Modern Marketing Research Combinatorial Computations: Shapley Value versus TURF tools

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2 authors:


Stan Lipovetsky
independent consultant


Michael Conklin
GfK Group

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MARKETING DECISION ANALYSIS BY TURF AND SHAPLEY VALUE

W. MICHAEL CONKLIN* and STAN LIPOVETSKY†

GfK Custom Research Inc., 8401 Golden Valley Road
Minneapolis, MN 55427, USA
**mconklin@Gfkcustomresearch.com*
†lipovetsky@Gfkcustomresearch.com

We consider a problem of marketing decisions for the choice of a product with maximum customer appeal. A widely used technique for this purpose is TURF, or Total Unduplicated Reach and Frequency, which evaluates a union of the events defined by the sample proportion of many products, or flavors of one product. However, when using TURF, it is often impossible to distinguish between subsets of different flavor combinations with practically the same level of coverage. An appropriate tool can be borrowed from cooperative game theory, namely, the Shapley Value, that permits the ordering of flavors by their strength in achieving maximum consumers' reach and provides more stable results than TURF. We describe marketing strategy reasons for using these techniques in the identification of the preferred combinations in media or product mix.

Keywords: Product lines; TURF; cooperative games; Shapley Value; marketing research.

1. Introduction

Resource allocation for product lines is a well known problem in marketing research. Although a large variety of flavors in a product line could potentially be more appealing to consumers, there are significant costs associated with such extensions. Packaging, advertising and inventory management costs all go up as the number of flavors increases. As a result, marketing managers need to find a set of flavors that provides the best combination of flavors for a fixed number of products that maximizes total sales or profit. The difficulty in solving the problem is that the incremental value of a particular product variant depends on which other variants are already in the line. An example of this problem can be taken from the world of fast moving consumer goods, such as soft drinks. A producer may wish to choose a limited number of flavors to include in the product line. The incremental value of the cola flavor differs depending on the other flavors that are present in the line. If the line already includes a cola then an additional cola flavor does not improve the overall product line sales. However, if there is not a cola in the line then adding it may be very beneficial.

Total Unduplicated Reach and Frequency (TURF) analysis is typically used to recommend solutions to the marketer's dilemma. TURF is a simple combinatorial technique that estimates the customers coverage by the union of a chosen subset of the products. It was developed for media mix models to find the best combination of magazines to place ads in to achieve the maximum audience reach and frequency of exposure.[4,6,15,18,22] While TURF answers the question on the total coverage by a product line, in practical applications, its results are often ambiguous because there usually is a substantial overlap in the appeal of different product variants. TURF provides a seemingly feasible combination for a product line of a specified size, but there is no guarantee that a subset of those products will be a best solution for a product line of a smaller or larger size.

A useful complementary approach producing clear results in these situations is found in cooperative game theory. We can think of the product line variant as a way of building coalitions among players (variants, in our case) to maximize the total value (that is the union coverage in our case). In cooperative games with many players, a convenient decision tool is the Shapley Value imputation. This method of arbitration in many players coalitions was introduced by L. Shapley,[27] and is described in many works on cooperative games.[14,17,23,25,28] The Shapley partitioning is based on the axioms of symmetry (or anonymity) of players, a dummy (or zero value player), and additivity. The latter two axioms are also known in the form of carriers (or effectiveness), and linearity (or aggregation). These axioms can even be weakened,[1,2,13,24,29] and Shapley Value can be used in many practical situations when the assumption of linear additive inputs can be applied as a first approximation to probably a more complicated scheme of margins partitioning. Game theory finds wide applications in marketing research and its related fields.[5,7–12,16,19–21,26,30] In our problem, the flavors take the role of the players, and the value of the game is the coverage of the customers. The Shapley Value, hereafter referred to as SV, was developed to provide the ordering of participants in a multi-player cooperative game, where it represents the worth of each player over all possible combinations of them. Applying this imputation tool to the product line variant problem, we get the ordering of the products that SV assigns to each of them. We use SV in the assumption of the transferable utility, and determine each flavor shared in the total reach of the product appeal.

