NON-PARAMETRIC BAYESIAN METHOD FOR IMAGE REGION SEGMENTATION

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Image Segmentation

- Image segmentation refers to the act of grouping pixels that share certain characteristics like color, intensity, texture, etc.
- The main goal is to be able to identify different objects in an image.
- Each region represent an object or part of an object.

Image Segmentation

- Image segmentation can be done in different ways, two of the most popular are:
  - Region Segmentation
  - Edge Detection

- The method proposed is a region segmentation method, but part of the research was to implement classic region segmentation algorithms and edge detection algorithms and compare results.

Image Segmentation

- The proposed method consists in three steps:
  - **Clustering step:** Associate pixels to a specific cluster based on their color.
  - **Split step:** Separate group of pixels which are connected and associated to the same cluster.
  - **Merge step:** Eliminate those objects composed of a number of pixels bellow of a threshold value and associate those pixels to the nearest object.

Clustering Step

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Clustering Step

- Associate each pixel to a specific cluster.
- Each cluster represents a Gaussian distribution with parameters μ_k and Δ_k.
- Also, each Gaussian as a mixing proportion parameter π_k associated to it.

Clustering Step

- The probability density function is given by

$$f(x_i|\theta) = \sum_{j=1}^{N} \pi_j \mathcal{N}(x_i|\mu_j, \Delta_j^{-1})$$

- If we consider that each observation (pixel) is generated by only one Gaussian function, then, the probability density function can be written as

$$f(x_i|\theta) = \prod_{j=1}^{N} \left[\pi_j \mathcal{N}(x_i|\mu_j, \Delta_j^{-1})\right]^{c_{ij}}$$

where $c_{ij} = 1$ if pixel i is associated to cluster j, otherwise $c_{ij} = 0$.

Clustering Step

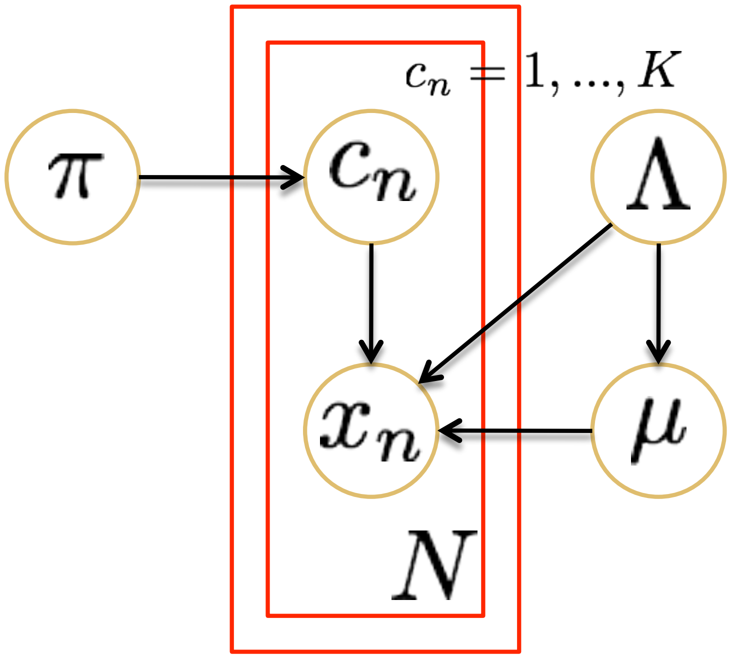
- The problem consists in estimate the values of the parameters for each Gaussian.

- One way to estimate the parameters in each iteration is using Gibbs sampling.

- It is required the conditional distribution of each parameter given the values of the other parameters.

Clustering Step

- The dependencies between parameters are illustrated with the following graphical model (Finite model)



$$P(X, C, \pi, \mu, \Lambda) = p(X|C, \mu, \Lambda)p(\mu|\Lambda)p(\Lambda)p(C|\pi)p(\pi)$$

Clustering Step

- In order to obtain the conditional distributions of each parameter, we need to define prior distributions for each parameter as well, given by:

$$p(c_{ij}) \sim Mult(\pi_1, ..., \pi_K)$$

$$p(\pi) \sim Dir(\alpha_0) \sim Dir(\alpha_{0_1}, ..., \alpha_{0_K})$$

$$p(\mu_j, \Lambda_j) \sim \mathcal{N}(\mu_j | m_0, (\beta_0 \Lambda_j)^{-1}) \mathcal{W}(\Lambda_j | W_0, v_0)$$

The values of m_0, α_0, β_0, W_0, v_0 are called hyperparameters and must be initialized before the sampling.

Clustering Step

- According to our graphical model, the posterior distributions are given by

$$q(C) \sim p(X|C, \mu, \Sigma)p(C|\pi)$$

$$q(\pi) \sim p(C|\pi)p(\pi)$$

$$q(\mu) \sim p(X|C, \mu, \Lambda)p(\mu|\Lambda)$$

$$q(\Lambda) \sim p(X|C, \mu, \Lambda)p(\Lambda)$$

Clustering Step

- According to our graph model, the posterior distributions are given by

$$q(C) \sim p(X|C, \mu, \Sigma)\boxed{p(C|\pi)} \leftarrow \text{Multinomial}$$

$$q(\pi) \sim p(C|\pi)\boxed{p(\pi)} \leftarrow \text{Dirichlet}$$

$$q(\mu) \sim p(X|C, \mu, \Lambda)\boxed{p(\mu|\Lambda)} \leftarrow \text{Normal}$$

$$q(\Lambda) \sim p(X|C, \mu, \Lambda)\boxed{p(\Lambda)} \leftarrow \text{Wishart}$$

Clustering Step

- The posterior distribution of the mixing proportions is a Dirichlet distribution with concentration parameter α /K.

$$q(\pi) \propto Dir(N_1 + \alpha/K, N_2 + \alpha/K, ..., N_k + \alpha/K)$$

where

$$\mathbb{E}[\pi_j] = \frac{N_j + \alpha/K}{N + \alpha}$$

- The prior distribution over the latent variables is a multinomial, then we have

$$p(c_{ij} = 1) = \pi_j$$

Clustering Step

- If that is true, then

$$p(c_{ij} = 1) = \frac{N_j + \alpha/K}{N + \alpha}$$

- In order to use Gibbs sampling, we need the conditional prior for a single indicator given all others.

Clustering Step

- If that is true, then

$$p(c_{ij} = 1) = \cancel{\frac{N_j \cdots K}{\cdots}}$$

- In order to use Gibbs sampling, we need the conditional prior for a single indicator given all others.

Clustering Step

- If that is true, then

$$p(c_{ij} = 1) = \frac{N_j \,[\text{illegible}]\, K}{[\text{illegible}]}$$

- In order to use Gibbs sampling, we need the conditional prior for a single indicator given all others.

- We have to remove the observation corresponding to the current indicator from the data set, then the prior is given by

Clustering Step

- If that is true, then

$$p(c_{ij} = 1) = \cancel{\frac{N_j \cdots K}{\cdots}}$$

- In order to use Gibbs sampling, we need the conditional prior for a single indicator given all others.

- We have to remove the observation corresponding to the current indicator from the data set, then the prior is given by

$$p(c_{ij} = 1) = \frac{N_{-i,j} + \alpha/K}{N - 1 + \alpha}$$

Clustering Step

- If we take $K \to \infty$ the prior distribution of the latent variables is defined by

$$p(c_{ij} = 1) = \begin{cases} \frac{N_{-i,j}}{N-1+\alpha} & \text{components where } N_{-i,j} > 0 \\ \frac{\alpha}{N-1+\alpha} & \text{all other components combined} \end{cases}$$

- The prior probability of certain cluster is proportional to the number of observations associated to it.

- The probability to assign certain observation to a new cluster is proportional to the concentration parameter.

Clustering Step

- For the components that has associated at least one observation is easy to see the last statement, because the probability for such components is given by

$$p(c_{ij} = 1) = \lim_{K \to \infty} \frac{N_{-i,j} + \alpha/K}{N - 1 + \alpha}$$
$$= \frac{N_{-i,j}}{N - 1 + \alpha}$$

- For the rest of the components that have zero observations associated to it, is no so easy to see, but is not also difficult.

- Suppose there are L components with at least one observation associated to it.

Clustering Step

- Let Q be the set of components with zero observations associated to them, the size of Q is given by

$$|Q| = K - L$$

- The probability for an observation to be associated to one component of set Q is expressed by

$$\begin{aligned}
p(c_{iQ} = 1) &= \sum_{k=1}^{|Q|} \lim_{K \to \infty} \frac{\alpha / K}{N - 1 + \alpha} \\
&= \frac{\alpha}{N - 1 + \alpha} \lim_{K \to \infty} \frac{|Q|}{K} \\
&= \frac{\alpha}{N - 1 + \alpha} \lim_{K \to \infty} \frac{K - L}{K} \\
&= \frac{\alpha}{N - 1 + \alpha}
\end{aligned}$$

Clustering Step: Illustration

- To illustrate the clustering step, we used a set of 300 observations generating three identifiable clusters.

