

# MODELOS DE DECISIÓN

- NO RESTRINGIDOS

MAX:  $f(X)$                       o                      MIN:  $f(X)$

Ejemplo:                      MIN :  $3 \cdot x_1 + \frac{2}{x_1} + \ln x_2 + 4 \cdot x_1 \cdot x_2$

- RESTRINGIDOS  
(PROGRAMAS MATEMÁTICOS)

# PROGRAMACIÓN MATEMÁTICA

$$\text{MAX:} \quad Z = f(x)$$

Sujeto a:

$$\left\{ \begin{array}{l} g_1(x) \leq b_1 \\ g_2(x) \leq b_2 \\ \dots\dots\dots \\ g_m(x) \leq b_m \end{array} \right.$$

# PROGRAMACIÓN LINEAL

Maximizar

$$\sum c_j x_j$$

sujeto a un conjunto  
de restricciones

$$\sum a_{ij} x_j \leq b_i$$

siendo

$$x_j \geq 0$$

# FUNCIÓN OBJETIVO

Maximizar

$$Z = \underbrace{\sum c_j x_j}_{\text{FUNCIONAL}}$$

FUNCIONAL

Ejemplo:  $Z = 6x_1 + 8x_2 + 3x_3$

Coeficientes del funcional

# FUNCIÓN OBJETIVO

Maximizar

$$Z = \underbrace{\sum c_j x_j}_{\text{FUNCIONAL}}$$

FUNCIONAL

Ejemplo:  $Z = 6x_1 + 8x_2 + 3x_3$

Variables de decisión

# RESTRICCIONES

Conjunto de inecuaciones o ecuaciones

**CONDICIONES  
DE VÍNCULO**

$$\left\{ \begin{array}{l} \sum a_{ij} x_j \leq b_i \\ \sum a_{ij} x_j \geq b_i \\ \sum a_{ij} x_j = b_i \end{array} \right.$$

# CONDICIONES DE VÍNCULO

Ejemplo:  $12x_1 + 9x_2 + 4x_3 \leq 500$

COEFICIENTES TECNOLÓGICOS

RHS

# CONDICIONES DE LAS VARIABLES

$$X_j$$

- NO NEGATIVIDAD
- CONTINUIDAD



# CONDICIONES DE LOS TÉRMINOS INDEPENDIENTES ("RHS")

$b_i$

- NO NEGATIVIDAD  
(en su forma estándar)

INECUACIÓN  $\longrightarrow$  ECUACIÓN

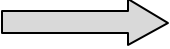
$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \leq 500$$

# VARIABLES “SLACKS”

$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \leq 500$$

$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 + x_4 = 500$$

DE HOLGURA

INECUACIÓN  ECUACIÓN

$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \leq 100$$

# VARIABLES “SLACKS”

$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \leq 100$$

$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 - x_4 = 100$$

|  
SUPERFLUA

# FORMA NATURAL

**MAX:**  $6 \cdot x_1 + 8 \cdot x_2 + 3 \cdot x_3$

**Sujeto a:**

$$\left\{ \begin{array}{l} 12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \leq 500 \\ 3 \cdot x_1 + 15 \cdot x_2 + 6 \cdot x_3 \leq 700 \\ 2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \leq 100 \\ 7 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 = 200 \end{array} \right.$$

**siendo:**  $x_j \geq 0$

# FORMAS DE FORMULACIÓN DE UN MODELO DE PL

- **FORMA NATURAL:** Restricciones de “ $\leq$ ”, “ $\geq$ ” y “ $=$ ”
- **FORMA CANÓNICA**
  - De MAX: Todas las restricciones de “ $\leq$ ”
  - De MIN: Todas las restricciones de “ $\geq$ ”
- **FORMA ESTÁNDAR**
  - Todas las restricciones de “ $=$ ”

# FORMA ESTÁNDAR

**MAX:**  $6 \cdot x_1 + 8 \cdot x_2 + 3 \cdot x_3$

**Sujeto a:**

$$\left\{ \begin{array}{lcl} 12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 + x_4 & = & 500 \\ 3 \cdot x_1 + 15 \cdot x_2 + 6 \cdot x_3 + x_5 & = & 700 \\ 2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 - x_6 & = & 100 \\ 7 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 & = & 200 \end{array} \right.$$

**siendo:**  $x_j \geq 0$



# FORMA CANÓNICA DE MAX

## NATURAL

$$\text{MAX: } 6 \cdot x_1 + 8 \cdot x_2 + 3 \cdot x_3$$

$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \leq 500$$

$$3 \cdot x_1 + 15 \cdot x_2 + 6 \cdot x_3 \leq 700$$

$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \leq 100$$

$$7 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 = 200$$

$$x_j \geq 0$$

## CANÓNICA

$$\text{MAX: } 6 \cdot x_1 + 8 \cdot x_2 + 3 \cdot x_3$$

$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \leq 500$$

$$3 \cdot x_1 + 15 \cdot x_2 + 6 \cdot x_3 \leq 700$$

$$-2 \cdot x_1 - 2 \cdot x_2 - 3 \cdot x_3 \leq -100$$

$$7 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 \leq 200$$

$$-7 \cdot x_1 - 4 \cdot x_2 - 3 \cdot x_3 \leq -200$$

$$x_j \geq 0$$

# FORMA CANÓNICA DE MIN

## NATURAL

$$\text{MAX: } 6 \cdot x_1 + 8 \cdot x_2 + 3 \cdot x_3$$

$$12 \cdot x_1 + 9 \cdot x_2 + 4 \cdot x_3 \leq 500$$

$$3 \cdot x_1 + 15 \cdot x_2 + 6 \cdot x_3 \leq 700$$

$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \leq 100$$

$$7 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 = 200$$

$$x_j \geq 0$$

## CANÓNICA

$$\text{MIN: } -6 \cdot x_1 - 8 \cdot x_2 - 3 \cdot x_3$$

$$-12 \cdot x_1 - 9 \cdot x_2 - 4 \cdot x_3 \geq -500$$

$$-3 \cdot x_1 - 15 \cdot x_2 - 6 \cdot x_3 \geq -700$$

$$2 \cdot x_1 + 2 \cdot x_2 + 3 \cdot x_3 \leq 100$$

$$7 \cdot x_1 + 4 \cdot x_2 + 3 \cdot x_3 \leq 200$$

$$-7 \cdot x_1 - 4 \cdot x_2 - 3 \cdot x_3 \geq -200$$

$$x_j \geq 0$$

# FORMA CANÓNICA

**MAX:**  $c_1 x_1 + c_2 x_2 + c_3 x_3 + \dots + c_k x_k$

**Sujeto a:**

$$\left\{ \begin{array}{l} a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + \dots + a_{1k} x_k \leq b_1 \\ a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + \dots + a_{2k} x_k \leq b_2 \\ a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + \dots + a_{3k} x_k \leq b_3 \\ \dots \\ a_{m1} x_1 + a_{m2} x_2 + a_{m3} x_3 + \dots + a_{mk} x_k \leq b_m \end{array} \right.$$

**siendo:**  $x_j \geq 0$

# FORMA ESTÁNDAR

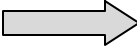
$$\text{MAX:} \quad c_1 x_1 + c_2 x_2 + c_3 x_3 + \dots + c_k x_k$$

Sujeto a

$$\left\{ \begin{array}{l} a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + \dots + a_{1k} x_k + x_{k+1} = b_1 \\ a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + \dots + a_{2k} x_k + x_{k+2} = b_2 \\ a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + \dots + a_{3k} x_k + x_{k+3} = b_3 \\ \dots\dots\dots \\ a_{m1} x_1 + a_{m2} x_2 + a_{m3} x_3 + \dots + a_{mk} x_k + x_n = b_m \end{array} \right.$$

siendo  $x_j \geq 0$

# FORMA MATRICIAL EXPANDIDA

<b>X1</b>	<b>X2</b>	<b>X3</b>		<b>RHS</b>
6	8	3		MAX
12	9	4	$\leq$	500
3	15	6	$\leq$	700
2	2	3	$\geq$	100
7	4	3	$=$	200

En un taller metalúrgico se fabrican dos tipos de piezas A y B, que deben seguir los siguientes procesos:

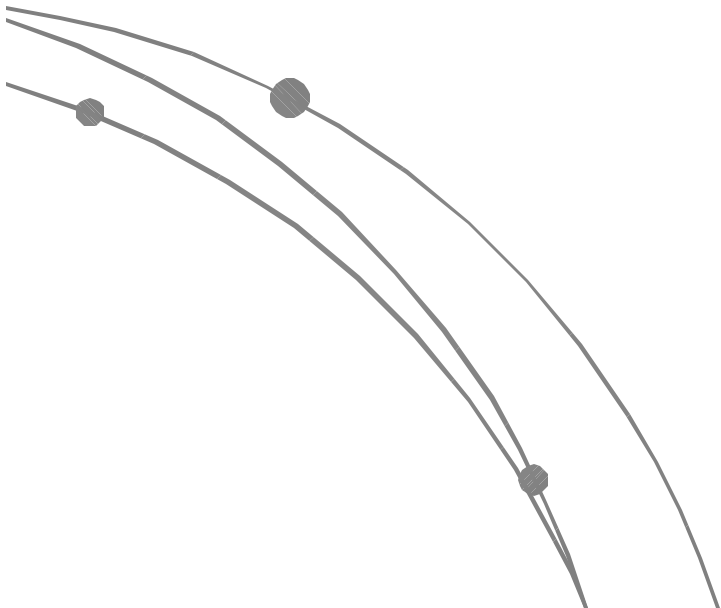
1. Estampado en hojas metálicas
2. Soldado
3. Pintado

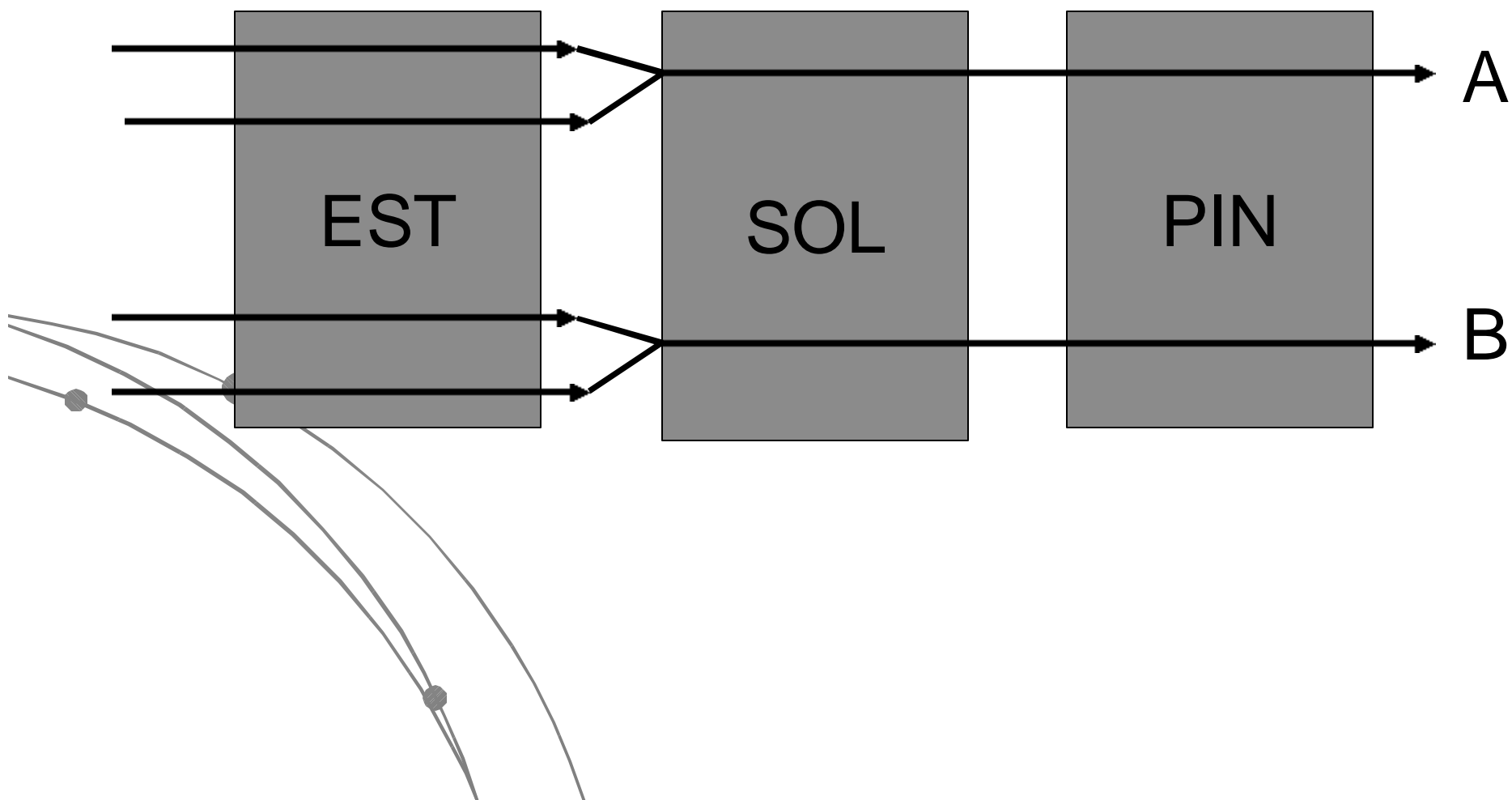
La operación de estampado consiste en preparar partes idénticas que luego serán soldadas de a pares, formando la pieza A. El mismo proceso se realiza para la pieza B.

La utilidad unitaria es de \$ 4 para la pieza A y \$ 3 para la pieza B. Se desea establecer el programa semanal de producción que maximice la utilidad del taller con respecto a las piezas consideradas.

Los insumos de equipos son los siguientes, para la realización de cada una de las operaciones (expresados en segundos por pieza):

<b>Operación</b>	<b>A</b>	<b>B</b>	<b>Tiempo disponible (seg./semana)</b>
<b>Estampado de c/parte</b>	<b>3</b>	<b>8</b>	<b>48.000</b>
<b>Soldado</b>	<b>12</b>	<b>6</b>	<b>42.000</b>
<b>Pintado</b>	<b>9</b>	<b>9</b>	<b>36.000</b>





