# CS 5/7320
# Artificial Intelligence

# Reinforcement Learning
## AIMA Chapter 17+22

Slides by Michael Hahsler
with figures from the AIMA textbook.

# Remember Chapter 16:
## Making Simple Decisions

For a decision that we make frequently and making it once does not affect the future decisions (**episodic environment**), we can use the **Principle of Maximum Expected Utility (MEU).**

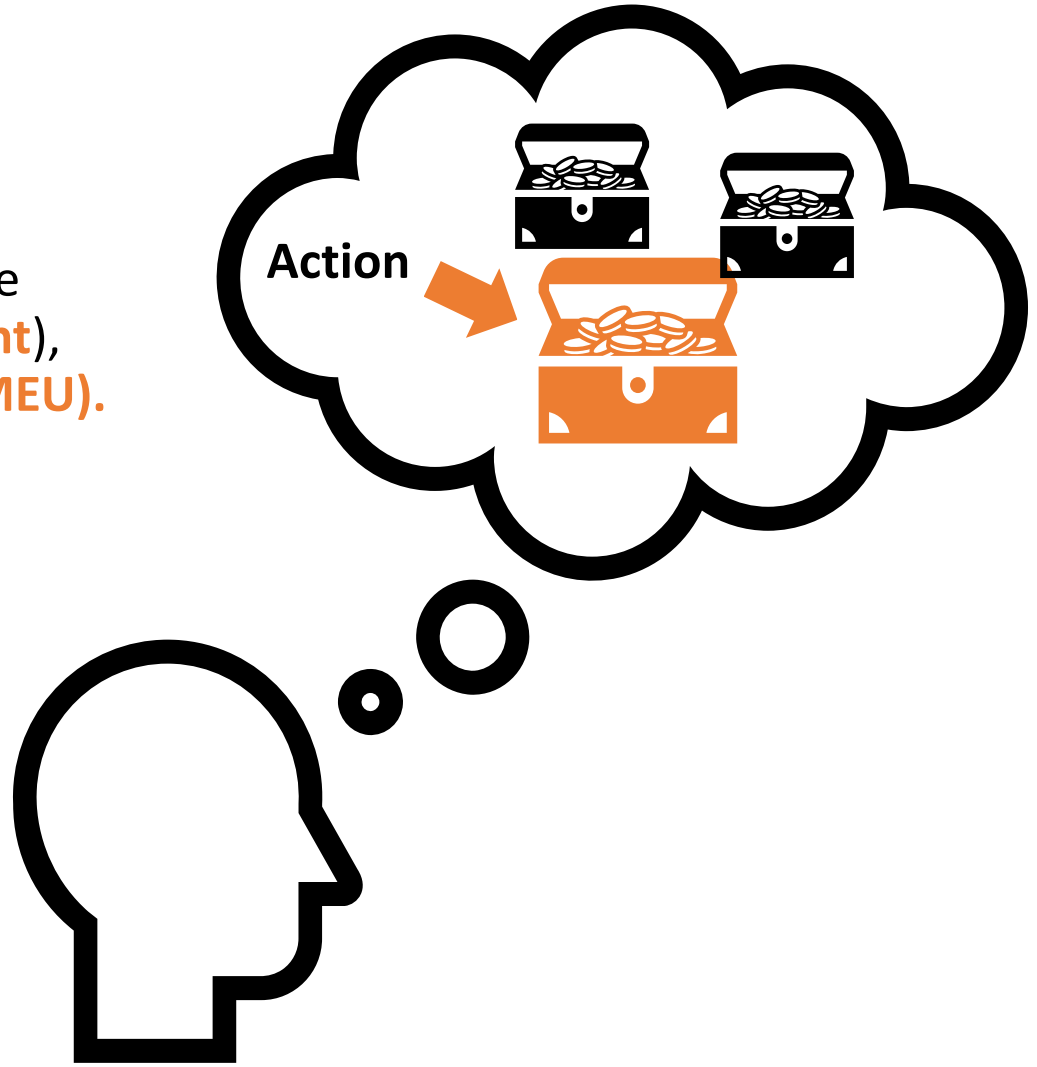
Given the expected utility of an action

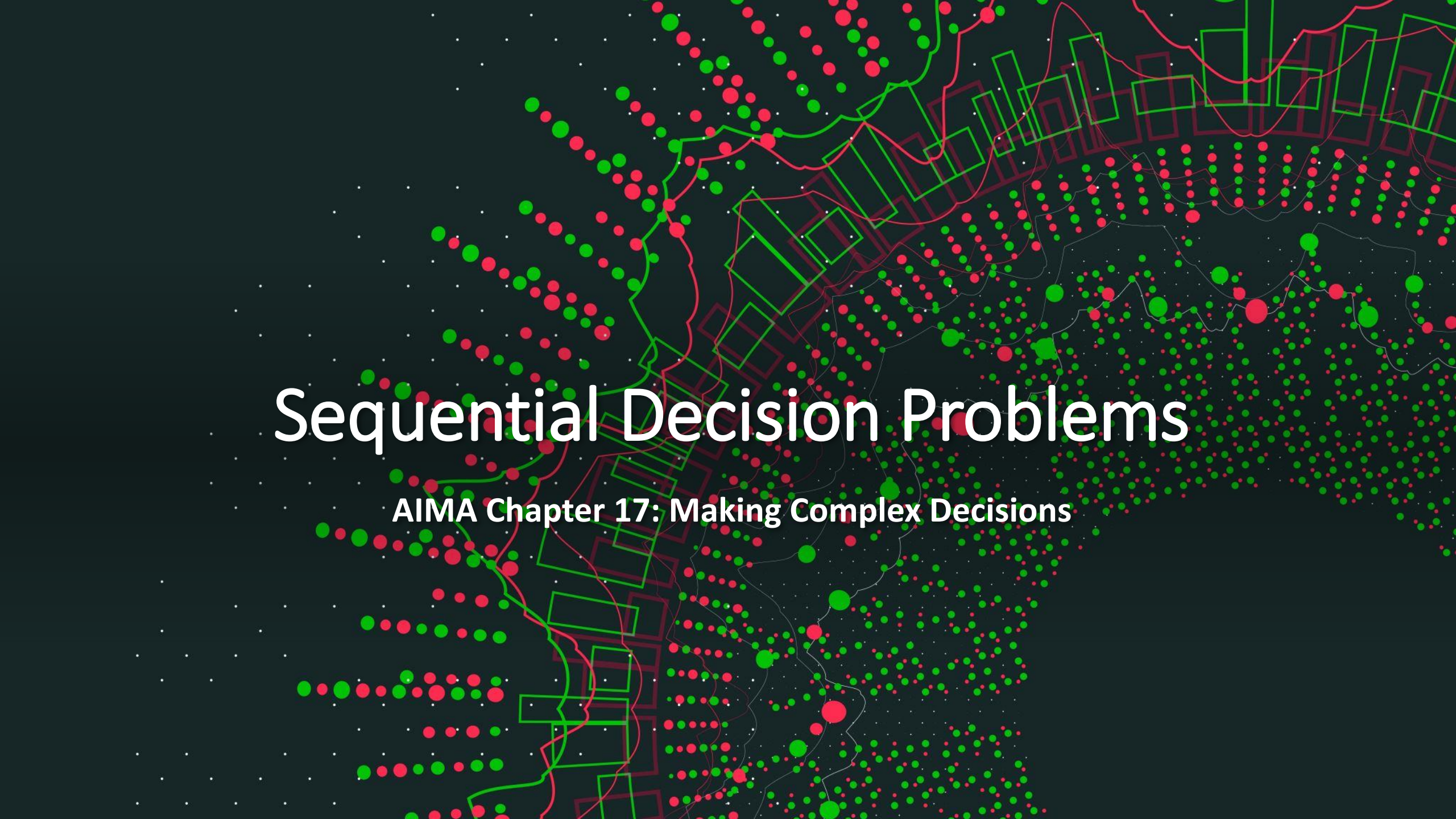
$$EU(a) = \sum_{s'} P(Result(a) = s')U(s')$$

choose action that maximizes the expected utility:

$$a^* = \text{argmax}_a \, EU(a)$$

Now we will talk about decision making in **sequential environments**.