- The hyperparameters used for this example are listed bellow

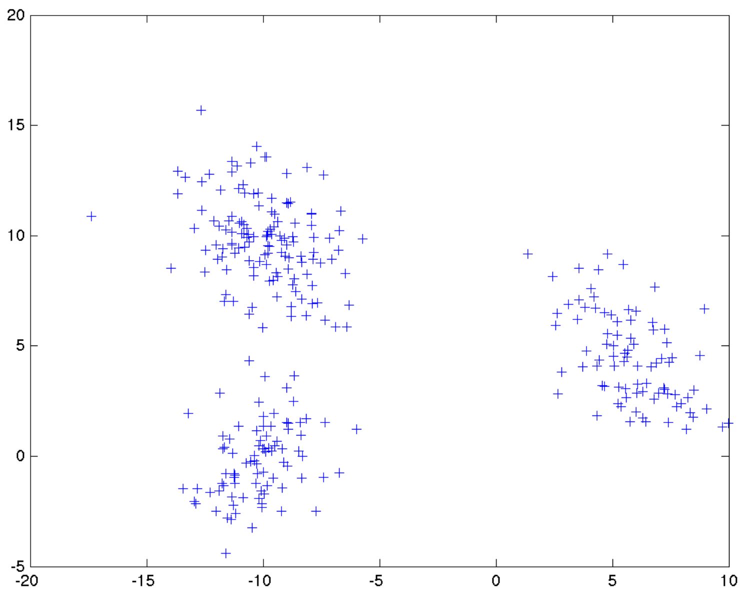
$$\alpha = 3.5$$

$$W_0 = 0.01 * d * I$$

$$v_0 = d$$

$$m_0 = \overline{x}$$

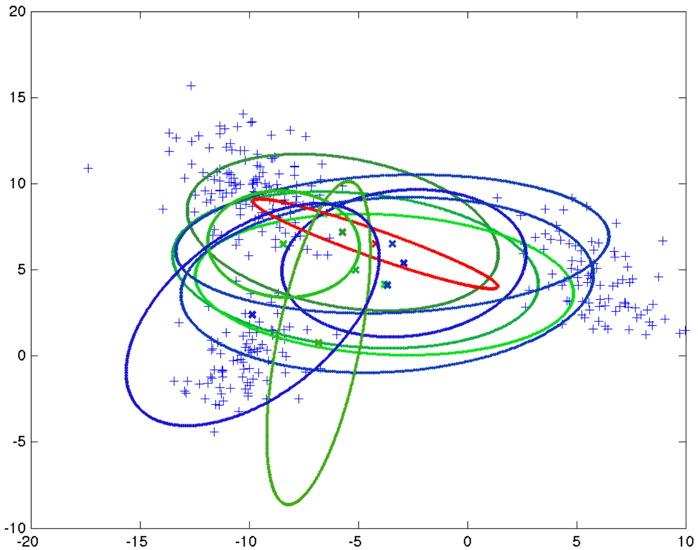
$$\beta_0 = 1$$



Data set with 300 observations

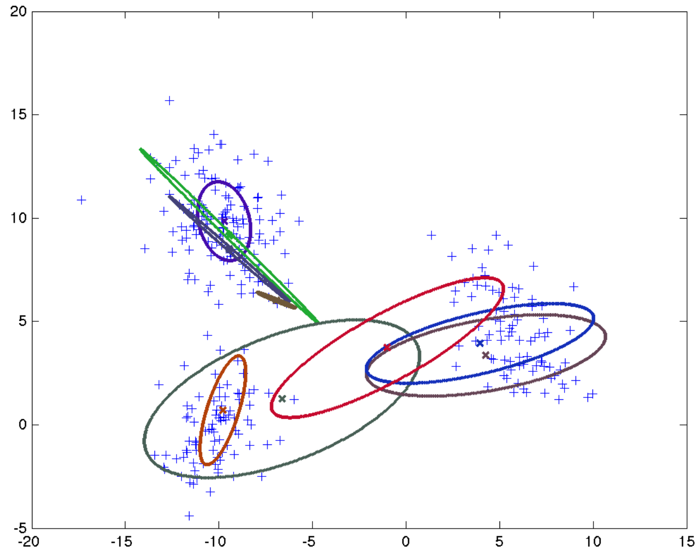
Clustering Step: Illustration

After 1 iteration



9 clusters

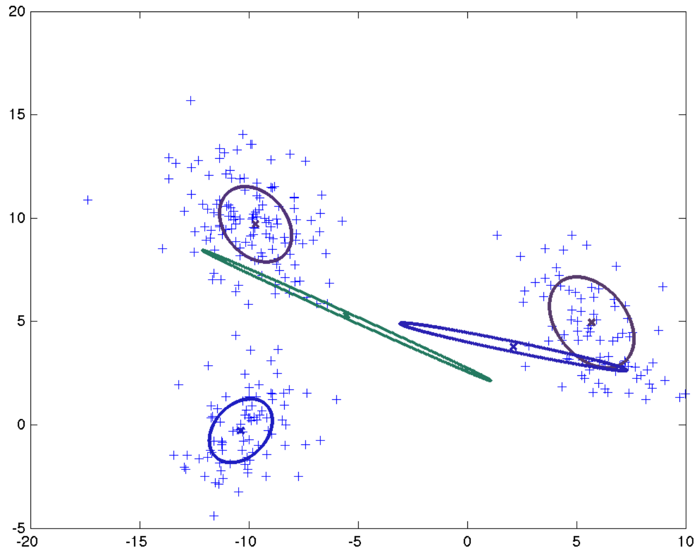
After 5 iterations



9 clusters

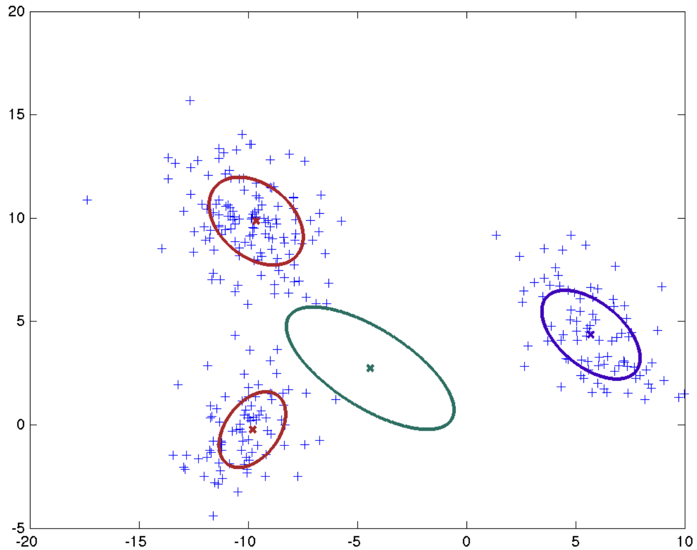
Clustering Step: Illustration

After 25 iterations



6 clusters

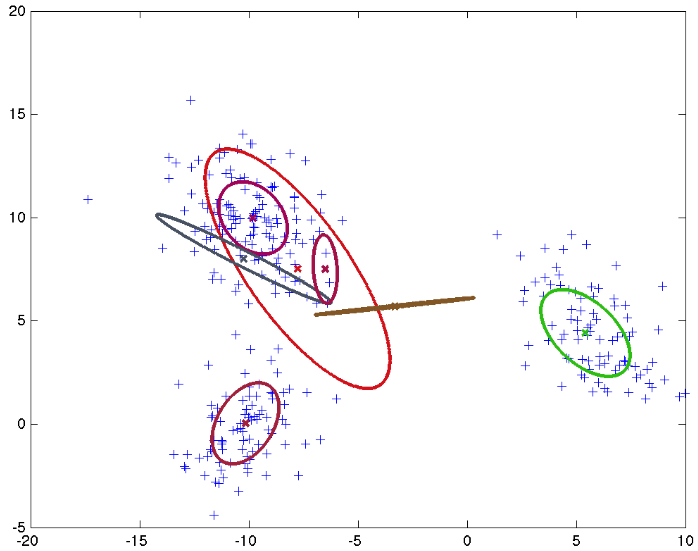
After 50 iterations



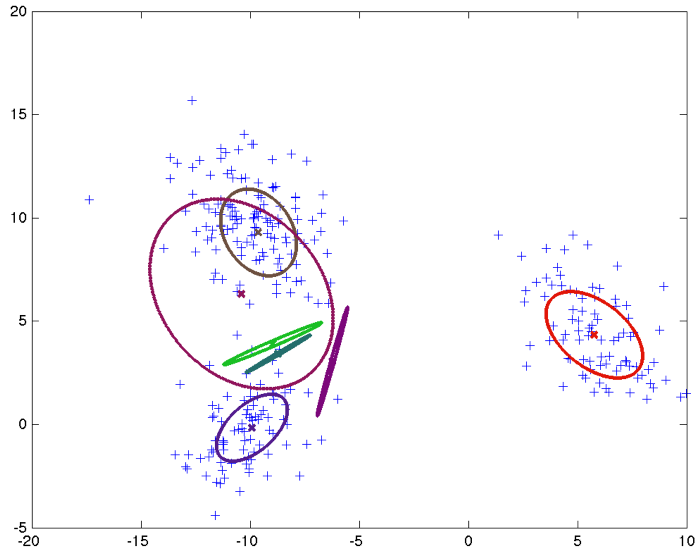
5 clusters

Clustering Step: Illustration

After 100 iterations	After 150 iterations



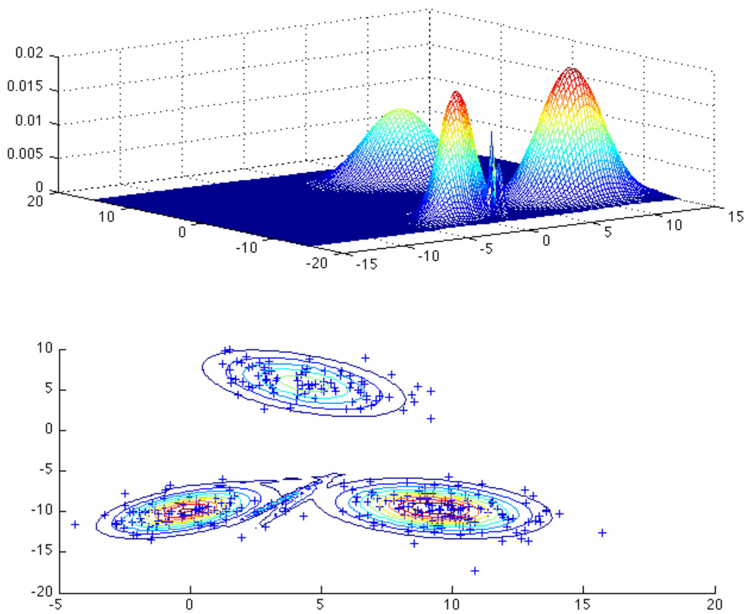
6 clusters



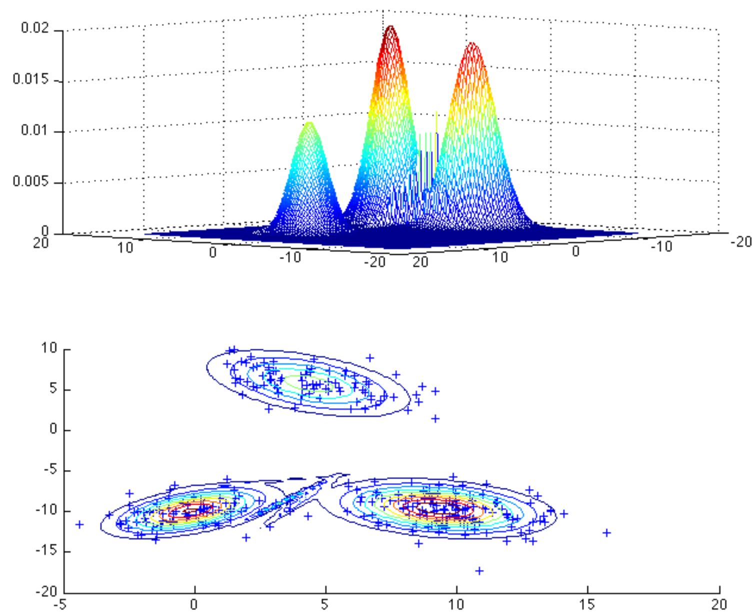
6 clusters

Clustering Step: Illustration

PDF after 150 observations



PDF after 150 observations



Probability density function using a non-parametric Bayesian mixture model wit concentration parameter α =3.5 for a set of 300 observations.

