Discussion

CS 5/7320
Artificial Intelligence

# Solving problems by searching

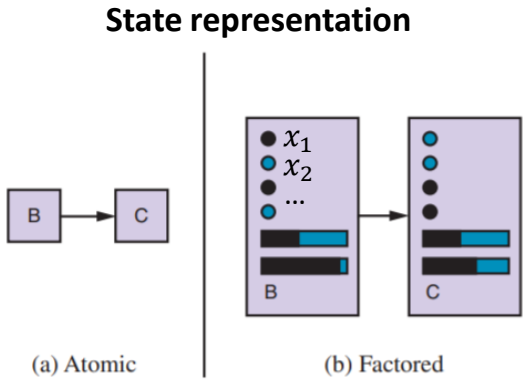
AIMA Chapter 3

Slides by Michael Hahsler
based on slides by Svetlana Lazepnik
with figures from the AIMA textbook.


State Space for Search

# State Space

- Number of different states the agent and environment can be in.
- **Reachable states** are defined by the initial state and the transition model. Not all states may be reachable from the initial state.
- **Search tree** spans the state space. Note that a single state can be represented by several search tree nodes if we have redundant paths.
- State space size is an indication of problem size.



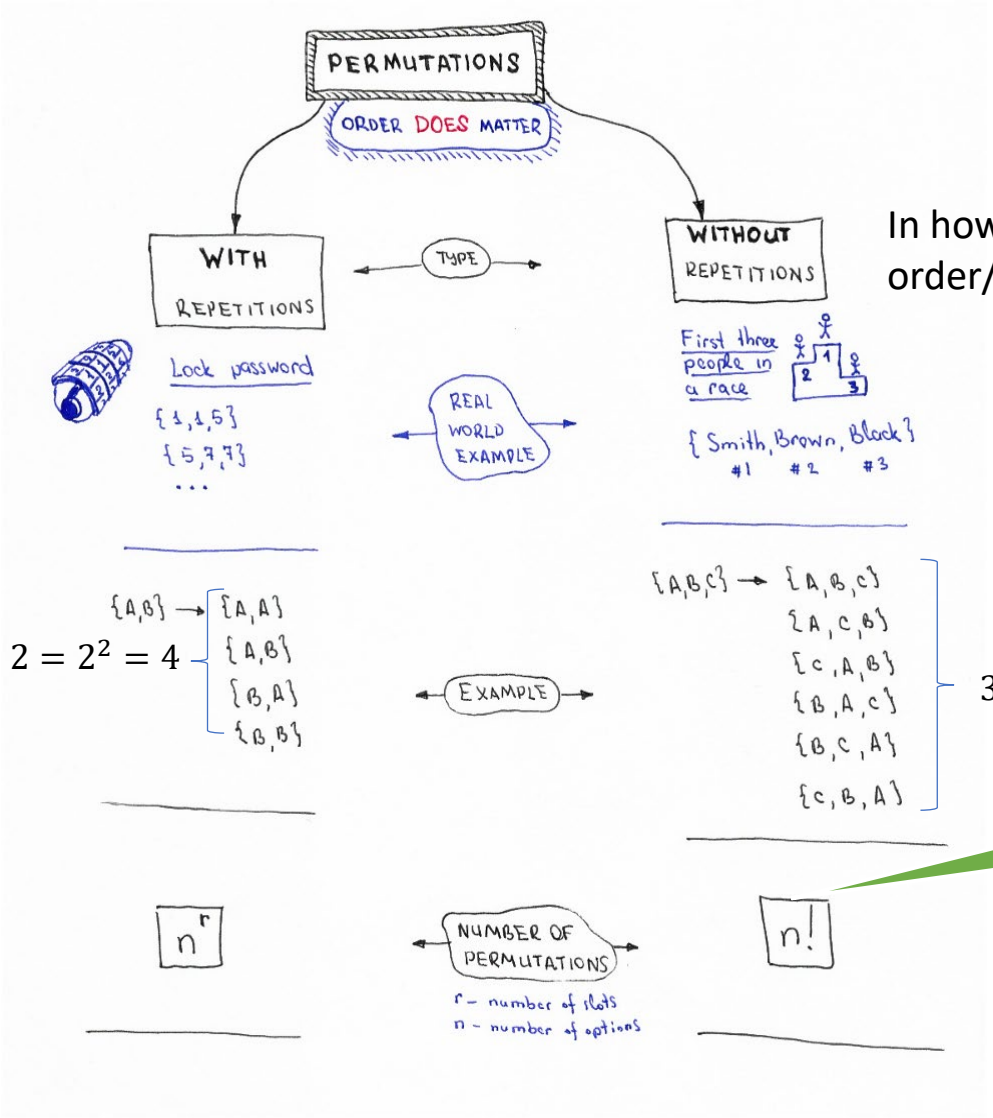

The state consists of variables called fluents that represent conditions that can change over time.

## State Space Size Estimation

- Even if the used algorithm represents the state space using atomic states, we may know that internally they have a factored representation that can be used to estimate the problem size.
- The basic rule to calculate (estimate) the state space size for factored state representation with $n$ fluents (variables) is:

$$|x_1| \times |x_2| \times \cdots \times |x_n|$$

where $|\cdot|$ is the number of possible values.




In how many ways can we order/arrange n objects?

$2 \times 2 = 2^2 = 4$

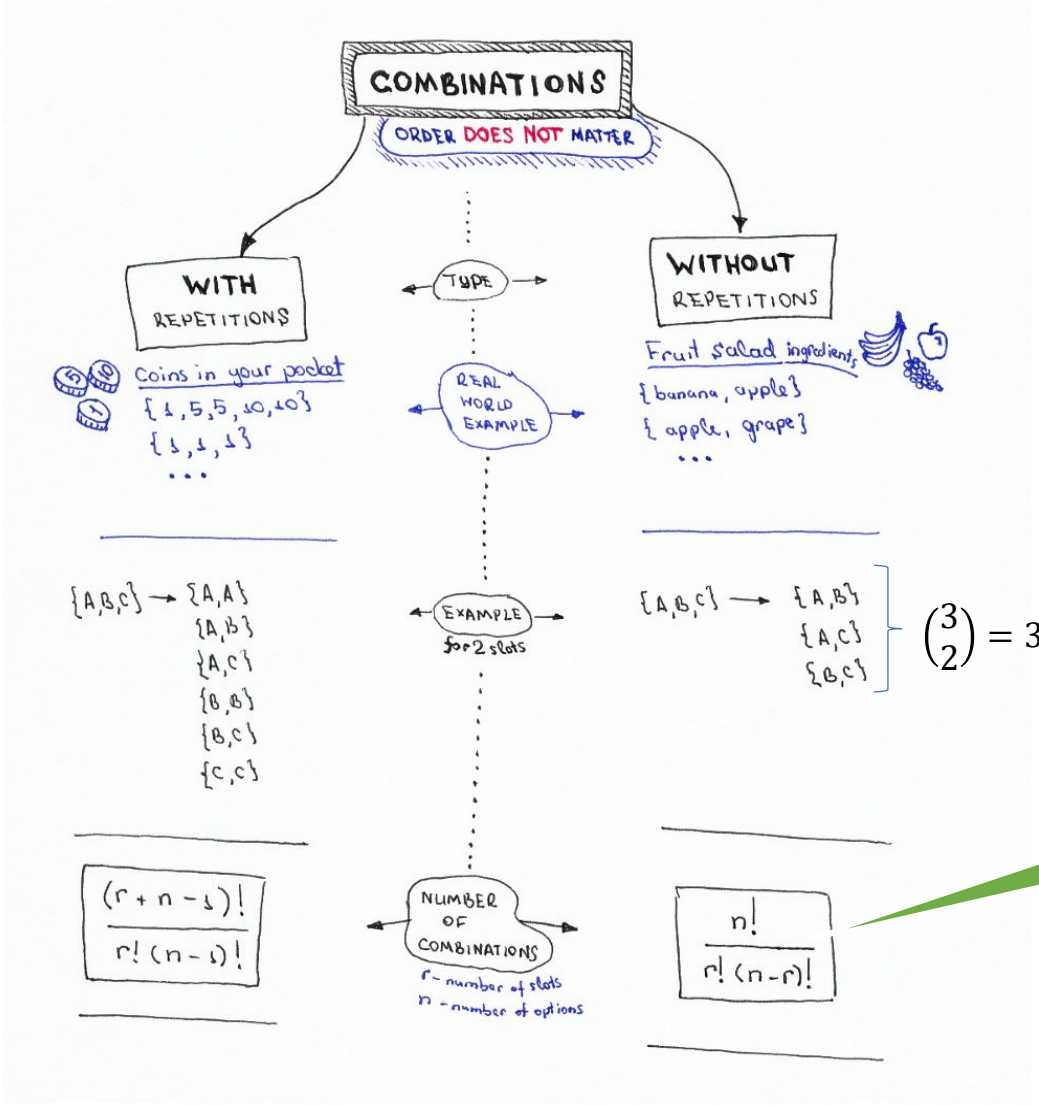
$3 \times 2 \times 1 = 6$

**Factorial**: $n! = n \times (n-1) \times \cdots \times 2 \times 1$

**#Python**
**import math**

**print (math.factorial(23))**

Source: Permutations/Combinations Cheat Sheets by Oleksii Trekhleb
https://itnext.io/permutations-combinations-algorithms-cheat-sheet-68c14879aba5




$\binom{3}{2} = 3$

**Binomial Coefficient:** $\binom{n}{r} = C(n,r) = {}_nC_r$

Read as "n choose r" because it is the number of ways can we choose $r$ out of $n$ objects?