The SV can serve as a complementary tool to TURF, giving the order of the flavors appeal that is especially useful when the TURF results correspond to many different lines with practically the same coverage. But the SV also provides a solution that is closer to the actual marketing situation. In most real world problems, the marketers do not have complete control over all of the variables. While a marketer may introduce a product line of size m, there is no guarantee that m variants will be available when a consumer is making his choice. A specific retail outlet may choose to carry fewer than m variants due to shelf space limitations. Or, there may be varying product variants available due to out-of-stock situations. The SV includes all these possibilities in its solution set. Another advantage of the SV consists in comparisons with all variants even if they are not in the final product line. Comparing across all possible combinations of variants we incorporate a possibility of competitive action in the analysis. Thus, calculating the SV across all possible combinations of potential product variants, we assure the best possible position even if competitors introduce some of the variants that were declined to produce.

Another important issue is the stability of SV in comparison to TURF results from a statistical point of view. TURF frequencies, being close to one another for many real data and having big standard deviations, are usually statistically indistinguishable. Bootstrap sampling shows[5–8] that the volatility of TURF output is high, so a chosen TURF solution could actually be an arbitrary decision. On the other hand, the SV being the average across the possible combinations, regularly yields stable bootstrapping output.

This paper is organized as follows. In Sec. 2, we consider TURF and Shapley Value techniques, Sec. 3 describes numerical results, and Sec. 4 summarizes.

2. TURF and Shapley Value Imputation

A data for customer preference analysis is presented by a matrix with N rows (number of customers) and n columns (total number of product variants, or flavors of a product). The elements of this matrix equal 0 or 1 for the non-chosen or chosen product respectively. The columns in this matrix represent the product variants $x_1, x_2, \ldots, x_n$. Each customer can choose any product variants, from zero to all n. For a product line size m (it can run from 1 to n) the TURF procedure counts the union of the purchase events for every possible combination from n products taken m at a time. In other words, the TURF procedure counts through each possible combination of flavors and counts how many consumers would buy at least one of them. Given a desired product line size m we can write the TURF objective as the maximum proportion estimated by the union of m flavors among all possible subsets M of them (the number of these subsets equals the number of combinations from n taken by m):

$$TURF = \max_{M}(\,\upsilon\,(M)) \tag{1}$$

where the value function

$$\upsilon(M) = p(x_1 \cup x_2 \cup \cdots \cup x_m) \tag{2}$$

estimates the union portion of the consumers attracted by the product line M that consists of the subset of m variables x_j taken from all n of them. Maximizing overall coverage from the flavors of the product corresponds to the availability of the product, thus, to the maximum objective of the profit obtained.

TURF can be conceptualized using Venn diagrams where the purchase intention for products a, b, c, etc. can be represented by circles' area corresponded to numbers

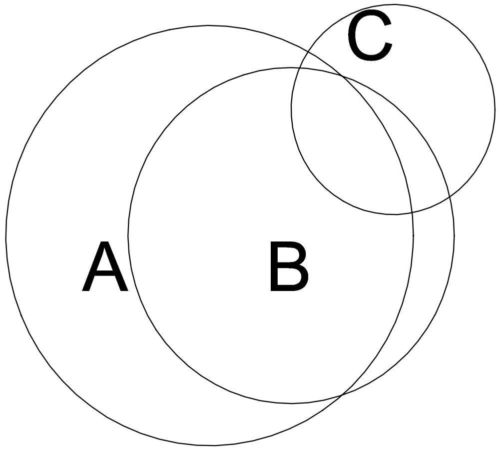

Fig. 1. Purchase interest for three products.

of consumers interested in the product. For instance, the best solution with two products corresponds to choosing the combination that maximizes the union of two sets. If products a and c mostly appeal to different sets of consumers the number of purchasers is maximized when these two products' line is utilized. However, many potential product line combinations can have very similar TURF scores because the concept appeal is much more important than variant differences. In Fig. 1, we see purchase intention for products A, B, and C, where each circle's area represents the number of people interested in the product. If a manager looks for two variants, the best solution corresponds to choosing the combination that maximizes the union of two sets. The number of purchasers is maximized when the two product line of A and C is utilized. This is because A and C mostly appeal to different sets of consumers. Since A and B, or B and C, overlap substantially, the incremental gain from adding C to A is much greater than any other combination of the two products. However, many potential product line combinations can have very similar TURF scores. Real data tends to look as it is shown in Fig. 2, because the concept appeal is much more important than the variant differences.

Let us consider Shapley Value application to assigning a value to each individual flavor. The Shapley Value is defined as each kth participant input to a coalition

$$S_k = \sum_{all\ M} \gamma_n(M)\big[\upsilon(M \cup \{k\}) - \upsilon(M)\big] \tag{3}$$

where weights of entering into a coalition M defined as

$$\gamma_n(M) = \frac{m!(n-m-1)!}{n!}. \tag{4}$$

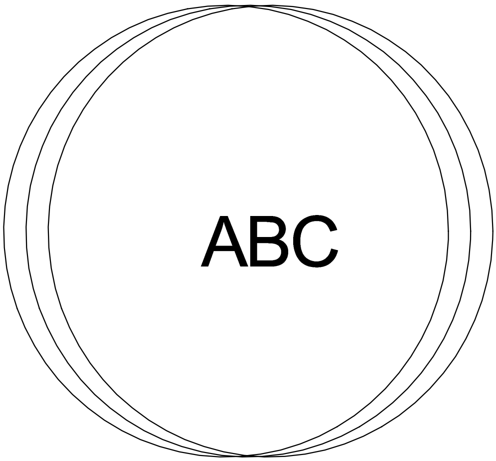

Fig. 2. Practical problems with TURF.

In (3) and (4), n is the total number of all the participants, m is the number of participants in the Mth coalition, and $\upsilon(.)$ is the characteristic function used for utility estimation for each coalition. By $M \cup \{k\}$ a set of participants which includes the kth participant is denoted, when M means a coalition without the kth participant. In our case, the participants of the coalition game are the flavors, or variants of a product.

Let us consider the characteristic function for our problem, denoting the product variants as $a, b, c, \ldots, z$, so z corresponds to the last nth variant. From the TURF output, we get frequencies reached in each set of variants. We will use the capital letters to denote these frequencies reached in each corresponded union — for instance, ABC denotes proportion for the union of a, b, and c variants:

$$ABC = p\,(a \cup b \cup c)\,. \tag{5}$$

Then the characteristic functions in (3) can be defined via the proportions of the kind (5) estimated by TURF. For easy exposition of the results, consider a case of $n = 5$ when the characteristic functions for the variant a are

$$\upsilon(0) = 0, \quad \upsilon(a) = A, \quad \upsilon(ab) = AB, \ldots, \quad \upsilon(abcde) = ABCDE, \tag{6}$$

and other variants b, c, d, and e characteristic functions are defined similarly. Such characteristic functions satisfies the axioms required by Shapley Value imputation. First, symmetry of players: all variants/flavors of the product are considered to be equal by their possible influence on the output. Second, zero value player: characteristic function for a dummy player is $\upsilon(0) = 0$ due to the definition of the reach value. Third, additivity: considering just the main inputs to the reach we work in the domain of the linear behavior of the union of the subsets.

The weights of imputation (4) in this case of $n = 5$ are:

$$\gamma\,(0) = \gamma\,(4) = 0.20, \quad \gamma\,(1) = \gamma\,(3) = 0.05, \quad \gamma\,(2) = 0.033. \tag{7}$$

Then SV_a for the variant a in explicit form can be written from (3) as

$$\begin{aligned} SV_a = {} & 0.2A + 0.05\,((AB - B) + (AC - C) + (AD - D) + (AE - E)) \\ & + 0.033\,((ABC - BC) + (ABD - BD) + (ABE - BE) \\ & + (ACD - CD) + (ACE - CE) + (ADE - DE)) \\ & + 0.05\,((ABCD - BCD) + (ABDE - BDE) + (ABCE - BCE) \\ & + (ACDE - CDE)) + 0.2\,(ABCDE - BCDE)\,. \end{aligned} \tag{8}$$

Various items in the sum (8) correspond to margins from the variant a to all the coalitions, and Shapley Value corresponds to the mean margin from the variant a averaged by its possible participation in all coalitions. Using analogous formulas for each of the other variants b, c, d, and e, and adding all of them, we can see that the total of all Shapley Values equals the maximum reach value:

$$SV_a + SV_b + SV_c + SV_d + SV_e = ABCDE. \tag{9}$$

It is a general property that the total of Shapley Values of all the participants equals the value of the coalition with all the participants together, or

$$\sum_{k=1}^{n} SV_k = \upsilon(all) \tag{10}$$

that in our problem equals the maximum proportion reached by all flavors together. Thus, the Shapley Values define the shares of the participants in their reach of the maximum value.

Regrouping the items in (3) or (8), we can represent Shapley Value formula as follows: the SV as follows:

$$\begin{aligned} SV_a = {} & \frac{1}{n-1}\,(A - \bar{1}) + \frac{1}{n-2}\left(\overline{A^*} - \bar{2}\right) + \frac{1}{n-3}\left(\overline{A^{**}} - \bar{3}\right) + \cdots \\ & + \frac{1}{n-(n-1)}\left(\overline{A^{*\cdots *}} - \overline{(n-1)}\right) + \frac{1}{n}\,(ABC\cdots Z)\,. \end{aligned} \tag{11}$$

It means that the Shapley Value for variant a can be obtained as a sum of mean values of the following deviations: of A from the mean value $\bar{1}$ of the first level coalitions with only one variant; of deviation $\overline{A*}$ (denotes mean value of coalitions with the variant a and one other variant — here the asterisk used for denoting any other variable) from the mean value $\bar{2}$ (averaging of all coalitions with two variants); of deviations $\overline{A**}$ (denotes mean value of coalitions with a and two other variants) from the mean $\bar{3}$ (averaging over all the coalitions with three participants); etc. to the item with $(n - 1)$ variants. The last item distributes the maximum possible value for all participants equally into each SV.

The formula (11) can be represented more generally as the Shapley Value imputation in the form more convenient for calculations:

$$S_k = \frac{1}{n} v\left(M_{all}\right) + \sum_{j=1}^{n-1} \frac{1}{n-j}\left(\bar{v}(M_{kj}) - \bar{v}(M_j)\right) \tag{12}$$

where $\bar{v}(M_{kj})$ is the average of the value function by attribute combinations of size j containing attribute k, and $\bar{v}(M_j)$ is the average of the value function by all attribute combinations of size j. The formula (12) gives an especially clear sense of the SV as a marginal input from each variant averaged by all possible coalitions. The important property of the formula (12) is the possibility to get the subsequent marginal inputs of coalitions of the 1st, 2nd, etc. levels to the total Shapley Value. If the data is available only on the several initial stages of coalitions with one, two and some other variants, it is possible to use (12) to estimate partial inputs and cumulative values to the total Shapley Value. Since each term is constructed by calculating a mean value of combinations with the product and a mean value of combinations without it, then we can estimate those means by sampling combinations. This can be easily done, and we suggest to incorporate such an approach whenever the number of products being evaluated is above ten.[5,6,8] Comparison of such cumulative values for each variant shows the stability of the SV from the partial data. This suggests an approach for reducing the computation time of the SV by limiting computation to the number of levels where stability is achieved. Plotting the partial values of the SV at each level of computation, we can consider the reach of SV by each variant. In Fig. 3, we see such a plot. Notice that some products could never reach stability because they have a unique appeal, and therefore always add to the SV at each level.

3. Numerical Results

To facilitate exposition of the results, for the first example, we consider a product line $m = 3$ that should be chosen from $n = 5$ flavors (denoted from a to e). This data for $N = 278$ respondents was taken from a real research study. The respondents indicate the flavors they choose (those could be from zero to all n flavors). The reach values (in total N) for a, b, c, d, and e flavors taken separately are 28.4%, 25.5%, 26.3%, 28.1%, and 25.9% respectively. So the priority ordering of the variants due to these margins is:

$$a \succ d \succ c \succ e \succ b. \tag{13}$$

We have two candidates for the best flavors in a and d with the highest margins. The results of TURF analysis for the lines with three flavors are shown in Table 1 where the counts and proportions for all possible 10 variants (combinations by 3 from 5) of the product line are presented. The additional last column presents the result for a line of all five flavors. The last row shows the bootstrapping output.

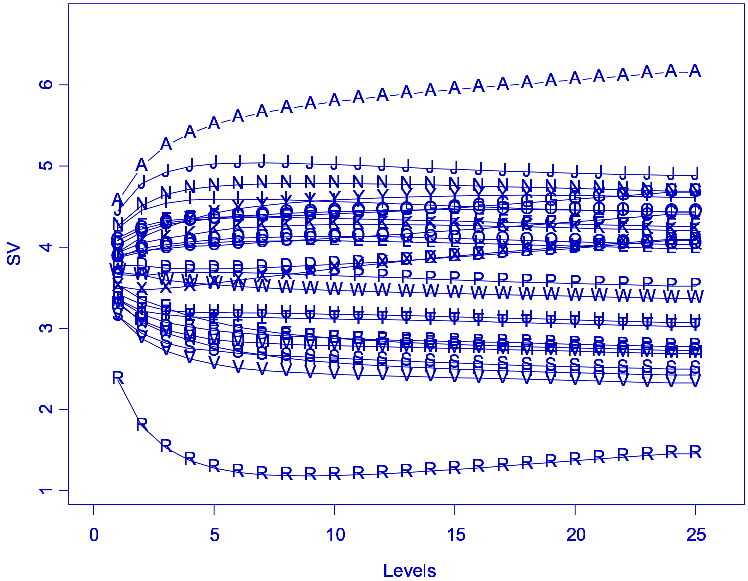

Fig. 3. Shapley values by computation level.

Table 1. Counts and percent of customers reached by different product lines.

Product Line	bcd	acd	bde	abd	ade	bce	ace	abc	abe	cde	abcde
Reached counts	119	119	119	118	118	117	115	114	113	111	132
Reached %	42.8	42.8	42.8	42.4	42.4	42.1	41.4	41.0	40.6	39.9	47.5
Boot-strap, %	26.8	25.8	25.8	15.2	14.2	12.4	3.4	1.8	1.0	0.6	—

The results in Table 1 are typical for marketing applications of TURF. There are three potential product lines — *bcd*, *acd*, and *bde*, that are indistinguishable by the reached appeal. For all of these three variants, the number of customers reached 119 which corresponds to proportion 119/278, or 42.8%. It is close to the maximum reach value achieved by all five flavors (Table 1, the last column) that equals 132/278, or 47.5% of the consumers. The best three variants include all five flavors from a to e, so we actually cannot choose the best three flavors. To make a decision based on such results is difficult because the TURF analysis yields no single best choice and actually all ten solutions are very close to one another. If we take into account the possibility of sampling error, the results of TURF become useless. We also perform bootstrapping on the TURF data to determine the frequency with which specific solutions appear. In the last row of Table 1, the results of 500 bootstrap iterations are shown. The TURF bootstrapping analysis only differentiates the worst combinations. Six out of the ten possible combinations are rated best more than 10 percent of the time. Only the four worst combinations are effectively ruled out. Three first combinations appeared as the best more than 25% of time, however, they include all five flavors.

The Shapley Value imputation for the same data yields the following percents:

$$SV_a = 9.8, \quad SV_b = 9.3, \quad SV_c = 9.1, \quad SV_d = 10.4, \quad SV_e = 8.9. \tag{14}$$

In accordance with Eqs. (9)–(10), these values are the shares of each flavor in their maximum reach, or

$$SV_a + SV_b + SV_c + SV_d + SV_e = 47.5 \tag{15}$$

where 47.5% is the value for proportion reached by all five flavors (see Table 1, the last column). The SV analysis also provides priority ordering of the variants due to the values in (14):

$$d \succ a \succ b \succ c \succ e, \tag{16}$$

that differs from the simple margins order in (13). We see by (16) that the best three variant product line consists of the flavors d, a, and b. It corresponds to using the first two and the last flavor from the margins ordering (13). In Table 1, the combination of variants (a, b, d) was in fourth place among the best solutions because TURF considers just three product lines while SV estimates over all possible combinations. SV ordering of the variants (16) is very stable. Bootstrapping 500 samples for the SV shows that differences of the SV means (their z-values in parentheses) are as follows: for d versus a flavors it is 0.544 (7.22), for a and b flavors 0.530 (8.23), for b and c flavors 0.317 (4.19), and for c and e flavors 0.184 (2.81). All these paired comparisons of order are significantly different ($p < 0.01$). These robust results can be easily understood by the specific structure of Shapley Value inputs (11), (12) as mean values by the variants of all possible coalitions.

Consider a practical example of twelve flavors of a product from a real project with $N = 301$ respondents. The flavors with their codes, individual reach percent, and the rank due to this percent are presented in Table 2. The aim of this problem was to choose a line with half the total number of flavors. From Table 2, we see that probably the best candidates to the line with m flavors are those of the first m ranked flavors with maximum percent of reach values. So the first six flavors in the order of their possible inclusion to the line could be:

$$e \succ g \succ a \succ b \succ f \succ j. \tag{17}$$

Applying TURF to find a line for $m = 6$, we actually obtain three lines *abcfgk, abcfgj*, and *abcefg* with the same coverage of maximum reach 98.01%. Among the flavors in these three lines, the flavors c and k are presented and those are not among the simple choices (17). These lines contain eight flavors, so it is not clear which of them to choose for the line with six flavors. If we take into account the lines with

Table 2. Flavors and their individual reach levels.

Flavor	Notation	% Reach	Rank
Cherry	a	59.80	3
Raspberry	b	59.14	4
Grape	c	42.52	7
Orange	d	30.56	9
Watermelon	e	66.11	1
Apple	f	58.47	5
Strawberry	g	61.79	2
Root Beer	h	23.59	11
Cola	i	21.26	12
Kiwi	j	44.52	6
Sour Berry	k	38.54	8
Lemon	l	28.90	10

the next reach level of practically the same value 97.67% we should consider eight different lines of various combinations of 6 flavors taken from 12 original flavors. The next reach level of 97.34% corresponds to 20 various lines, and their variants include all 12 original flavors. Among all 924 possible combinations from 12 by 6, most of them (889 combinations) have the reach level above 90%, while the total reach by all 12 flavors equals 99.0%, or 298 respondents. So it is hardly possible to say what to prefer for the line of size six.

In such situation, the Shapley Value is an especially valuable tool producing clear results — those presented in Table 3. In Table 3, we see the SV ordered from its maximum to minimum, presented in counts, when their total equals the number of respondents (301), and in percent to total. Due to SV, the six main flavors with high strength of coverage are:

$$e \succ a \succ g \succ b \succ f \succ j \tag{18}$$

that support the ordering in (17) and has a clear sense of the preferred choice found by ordering across all possible lines. The last column in Table 3 presents the cumulative coverage that can be reached by subsequent extending of the line size due to SV order. For the size $m = 6$, the reach value is 97.3%. Although it is slightly less than maximum reach 98.01% suggested by TURF (the mentioned three lines *abcfgk, abcfgj*, and *abcefg*) we can be sure that it is not an arbitrary solution with a volatile change of the line due to sampling error, as it usually is with TURF solutions.

An important difference between TURF and SV — the former tends to add the "specialty-niche" item to the product line, while the latter takes into account both the incremental reach and the overall appeal of the flavor. A "specialty-niche" flavor will only have a strong Shapley Value if it has wide enough appeal. We performed the following simulation study. We considered three flavors with high overlap (all correlations equal 0.9) and high overall appeal (purchase probability equals 50%), and one additional flavor with unique appeal (negative correlation with each other flavor, equals −0.9). We varied the overall appeal of the "specialty-niche" flavor

Table 3. Shapley Value ordering and Cumulative Reach.

Flavor	Notation	Shapley Value	% Shapley Value	% Cumulative Reach
Watermelon	e	38.81	13.02	66.1
Cherry	a	36.38	12.21	86.4
Strawberry	g	36.36	12.20	90.7
Raspberry	b	34.94	11.73	95.0
Apple	f	33.93	11.39	96.7
Kiwi	j	23.51	7.89	97.3
Grape	c	22.93	7.70	98.3
Sour Berry	k	20.11	6.75	98.7
Orange	d	14.91	5.00	99.0
Lemon	l	14.43	4.84	99.0
Root Beer	h	11.55	3.88	99.0
Cola	i	10.13	3.40	99.0
Total	—	298	100	—

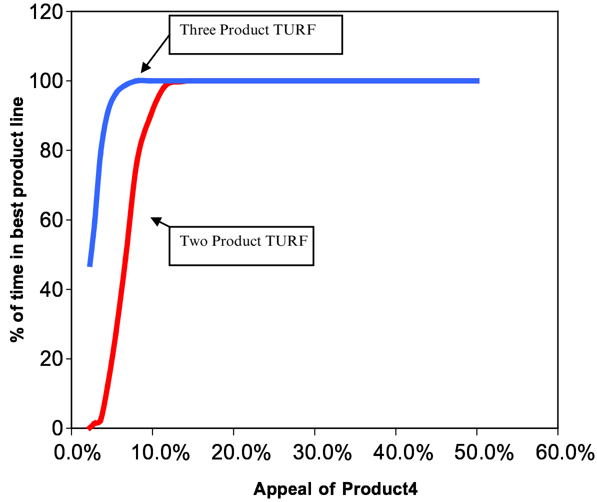

Fig. 4. Simulation results for TURF.