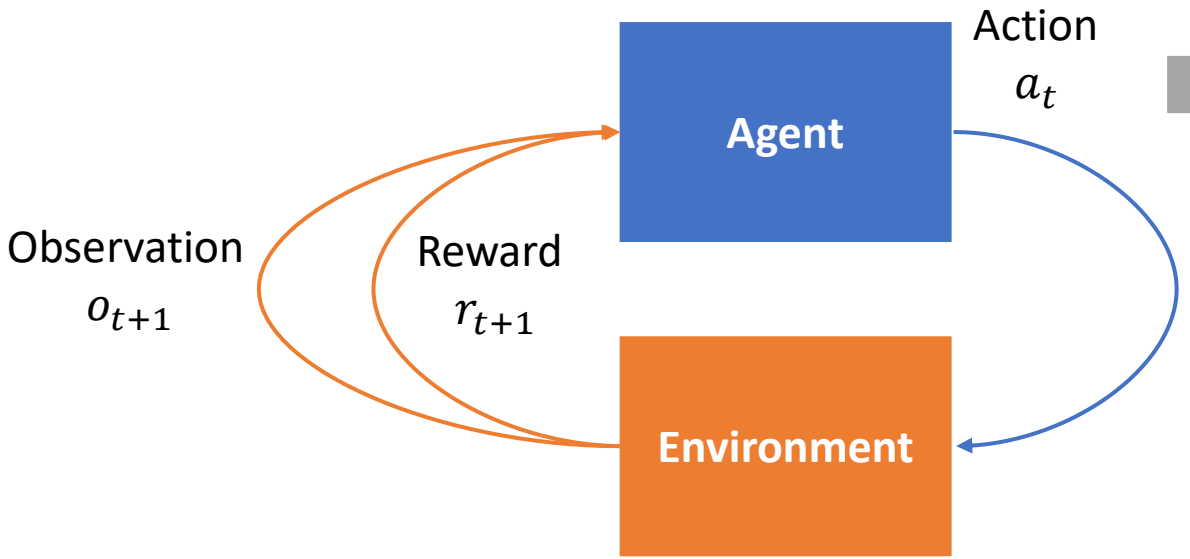


# Sequential Decision Problems

**AIMA Chapter 17: Making Complex Decisions**

# Sequential Decision Problems

- **Utility-based agent**: The agent's utility depends on a sequence of decisions that depend on each other.
- Sequential decision problems incorporate utilities, uncertainty, and sensing.




Sequence: $(o_0, r_0), a_0, (o_1, r_1), a_1, (o_2, r_2), a_2, \ldots$

**Goal:** Observations and rewards depend on the state of the system and the agent wants to maximize the expected discounted reward:

$$U = E\left[\sum_{t=0}^{T} \gamma^t r_t\right]$$

$\gamma$ … discounting factor

$T$ … time horizon may be infinity

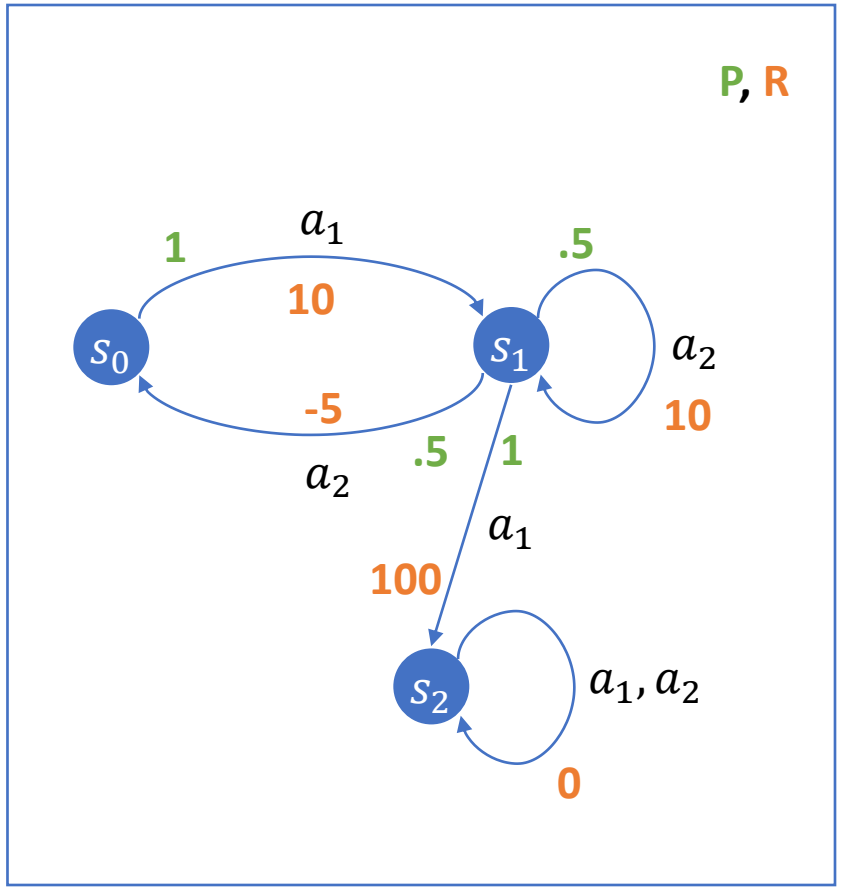
# Definition: Markov Decision Process (MDP)

- MDPs are sequential decision problems with
  - a fully observable ($o_t = s_t$), stochastic environment;
  - a Markovian transition model: future states do not depend on past states give the current state;
  - additive rewards.

- MDPs are discrete-time stochastic control processes defines by:
  - a finite set of **states** $S = \{s_0, s_1, s_2, \dots\}$ (initial state $s_0$)
  - a set of available **actions** $ACTIONS(s)$ in each state $s$
  - a **transition model** $P(s' \mid s, a)$ where $a \in ACTIONS(s)$
  - a **reward function** $R(s)$ where the reward depends on the current state (often $R(s, a, s')$ is used)

- Time horizon
  - **Infinite horizon**: non-episodic (continuous) tasks with no terminal state.
  - **Finite horizon**: episodic tasks. Episode ends after a number of periods or when a terminal state is reached. Episodes contain a sequence of several actions that affect each other.

