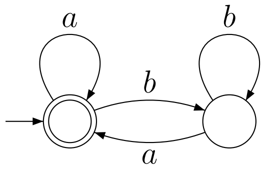
Exercises

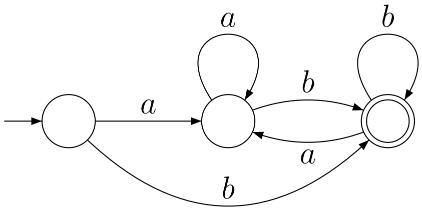
Exercise 1

Given the following automata:

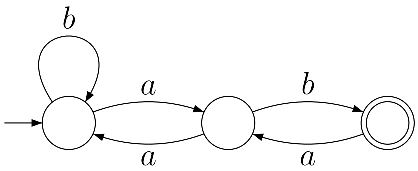
A_1



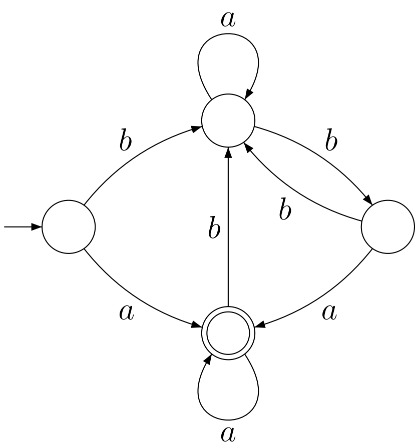
A_2



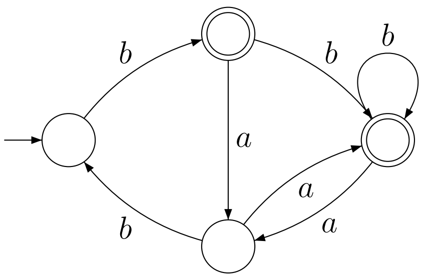
A_3



A_4

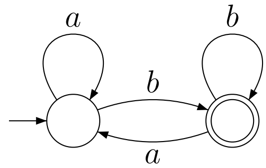


A_5



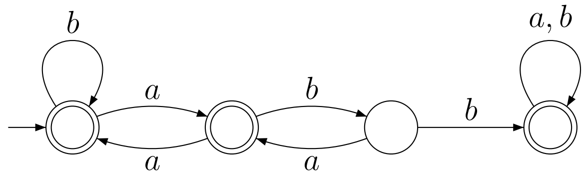
(a) Obtain a DFA for $\overline{L(A_1)}$

Answer:



(b) Obtain a DFA for $\overline{L(A_3)}$

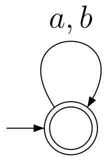
Answer:



(c) Obtain a DFA for $L(A_1) \cup L(A_2)$

Answer:

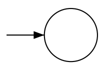
This construction returns a complete DFA where all states are final, and therefore equivalent to the following one:



(d) Obtain a DFA for $L(A_1) \cap L(A_2)$

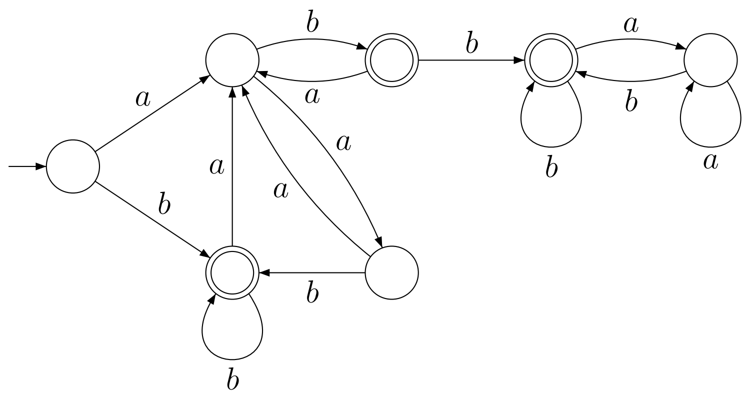
Answer:

This construction returns a complete DFA without final states, and therefore equivalent to the following one:



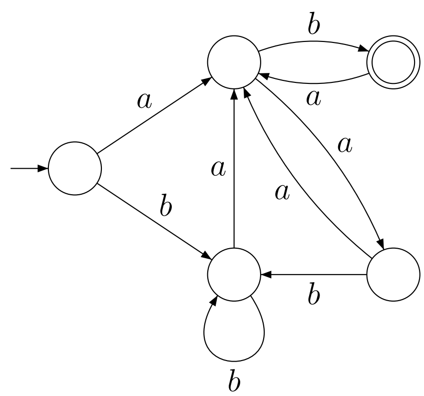
(e) Obtain a DFA for $L(A_2) \cup L(A_3)$

Answer:



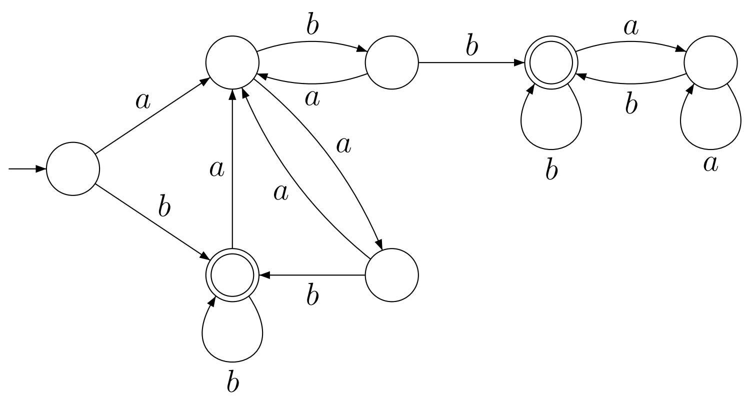
(f) Obtain a DFA for $L(A_2) \cap L(A_3)$

Answer:



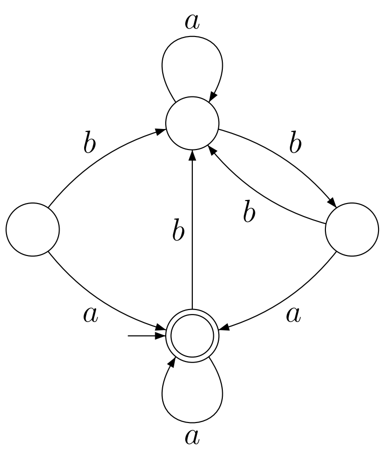
(g) Obtain a DFA for $L(A_2) - L(A_3)$

Answer:



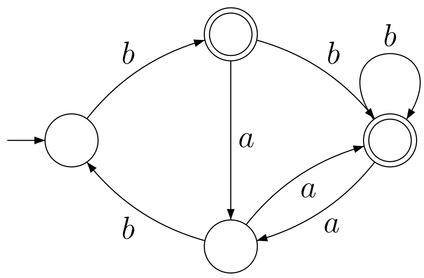
(h) Obtain a DFA for $(abba)^{-1}L(A_4)$

Answer:



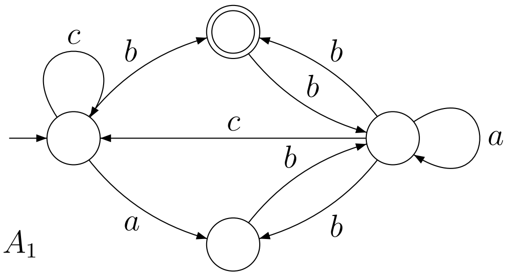
(i) Obtain a DFA for the language $(bbbab)^{-1}L(A_5)$

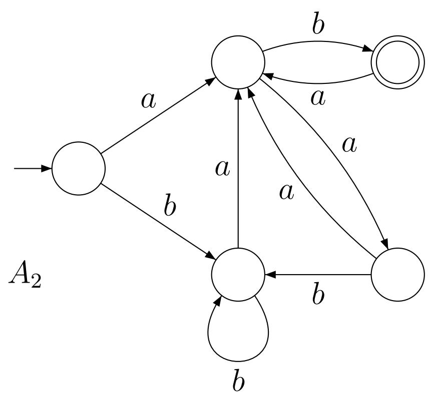
Answer:



Exercise 2

Given the automata:





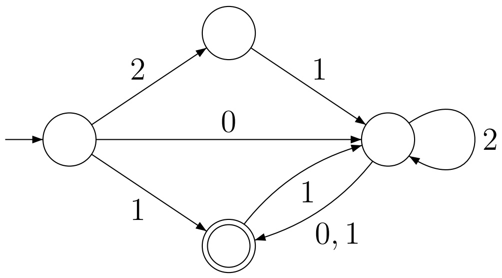
and the following homomorphisms:

$$h : \{a,b,c\} \to \{0,1,2\}^* \quad \begin{cases} h(a) = 00 \\ h(b) = 1 \\ h(c) = \lambda \end{cases} \qquad g : \{0,1,2\} \to \{a,b,c\}^* \quad \begin{cases} g(0) = ab \\ g(1) = bbb \\ g(2) = a \end{cases} \qquad f : \{0,1,2\} \to \{a,b\}^* \quad \begin{cases} f(0) = ab \\ f(1) = bab \\ f(2) = \lambda \end{cases}$$

(a) Obtain a DFA for the language $g^{-1}(L(A_1))$

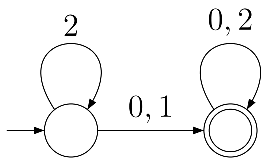
Answer:

Note that A_1 is non-deterministic. In order to apply the construction explained in theory, the automaton must be deterministic.



(b) Obtain a DFA for $f^{-1}(L(A_2))$

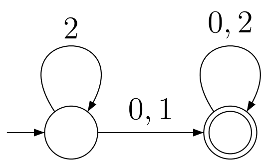
Answer:



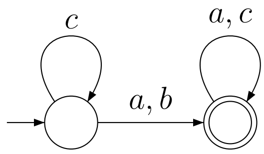
(c) Obtain a DFA for the language $h^{-1}(f^{-1}(L(A_2)))$

Answer:

We consider the automaton for $f^{-1}(L(A_2))$ (shown in part b of this exercise)



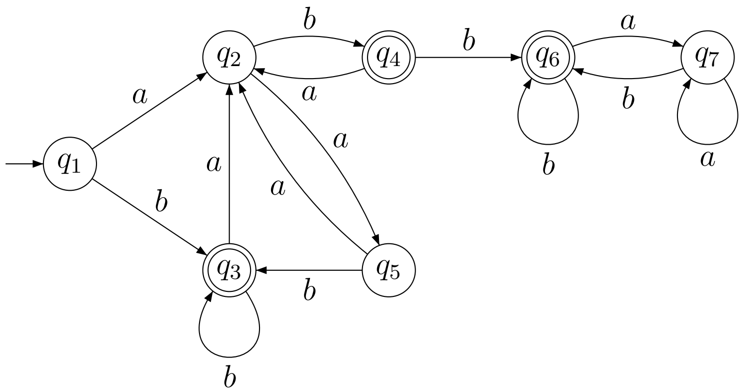
the following automaton accepts $h^{-1}(f^{-1}(L(A_2)))$:



Exercise 3

For each one of the following automata, obtain the minimal equivalent DFA:

(a)



Answer:

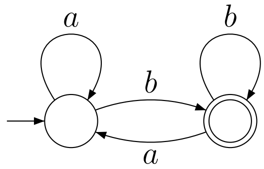
First state partition distinguish between final and non-final states:

$$\pi_0 = \{\{q_1, q_2, q_5, q_7\}, \{q_3, q_4, q_6\}\}$$

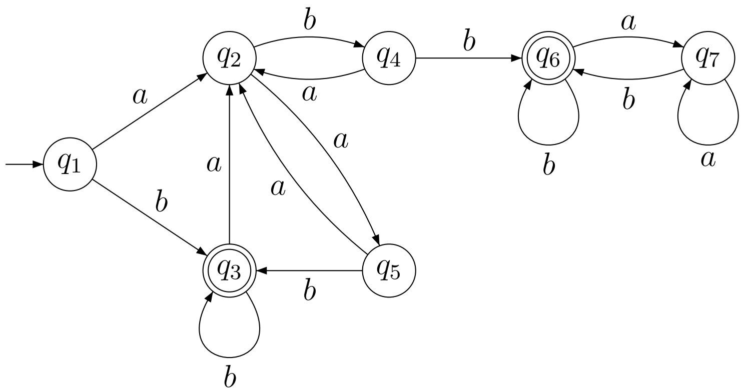
Taking into account this first partition:

		a	b
B_1	q_1	$q_2 \in B_1$	$q_3 \in B_3$
	q_2	$q_5 \in B_1$	$q_4 \in B_3$
	q_5	B_1	B_3
	q_7	B_1	B_3
B_3	q_3	B_1	B_3
	q_4	B_1	B_3
	q_6	B_1	B_3

For each block of the partition, the behavior of each state is the same, thus, the partition is not refined. The minimal DFA is the following one:



(b)



Answer:

First partition of states distinguish between final and non-final states:

$$\pi_0 = \{\{q_1, q_2, q_4, q_5, q_7\}, \{q_3, q_6\}\}$$

Taking into account this first partition:

		a	b
B_1	q_1	$q_2 \in B_1$	$q_3 \in B_3$
	q_2	$q_5 \in B_1$	$q_4 \in B_1$
	q_4	B_1	B_3
	q_5	B_1	B_3
	q_7	B_1	B_3
B_3	q_3	B_1	B_3
	q_6	B_1	B_3

Note that state q_2 behaves in a different way than the remaining states in its block, therefore, the partition is refined:

$$\pi_1 = \{\{q_1, q_4, q_5, q_7\}, \{q_2\}, \{q_3, q_6\}\}$$

		a	b
B_1	q_1	$q_2 \in B_2$	$q_3 \in B_3$
	q_4	$q_2 \in B_2$	$q_6 \in B_3$
	q_5	B_2	B_3
	q_7	B_1	B_3
B_2	q_2	$--$	$--$
B_3	q_3	B_2	B_3
	q_6	B_1	B_3

Note that B_2 block is a singleton, therefore it is not necessary to analyze the behavior of the state. In this interation, the state q_7 behaves in a different way than the remaining states in its block, and the same happens with state q_3. The partition that results is:

$$\pi_2 = \{\{q_1, q_4, q_5\}, \{q_2\}, \{q_3\}, \{q_6\}, \{q_7\}\}$$

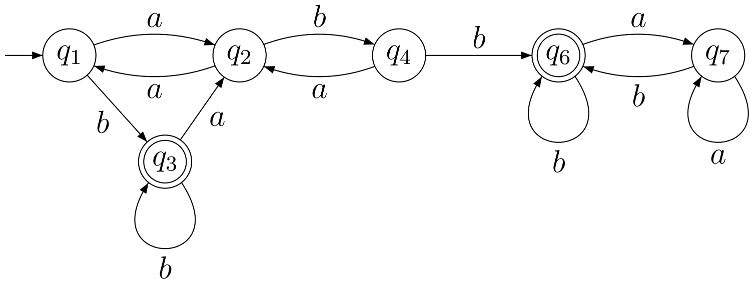
		a	b
B_1	q_1	B_2	B_3
	q_4	B_2	B_6
	q_5	B_2	B_3
B_2	q_2	$--$	$--$
B_3	q_3	$--$	$--$
B_6	q_6	$--$	$--$
B_7	q_7	$--$	$--$

In this iteration, the state q_4 is distinguished from the rest. The refined partition is shown:

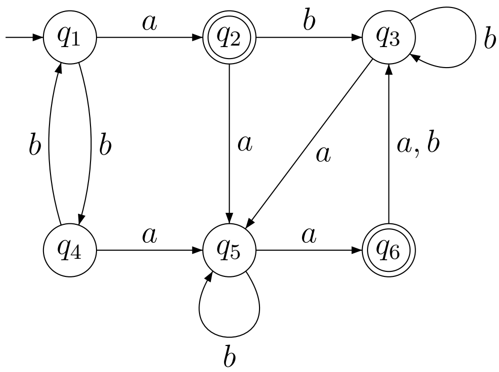
$$\pi_3 = \{\{q_1, q_5\}, \{q_2\}, \{q_3\}, \{q_4\}, \{q_6\}, \{q_7\}\}$$

		a	b
B_1	q_1	B_2	B_3
	q_5	B_2	B_3
B_2	q_2	$--$	$--$
B_3	q_3	$--$	$--$
B_4	q_4	$--$	$--$
B_6	q_6	$--$	$--$
B_7	q_7	$--$	$--$