from 5% to 50%, and for each point generated 500 simulated studies with 250 respondents each. Then the percent of times the "specialty-niche" flavor is in the top set chosen by TURF was calculated. Also the average Shapley Value of all four flavors was evaluated. Simulation results for TURF are presented in Fig. 4. We see that TURF always chooses the niche product in a three-product line when overall appeal becomes higher than 8.1%. It means that TURF only looks at uniqueness of appeal. In Fig. 5, the results of Shapley Value simulation are presented. In this graph, we see that the niche product has a better Shapley Value once its overall appeal overcomes 21%. These results show that SV provides better handling of niche flavors, optimizes over breadth of appeal and uniqueness of appeal, provides

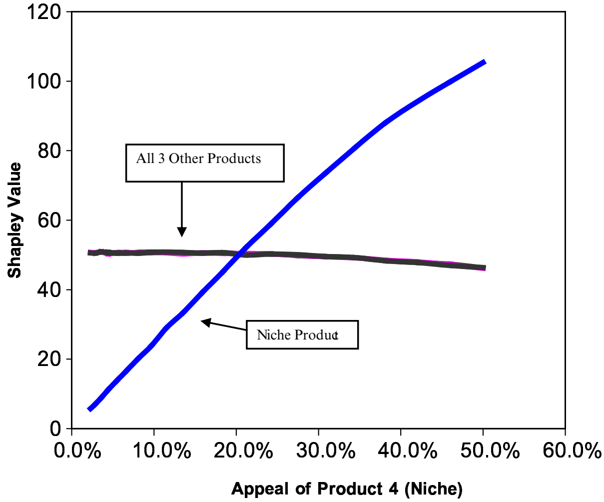

Fig. 5. Simulation results for Shapley Value.

information about each flavor rather than just about their combinations, and has very stable results.

Thus, the Shapley Value is a measure of the relative strength of individual items. There are two aspects of that strength. To achieve a high Shapley Value, an item must appeal to a relatively large number of consumers and some of those consumers have a unique attraction to the item. In other words, many consumers must intend to buy the item and some subset of those consumers must intend to buy only that item, or only a small subset of items including that item. If an item appeals to a relatively small number of consumers, then its Shapley Value will be small because many lines that do not include the item will have higher purchase interest. And if two items reach exactly the same people, they will have the same Shapley Value, so their relative strength will be equal.

4. Summary

We considered TURF analysis for product line variant decisions and proposed an additional decision rule, the Shapley Value. TURF gives the reach level of coverage in a product bundle, while the SV assigns a strength ordering to each product. The TURF solution cannot guarantee availability of chosen variants. It cannot control competitive introductions of variants we choose not to produce and the actual set consumers choose from is not a fixed size. Therefore, it is better to maximize the strength of the product line over all possible combinations and not just combinations of one particular size. Shapley Value Analysis provides an overall "value" for each potential product in the line. These products can be arrayed from the most valuable to the least valuable. Often, there are several high value products that separate themselves from the remainder of the potential products. These products become the clear best candidates for membership in the product line. The ease with which the best products to include in a product line can be identified with a Shapley Value is in sharp contrast to the typical results from TURF where there may be a dozen or two different product lines that are evaluated as being equal. While TURF evaluates combinations of products into a line of a specific size, the Shapley Value evaluates each product across all possible product lines and product line sizes. It means that Shapley Value corresponds to a global maximum of the coverage estimated via a smoothed objective, rather than via the numerous volatile local maxima of the TURF solutions. This also means that Shapley Value Analysis explicitly includes in its calculations situations like out of stocks, where the full product line is not available, and competitive actions, where competitors produce products that a decision maker has chosen not to introduce.

Shapley Value of a product can be interpreted as the incremental number of individual consumers that will buy any of this product in the product line. Total of all SV equals the maximum possible coverage, so individual values define shares that each variant (flavor) contributes to the total appeal. Shapley Values represent a measure of the relative importance of each product, and it provides a preferred ordering of the product line variants. This has further implications for business strategy that are not evident from TURF analysis, because the product variant with the highest Shapley Value provides the greatest gain over all possible product combinations. This implies that it is preferable to have that variant always in the available choice set when consumers are purchasing. The strategy implications of this statement are that the marketer should expend more effort to ensure that the product variant with the highest Shapley Value has the minimum possible out-of-stock probability. Thus, the management of inventory should give preference to the highest Shapley Value variant. Following this strategy ensures that no matter what combination of product variants and competitive variants that are on the shelf at any time, the variants with the highest Shapley Values are the best decision. Shapley Value, being a tool for averaging over coalitions, can help in solving numerous theoretical and practical problems of decision making for management and operations research in marketing, advertising, and other related fields.

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