This is different from the previous definition of an **episodic** environment!



# Example: 4x3 Grid World

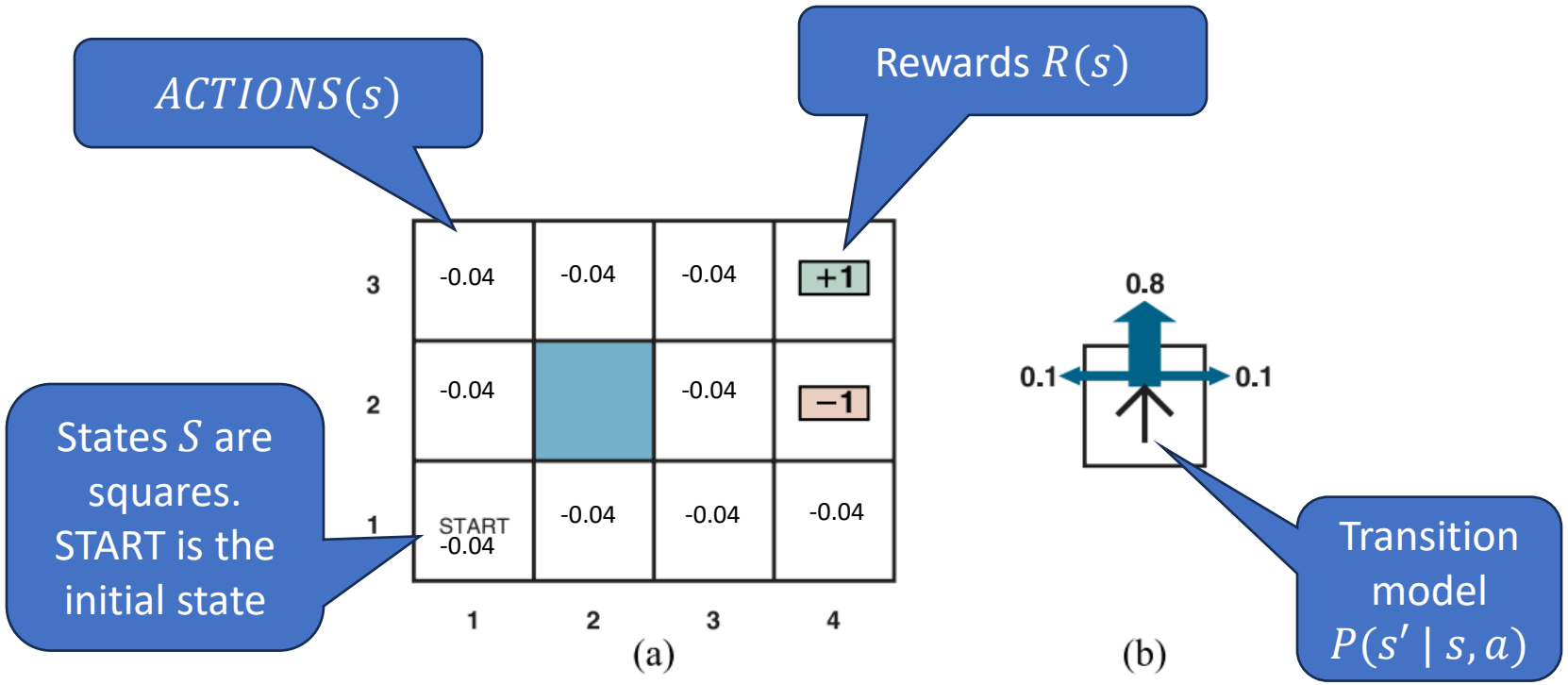



**Figure 17.1** (a) A simple, stochastic $4 \times 3$ environment that presents the agent with a sequential decision problem. (b) Illustration of the transition model of the environment: the "intended" outcome occurs with probability 0.8, but with probability 0.2 the agent moves at right angles to the intended direction. A collision with a wall results in no movement. Transitions into the two terminal states have reward +1 and –1, respectively, and all other transitions have a reward of –0.04.

**Goal**

For each square: determine what direction should we try to go to maximize the total utility?

This is called a **policy** written as the function

$$\pi: S \rightarrow ACTIONS(S)$$

# Optimal Policy

- A policy $\boldsymbol{\pi} = \{\pi(s_0), \pi(s_1), \dots\}$ defines for each state which action to take.
- The expected utility of being in state $s$ under policy $\pi$ can be calculated as the sum:

$$U^{\pi}(s) = E_{\pi}\left[\sum_{t=0}^{\infty} \gamma^t R(s_t) \,|s_0 = s\right]$$

$\gamma$ is a discounting factor to give more weight to immediate rewards.

