CS 5/7320
Artificial Intelligence

Reinforcement Learning
AIMA Chapter 17+22

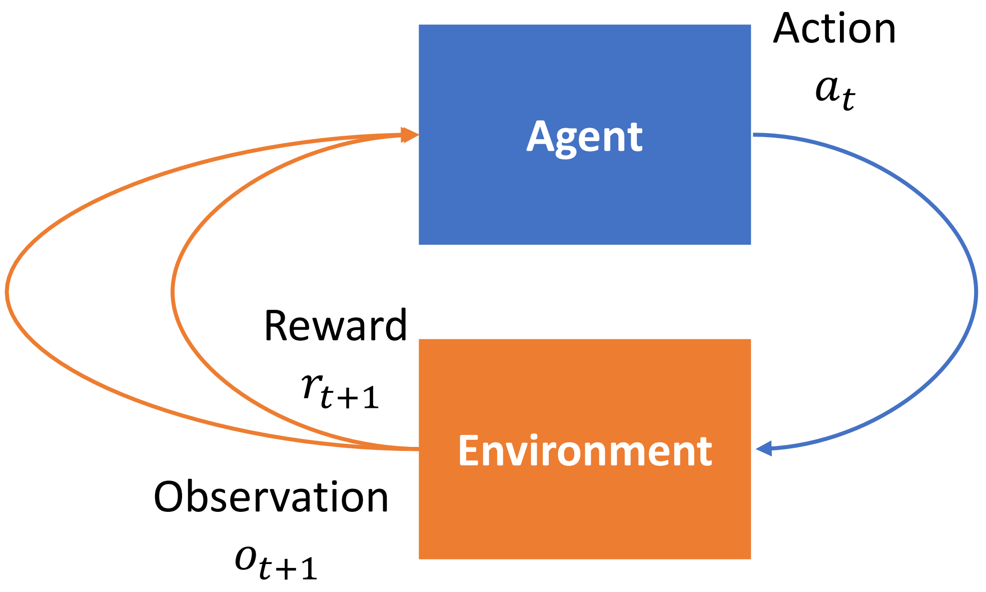
Slides by Michael Hahsler
with figures from the AIMA textbook.

Sequential Decision Problems

AIMA Chapter 17: Making Complex Decisions

Sequential Decision Problems

- **Utility-based agent**: The agent's utility depends on a sequence of decisions spread out over time.
- Sequential decision problems incorporate utilities, uncertainty, and sensing.



Sequence: $r_0, o_0, a_0, r_1, o_1, a_1, r_2, o_2, a_2, , \ldots$

Observations and rewards depend on the state of the system and the agent wants to maximize (discounted) expected reward over time

$$U = E\left[\sum_{t=0}^{\infty} \gamma^t r_t\right]$$

Markov Decision Process (MDP)

- Models a fully observable environment: The agent's observation is the state $o_t = s_t$.
- An MDP defines a sequential decision problem with
  - a finite set of states S (initial state s_0)
  - a set of available actions $ACTIONS(s)$ in each state s
  - a transition model $P(s' \mid s, a)$ where $a \in ACTIONS(s)$
  - a reward function $R(s)$ where the reward depends on the current state.

- The goal is to find an **optimal policy $\boldsymbol{\pi}^*$** that prescribes for each state the optimal action $\pi(s)$ to maximize the expected utility over time.