Clustering Step

- What happens if we change the value of the concentration parameter?

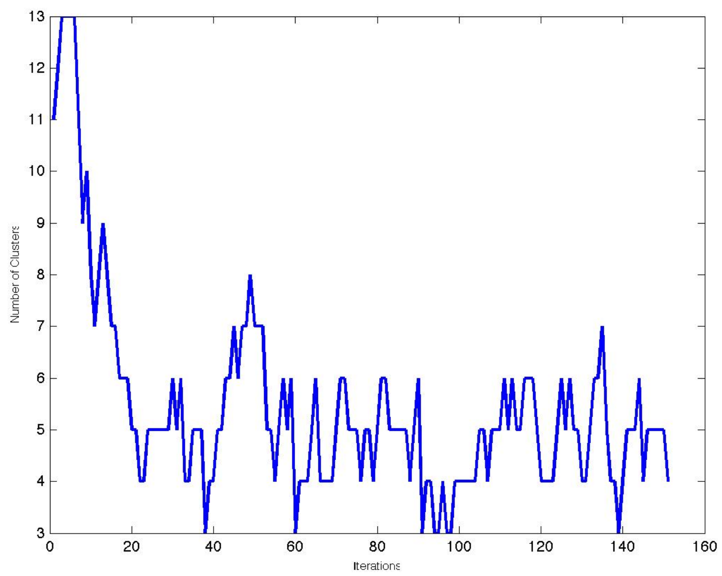
  - If we increase the concentration parameter it is expected that the number of observable components will be bigger, but at the end the number of components will oscillate around the expected value.

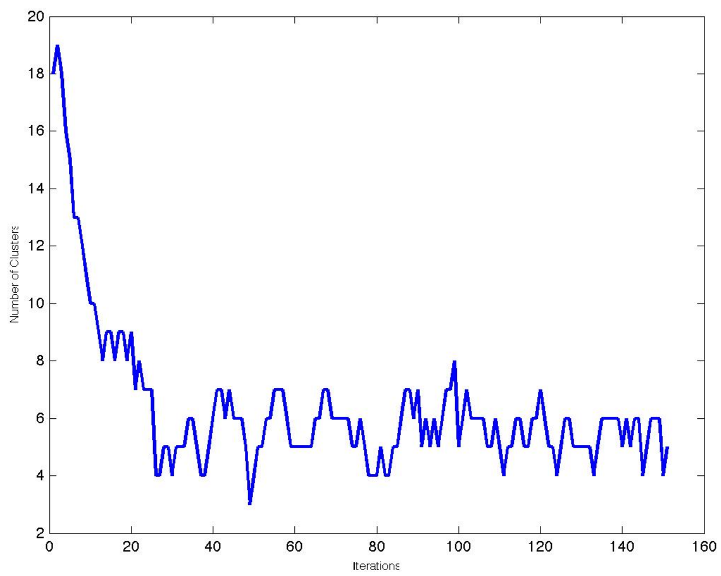
  - On the other hand, if we decrease the concentration parameter it is will be more difficult to create new components.

Clustering Step

With α = 3.5 and 150 iterations

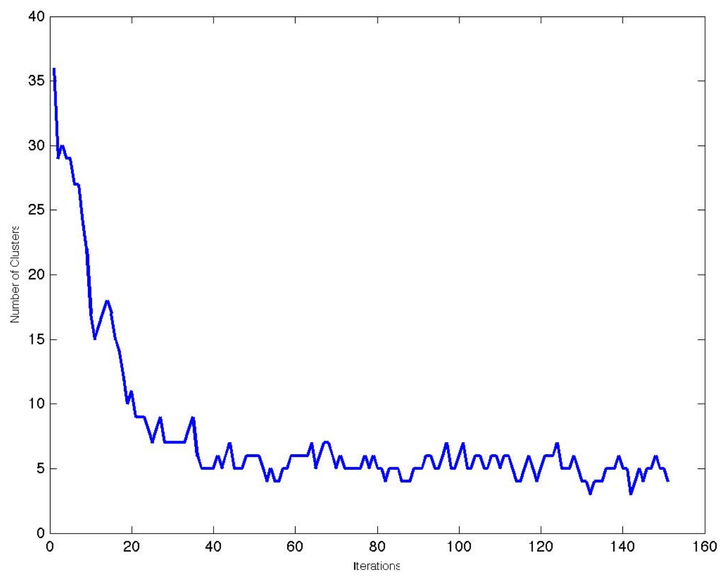
With α = 5 and 150 iterations



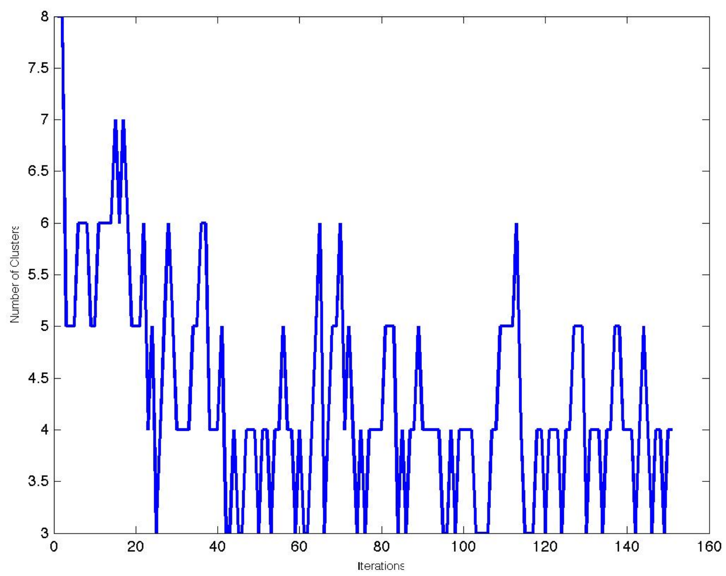


Clustering Step

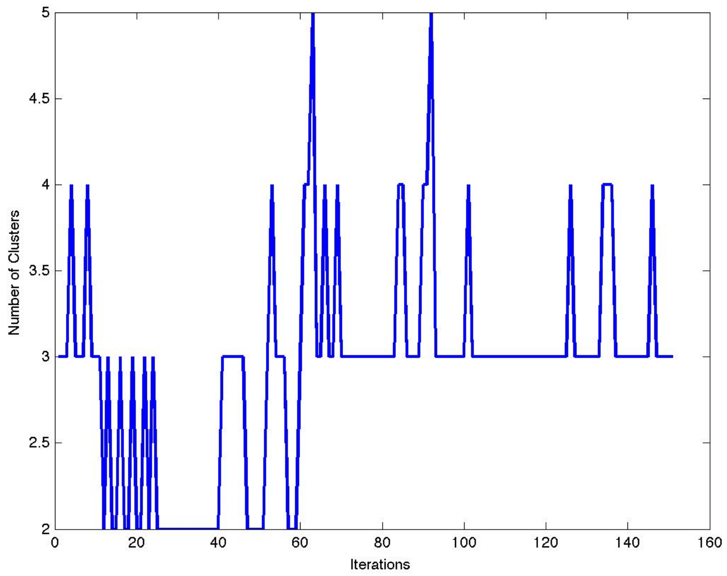
With α = 10 and 150 iterations



With α = 1.5 and 150 iterations



Clustering Step



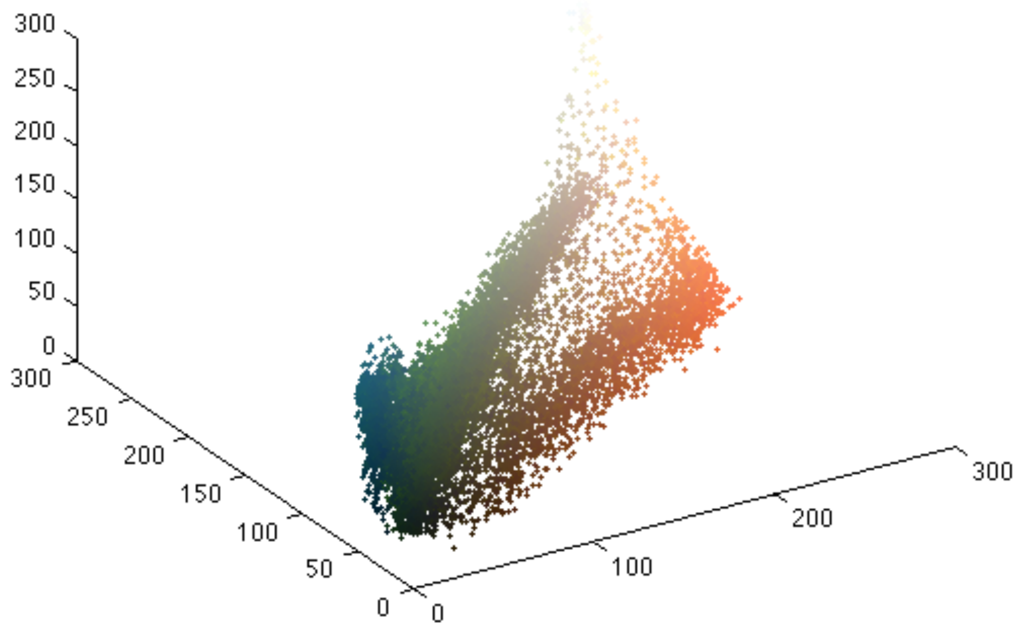
- There are a total of three identifiable clusters.
- Using $\alpha = 0.5$ The initial number of clusters is bellow three, but after around 60 iterations, the number of cluster established around three.

