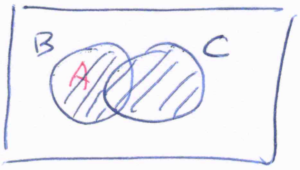
Problemes Tema 2 bloc 1

8) Si A, B, C són conjunts qualsevol indiqueu quines de les següents afirmacions són correctes, justificant les respostes:

a) Si $A = B \setminus C$, aleshores $B = A \cup C$



$A = B - C$



$A \cup C = B \cup C \neq B$

No és cert que $B = A \cup C$. Per exemple

$B = \{1, 2, 3\}$

$C = \{3, 4, 5\}$

$A = B \setminus C = \{1, 2\}$

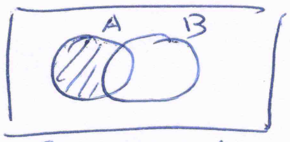
$A \cup C = \{1, 2, 3, 4, 5\}$

Es pot comprovar que $A \cup C = B \cup C$

$$A \cup C = (B \setminus C) \cup C \overset{\text{def} \setminus}{=} (B \cap C^c) \cup C \overset{\text{dist.}}{=} (B \cup C) \cap (C^c \cup C) =$$

$$\overset{\text{prop comp.}}{=} (B \cup C) \cap E \overset{\text{E neutre} \cap}{=} B \cup C$$

b) $(A \cup B) \setminus B = A$



$(A \cup B) \setminus A$

No és cert, en general

$A = \{1, 2, 3, 4\}$

$B = \{3, 4, 5\}$

$(A \cup B) \setminus B = \{1, 2\} \neq A$

c) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$

Sí és correcte:

$$(A \setminus B) \cup (A \setminus C) \overset{\text{def} \setminus}{=} (A \cap B^c) \cup (A \cap C^c) \overset{\text{dist}}{=} A \cap (B^c \cup C^c) =$$

$$\overset{\text{L. Morgan}}{=} A \cap (B \cap C)^c \overset{\text{def} \setminus}{=} A \setminus (B \cap C)$$

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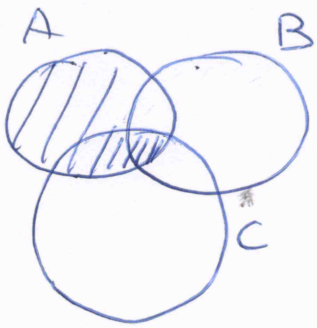
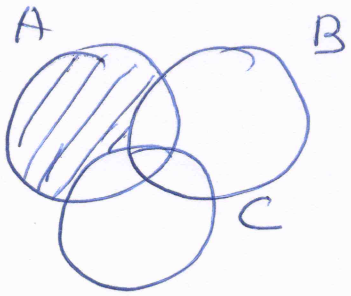
d) $A \setminus (B \setminus C) = (A \setminus B) \setminus C$



$A \setminus (B \setminus C)$



$(A \setminus B) \setminus C$

En seneral $A \setminus (B \setminus C) \neq (A \setminus B) \setminus C$

$A = \{1, 2, 3, 4, 5\}$

$B = \{3, 5, 7, 9\}$

$C = \{3, 4, 5, 6\}$

$A \setminus (B \setminus C) = A \setminus \{5, 7, 9\} = \{1, 2, 3, 4\}$

$(A \setminus B) \setminus C = \{1, 2, 4\} \setminus C = \{1, 2\}$