CS 5/7320
Artificial Intelligence

Reinforcement Learning
AIMA Chapter 17+22

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with figures from the AIMA textbook.

Remember Chapter 16:
Making Simple Decisions

For a decision that we make frequently and making it once does not affect the future decisions (**episodic environment**), we can use the **Principle of Maximum Expected Utility (MEU).**

Given the expected utility of an action

$$EU(a) = \sum_{s'} P(Result(a) = s')U(s')$$

choose action that maximizes the expected utility:

$$a^* = \text{argmax}_a \, EU(a)$$

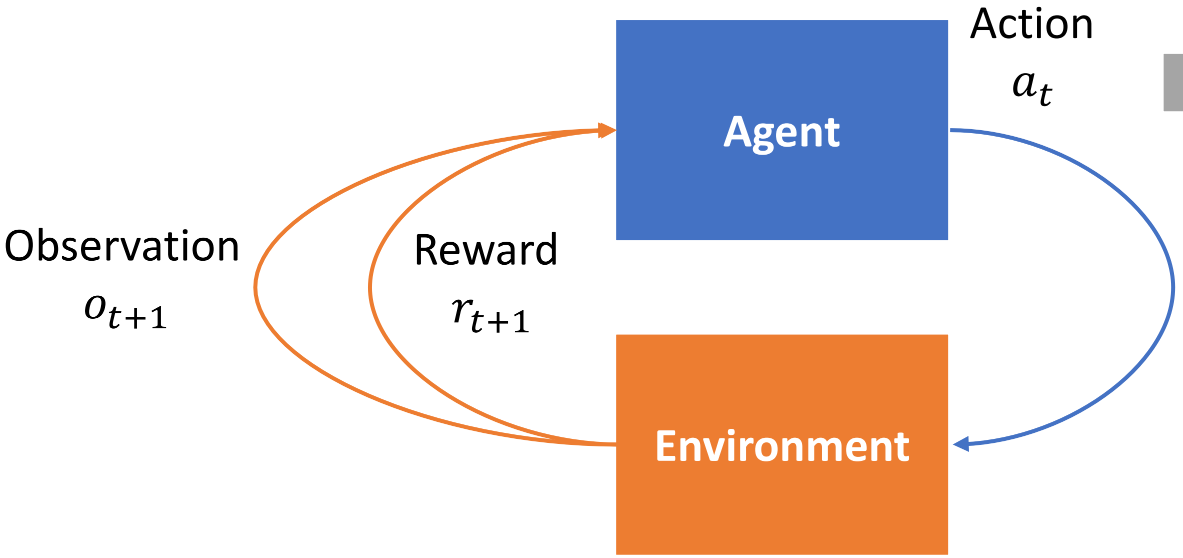
Now we will talk about decision making in **sequential environments.**

Sequential Decision Problems

AIMA Chapter 17: Making Complex Decisions

Sequential Decision Problems

- **Utility-based agent**: The agent's utility depends on a sequence of decisions that depend on each other.
- Sequential decision problems incorporate utilities, uncertainty, and sensing.



Sequence: $(o_0, r_0), a_0, (o_1, r_1), a_1, (o_2, r_2), a_2, \ldots$

Goal: Observations and rewards depend on the state of the system and the agent wants to maximize the expected discounted reward:

