Chapter 1

Image Denoising

Given a noisy binary image Y with pixel values $y_i \in \{-1, +1\}$. The goal is to recover the original image X using the information given by the noisy image. The prior knowledge can be captured using Markov random field as shown in figure 1.1.

A distribution P is a log-linear model over a Markov network $\mathcal{H}$ if is associated with:

- a set of features $\mathcal{F} = \{f_1(D_1), ..., f_k(D_k)\}$, where each D_i is a clique in H.
- a set of weights $w_1, ..., w_k$, such that

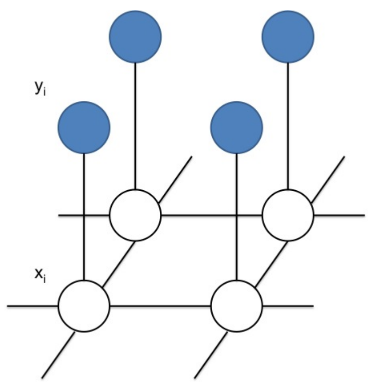

Figure 1.1: Markov rando field for image denoising, where x_i represent the pixels of the noise free image, and y_i the corresponding pixel in the noisy image.

$$P(X_1, ..., X_n) = \frac{1}{Z} \exp \left[-\sum_{i=1}^{k} w_i f_i(D_i) \right]$$

where $\sum_{i=1}^{k} w_i f_i(D_i)$ is the energy function. For the image denoising example, the cliques of the graph are given by the connections between each pixel x_i and its neighbors and by the connection of pixel x_i and the corresponding pixel y_i in the noisy image. According to this, the energy function of our model can be expressed by:

$$E(x, y) = h \sum_{ij} x_{ij} - \beta \sum_{i,j \in Nb(i)} x_i x_j - \eta \sum_i x_i y_i$$

where $Nb(i)$ are the indexes of the pixels that are neighbors of pixel i. Then, the distribution P for our model is given by

$$P(x, y) = \frac{1}{Z} \exp(-E(x, y))$$

Models where variables can have two states $\{-1, +1\}$ and interact with its neighbors are called *Ising models.* which have been widely studied in statistical physics as a model for the energy of a physical system involving a system of interacting atoms.

For the case of image restoration we want to obtain the model of minimum energy that give us the maximum probability. In order to achieve this we used an iterative algorithm called *"Iterative Conditional Modes"*. which is explained bellow.

1.1 Iterative Conditional Modes

The Iterative Conditional Modes (ICM) algorithm proposed by Kittler and Föglein in 1984. This algorithm tries to minimize the energy of each node individually in every iteration, when there is no change in the values of the nodes from one iteration to another we can say that the algorithm has converged to a local optimum.

Because neighboring pixels tend to have the same values, it is expected that there will be a strong correlation between them. If there a re single black pixel surrounded by white pixels, is highly probable that this is a flipped pixel. also there will be a strong correlation between each pair x_i, y_i, assuming there is a small level of noise.

Then, the ICM implementation is similar to the gradient descent method, only that ICM tries to minimize each node individually trying to minimize the energy of the model. Because it consists in a greedy behavior, the algorithm can converge to a local optimum, instead to the global one.

The energy of a single node in iteration k is given by:

$$E\left(x_i|y_i\right) = hx_i - \beta \sum_{j \in Nb(i)} x_i x_j^{(k)} - \eta x_i y_i$$

Thus, the new value of node x_i is obtained according to the following equation:

$$x_i^{(k+1)} = \arg \min_{x_i \in \{-1,+1\}} E(x_i|y_i)$$

When a pixel is flipped the term $-\eta x_i y_i$ will be positive, increasing the energy of that pixel. Then, why change it? The answer is given by the term $-\beta \sum_{j \in Nb(i)} x_i x_j^{(k)}$ if this term makes the energy to decrease when the pixel is flipped, then the value of that pixel is changed.

1.2 Results

The following results were obtained using $h = 0, \beta = 1.0$ and $\eta = 2.1$. Each image was corrupted changing 10% of the pixels. The noisy images are shown in column on the left and the resulting images after applying the ICM algorithm in column on the right. The initial approximation for each pixel was $x_i = y_i$.

The noise used for testing is a "salt & pepper" noise, created just by flipping the 10% of the pixels. The images shown bellow is the result after 5 iterations of the ICM algorithm, again, because the behavior is similar to the gradient descent, in few iterations the algorithm reaches a local optimum.

Figure 1.2: ICM for image restoration where 99.75% of the pixels agree with the orginal image.

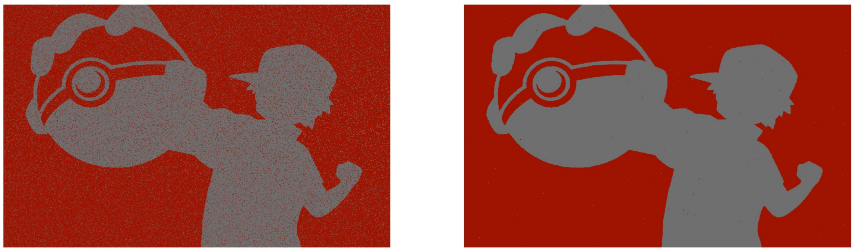

Figure 1.3: ICM for image restoration where 99.85% of the pixels agree with the orginal image.

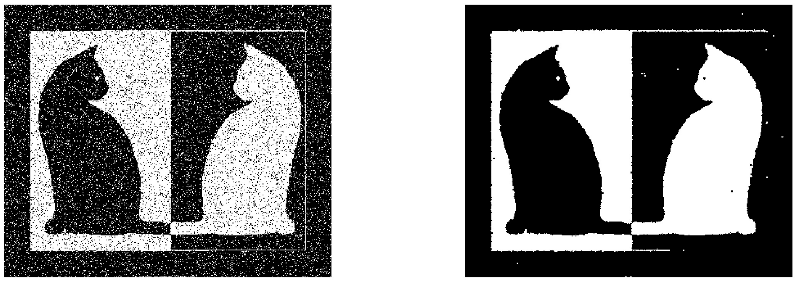

Figure 1.4: ICM for image restoration where 99.26% of the pixels agree with the orginal image.

Figure 1.5: ICM for image restoration where 91.77% of the pixels agree with the orginal image.