Clustering Step

- Using RGB values as points in a 3-dimensional space.
- Associate each point to the cluster which is more probably to belong to.
- There will be one cluster for red pixels, another for purple, another for yellow, etc.
- Try to associate pixels with similar color to the same cluster.

Clustering Step

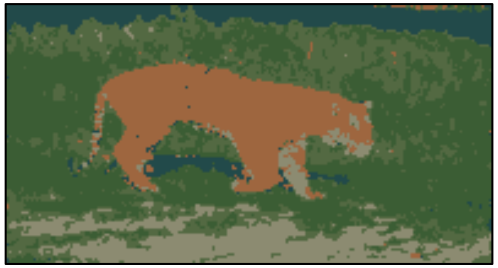
- Consider the following image



Representation of an image in the 3-dimensional space. In the left the original image showing a tiger in a nature environment. At right the representation of the image in a 3-dimesional space, where each dimension corresponds to a color level.

Clustering Step

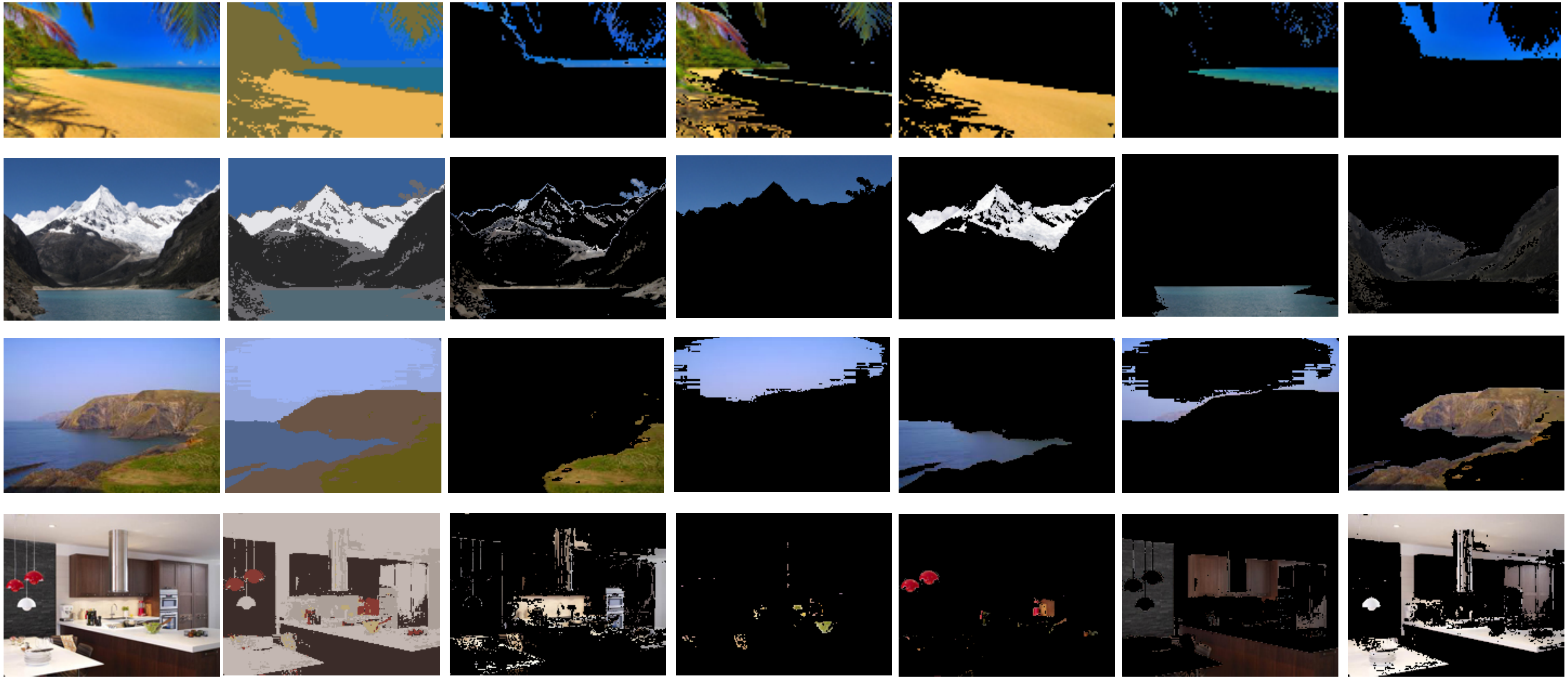
- After applying the clustering algorithm using 5 Gaussians we obtain



- Now we can identify easily the pixels that are associated to each cluster

Clustering Step

Following we show some examples of clustering using 5 Gaussian functions.



Clustering step using 5 Gaussian functions. At left the original image, then the segmented image showing the 5 clusters, finally 5 images displaying the pixels from each one of the clusters.

Split Step

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Split Step

- Isolate group of pixels that are connected and belong to the same cluster.

- Use a flood fill algorithm starting in any vertex to identify the region connected to that vertex.

- It can be used any graph search algorithm as DFS (Depth First Search) or BFS (Breadth First Search). The complexity of these algorithms is $O(V + E)$.

Split Step

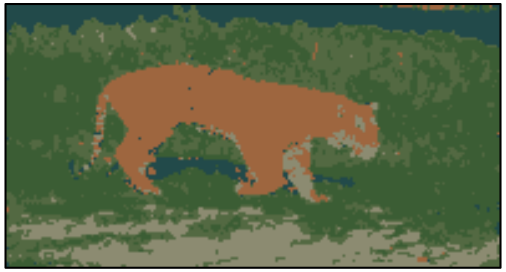
- For our case we used a BFS to implement the flood fill algorithm using just a queue.

- The flood fill algorithm compare the current pixel with each one of its 8 neighbors.

- If the neighbor pixel and the current pixel are associated to the same cluster, then they are part of the same object.

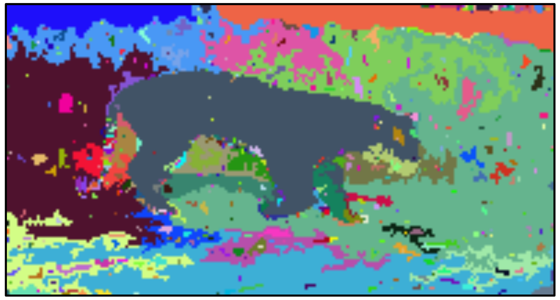
Split Step

- Using the following image

Original image at left. Resulting image after clustering step at right.

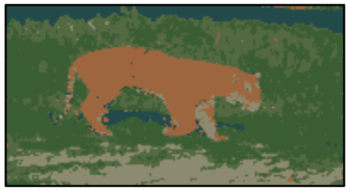
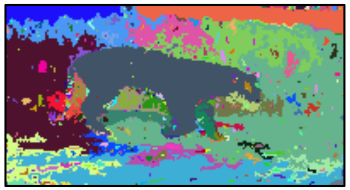
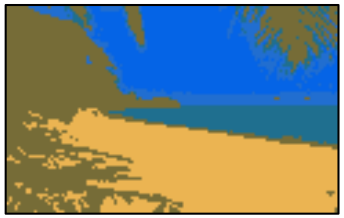
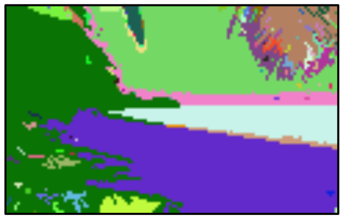
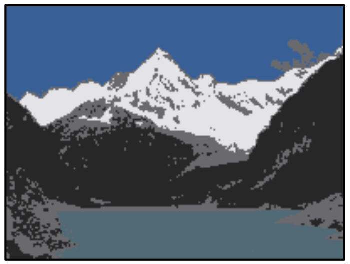
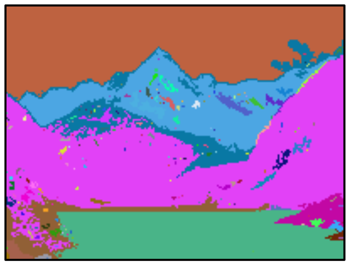
- After the split step, we get the image is compounded by 507 isolated areas or objects.



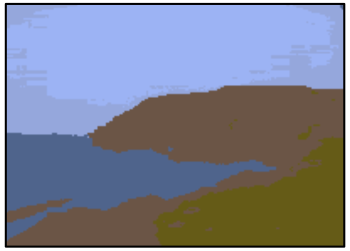
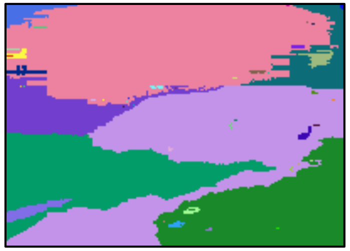
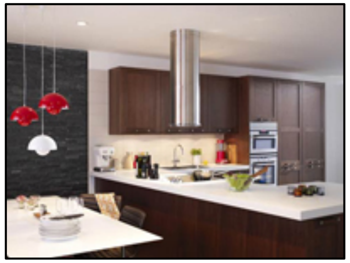
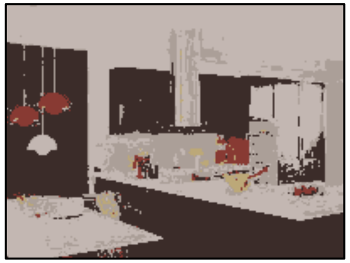
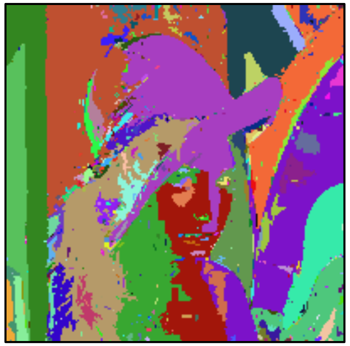
Resulting image after split step

Split Step

- Applying the split step in the same test images used before, we obtain

Original Image	**Clustering Step**	**Split Step**	**Objects**
			507
			128
			197

Split Step

Original Image	Clustering Step	Split Step	Objects
			48
			470
			718

Merge Step

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Merge Step

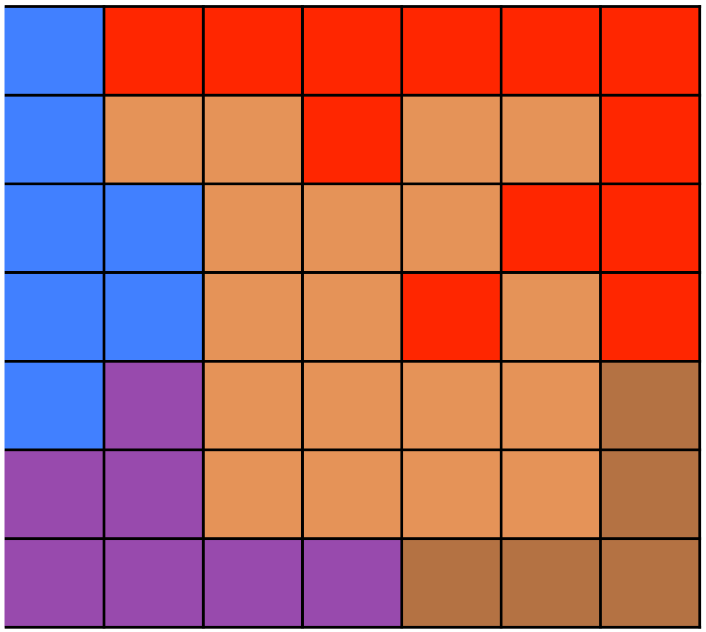
- Eliminate small objects.
- Those objects with a number of pixels bellow of certain threshold value are eliminated.
- The pixels of the removed objects are associated to the most similar neighbor not removed object.

Merge Step

- The objects are eliminated in the same way they were founded, using a flood fill algorithm.
- A simple graph search algorithm can be used to implement the flood fill.
- If an object has more than one not removed neighbor, the pixels from this object are associated to the neighbor with the most similar color.

Merge Step

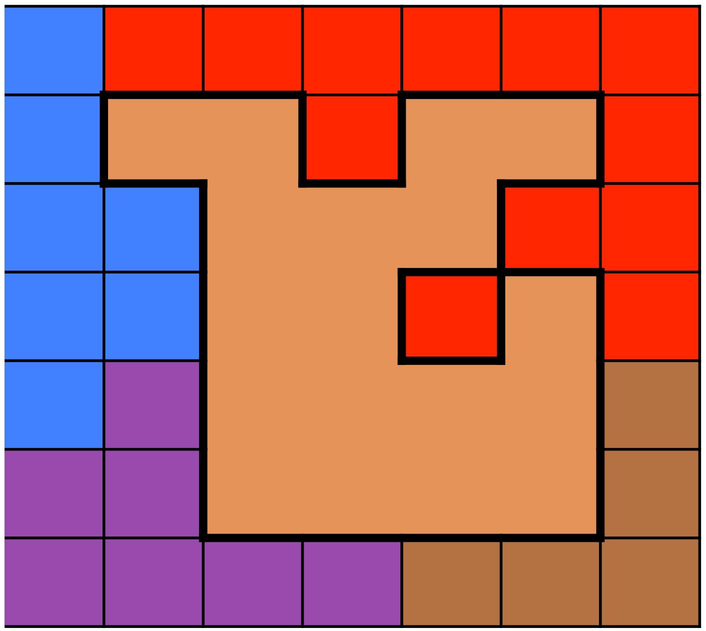
Consider the following portion of an image



Merge Step

Consider the following portion of an image

Identify the object to be removed

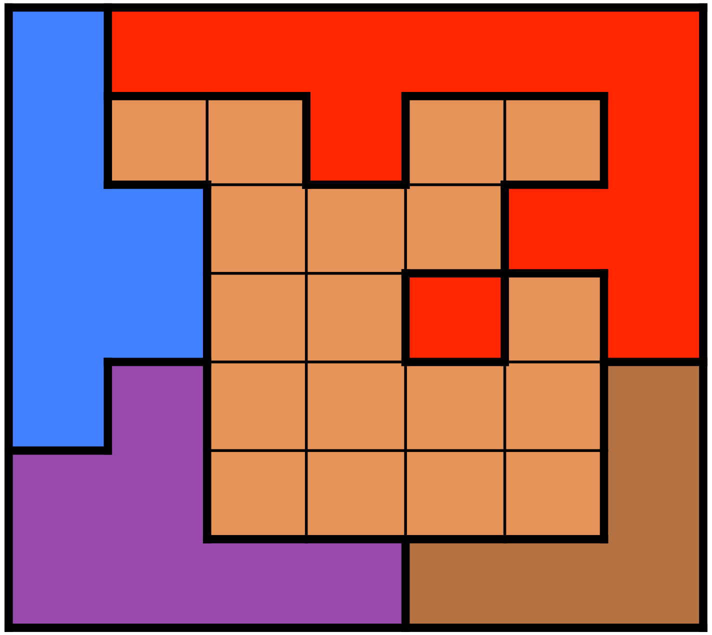


Merge Step

Consider the following portion of an image

Identify the object to be removed

Identify its neighbors

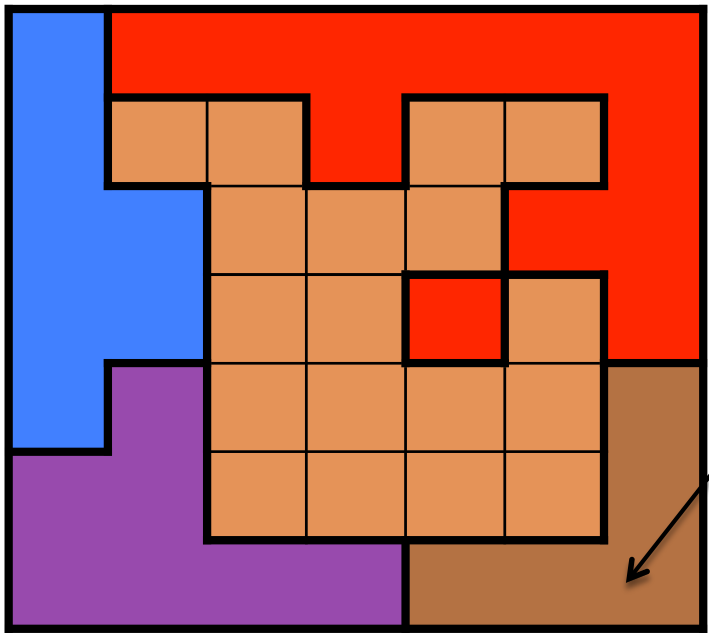


Merge Step

Consider the following portion of an image

Identify the object to be removed

Identify its neighbors



Neighbor with the most similar color

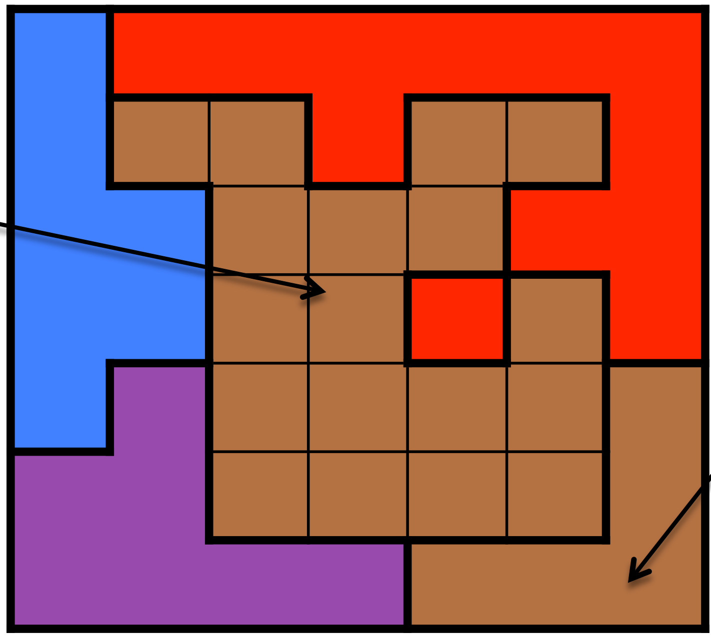
Merge Step

Consider the following portion of an image

Identify the object to be removed

Identify its neighbors

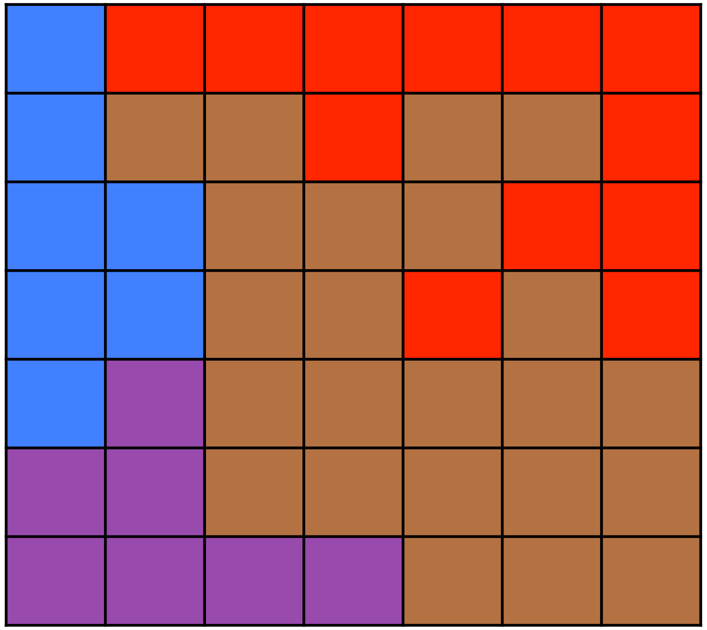
Pixels are associated to the object with the most similar color



Neighbor with the most similar color

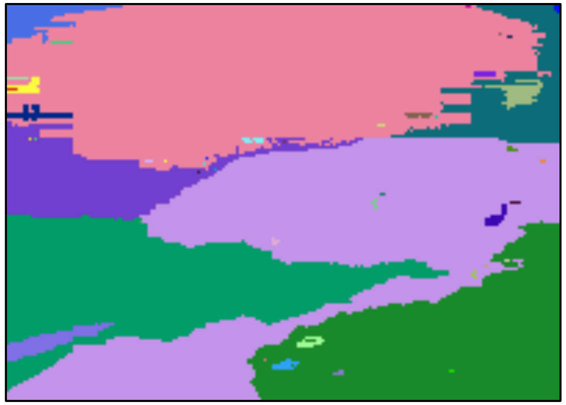
Merge Step

Resulting image!!



Merge Step

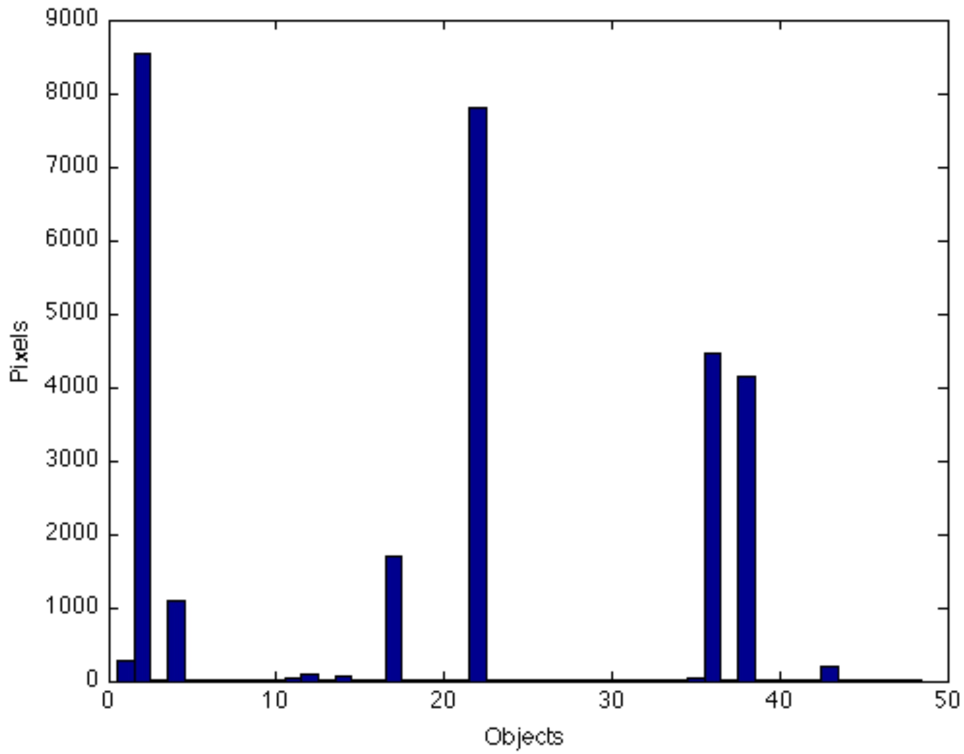
Consider the following image



In the left the original image, in the right the resulting image after the split step.

After the split step we have a total of 48 objects in the image, some of them bigger than others. Following is a histogram showing the number of pixels in each object.

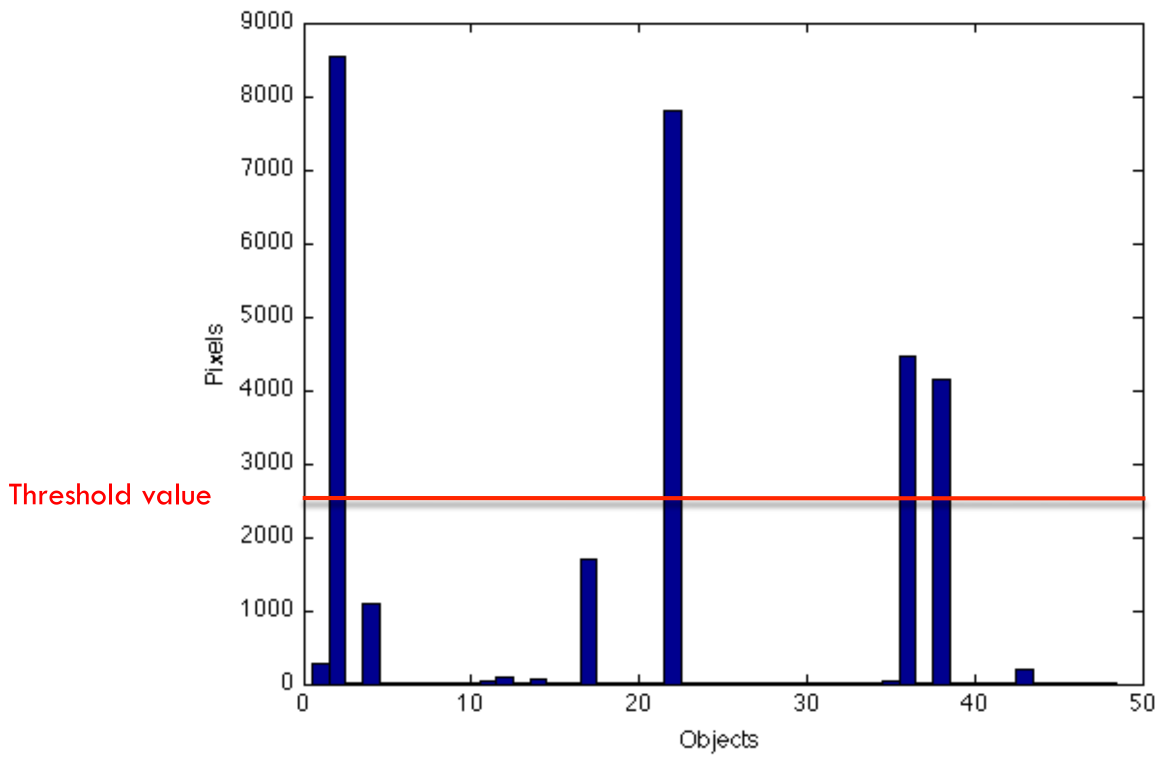
Merge Step



Histogram of objects vs. pixels, where objects occupying a big amount of pixels are clearly identified. The merge step add the pixels from small objects to bigger and near objects.

Merge Step

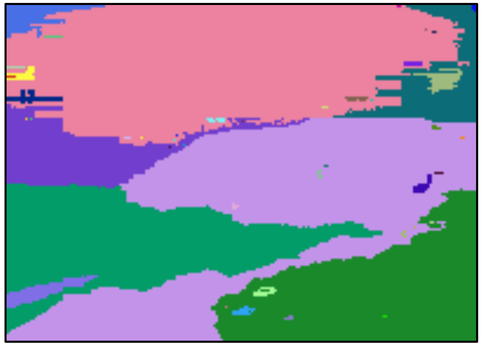
We can see the threshold value as a line, an those bins bellow that line are removed and their pixels pass to be part of bigger objects.



Objects with number of pixels bellow the threshold values are removed ant their pixels

Merge Step

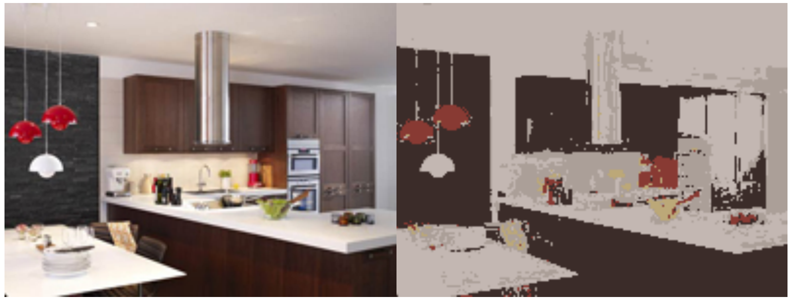
After applying the merge step with a threshold value TH = 100, we obtain

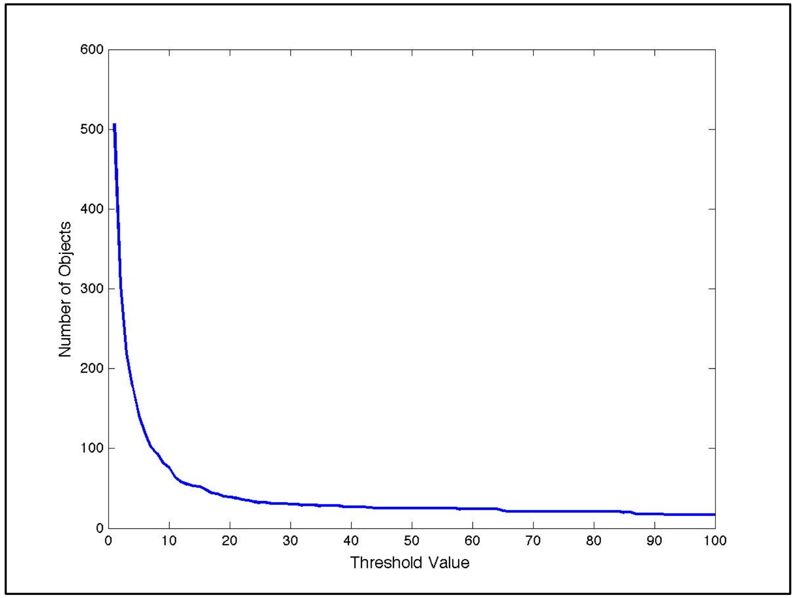
Original Image	**Split Step**	**Merge Step**	**Objects**
			8

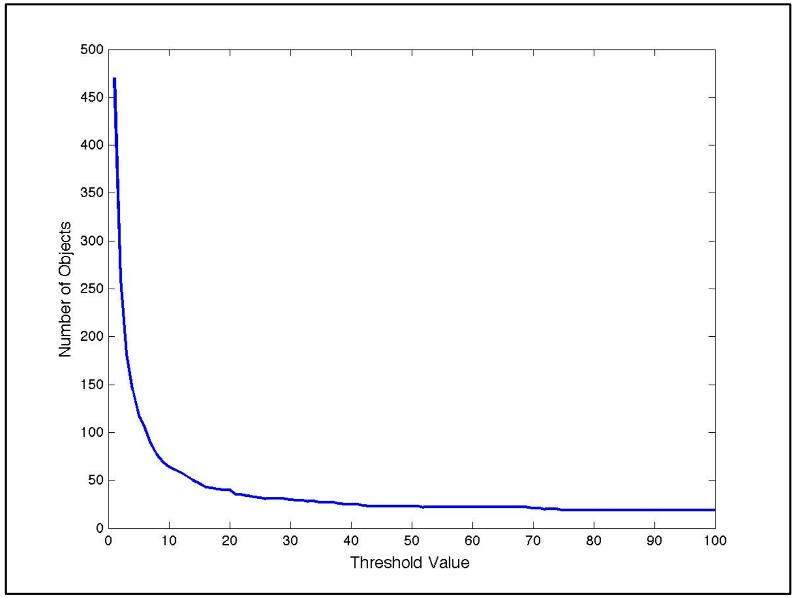
The image at the left is the original image, the second image is the result after applying the split step, the third image is the resulting image after the merge step, the last column indicate the number of objects founded after the merge step.