$$U = E\left[\sum_{t=0}^{T} \gamma^t r_t\right]$$

γ … discounting factor
T … time horizon may be infinity

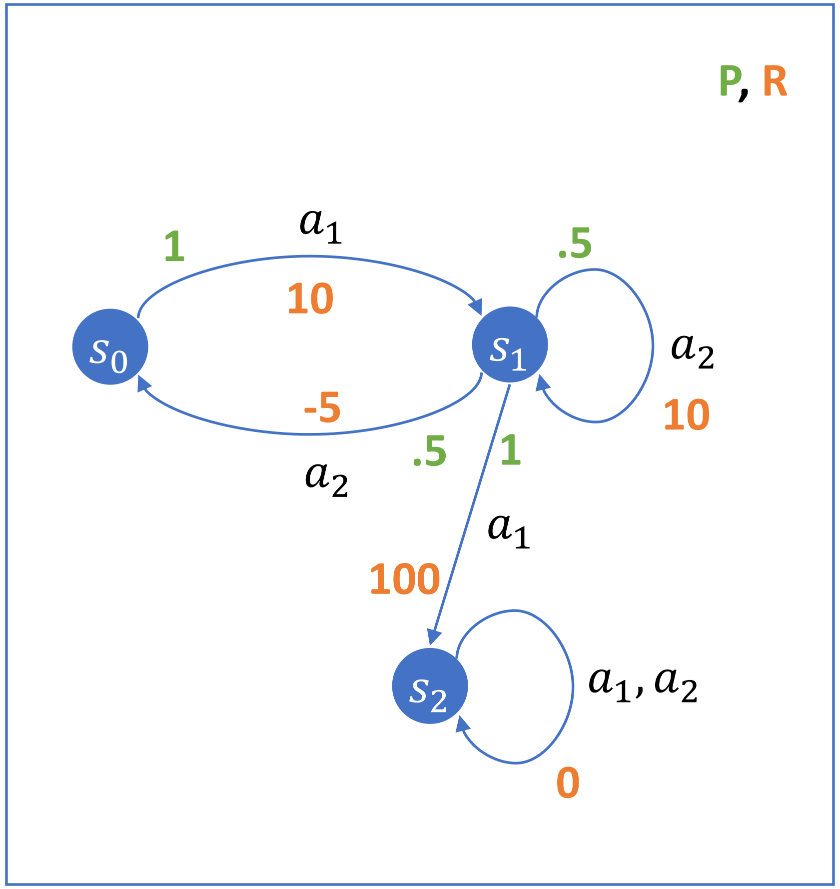
Definition: Markov Decision Process (MDP)

- MDPs are sequential decision problems with
  - a fully observable ($o_t = s_t$), stochastic environment;
  - a Markovian transition model: future states do not depend on past states give the current state;
  - additive rewards.

- MDPs are discrete-time stochastic control processes defines by:
  - a finite set of **states** $S = \{s_0, s_1, s_2, \dots\}$ (initial state s_0)
  - a set of available **actions** $ACTIONS(s)$ in each state s
  - a **transition model** $P(s' \mid s, a)$ where $a \in ACTIONS(s)$
  - a **reward function** $R(s)$ where the reward depends on the current state (often $R(s, a, s')$ is used)

- Time horizon
  - **Infinite horizon**: non-episodic (continuous) tasks with no terminal state.
  - **Finite horizon**: episodic tasks. Episode ends after a number of periods or when a terminal state is reached. Episodes contain a sequence of several actions that affect each other.

This is different from the previous definition of an **episodic** environment!