Example: 4x3 Grid World

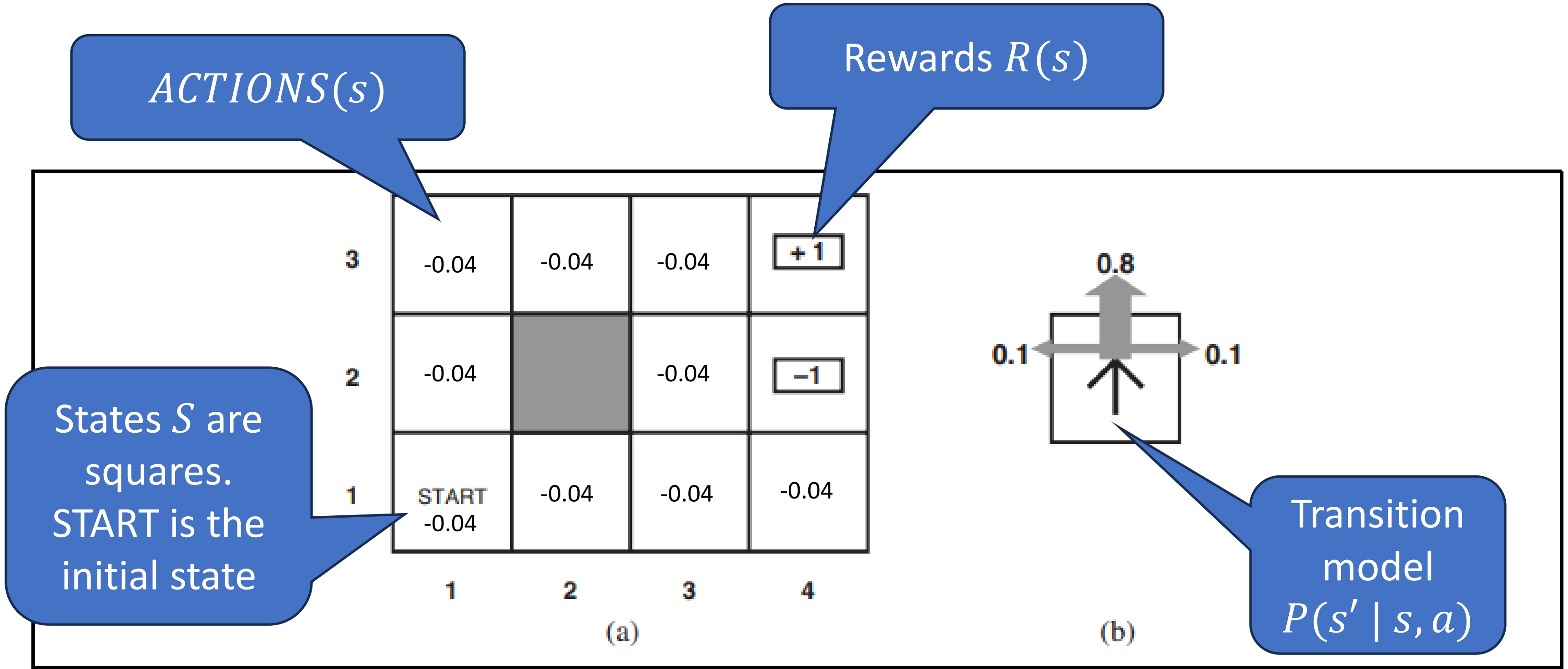
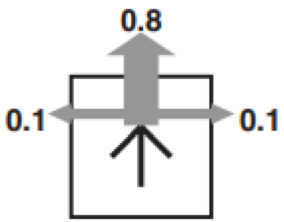


Figure 17.1 (a) A simple 4×3 environment that presents the agent with a sequential decision problem. (b) Illustration of the transition model of the environment: the "intended" outcome occurs with probability 0.8, but with probability 0.2 the agent moves at right angles to the intended direction. A collision with a wall results in no movement. The two terminal states have reward +1 and –1, respectively, and all other states have a reward of –0.04.

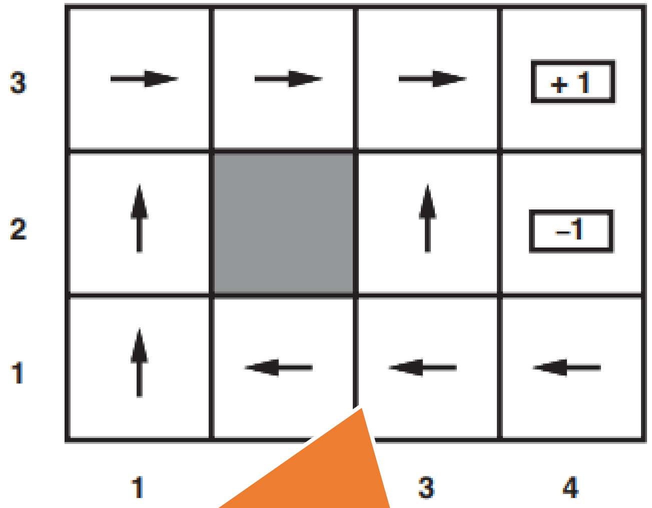
Goal: What direction should we go in each square?

$$\pi(s)$$

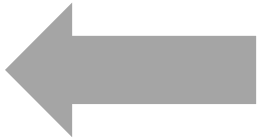
Solution: 4x3 Grid World



Optimal action in each state (policy π^*)



Why is it optimal to walk away from the +1 square?



Always move to higher utility states

Value of being in a state $U(s)$ (given that we will follow π^*)

	1	2	3	4
3	0.812	0.868	0.918	+ 1
2	0.762		0.660	–1
1	0.705	0.655	0.611	0.388

$\gamma = 1$

Question: How to we find the optimal value function/optimal policy?