The number of object decrease depending on the threshold value, resulting in bigger and more easily identifiable regions

Merge Step

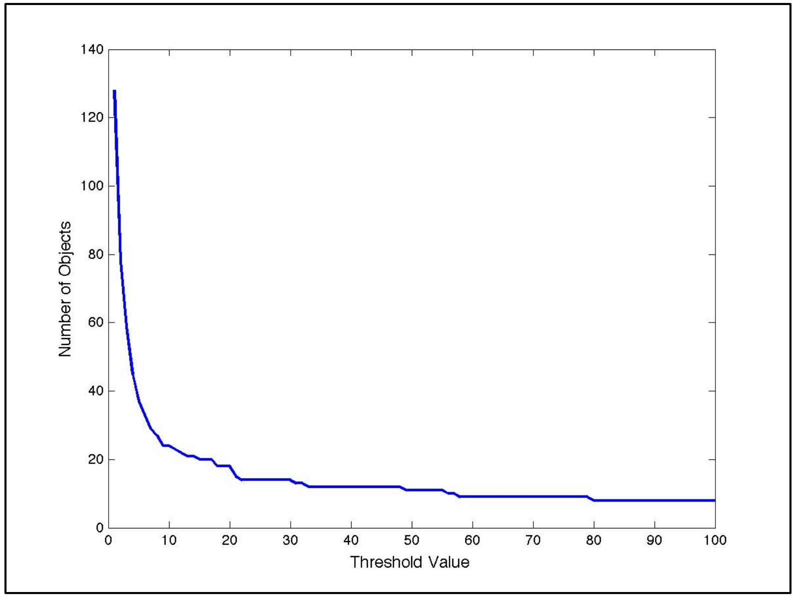


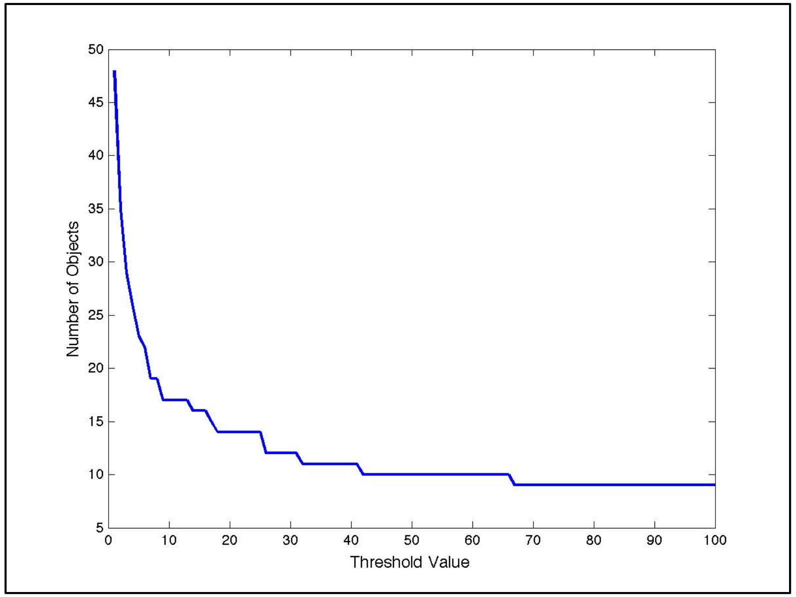




The image at the right corner represents the original image, in the left corner is the resulting image after the clustering step. The graph shows a relation between the threshold value and the number of objects after the merge step.

Merge Step





The image at the right corner represents the original image, in the left corner is the resulting image after the clustering step. The graph shows a relation between the threshold value and the number of objects after the merge step.

Results

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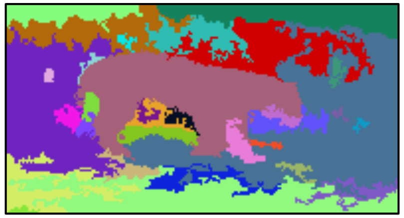
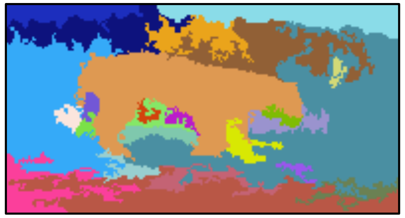
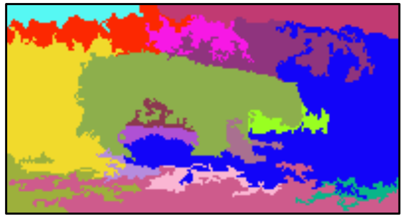
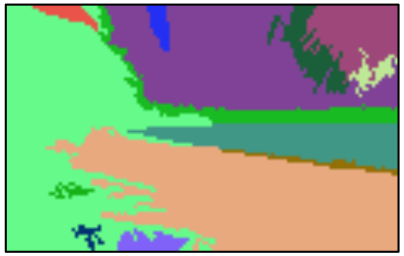
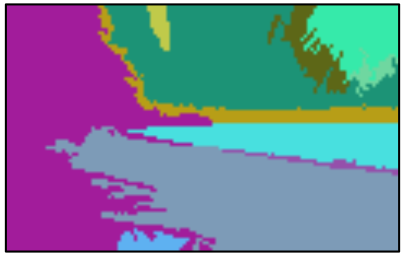
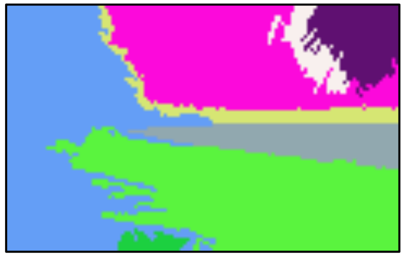
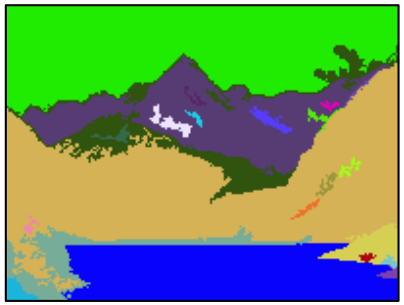
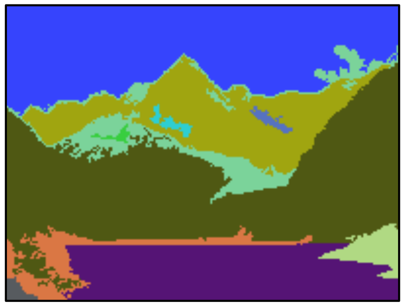
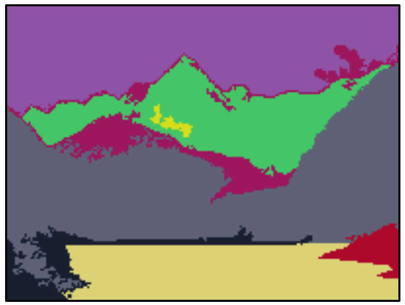
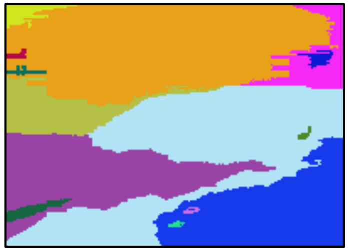
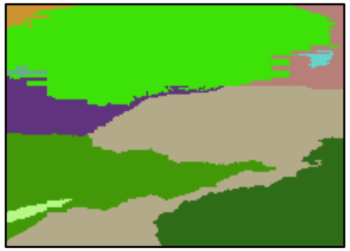
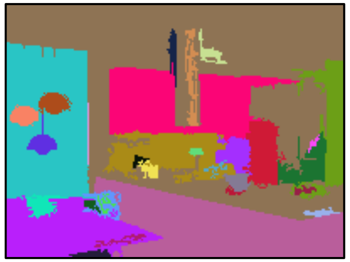
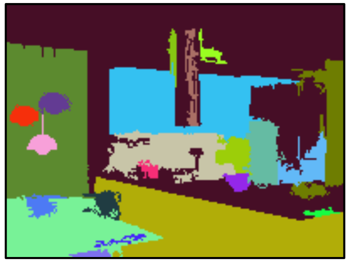
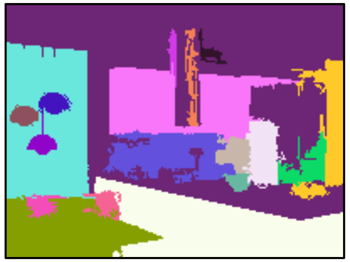
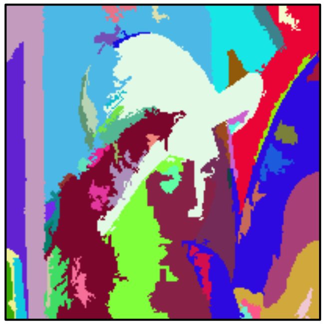
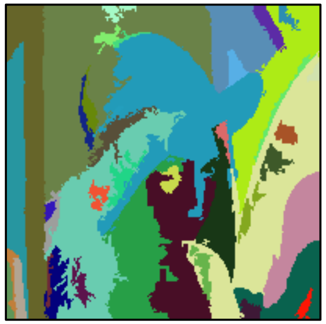
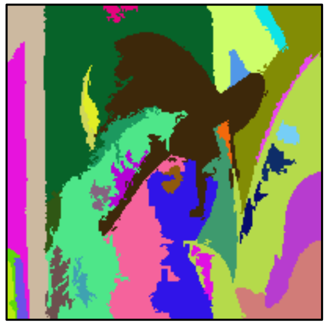
Results