Example: 4x3 Grid World

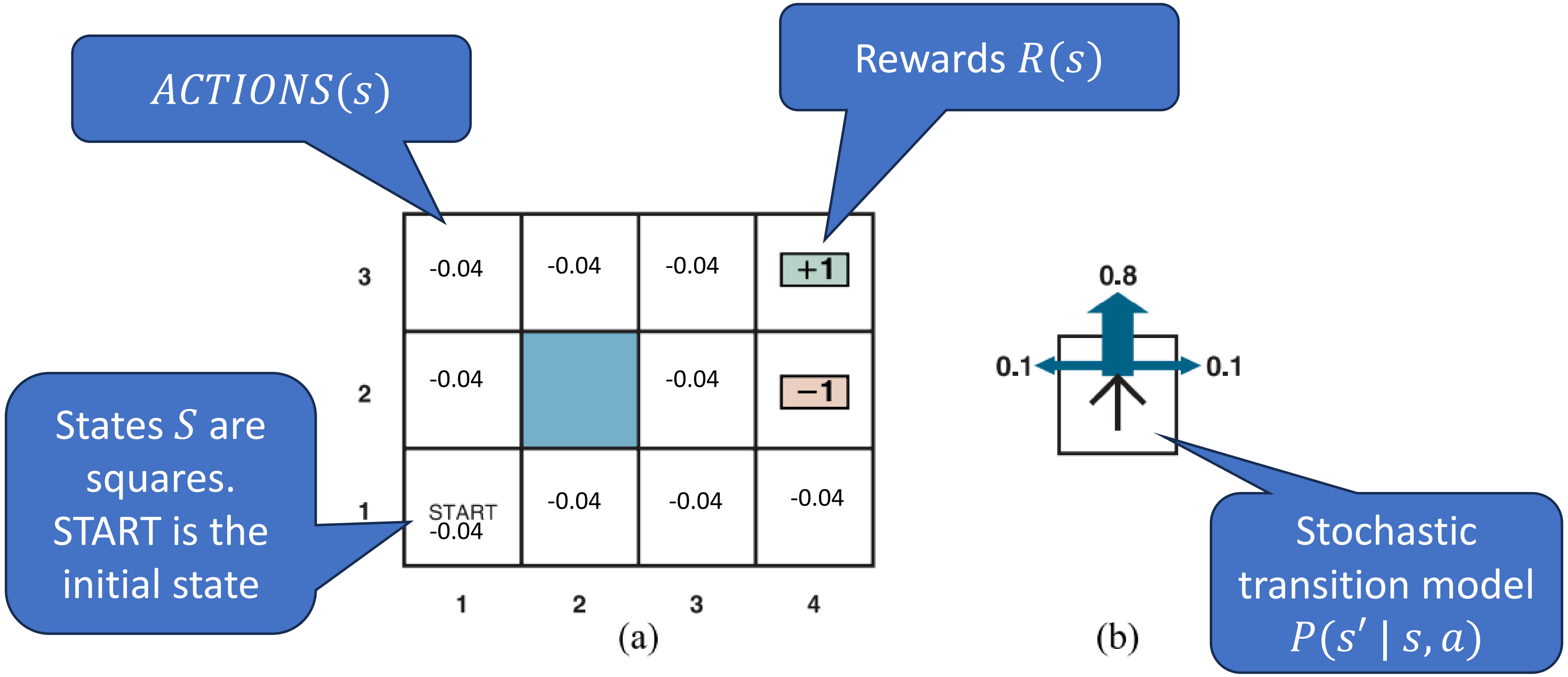


Figure 17.1 (a) A simple, stochastic 4×3 environment that presents the agent with a sequential decision problem. (b) Illustration of the transition model of the environment: the "intended" outcome occurs with probability 0.8, but with probability 0.2 the agent moves at right angles to the intended direction. A collision with a wall results in no movement. Transitions into the two terminal states have reward +1 and –1, respectively, and all other transitions have a reward of –0.04.

Goal

For each square: determine what direction should we try to go to maximize the total utility?

This is called a **policy** written as the function

$$\pi: S \rightarrow ACTIONS(S)$$

Optimal Policy

- A policy $\boldsymbol{\pi} = \{\pi(s_0), \pi(s_1), \dots\}$ defines for each state which action to take.
- The expected utility of being in state s under policy π can be calculated as the sum:

$$U^{\pi}(s) = E_{\pi}\left[\sum_{t=0}^{\infty} \gamma^t R(s_t) \,|s_0 = s\right]$$

γ is a discounting factor to give more weight to immediate rewards.

E_π is the expectation over sequences that can be created by following π.

- The goal of solving an MDP is to find an optimal policy $\boldsymbol{\pi}$ that maximizes the expected future utility for each state

$$\pi^*(s) = \operatorname*{argmax}_{\pi} U^{\pi}(s) \quad \text{for all } s \in S$$

- The recursive **Bellman equation** holds for the optimal value function U ("Bellman optimality condition"):

$$U^{\pi^*}(s) = R(s) + \gamma \max_{a \in A} \underbrace{\sum_{s'} P(s'|s,a)}_{\text{Expectation}} \; U^{\pi^*}(s')$$

Immediate Reward

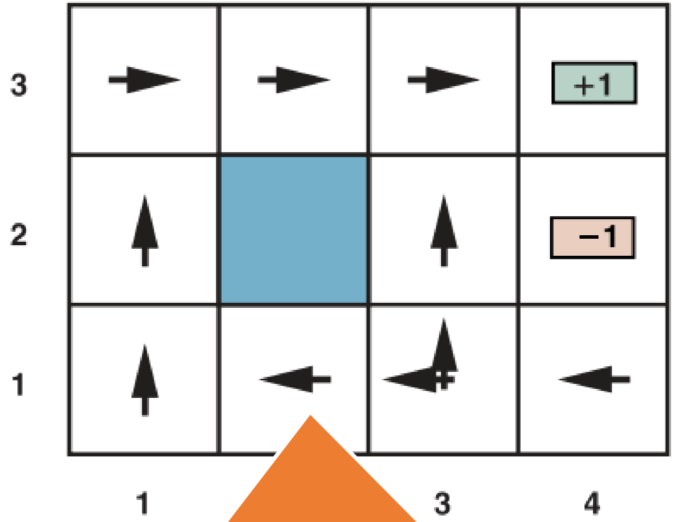
π^* uses the best action

Expectation

Utility of the next state

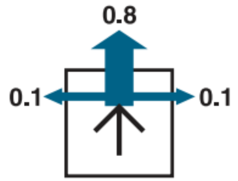
Solution: 4x3 Grid World

Optimal action in each state (policy π^*)

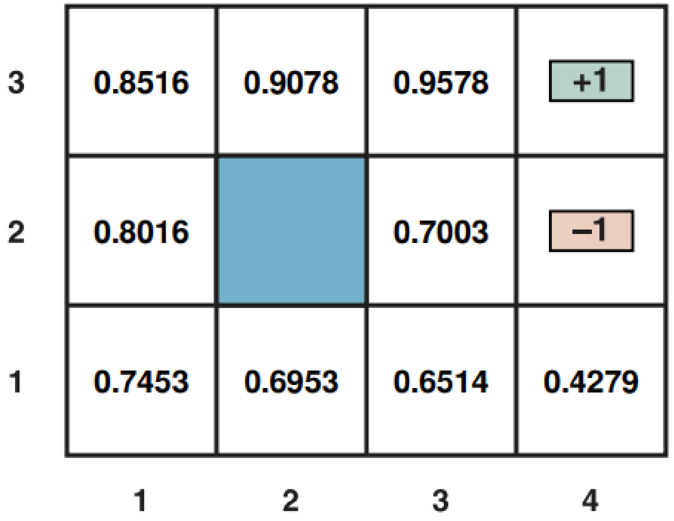


Why is it optimal to walk away from the +1 square?

Always pick the action with the highest expected utility.



Value of being in a state $U(s)$ (given that we will follow π^*)



$\gamma = 1$

How to we find the optimal value function/optimal policy?

Policy Iteration

Value Iteration

Q-Function

- $Q(s,a)$ is called the state-action value function. It gives the expected utility of action a in state s.

$$Q(s,a) = \underbrace{R(s)}_{\text{Immediate Reward}} + \gamma \sum_{s'} \underbrace{P(s'|s,a)[U(s')]}_{\text{Expected utility of the next state}}$$

- Relationship with the state value function: $U(s) = \max_a Q(s,a)$
- The Q-function is often used for convenience in algorithms.

Value Iteration: Estimate the Optimal Value Function U^{π^*}

Algorithm: Start with a $U(s)$ vector of 0 for all states and then update (Bellman update) the vector iteratively until it converges to the unique optimal solution U^{π^*}.

function VALUE-ITERATION(mdp, ϵ) **returns** a utility function
  inputs: mdp, an MDP with states S, actions $A(s)$, transition model $P(s' \mid s, a)$,
        rewards $R(s, a, s')$, discount γ
      ϵ, the maximum error allowed in the utility of any state
  local variables: U, U', vectors of utilities for states in S, initially zero
          δ, the maximum relative change in the utility of any state

  repeat
    $U \leftarrow U'$; $\delta \leftarrow 0$
    for each state s **in** S **do**
      $U'[s] \leftarrow \max_{a \in A(s)}$ Q-VALUE(mdp, s, a, U)
      if $|U'[s] - U[s]| > \delta$ **then** $\delta \leftarrow |U'[s] - U[s]|$
  until $\delta \leq \epsilon(1-\gamma)/\gamma$
  return U

Update with the value of the best action in state s.

Uses a proxy for policy loss
$\|U^\pi - U\|_\infty$ as the stopping criterion

U converges to U^{π^*}
and we can extract π^*

Policy Iteration: Learn the Optimal Policy π^*

Policy iteration tries to directly find the optimal policy by iterating policy evaluation and improvement.

function POLICY-ITERATION(mdp) **returns** a policy
  inputs: mdp, an MDP with states S, actions $A(s)$, transition model $P(s' \mid s, a)$
  local variables: U, a vector of utilities for states in S, initially zero
    π, a policy vector indexed by state, initially random

  repeat
    $U \leftarrow$ POLICY-EVALUATION(π, U, mdp)
    $unchanged? \leftarrow$ true
    for each state s **in** S **do**
      $a^* \leftarrow \underset{a \in A(s)}{\text{argmax}}$ Q-VALUE(mdp, s, a, U)
      if Q-VALUE(mdp, s, a^*, U) $>$ Q-VALUE($mdp, s, \pi[s], U$) **then**
        $\pi[s] \leftarrow a^*$; $unchanged? \leftarrow$ false
  until $unchanged?$
  return π

Calculate U given current policy (either solve an LP or value iteration with fixed policy)

Greedy policy Improvement

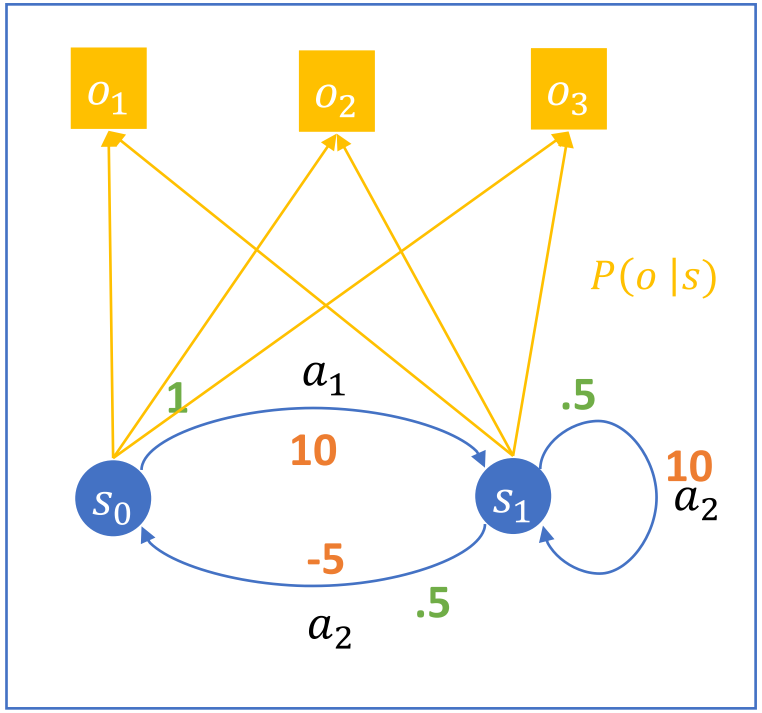
π converges to π^*
(and U converges to U^{π^*})

Partially Observable Markov Decision Model (POMDP)

- If the environment is **partially observable**, then $o_t \neq s_t$ and the model is expanded by
  - a **sensor model** $P(o \mid s)$ for receiving observation o given being in state s.

- This makes things a lot more complicated, and we have to work with **belief states**. A belief state is a distribution over states.
  Example: For a problem with three states, the belief state $b = (.2, .8, 0)$ means the agent beliefs that it is with 20% in state 1 and 80% in state 2 but not in state 3.

- An MDP that uses belief states instead of system states is called a **belief MDP**.
  Issue: the probabilities in belief states are continuous, and the number of different belief states is infinite.
- The solution of a POMDP is a policy with the optimal actions for sets of belief states (i.e., ranges of belief).
- For all but tiny problems, POMDPs can only be solved **approximately** (e.g., by grid-based methods).



Reinforcement Learning

AIMA Chapter 22

Reinforcement Learning (RL)

- RL assumes that the problem can be modeled by an **MDP**.

- What if we do not know the exact transition model $P(s' \mid s, a)$?

  Now we cannot solve the MDP (estimate the state utility function/policy) because we cannot predict what the future states after an action will be!

- The agent needs to explore (try actions) and **use the reward signal to update its estimate of the utility of states and actions**. This is a learning process where the reward provides positive reinforcement.

Q-Learning

- Q-Learning learns the state-action value function from interactions with the environment (percepts).
- This agent function learns a **Q-table** for the state-action function Q.