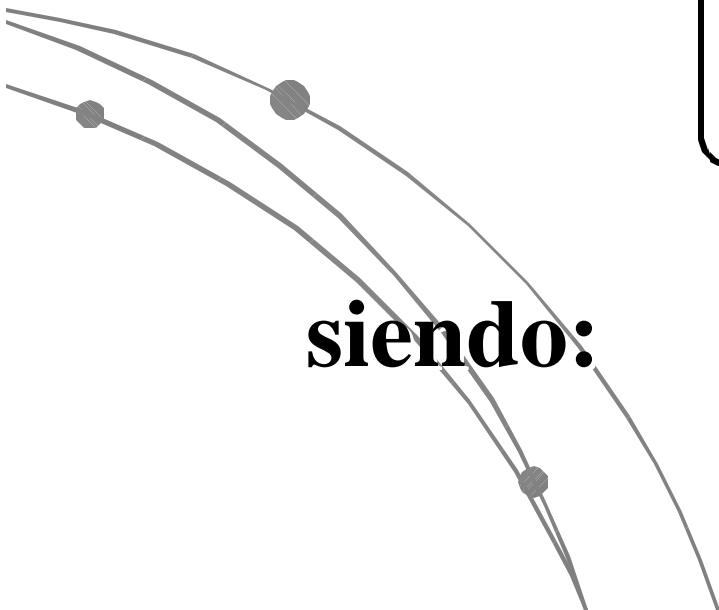


$$\text{MAX: } Z = 4 x_1 + 3 x_2$$

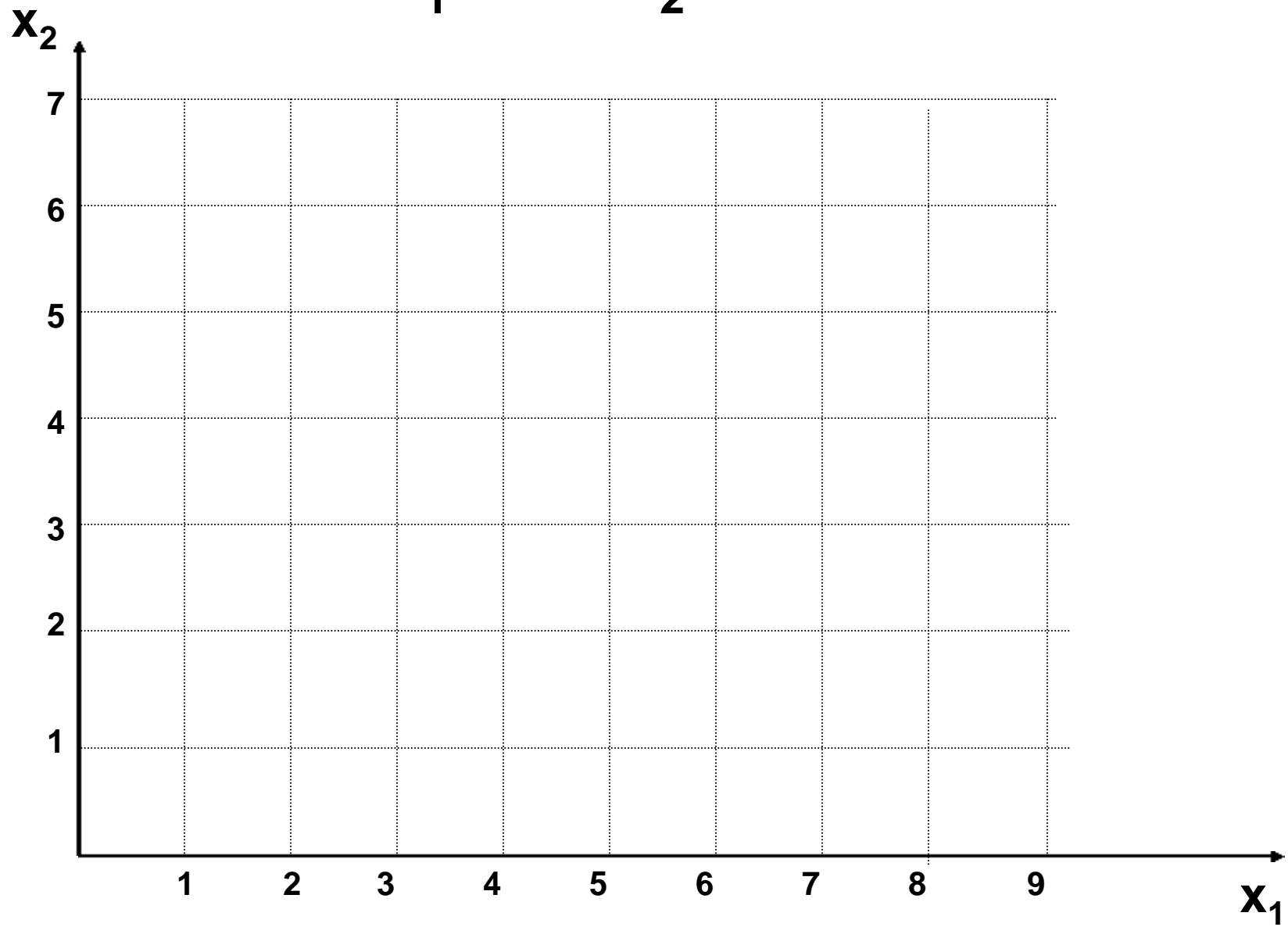
**Sujeto a:**

$$\left\{ \begin{array}{l} 6 x_1 + 16 x_2 \text{ £ } 48000 \\ 12 x_1 + 6 x_2 \text{ £ } 42000 \\ 9 x_1 + 9 x_2 \text{ £ } 36000 \end{array} \right.$$

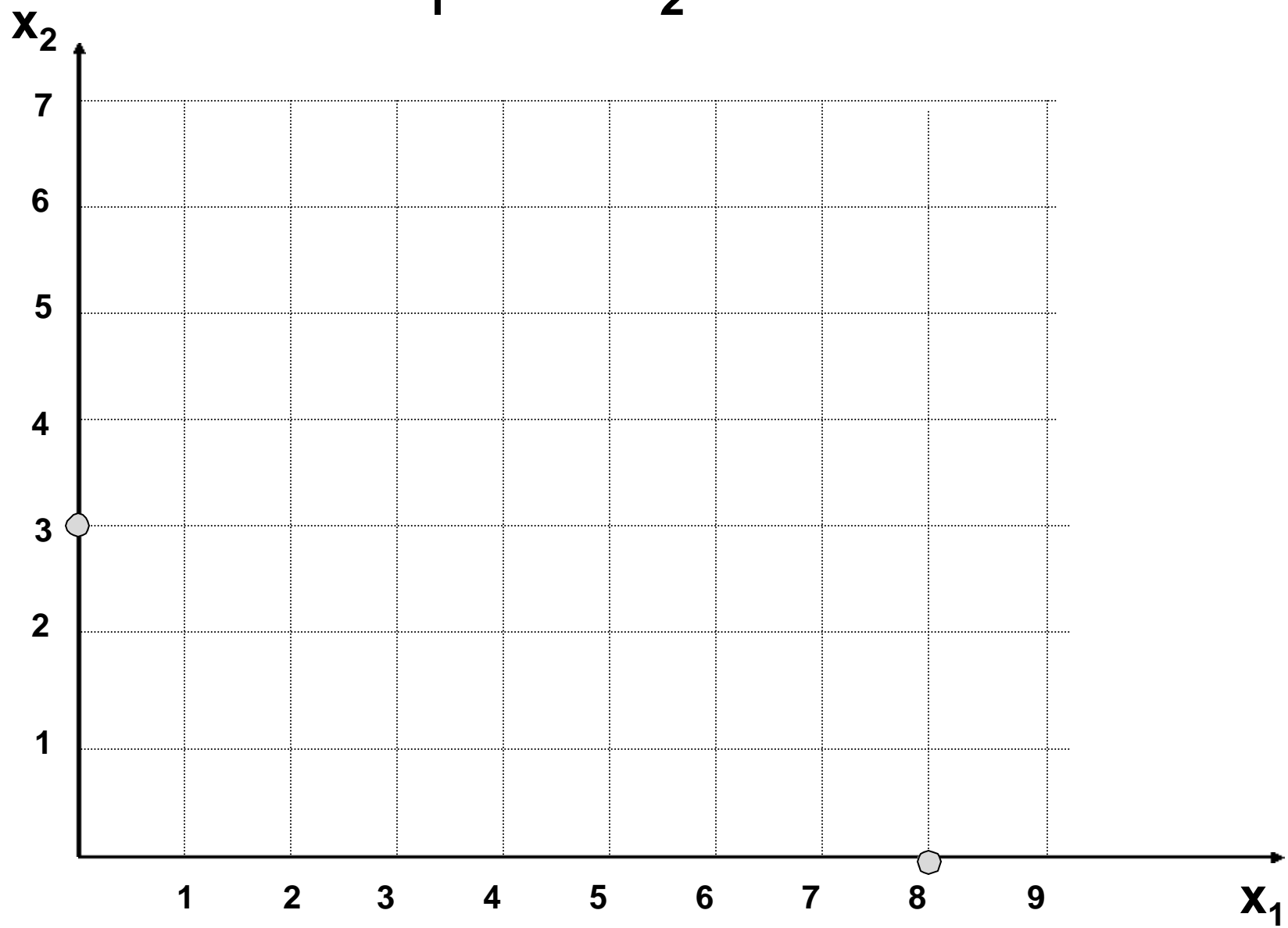
**siendo:**  $x_1, x_2 \geq 0$  y continuas



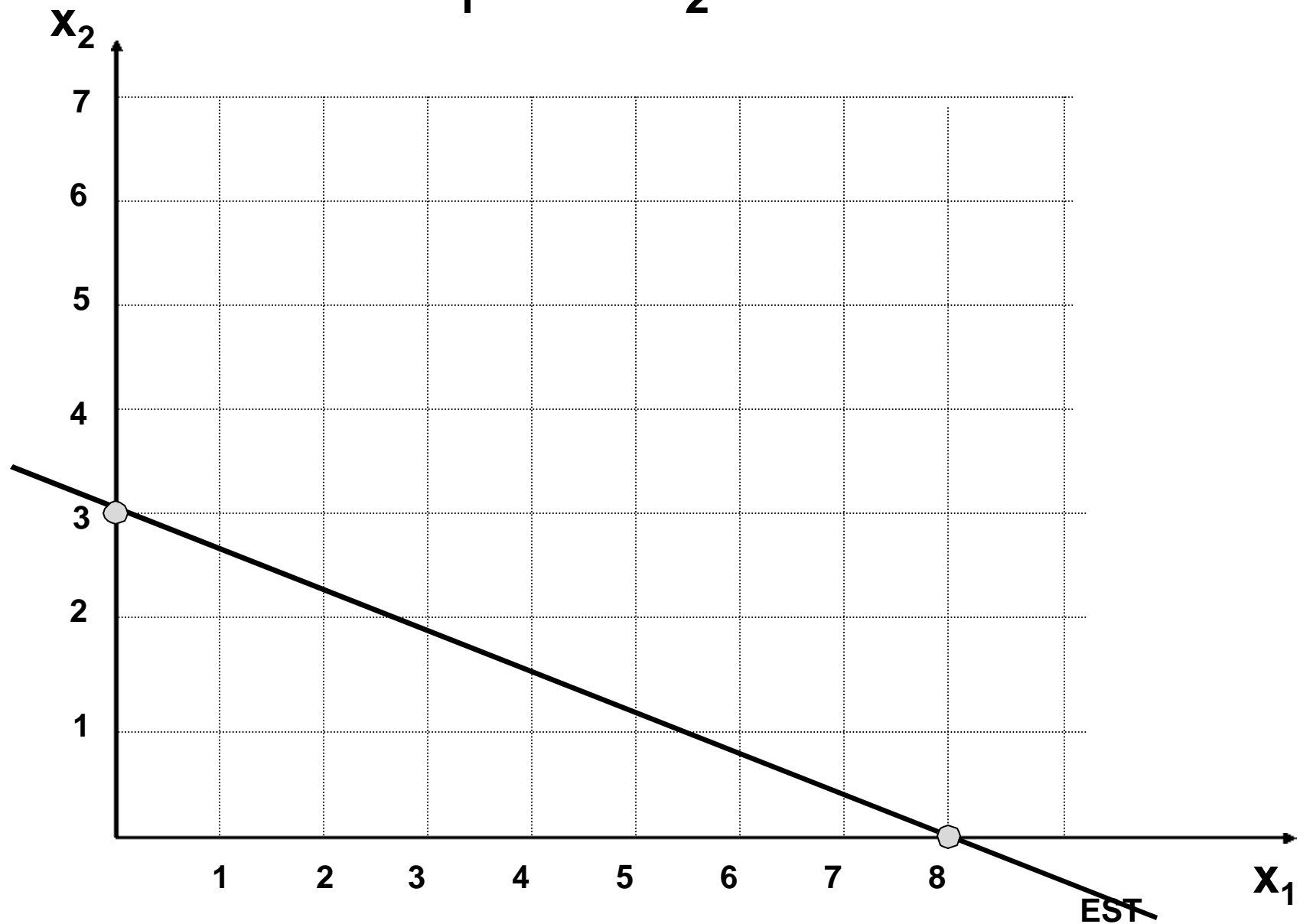
$$6x_1 + 16x_2 \leq 48000$$



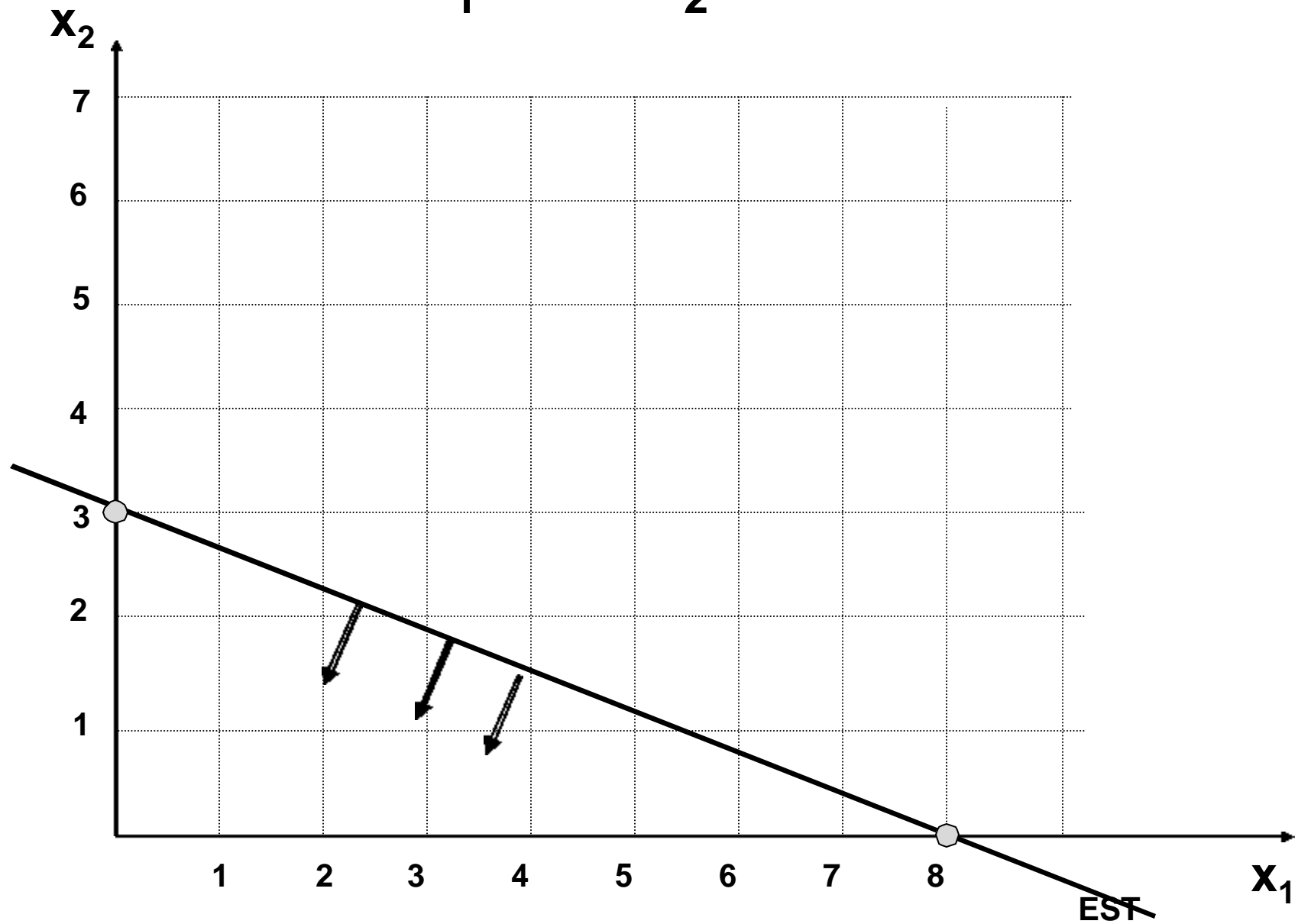
$$6x_1 + 16x_2 = 48000$$



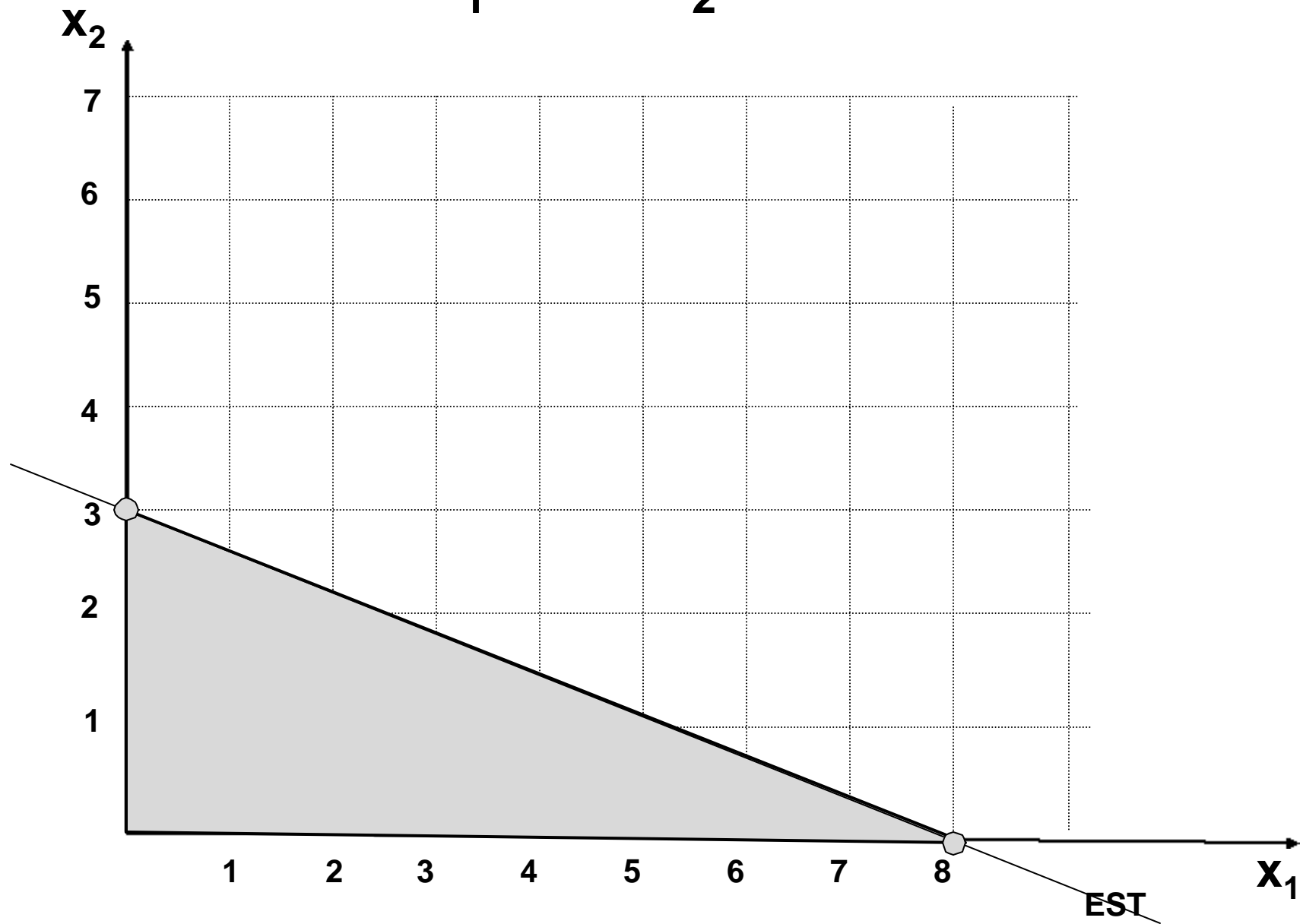
$$6x_1 + 16x_2 = 48000$$



$$6x_1 + 16x_2 \leq 48000$$

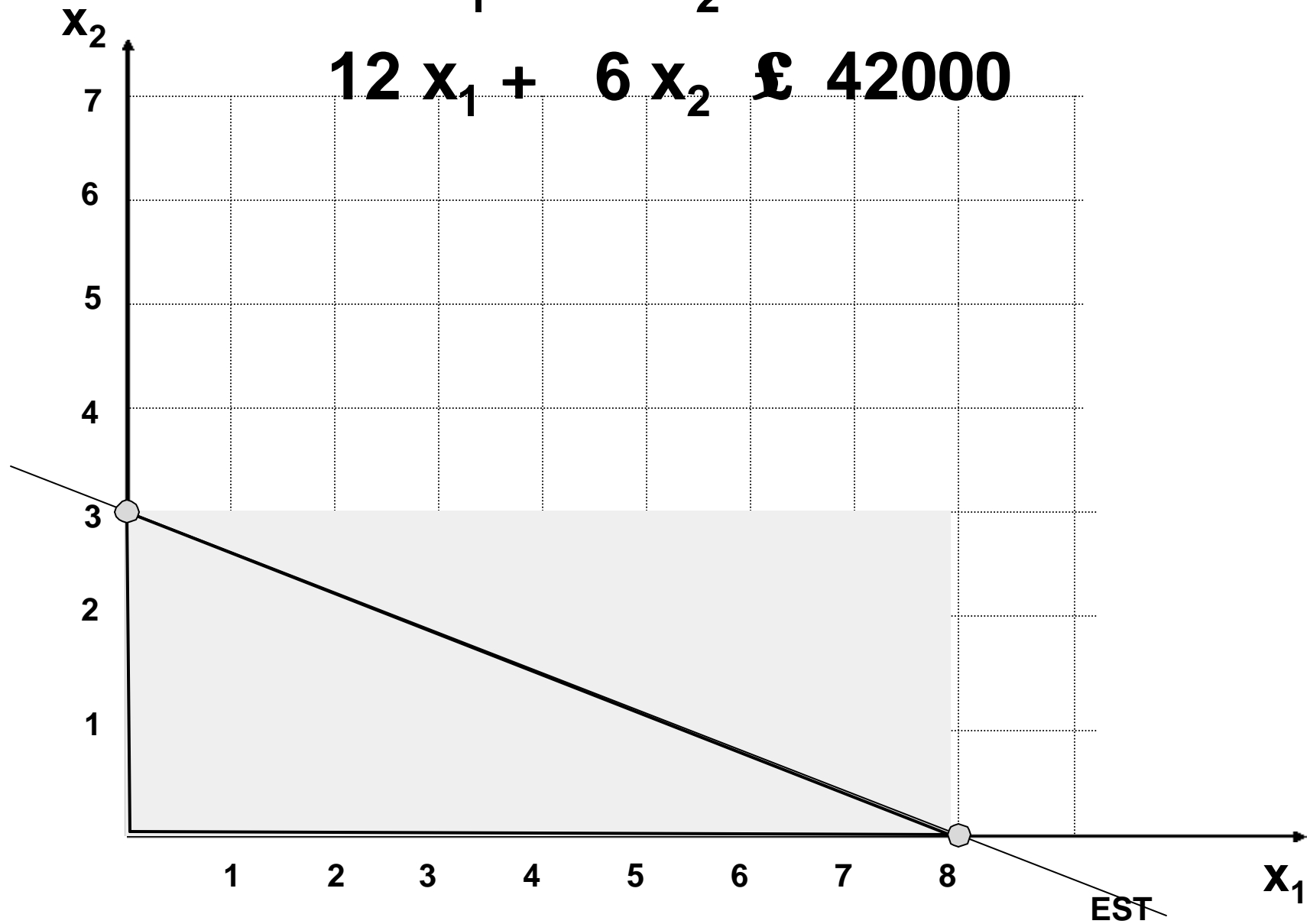


$$6x_1 + 16x_2 \leq 48000$$



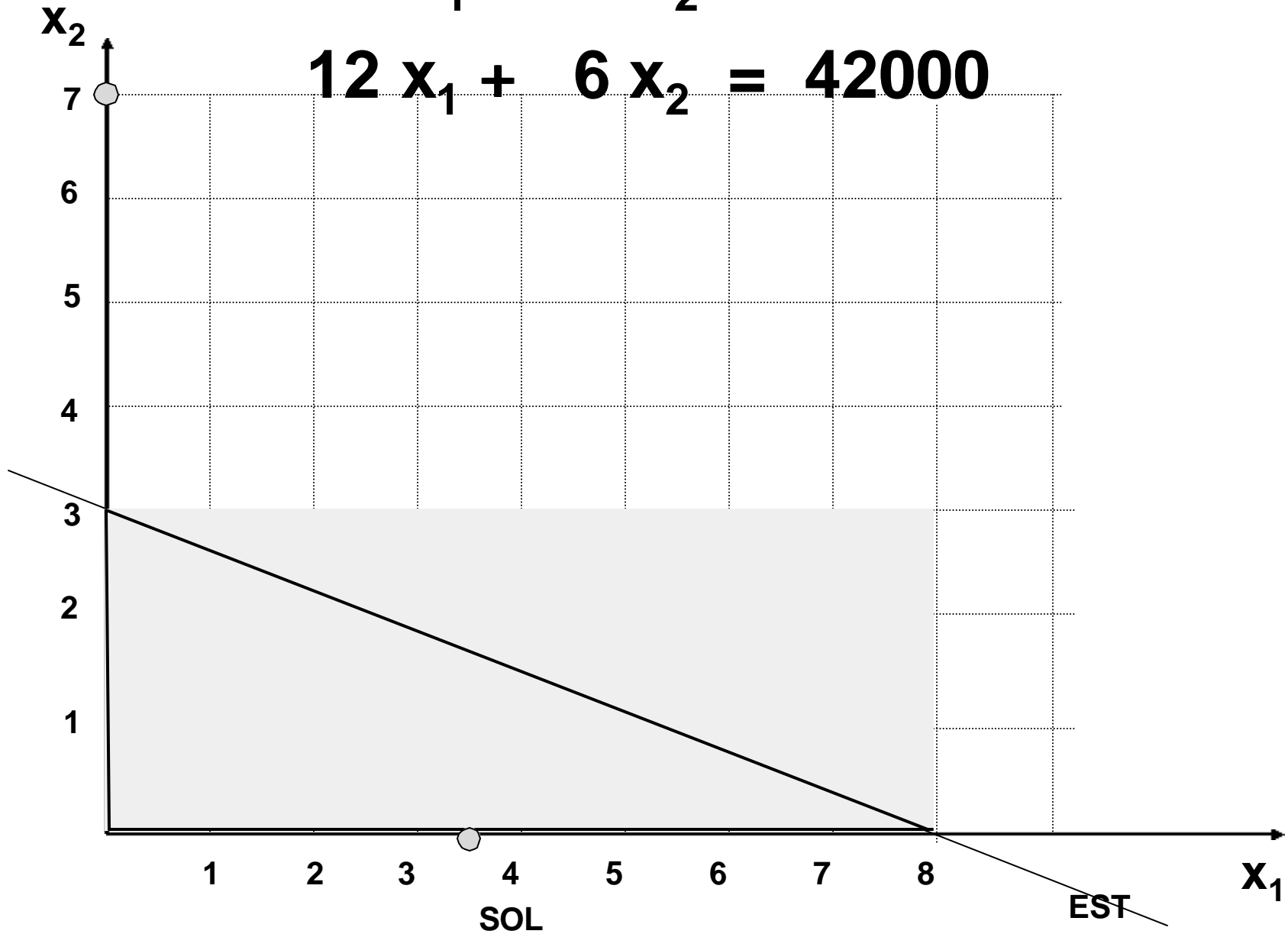
$$6x_1 + 16x_2 \leq 48000$$

$$12x_1 + 6x_2 \leq 42000$$



$$6x_1 + 16x_2 \leq 48000$$

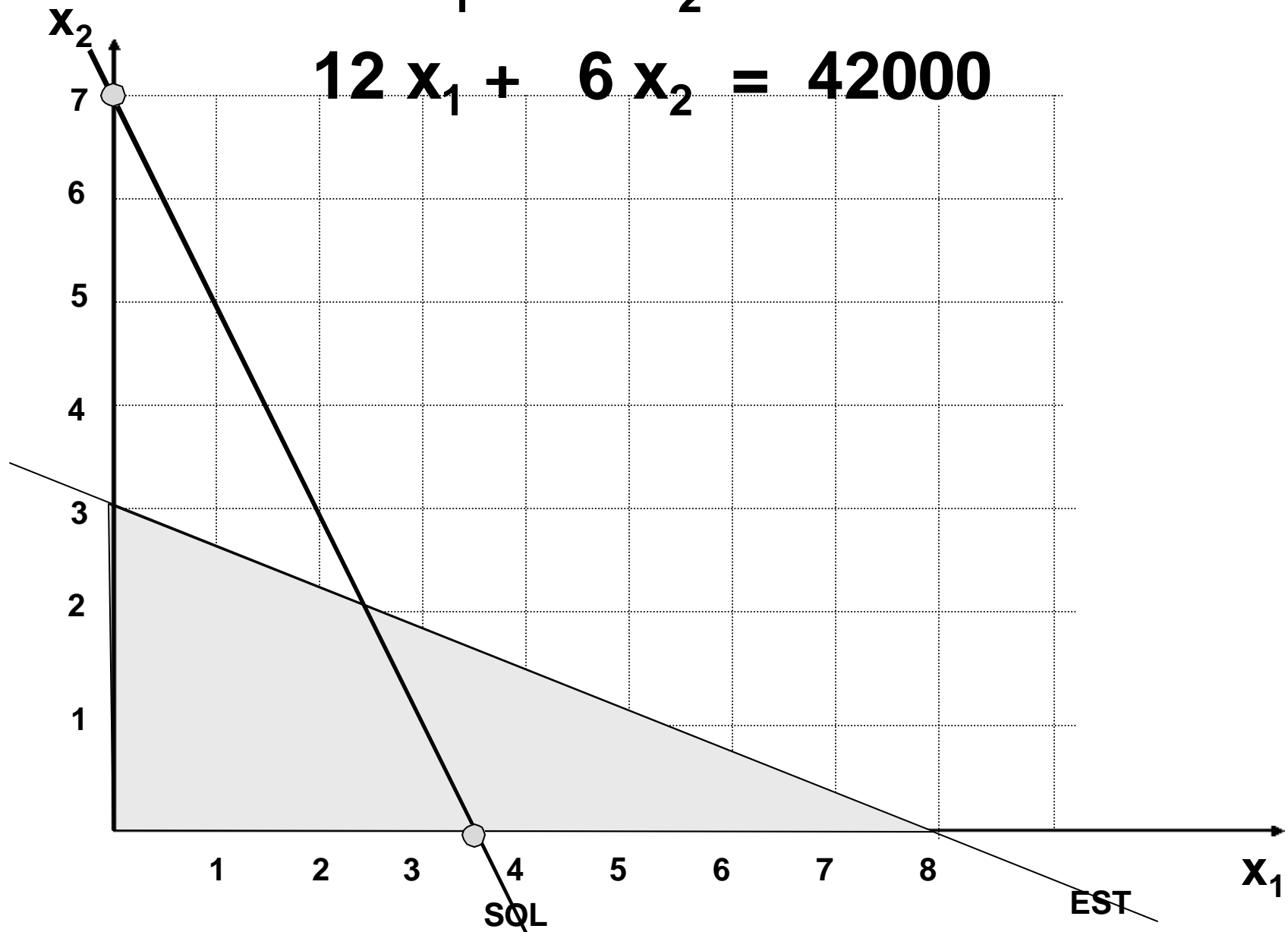
$$12x_1 + 6x_2 = 42000$$





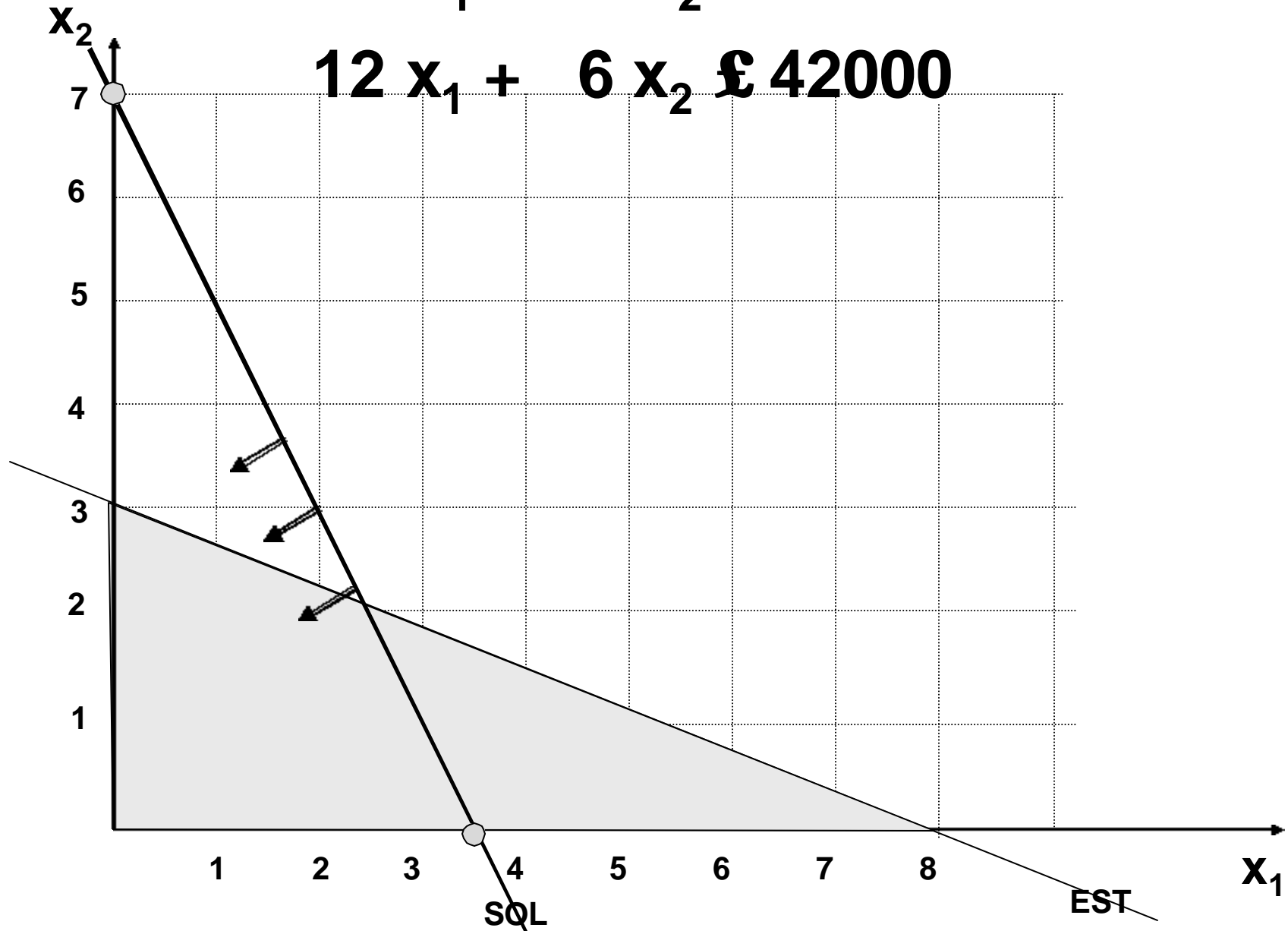
$$6x_1 + 16x_2 \leq 48000$$

$$12x_1 + 6x_2 = 42000$$