$E_{\pi}$ is the expectation over sequences that can be created by following $\pi$.

- The goal of solving an MDP is to find an optimal policy $\boldsymbol{\pi}$ that maximizes the expected future utility for each state

$$\pi^*(s) = \underset{\pi}{\operatorname{argmax}}\, U^{\pi}(s) \quad \text{for all } s \in S$$

- The recursive **Bellman equation** holds for the optimal value function $U$ ("Bellman optimality condition"):

$$U^{\pi^*}(s) = R(s) + \gamma \max_{a \in A} \sum_{s'} P(s'|s,a)\; U^{\pi^*}(s')$$

Immediate Reward

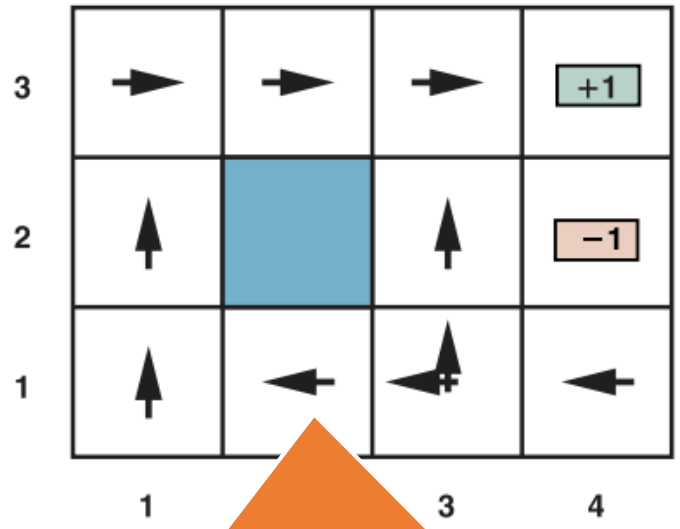
$\pi^*$ uses the best action

Expectation

Utility of the next state

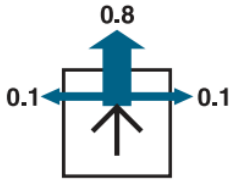
# Solution: 4x3 Grid World

**Optimal action in each state (policy $\pi^*$)**




Why is it optimal to walk away from the +1 square?

Always pick the action with the highest expected utility.




**Value of being in a state $U(s)$ (given that we will follow $\pi^*$)**

| | | | | |
|---|---|---|---|---|
| 3 | 0.8516 | 0.9078 | 0.9578 | +1 |
| 2 | 0.8016 | | 0.7003 | −1 |
| 1 | 0.7453 | 0.6953 | 0.6514 | 0.4279 |
| | 1 | 2 | 3 | 4 |

$\gamma = 1$

How to we find the optimal value function/optimal policy?

Policy Iteration

Value Iteration

# Q-Function

- $Q(s, a)$ is called the state-action value function. It gives the expected utility of action $a$ in state $s$.

$$Q(s, a) = R(s) + \gamma \sum_{s'} P(s'|s, a)[\, U(s')]$$

Immediate Reward

Expected utility of the next state

- Relationship with the state value function: $U(s) = \max_a Q(s, a)$

- The Q-function is often used for convenience in solving MDPs.

# Value Iteration: Estimate the Optimal Value Function $U^{\pi^*}$

**Algorithm**: Start with a $U(s)$ vector of 0 for all states and then update (Bellman update) the vector iteratively until it converges to the unique optimal solution $U^{\pi^*}$.

**function** VALUE-ITERATION($mdp, \epsilon$) **returns** a utility function
  **inputs**: $mdp$, an MDP with states $S$, actions $A(s)$, transition model $P(s' \mid s, a)$,
    rewards $R(s, a, s')$, discount $\gamma$
    $\epsilon$, the maximum error allowed in the utility of any state
  **local variables**: $U$, $U'$, vectors of utilities for states in $S$, initially zero
    $\delta$, the maximum relative change in the utility of any state

  **repeat**
    $U \leftarrow U'$; $\delta \leftarrow 0$
    **for each** state $s$ **in** $S$ **do**
      $U'[s] \leftarrow \max_{a \in A(s)}$ Q-VALUE($mdp, s, a, U$)
      **if** $|U'[s] - U[s]| > \delta$ **then** $\delta \leftarrow |U'[s] - U[s]|$
  **until** $\delta \leq \epsilon(1-\gamma)/\gamma$
  **return** $U$

**Update with the value of the best action in state s.**

**Uses a proxy for policy loss** $\|\boldsymbol{U^{\pi}} - \boldsymbol{U}\|_{\infty}$ as the stopping criterion

$U$ converges to $U^{\pi^*}$ and we can extract $\pi^*$

# Policy Iteration: Learn the Optimal Policy $\pi^*$

Policy iteration tries to directly find the optimal policy by iterating policy evaluation and improvement.

**function** POLICY-ITERATION($mdp$) **returns** a policy
  **inputs**: $mdp$, an MDP with states $S$, actions $A(s)$, transition model $P(s' \mid s, a)$
  **local variables**: $U$, a vector of utilities for states in $S$, initially zero
    $\pi$, a policy vector indexed by state, initially random

  **repeat**
    $U \leftarrow$ POLICY-EVALUATION($\pi, U, mdp$)
    $unchanged? \leftarrow$ true
    **for each** state $s$ **in** $S$ **do**
      $a^* \leftarrow \underset{a \in A(s)}{\text{argmax}}$ Q-VALUE($mdp, s, a, U$)
      **if** Q-VALUE($mdp, s, a^*, U$) $>$ Q-VALUE($mdp, s, \pi[s], U$) **then**
        $\pi[s] \leftarrow a^*$; $unchanged? \leftarrow$ false
  **until** $unchanged?$
  **return** $\pi$

Calculate U given current policy (either solve an LP or value iteration with fixed policy)

Greedy policy Improvement

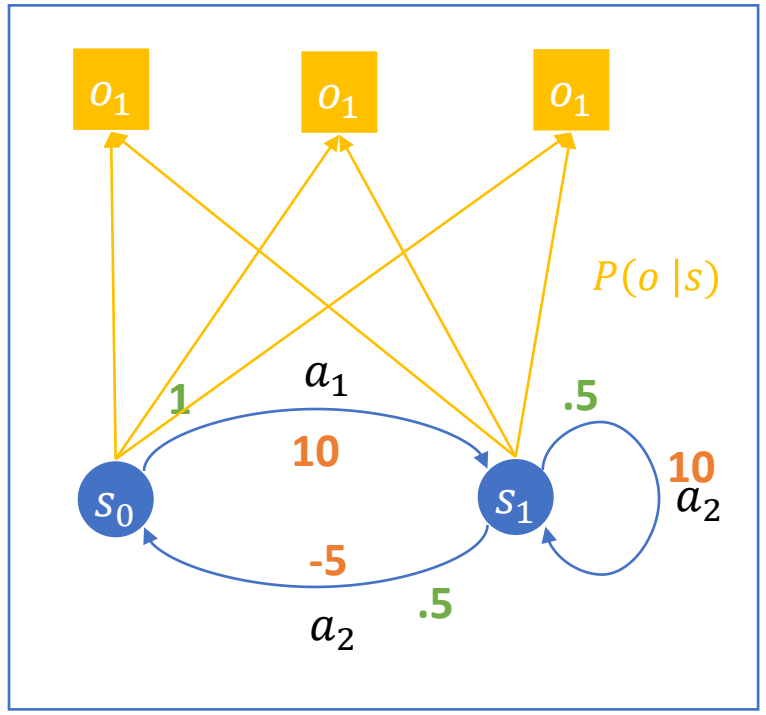
$\pi$ converges to $\pi^*$
(and $U$ converges to $U^{\pi^*}$)

# Partially Observable Markov Decision Model (POMDP)

- If the environment is **partially observable**, then $o_t \neq s_t$ and the model is expanded by
  - a **sensor model** $P(o \mid s)$ for receiving observation $o$ given being in state $s$.

- This makes things a lot more complicated, and we have to work with **belief states**. A belief state is a distribution over states.
  Example: For a problem with three states, the belief state $b = (.2, .8, 0)$ means the agent beliefs that it is with 20% in state 1 and 80% in state 2 but not in state 3.

- An MDP that uses belief states instead of system states is called a **belief MDP**.
  Issue: the probabilities in belief states are continuous, and the number of different belief states is infinite.
- The solution of a POMDP is a policy with the optimal actions for sets of belief states (i.e., ranges of belief).
- For all but tiny problems, POMDPs can only be solved **approximately** (e.g., by grid-based methods).



# Reinforcement Learning

**AIMA Chapter 22**

# Reinforcement Learning (RL)

- RL assumes that the problem can be modeled by an **MDP**.

- What if we do not know the exact transition model $P(s' \mid s, a)$?

  Now we cannot solve the MDP (estimate the state utility function/policy) because we cannot predict what the future states after an action are!

- The agent needs to explore (try actions) and **use the reward signal to update its estimate of the utility of states and actions**. This is a learning process where the reward provides positive reinforcement.

# Q-Learning

- Q-Learning learns the state-action value function from interactions with the environment (percepts).
- This agent function learns a table for the state-action function $Q$.

**function** Q-LEARNING-AGENT(*percept*) **returns** an action

**inputs**: *percept*, a percept indicating the current state $s'$ and reward signal $r$

**persistent**: $Q$, a table of action values indexed by state and action, initially zero

$N_{sa}$, a table of frequencies for state–action pairs, initially zero

$s$, $a$, the previous state and action, initially null

**if** $s$ is not null **then**

increment $N_{sa}[s, a]$

$Q[s, a] \leftarrow Q[s, a] + \alpha(N_{sa}[s, a])(r + \gamma \max_{a'} Q[s', a'] - Q[s, a])$

$s, a \leftarrow s', \text{argmax}_{a'} f(Q[s', a'], N_{sa}[s', a'])$

**return** $a$

New episode has no s.

Learning rate

Make $Q[s, a]$ a little more similar to the received reward + the best Q-value of the successor state.

$f$ is the exploration function and decides on the next action. As N increases it can exploit good actions more.

# Value Function Approximation

- $U$ $(Q)$ needs to store and estimate one entry for each state (state/action combination).
- Issues and solutions
  - Too many entries to store → lossy compression
  - Many combinations are rarely seen → generalize to unseen entries

- **Idea**: Estimate the state value by learning an approximation function $\widehat{U}(s) = g_\theta(s)$ based on features of $s$.
- 4x3 Grid World Example: Use a linear combination of state features $(x, y)$ and learn $\boldsymbol{\theta}$ from observed data.

$U(s)$

| | | | | |
|---|---|---|---|---|
| 3 | 0.8516 | 0.9078 | 0.9578 | +1 |
| 2 | 0.8016 | | 0.7003 | −1 |
| 1 | 0.7453 | 0.6953 | 0.6514 | 0.4279 |
| | 1 | 2 | 3 | 4 |

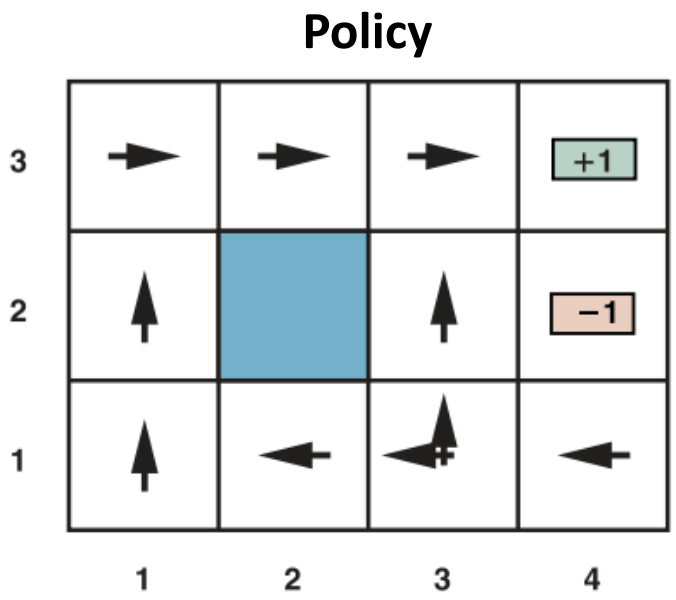
Learn $\theta$ from observed interactions with the environment to approximate $U(s)$

$$\widehat{U}_\theta(x, y) = \theta_0 + \theta_1 x + \theta_2 y$$

**Notes**:

- $\boldsymbol{\theta}$ can be updated iteratively after each new observed utility.
- We typically need non-linear approximators that can be incrementally updated (online learning). → Deep ANNs called Value Net

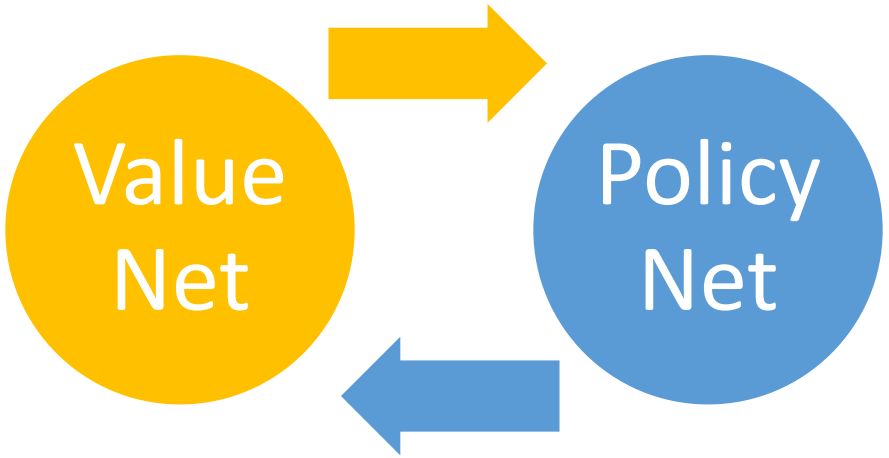
# Policy Nets: Policy Approximation

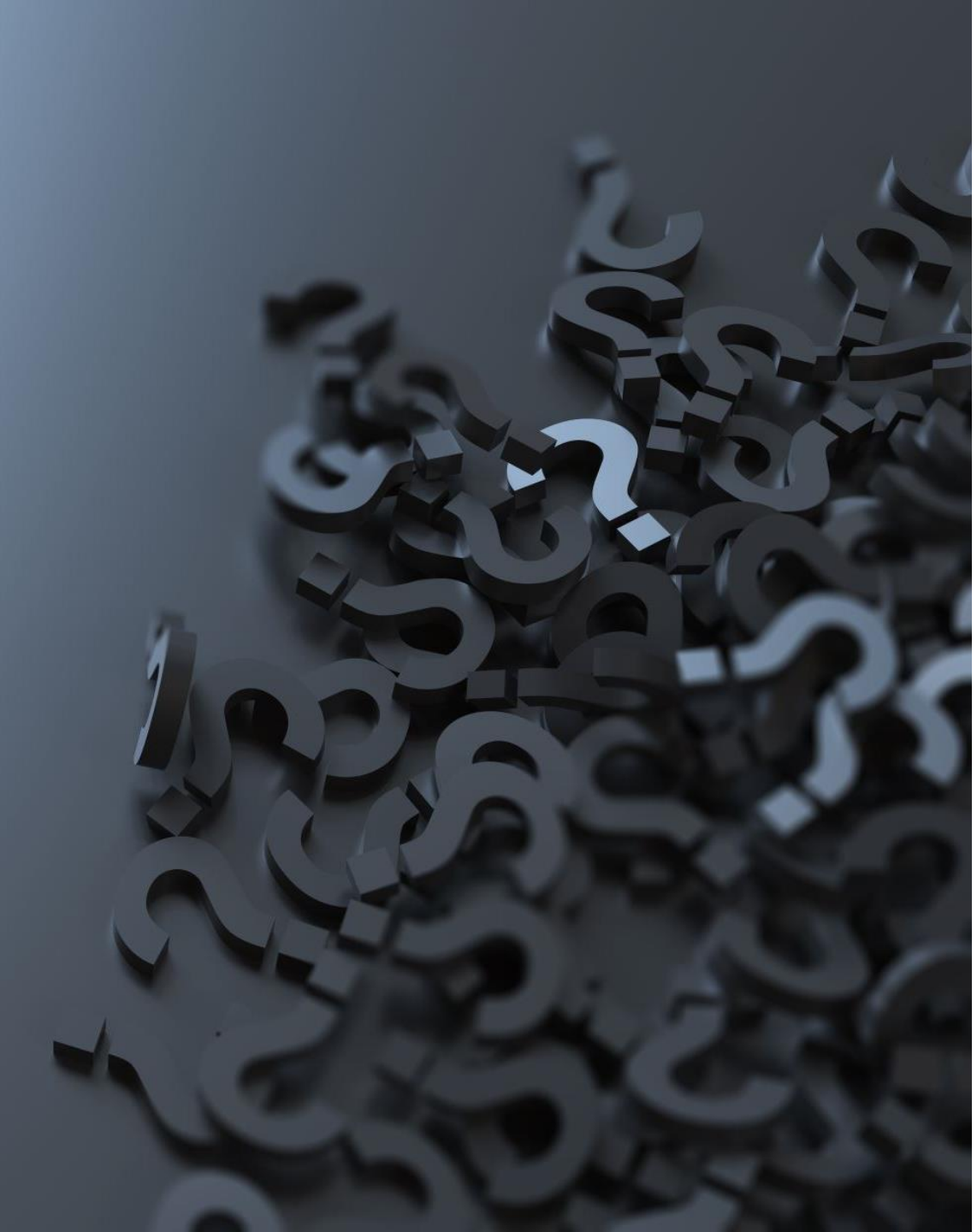



- Policy tables are also large and generalization may be helpful.
- Use a neural network to learn to represent a policy as a function

$$f: S \to A.$$

- Often used together with a Value Net to iteratively improve the Value Net and then the Policy Net.





# Summary

- Agents can learn the value of being in a state from **reward signals**.
- Rewards can be delayed (e.g., at the end of a game).
- Not being able to fully **observe the state** makes the problem more difficult (POMDP).
- **Unknown transition models** lead to the need of exploration by trying actions (model free methods like Q-Learning).
- All these problems are computationally very expensive and often can only be solved by **approximation**. State of the art is to use deep artificial neural networks for function approximation.