Q-Table

s	a	$Q(s,a)$

function Q-LEARNING-AGENT(*percept*) **returns** an action
inputs: *percept*, a percept indicating the current state s' and reward signal r
persistent: Q, a table of action values indexed by state and action, initially zero
N_{sa}, a table of frequencies for state–action pairs, initially zero
s, a, the previous state and action, initially null

if s is not null **then**
increment $N_{sa}[s,a]$
$Q[s,a] \leftarrow Q[s,a] + \alpha(N_{sa}[s,a])(r + \gamma \max_{a'} Q[s',a'] - Q[s,a])$
$s, a \leftarrow s', \operatorname{argmax}_{a'} f(Q[s',a'], N_{sa}[s',a'])$
return a

New episode has no s.

Learning rate

Make $Q[s,a]$ a little more similar to the received reward + the best Q-value of the successor state.

f is the exploration function and decides on the next action. As N increases it can exploit good actions more.

Value Function Approximation

- U (Q) needs to store and estimate one entry for each state (state/action combination).
- Issues and solutions
  - Too many entries to store → lossy compression
  - Many combinations are rarely seen → generalize to unseen entries

- **Idea**: Estimate the state value by learning an approximation function $\hat{U}(s) = g_\theta(s)$ based on features of s.
- **Example**: 4x3 Grid World with a linear combination of state features (x, y) and learn $\boldsymbol{\theta}$ from observed data.

$U(s)$

	1	2	3	4
3	0.8516	0.9078	0.9578	+1
2	0.8016		0.7003	−1
1	0.7453	0.6953	0.6514	0.4279

Learn $\boldsymbol{\theta}$ from observed interactions with the environment to approximate $U(s)$

$$\hat{U}_\theta(x, y) = \theta_0 + \theta_1 x + \theta_2 y$$

$\boldsymbol{\theta}$ can be updated iteratively after each new observed utility using gradient descent.

Traditional Q-Learning

s →

s	a	$Q(s,a)$
1	1	0.7
1	2	0.3
…	…	…

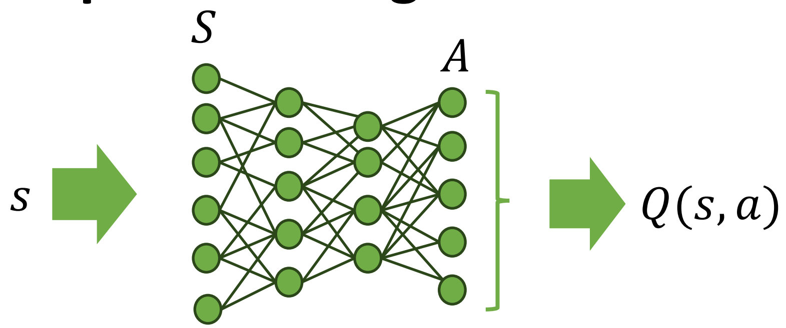
→ $Q(s,a)$

Q-Table

function Q-LEARNING-AGENT(*percept*) **returns** an action
  inputs: *percept*, a percept indicating the current state s' and reward signal r
  persistent: Q, a table of action values indexed by state and action, initially zero
    N_{sa}, a table of frequencies for state–action pairs, initially zero
    s, a, the previous state and action, initially null

  if s is not null **then**
    increment $N_{sa}[s,a]$
    $Q[s,a] \leftarrow Q[s,a] + \alpha(N_{sa}[s,a])(\underbrace{r + \gamma \max_{a'} Q[s',a']}_{\text{target}} - \underbrace{\boxed{Q[s,a]}}_{\text{prediction}})$
  $s, a \leftarrow s', \mathrm{argmax}_{a'} f(Q[s',a'], N_{sa}[s',a'])$
  return a