Special case for $r = 2$: $\binom{n}{2} = \frac{n(n-1)}{2}$

**#Python**
**import scipy.special**

**# the two give the same results**
**scipy.special.binom(10, 5)**
**scipy.special.comb(10, 5)**

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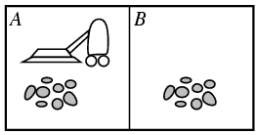
# Examples: What is the state space size?




**Dirt**

- **Permutation:** A and B are different rooms, order does matter!
- **With repetition:** Dirt can be in both rooms.
- There are 2 options (clean/dirty)

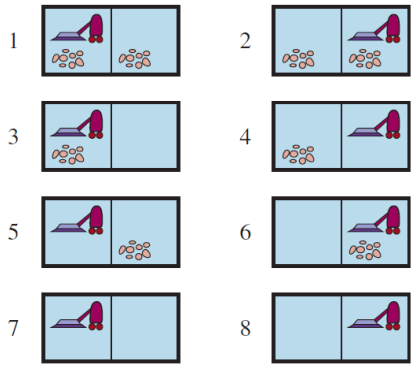
$\rightarrow 2^2$

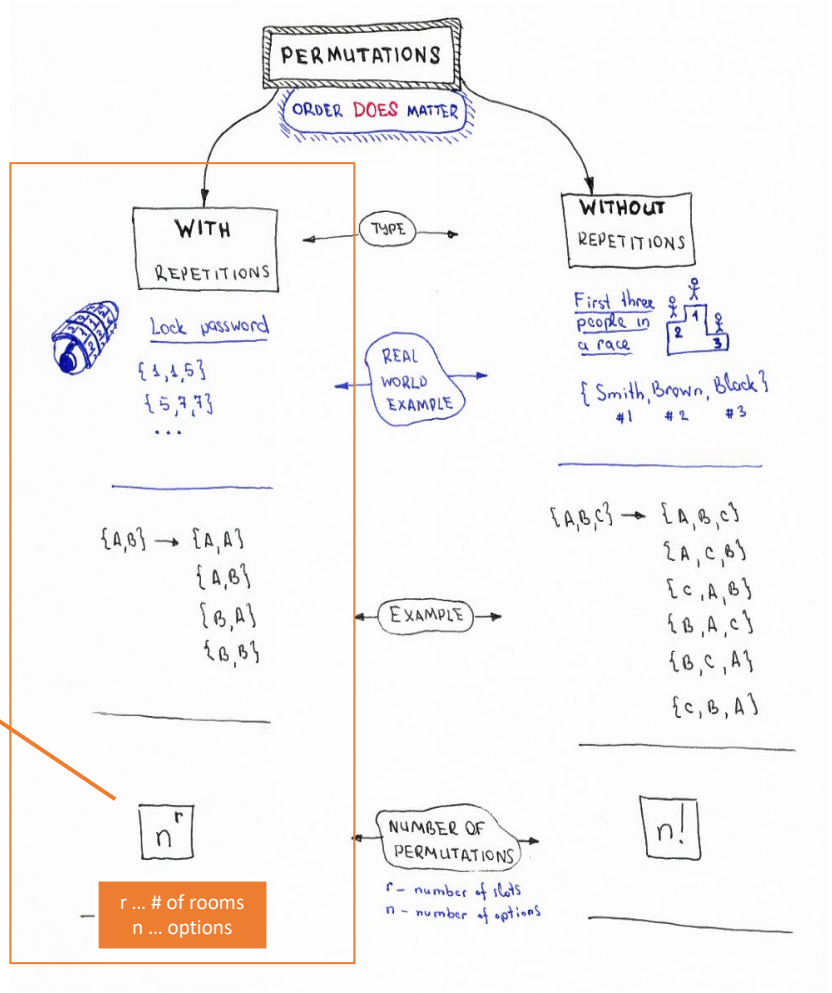
**Robot location**

- Can be in 1 out of 2 rooms.