Original Image	**After Merging (TH = 25)**	**After Merging (TH = 50)**	**After Merging (TH = 100)**
	 32/507 Objects (93.7% Red)	 25/507 Objects (95.0% Red)	 17/507 Objects (96.6% Red)
	 14/128 Objects (89% Red)	 11/128 Objects (91.4% Red)	 8/128 Objects (93.7% Red)
	 23/197 Objects (88.32% Red)	 11/197 Objects (94.4% Red)	 8/197 Objects (96% Red)

Results

Original Image	After Merging (TH = 25)	After Merging (TH = 50)	After Merging (TH = 100)
			
	14/48 Objects (70.8% Red)	10/48 Objects (79.2% Red)	9/48 Objects (81.25% Red)
			
	32/470 Objects (93.2% Red)	23/470 Objects (95.1% Red)	19/470 Objects (96.0% Red)
			
	55/718 Objects (92.3% Red)	43/718 Objects (94.0% Red)	34/718 Objects (95.3% Red)

Other Implemented Methods

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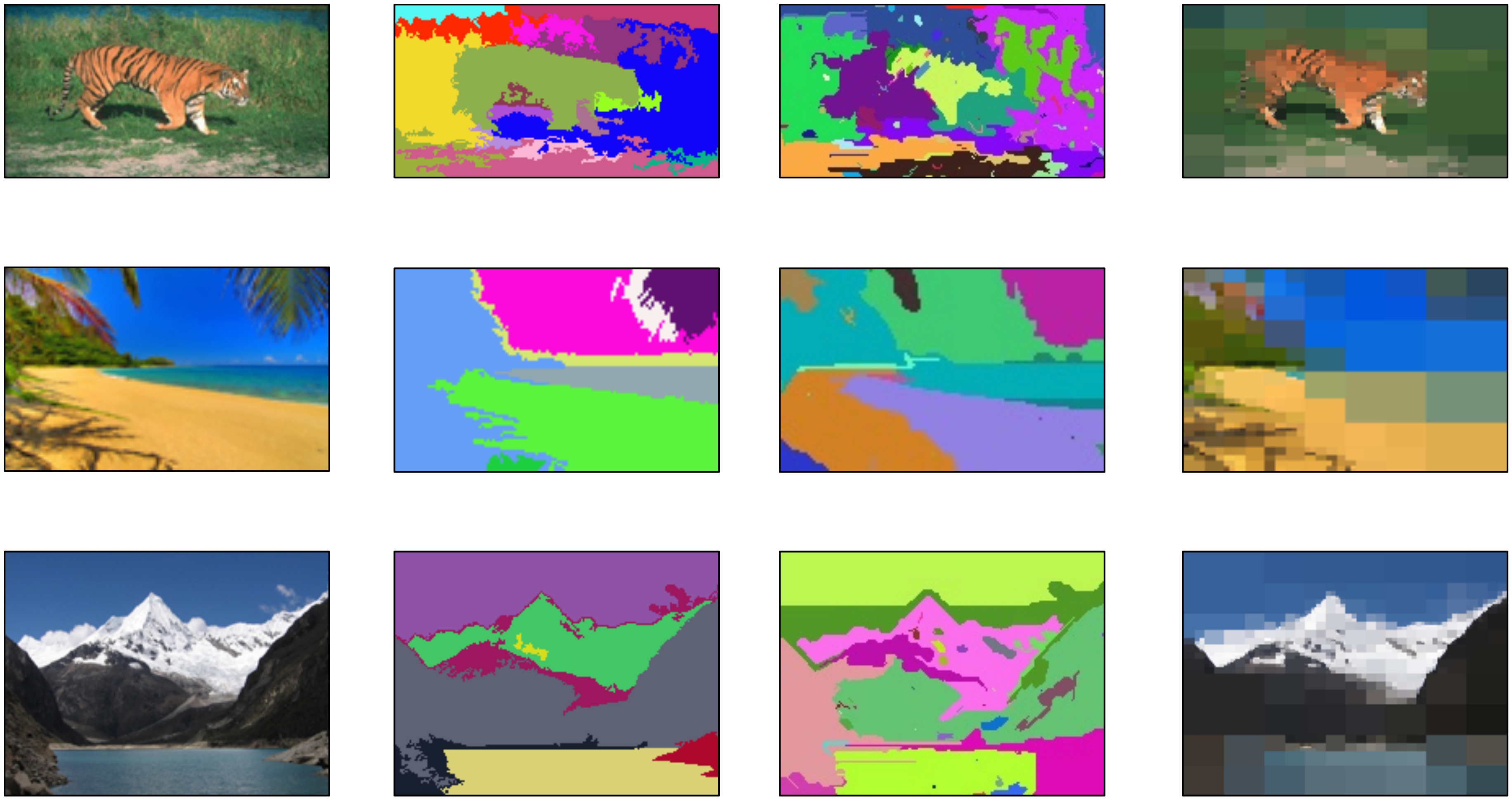
Other Implemented Methods

- Three region segmentation methods has been implemented so far, making emphasis in the method just described.
  - Non-parametric Bayesian Region Segmentation Method.
  - Minimum Spanning Tree Region Segmentation Method.
  - Quad Tree Region Segmentation Method
- Also different segmentation methods by edge detection were implemented, but they were omitted in this presentation.

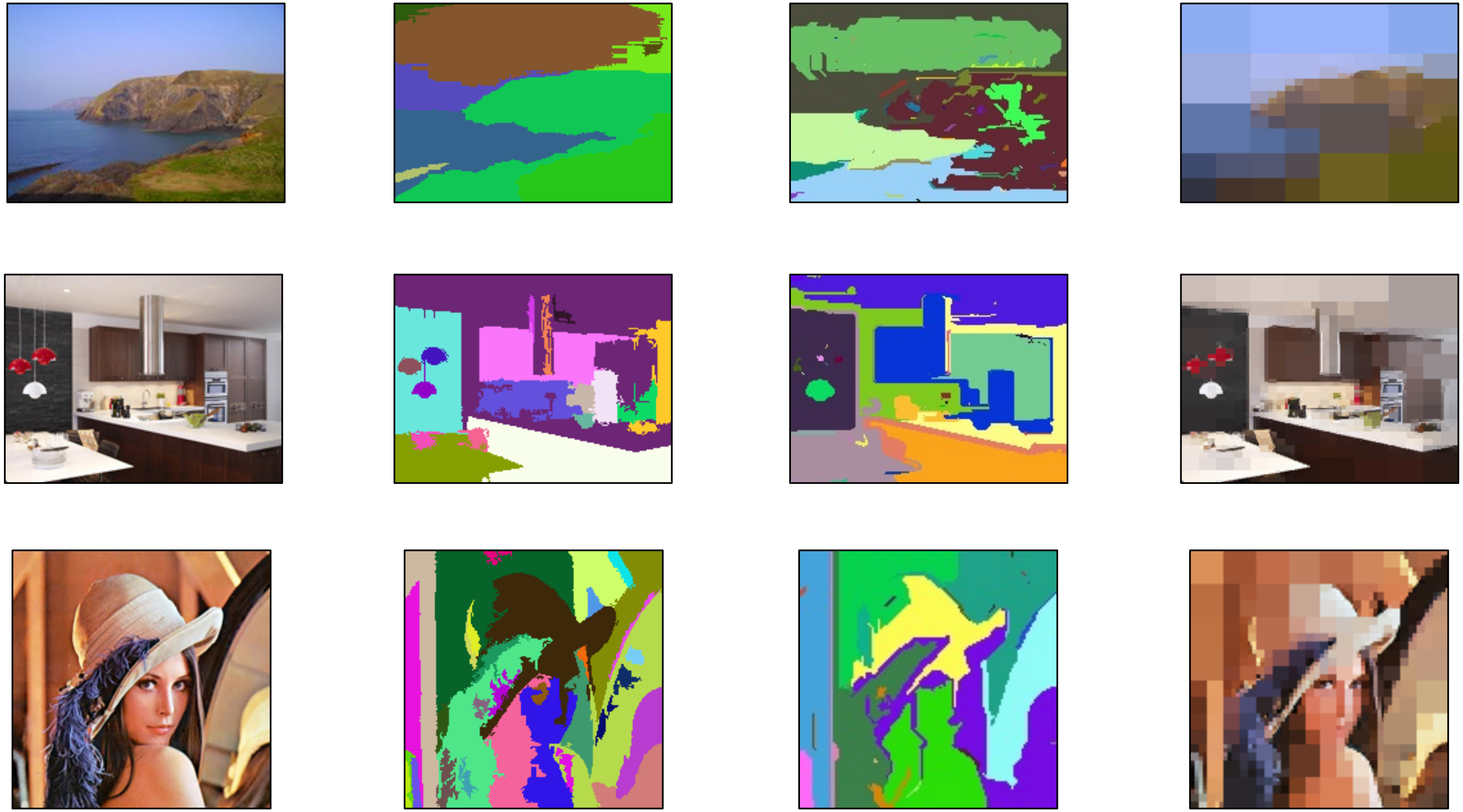
Other Implemented Methods

Original Image | **Proposed Method** | **MST Segmentation** | **Quad Tree Segmentation**



Other Implemented Methods

Original Image	Proposed Method	MST Segmentation	Quad Tree Segmentation



Actual Work

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Actual Work

- Recompilation of algorithms (for ACM competition).
  - Graphs
  - Geometry
  - Dynamic Programming
  - Sorting
  - Number Theory
  - Etc.
- Expected releasing date (beta version):

October 6th, 2011