Deep Q-Learning



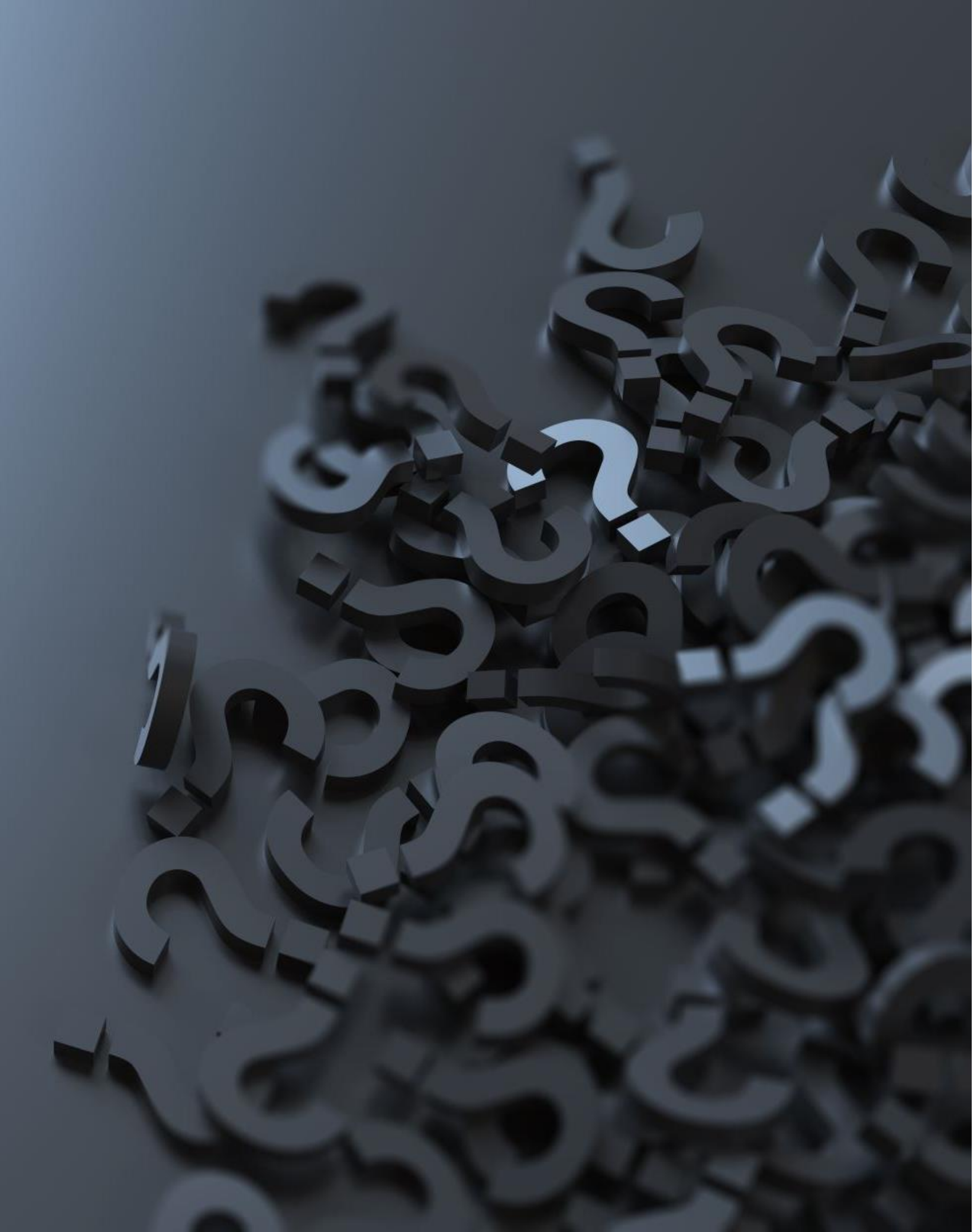
Deep Q-Network (DQN)

Target networks: It turns out that the Q-Network is unstable if the same network is used to estimate $Q(s,a)$ and also $Q(s',a')$. Deep Q-Learning uses a second target network for $Q(s',a')$ that is updated with the prediction network every C steps.

Experience replay: To reduce instability more, generate actions using the current network and store the experience $\langle s,a,r,s' \rangle$ in a table. Update the model parameters by sampling from the table.

Loss function: squared difference between prediction and target.

Volodymyr Mni et al., [Playing Atari with Deep Reinforcement Learning](), NIPS Deep Learning Workshop 2013.



Summary

- Agents can learn the value of being in a state from **reward signals**.
- Rewards can be delayed (e.g., at the end of a game).
- Not being able to fully **observe the state** makes the problem more difficult (POMDP).
- **Unknown transition models** lead to the need of exploration by trying actions (model free methods like Q-Learning).
- All these problems are computationally very expensive and often can only be solved by **approximation**. State of the art is to use deep artificial neural networks for function approximation.