Exercises

Exercise 1

Taking into account the following words over $\{0,1\}$:

$$x = 00011$$
$$y = 011000$$
$$z = 01010$$

compute the following operations:

(a) $|x|,\ |y|_1$

(b) x^R, y^R, z^R

(c) $xy,\ yz,\ z^2$

(d) $Pref(x),\ Suf(y),\ Seg(z),\ Pref(Pref(x)),\ Pref(Suf(z))$

Exercise 2

Taking into account the following languages over $\{0,1\}$:

$$L_1 = \{0, 01, 001\}$$
$$L_2 = \{\lambda, 01, 0011\}$$
$$L_3 = \{0x \ :\ x \in \{0,1\}^*\}$$
$$L_4 = \{x0 \ :\ x \in \{0,1\}^*\}$$
$$L_5 = \{x \in \{0,1\}^* \ :\ |x|_0 = |x|_1\}$$

(a) Enumerate the first 10 words of L_3 in canonical order

(b) Enumerate the first 10 words of L_4 in canonical order

(c) Enumerate the first 10 words of L_5 in canonical order

Exercise 3

Taking into account the languages described in Exercise 2, give a description of the languages output by the following operations:

(a) $L_1 \cup L_2, L_1 \cup L_3, L_2 \cup L_3, L_3 \cup L_4$

(b) $L_1 \cap L_2, L_1 \cap L_3, L_1 \cap L_4, L_2 \cap L_4, L_3 \cap L_4$

(c) $\overline{L_3},\ \overline{L_5}$

(d) $L_1 - L_2, L_2 - L_3, L_2 - L_4, L_3 - L_4$

(e) $L_1 \triangle L_2, L_1 \triangle L_3, L_3 \triangle L_4$

(f) $L_1L_2, L_4L_3, L_2L_3, L_3L_4, L_1^2, L_5^2, L_2^3, L_3^5$

(g) $L_1^*, L_4^*, L_1^+, L_3^+, L_5^*$

(h) L_2^R, L_3^R, L_5^R

(i) $Pref(L_1), Pref(L_4), Pref(L_3), Seg(L_1), Seg(L_4), Suf(L_2)$

(j) $0^{-1}L_1, 0^{-1}L_2, 0^{-1}L_3, 0^{-1}L_4, 1^{-1}L_1, 1^{-1}L_3, 1^{-1}L_4, (01)^{-1}L_1$

(k) $(01)^{-1}L_3, (01)^{-1}L_4$

Note that the languages L_3 and L_4 can be expressed as:

$$L_3 = \{0\}\{0,1\}^*$$
$$L_4 = \{0,1\}^*\{0\}$$

Hint: Consider the properties of the right quotient

Exercise 4

Consider the languages described in Exercise 2 and the following homomorphism:

$$h:\{0,1\} \to \{a,b,c\}^* \quad \begin{cases} h(0) = a \\ h(1) = bc \end{cases}$$

$$g:\{a,b,c\} \to \{0,1\}^* \quad \begin{cases} g(a) = 01 \\ g(b) = 10 \\ g(c) = \lambda \end{cases}$$

$$f:\{0,1\} \to \{0,1\}^* \quad \begin{cases} f(0) = 0 \\ f(1) = 011 \end{cases}$$

Give a description of the languages output by the following operations:

(a) $h(L_1), h(L_2), h(L_3), h(L_4)$

(b) $g^{-1}(L_1), g^{-1}(L_2), g^{-1}(L_3), g^{-1}(L_4)$

(c) $f(L_1), f(L_2), f(L_3), f^{-1}(L_1), f^{-1}(L_2), f^{-1}(L_3), f^{-1}(L_4)$