Value Iteration: Estimate the Value function

function VALUE-ITERATION(mdp, ϵ) **returns** a utility function
inputs: mdp, an MDP with states S, actions $A(s)$, transition model $P(s' \mid s, a)$,
rewards $R(s)$, discount γ
ϵ, the maximum error allowed in the utility of any state
local variables: U, U', vectors of utilities for states in S, initially zero
δ, the maximum change in the utility of any state in an iteration

repeat
$U \leftarrow U'$; $\delta \leftarrow 0$
for each state s **in** S **do**
$$U'[s] \leftarrow R(s) + \gamma \max_{a \in A(s)} \sum_{s'} P(s' \mid s, a)\, U[s']$$
if $|U'[s] - U[s]| > \delta$ **then** $\delta \leftarrow |U'[s] - U[s]|$
until $\delta < \epsilon(1-\gamma)/\gamma$
return U

Bellman update: Update a state with the reward + the expected utility of the state reached with the best action

U converges to U^{π^*} and we can extract π^*

Policy Iteration: Learn the optimal policy

function POLICY-ITERATION(mdp) **returns** a policy
  inputs: mdp, an MDP with states S, actions $A(s)$, transition model $P(s' \mid s, a)$
  local variables: U, a vector of utilities for states in S, initially zero
    π, a policy vector indexed by state, initially random

  repeat
    $U \leftarrow$ POLICY-EVALUATION(π, U, mdp)
    $unchanged? \leftarrow$ true
    for each state s **in** S **do**
      if $\max_{a \in A(s)} \sum_{s'} P(s' \mid s, a)\ U[s'] > \sum_{s'} P(s' \mid s, \pi[s])\ U[s']$ **then do**
        $\pi[s] \leftarrow \operatorname{argmax}_{a \in A(s)} \sum_{s'} P(s' \mid s, a)\ U[s']$
        $unchanged? \leftarrow$ false
  until $unchanged?$
  return π

Calculate U given current policy
(eighter solve an LP or iterative solution)

Policy
Improvement

π converges to π^*
(and U converges to U^{π^*})

Partially Observable Markov Decision Model (POMDP)

- If the environment is partially observable, then the model is expanded by
  - a **sensor model** $P(o \mid s)$ for receiving observation o given being in state s.

- This makes things a lot more complicated, and we have to work with **belief states**. A belief state is a distribution over states.
  Example: For a problem with three states, the belief state $b = (.2, .8, 0)$ means the agent beliefs that it is 20% in state 1 and 80% in state 2 but not in state 3.

- An MDP that uses belief states is called a **belief MDP**. Issue: belief states are continuous, and the number of different belief states is infinite.
- The solution of a POMDP is a policy with the optimal actions for sets of belief states (i.e., ranges of belief).
- For all but tiny problems, POMDPs can only be solved **approximately** (e.g., by grid-based methods).

Reinforcement Learning

AIMA Chapter 22

Reinforcement Learning

- Reinforcement learning assumes that the problem can be modeled by an MDP.

- What if we do not know the transition model $P(s' \mid s, a)$?

  Now we cannot solve the MDP (estimate the state utility function/policy) because we cannot predict future states!

- The agent needs to explore (try actions) and use the reward signal to update its estimate (=learn) of the utility of states and actions.

Q-Learning

- Q-Learning learns the state-action value function $Q(s, a)$ where $U(s) = \max_a Q(s, a)$.

function Q-LEARNING-AGENT(*percept*) **returns** an action
  inputs: *percept*, a percept indicating the current state s' and reward signal r'
  persistent: Q, a table of action values indexed by state and action, initially zero
    N_{sa}, a table of frequencies for state–action pairs, initially zero
    s, a, r, the previous state, action, and reward, initially null

  if TERMINAL?(s) **then** $Q[s, None] \leftarrow r'$
  if s is not null **then**
    increment $N_{sa}[s, a]$
    $Q[s, a] \leftarrow Q[s, a] + \alpha(N_{sa}[s, a])(r + \gamma \max_{a'} Q[s', a'] - Q[s, a])$
  $s, a, r \leftarrow s', \operatorname{argmax}_{a'} f(Q[s', a'], N_{sa}[s', a']), r'$
  return a

Make $Q[s, a]$ a little more similar to the received reward + the best Q-value of the successor state.

f is the exploration function and decides on the next action. As N increases it can exploit good actions more.

Function Approximation

- U (Q) needs to store and estimate one entry for each state (state/action combination)!
- Issues and solutions
  - Too many entries to store → lossy compression
  - Many combinations are rarely seen → generalize to unseen entries

- **Idea**: Estimate the state value by learning a approximation function $\widehat{U}(s) = g_\theta(s)$ based on features of s.
- 4x3 Grid World Example: Use a linear combination of state features (x, y) and learn $\boldsymbol{\theta}$ from observed data.

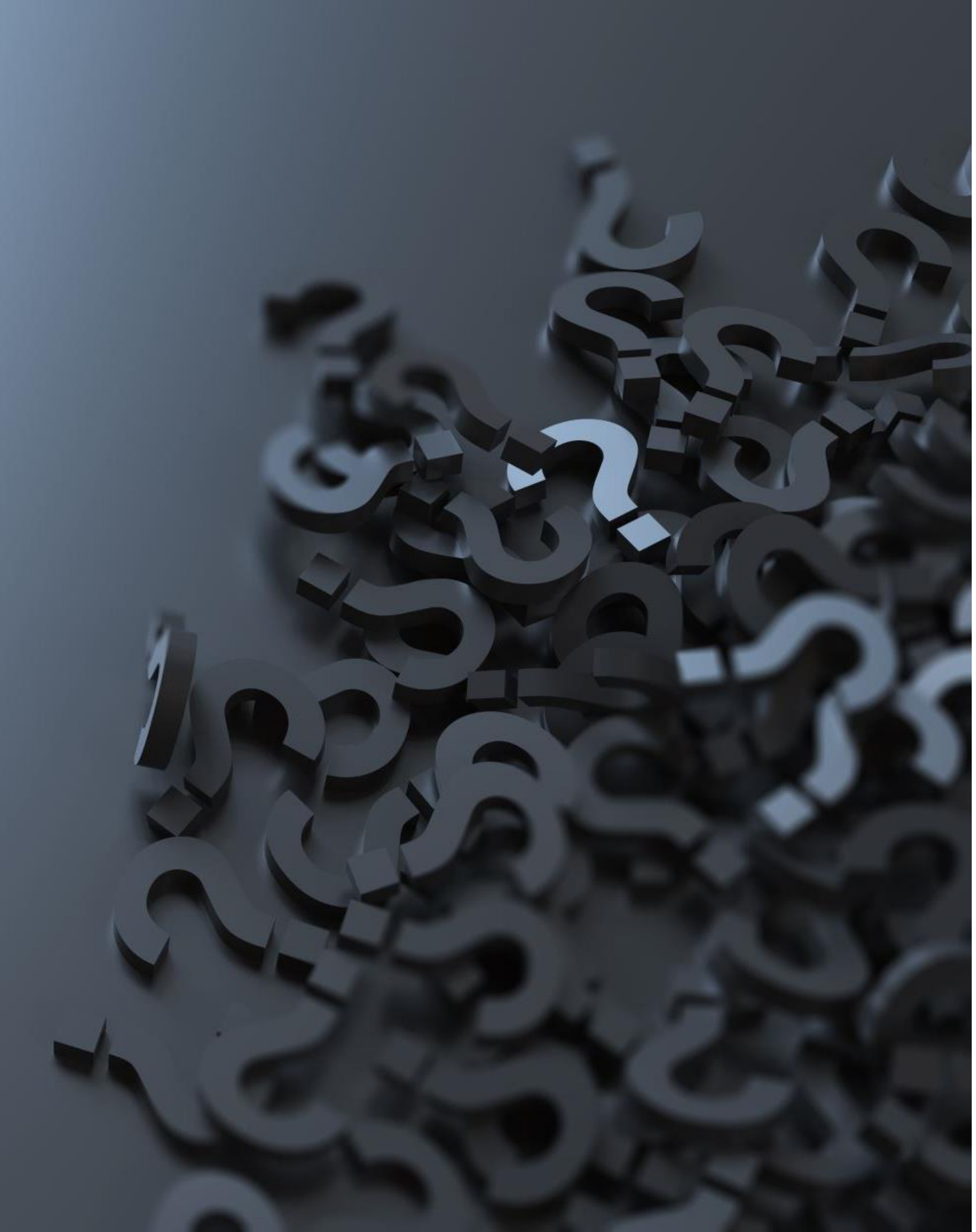
$$\widehat{U}_{theta}(x, y) = \theta_0 + \theta_1 x + \theta_2 y$$

Learn $\boldsymbol{\theta}$ from observed interactions with the environment to approximate $U(s)$

$U(s)$

3	0.812	0.868	0.918	+1
2	0.762		0.660	–1
1	0.705	0.655	0.611	0.388
	1	2	3	4

Notes:
We can also approximate the state-value function Q for Q-learning.
We typically need non-linear approximators that can be incrementally updated (online learning). → Deep ANNs



Summary

- Agents can learn the value of being in a state from **reward signals**.
- Rewards can be delayed (e.g., at the end of a game).
- Not being able to fully **observe the state** makes the problem more difficult (POMDP).
- **Unknown transition models** lead to the need of exploration by trying actions (model free methods like Q-Learning).
- All these problems are computationally very expensive and often can only be solved by **approximation**. State of the art is to use deep artificial neural networks for function approximation.