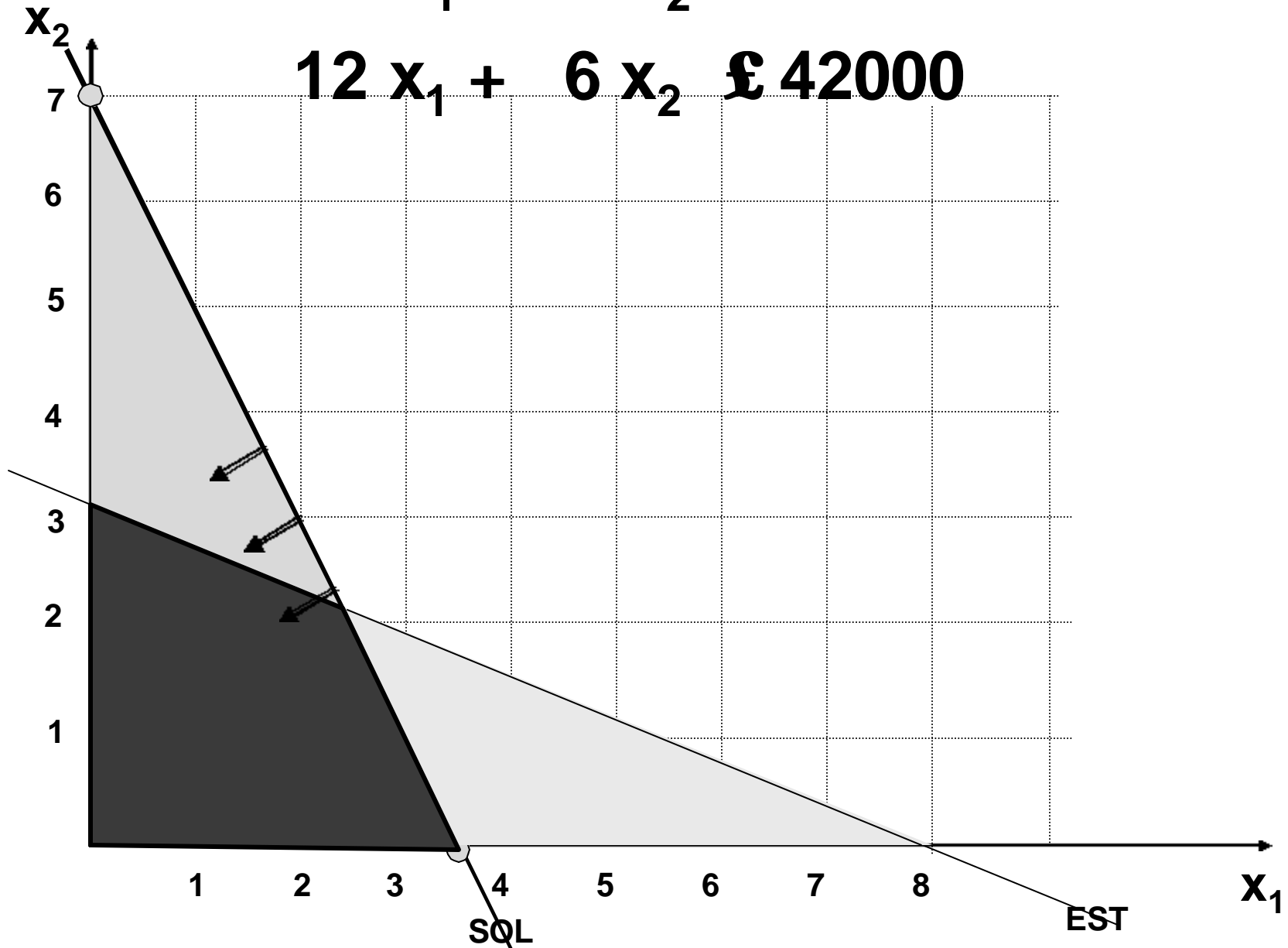
$$6x_1 + 16x_2 \text{ £ } 48000$$

$$12x_1 + 6x_2 \text{ £ } 42000$$



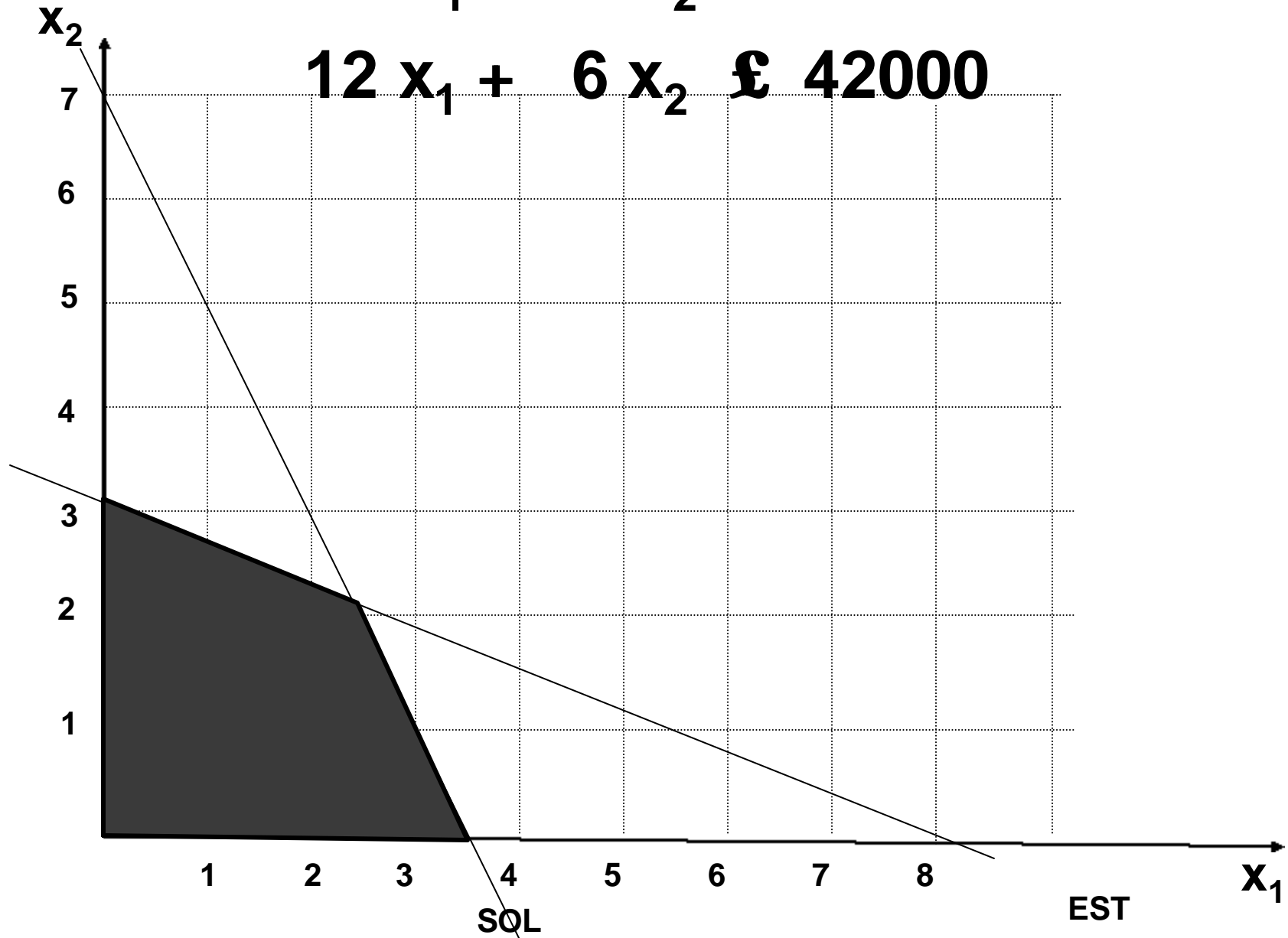
$$6x_1 + 16x_2 \quad \text{£} \quad 48000$$