This partition is not further refined. The minimal DFA is shown below:



(c)



Answer:

First state partition distinguish between final and non-final states:

$$\pi_0 = \{\{q_1, q_3, q_4, q_5\}, \{q_2, q_6\}\}$$

Taking into account this first partition:

		a	b
B_1	q_1	B_2	B_1
	q_3	B_1	B_1
	q_4	B_1	B_1
	q_5	B_2	B_1
B_2	q_2	B_1	B_1
	q_6	B_1	B_1

Note that states q_1 and q_5 behave differently than states q_3 and q_4, therefore, the partition is refined:

$$\pi_1 = \{\{q_1, q_5\}, \{q_3, q_4\}, \{q_2, q_6\}\}$$

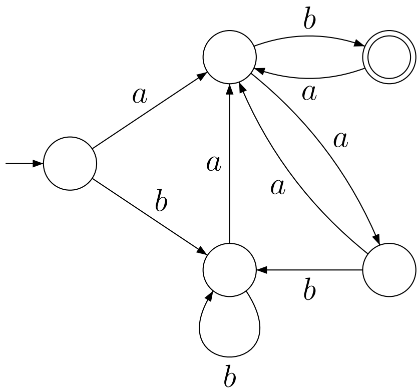
		a	b
B_1	q_1	B_2	B_3
	q_5	B_2	B_1
B_2	q_2	B_1	B_3
	q_6	B_3	B_3
B_3	q_3	B_1	B_3
	q_4	B_1	B_1

All blocks are refined and the resulting partition is:

$$\pi_1 = \{\{q_1\}, \{q_2\}, \{q_3\}, \{q_4\}, \{q_5\}, \{q_6\}\}$$

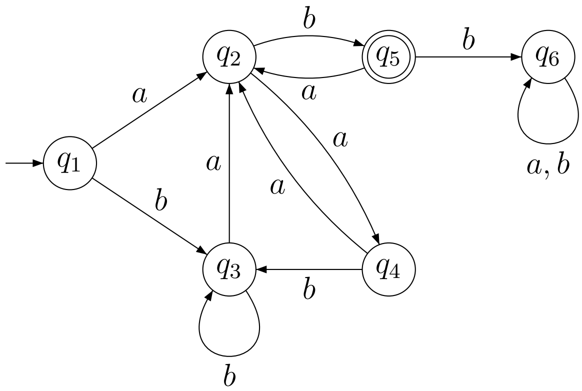
that cannot be refined again. Therefore, the automaton was already minimal.

(d)



Answer:

The complete automaton is shown below:



First state partition distinguish between final and non-final states:

$$\pi_0 = \{\{q_1, q_2, q_3, q_4, q_6\}, \{q_5\}\}$$

Taking into account this first partition:

		a	b
B_1	q_1	B_1	B_1
	q_2	B_1	B_5
	q_3	B_1	B_1
	q_4	B_1	B_1
	q_6	B_1	B_1
B_5	q_5	$--$	$--$

A new block with state q_2 is created. The resulting partition is shown:

$$\pi_1 = \{\{q_1, q_3, q_4, q_6\}, \{q_2\}, \{q_5\}\}$$

		a	b
B_1	q_1	B_2	B_1
	q_3	B_2	B_1
	q_4	B_2	B_1
	q_6	B_1	B_1
B_2	q_2	$--$	$--$
B_5	q_5	$--$	$--$

State q_6 is distinguished and the partition is:

$$\pi_2 = \{\{q_1, q_3, q_4\}, \{q_2\}, \{q_5\}, \{q_6\}\}$$

		a	b
B_1	q_1	B_2	B_1
	q_3	B_2	B_1
	q_4	B_2	B_1
B_2	q_2	$--$	$--$
B_5	q_5	$--$	$--$
B_6	q_6	$--$	$--$

In this iteration the partition is not refined. The minimal DFA for the language is shown below: