Computabilidad y Complejidad

Ejemplos de máquinas de Turing

Ejemplo 1

$$L(M) = \{a^n b^n : n \geq 1\}$$

$$M = (\{a, b\}, \{a, b, X, Y, B\}, \{q_0, q_1, q_2, q_3, q_4\}, f, B, q_0, \{q_4\})$$

	a	b	X	Y	B
q_0	(q_1, X, R)	——	——	(q_3, Y, R)	——
q_1	(q_1, a, R)	(q_2, Y, L)	——	(q_1, Y, R)	——
q_2	(q_2, a, L)	——	(q_0, X, R)	(q_2, Y, L)	——
q_3	——	——	——	(q_3, Y, R)	(q_4, B, R)
q_4	——	——	——	——	——

Ejemplo 1

	a	b	X	Y	B
q_0	(q_1, X, R)	——	——	(q_3, Y, R)	——
q_1	(q_1, a, R)	(q_2, Y, L)	——	(q_1, Y, R)	——
q_2	(q_2, a, L)	——	(q_0, X, R)	(q_2, Y, L)	——
q_3	——	——	——	(q_3, Y, R)	(q_4, B, R)
q_4	——	——	——	——	——

$Bq_0aaabbb \vdash\!\!- Xq_1aabbb \vdash\!\!- Xaq_1abbb \vdash\!\!- Xaaq_1bbb \vdash\!\!- Xaq_2aYbb \vdash\!\!-$

$Xq_2aaYbb \vdash\!\!- q_2XaaYbb \vdash\!\!- Xq_0aaYbb \vdash\!\!- XXq_1aYbb \vdash\!\!- XXaq_1Ybb \vdash\!\!-$

$XXaYq_1bb \vdash\!\!- XXaq_2YYb \vdash\!\!- XXq_2aYYb \vdash\!\!- Xq_2XaYYb \vdash\!\!- XXq_0aYYb \vdash\!\!-$

$XXXq_1YYb \vdash\!\!- XXXYq_1Yb \vdash\!\!- XXXYYq_1b \vdash\!\!- XXXYq_2YY \vdash\!\!- XXXq_2YYY \vdash\!\!-$

$XXq_2XYYY \vdash\!\!- XXXq_0YYY \vdash\!\!- XXXYq_3YY \vdash\!\!- XXXYYq_3Y \vdash\!\!- XXXYYYq_3B$

$\vdash\!\!- XXXYYYBq_4B$

Ejemplo 2

$$L(M) = \{wcw^r : w \in (a+b)^*\}$$

$$M = (\{a, b, c\}, \{a, b, c, B\}, \{q_0, q_a, q_{1a}, q_{2a}, q_b, q_{1b}, q_{2b}, q_r, q_t, q_f\}, f, B, q_0, \{q_f\})$$

	a	b	c	B
q_0	(q_a, B, R)	(q_b, B, R)	(q_t, B, R)	———
q_a	(q_a, a, R)	(q_a, b, R)	(q_{1a}, c, R)	———
q_{1a}	(q_{1a}, a, R)	(q_{1a}, b, R)	———	(q_{2a}, B, L)
q_{2a}	(q_r, B, L)	———	———	———
q_b	(q_b, a, R)	(q_b, b, R)	(q_{1b}, c, R)	———
q_{1b}	(q_{1b}, a, R)	(q_{1b}, b, R)	———	(q_{2b}, B, L)
q_{2b}	———	(q_r, B, L)	———	———
q_r	(q_r, a, L)	(q_r, b, L)	(q_r, c, L)	(q_0, B, R)
q_t	———	———	———	(q_f, B, R)
q_f	———	———	———	———

Ejemplo 3

$$\text{resta}(m, n) = \begin{cases} m - n & \text{si} \quad m \geq n \\ \\ 0 & \text{si} \quad m < n \end{cases}$$

$$M = (\{0, 1\}, \{0, 1, B\}, \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, f, B, q_0, \emptyset)$$

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	—
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	—
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6	—	—	—

Ejemplo 3

	0	1	B
q_0	(q_1, B, R)	(q_5, B, R)	——
q_1	$(q_1, 0, R)$	$(q_2, 1, R)$	——
q_2	$(q_3, 1, L)$	$(q_2, 1, R)$	(q_4, B, L)
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	(q_0, B, R)
q_4	$(q_4, 0, L)$	(q_4, B, L)	$(q_6, 0, R)$
q_5	(q_5, B, R)	(q_5, B, R)	(q_6, B, R)
q_6	——	——	——

$B q_0 000100 \vdash B q_1 00100 \vdash 0 q_1 0100 \vdash 00 q_1 100 \vdash 001 q_2 00 \vdash$

$00 q_3 110 \vdash 0 q_3 0110 \vdash B q_3 00110 \vdash q_3 B 00110 \vdash B q_0 00110 \vdash$

$B q_1 0110 \vdash 0 q_1 110 \vdash 01 q_2 10 \vdash 011 q_2 0 \vdash 01 q_3 11 \vdash$

$0 q_3 111 \vdash B q_3 0111 \vdash q_3 B 0111 \vdash B q_0 0111 \vdash B q_1 111 \vdash$

$1 q_2 11 \vdash 11 q_2 1 \vdash 111 q_2 B \vdash 11 q_4 1 \vdash 1 q_4 1 \vdash$

$B q_4 1 \vdash B q_4 B \vdash 0 q_6 B$

Ejemplo 4 $multiplicacion(x, y) = x \cdot y$ (con valores $x, y > 0$)

	0	1	#	B
q_{ini}	$(q_{ini2}, \#, R)$	——	——	——
q_{ini2}	$(q_{ini2}, 0, R)$	$(q_{ini2}, 1, R)$	——	$(q_{ini3}, 1, L)$
q_{ini3}	$(q_{ini3}, 0, L)$	$(q_{ini3}, 1, R)$	(q_6, B, R)	——
q_0	(q_6, B, R)	——	——	——
q_1	$(q_2, \#, R)$	$(q_4, 1, L)$	——	——
q_2	$(q_2, 0, R)$	$(q_2, 1, R)$	——	$(q_3, 0, L)$
q_3	$(q_3, 0, L)$	$(q_3, 1, L)$	$(q_1, \#, R)$	——
q_4	——	$(q_5, 1, R)$	$(q_4, 0, L)$	——
q_5	$(q_7, 0, L)$	——	——	——
q_6	$(q_6, 0, R)$	$(q_1, 1, R)$	——	——
q_7	——	$(q_8, 1, L)$	——	——
q_8	$(q_9, 0, L)$	——	——	(q_{10}, B, R)
q_9	$(q_9, 0, L)$	——	——	(q_0, B, R)
q_{10}	——	(q_{11}, B, R)	——	——
q_{11}	(q_{11}, B, R)	(q_{12}, B, R)	——	——
q_{12}	——	——	——	——