$\rightarrow 2$

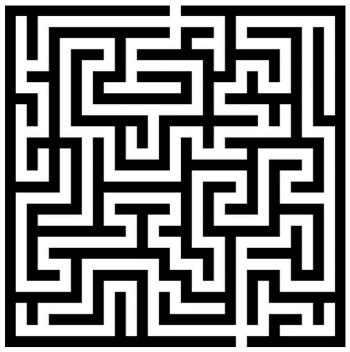
Total: $2 \times 2^2 = 2^3 = 8$



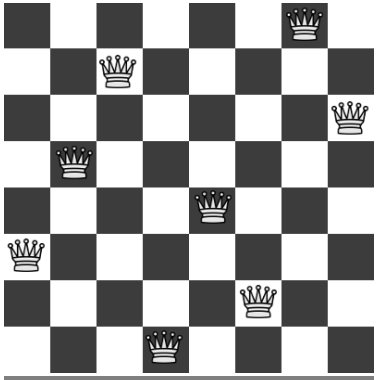



# Examples: What is the state space size?

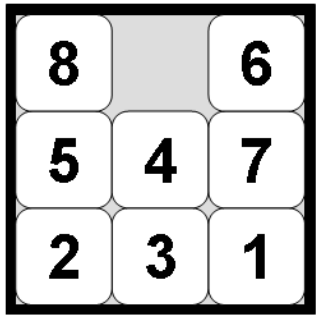
Often a rough upper limit is sufficient to determine how hard the search problem is.



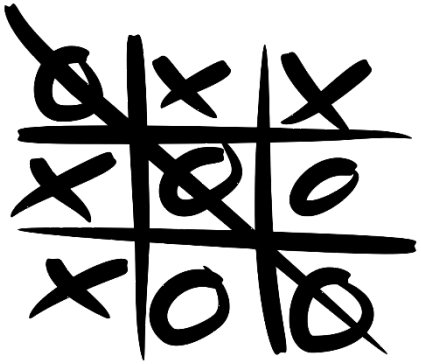
Maze



8-queens problem




8-puzzle problem



Tic-tac-toe

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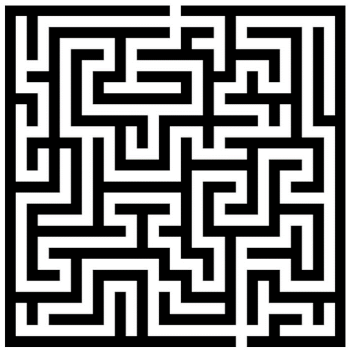
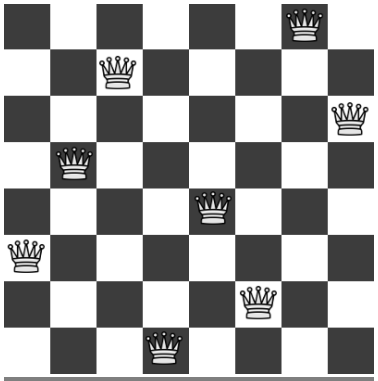


Maze

Positions the agent can be in.

n = Number of white squares.



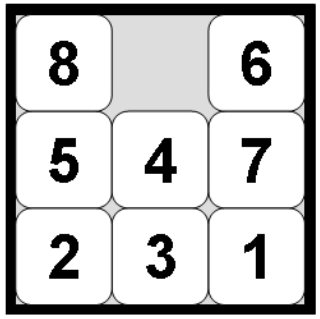
8-queens problem

All arrangements with 8 queens on the board.

$n < 2^{64} \approx 1.8 \times 10^{19}$

We can only have 8 queens:

$n = \binom{64}{8} \approx 4.4 \times 10^{9}$



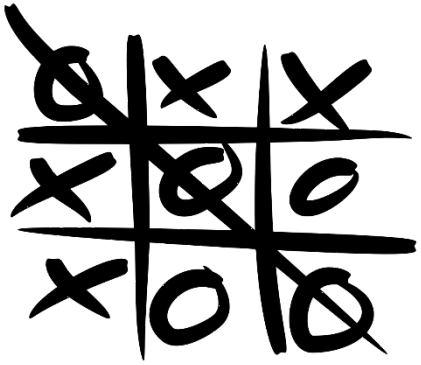

8-puzzle problem

All arrangements of 9 elements.

$n \leq 9!$

Half is unreachable:

$n = \frac{9!}{2} = 181{,}440$



Tic-tac-toe

All possible boards.

$n < 3^9 = 19{,}683$

Many boards are not legal (e.g., all x's)

Assignment

# Q&A