$$12x_1 + 6x_2 \quad \text{£} \quad 42000$$



$$6x_1 + 16x_2 \leq 48000$$

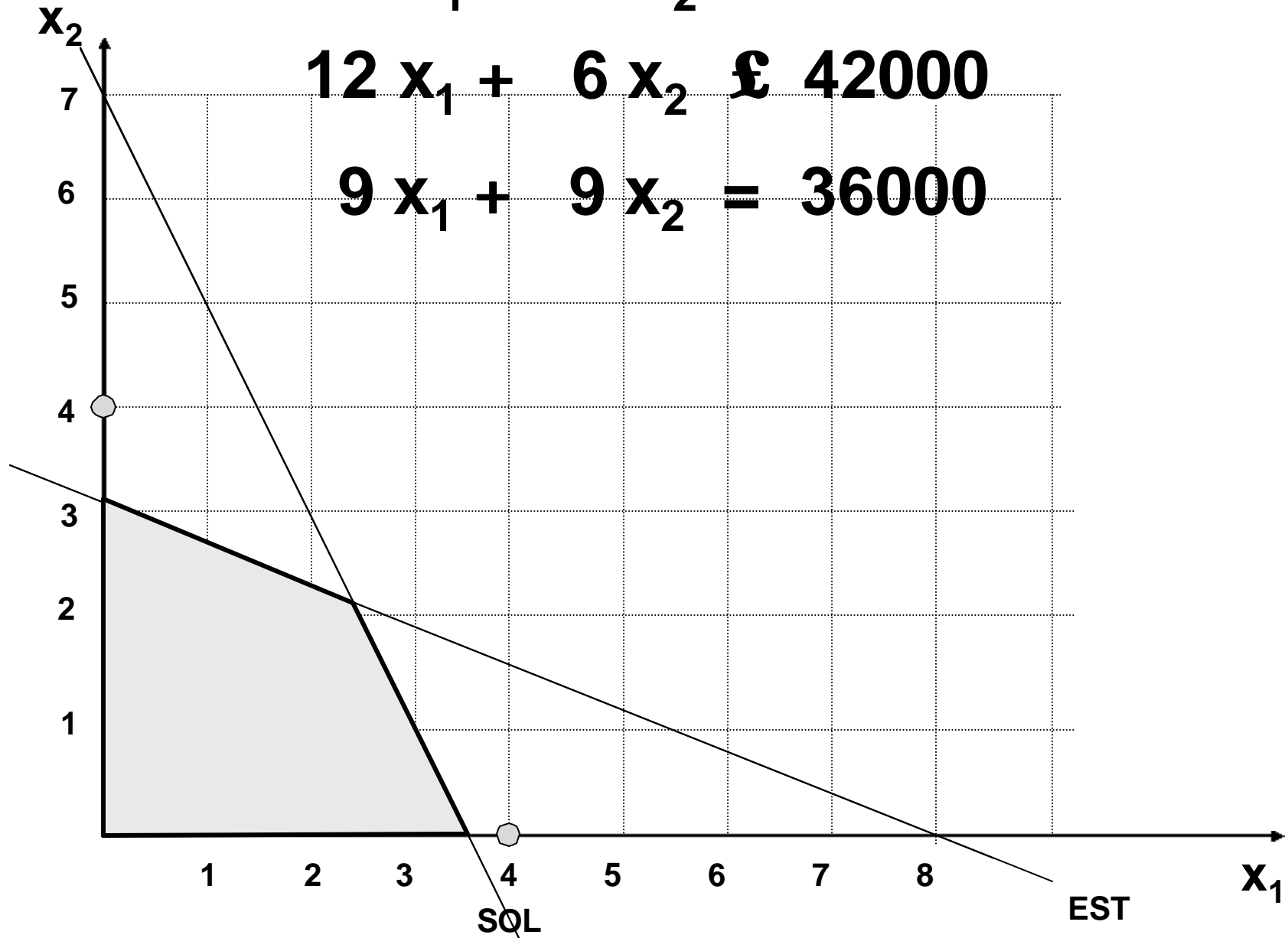
$$12x_1 + 6x_2 \leq 42000$$



$$6x_1 + 16x_2 \leq 48000$$

$$12x_1 + 6x_2 \leq 42000$$

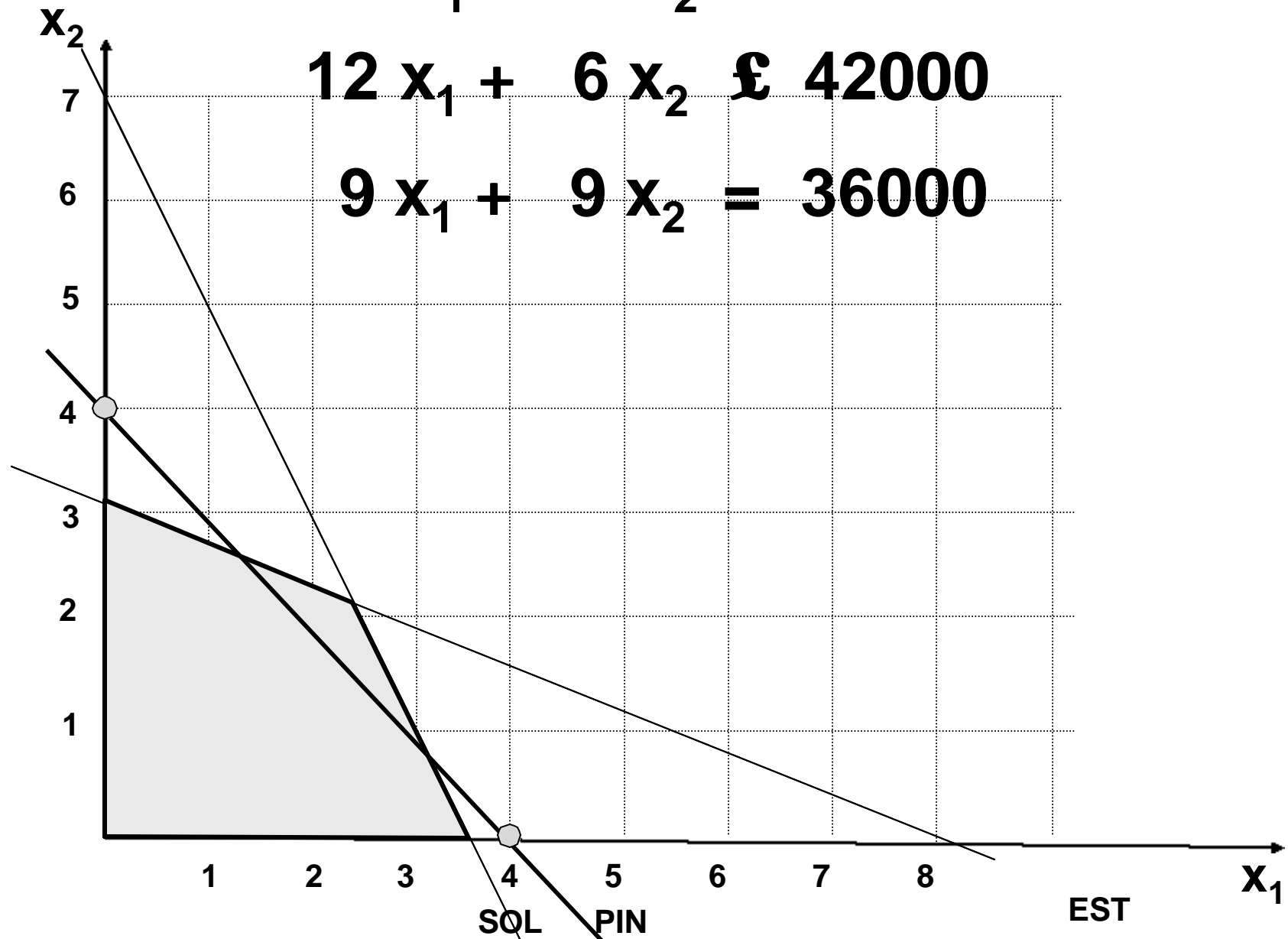
$$9x_1 + 9x_2 = 36000$$



$$6x_1 + 16x_2 \leq 48000$$

$$12x_1 + 6x_2 \leq 42000$$

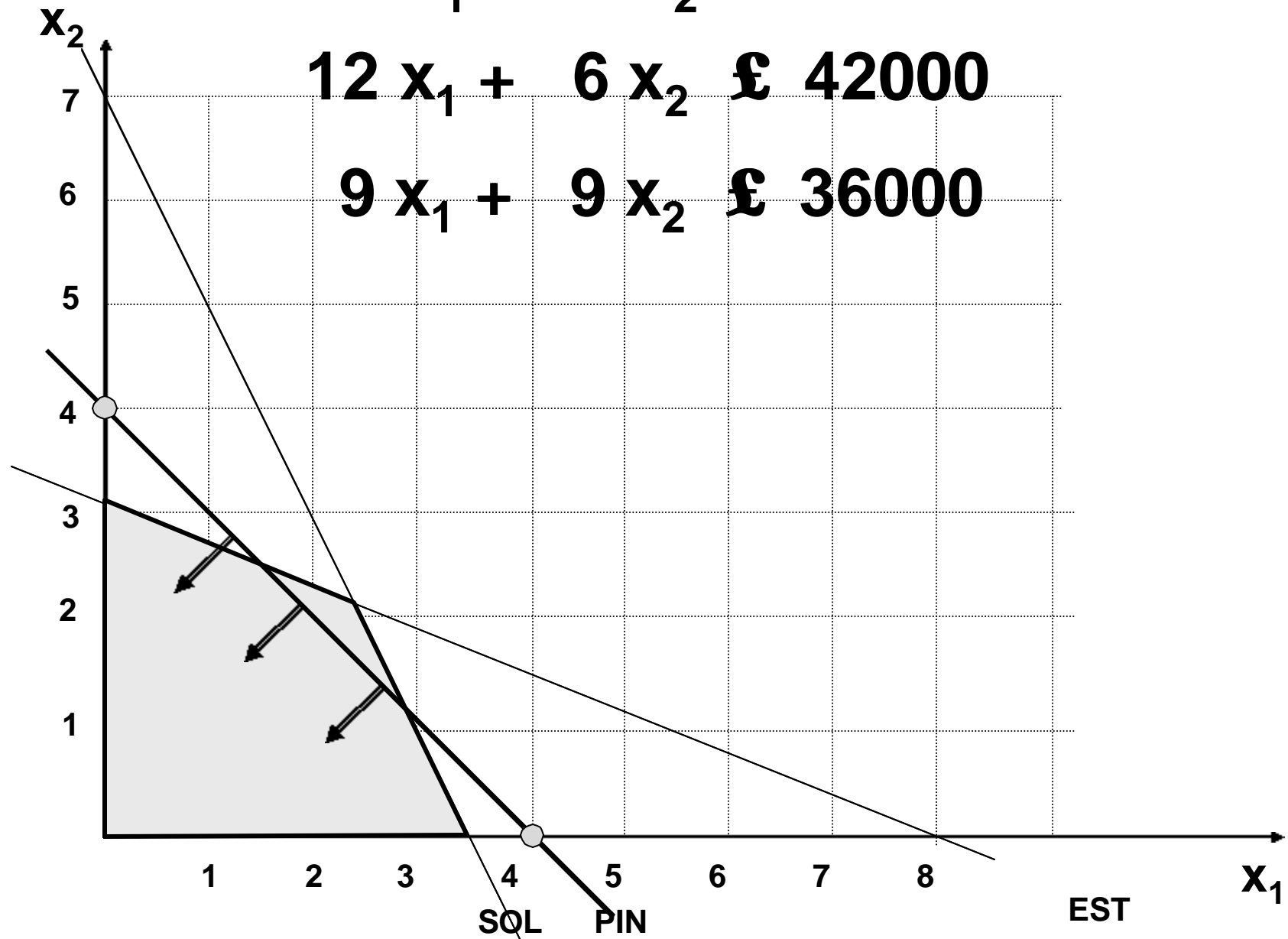
$$9x_1 + 9x_2 = 36000$$



$$6x_1 + 16x_2 \quad \text{£} \quad 48000$$

$$12x_1 + 6x_2 \quad \text{£} \quad 42000$$

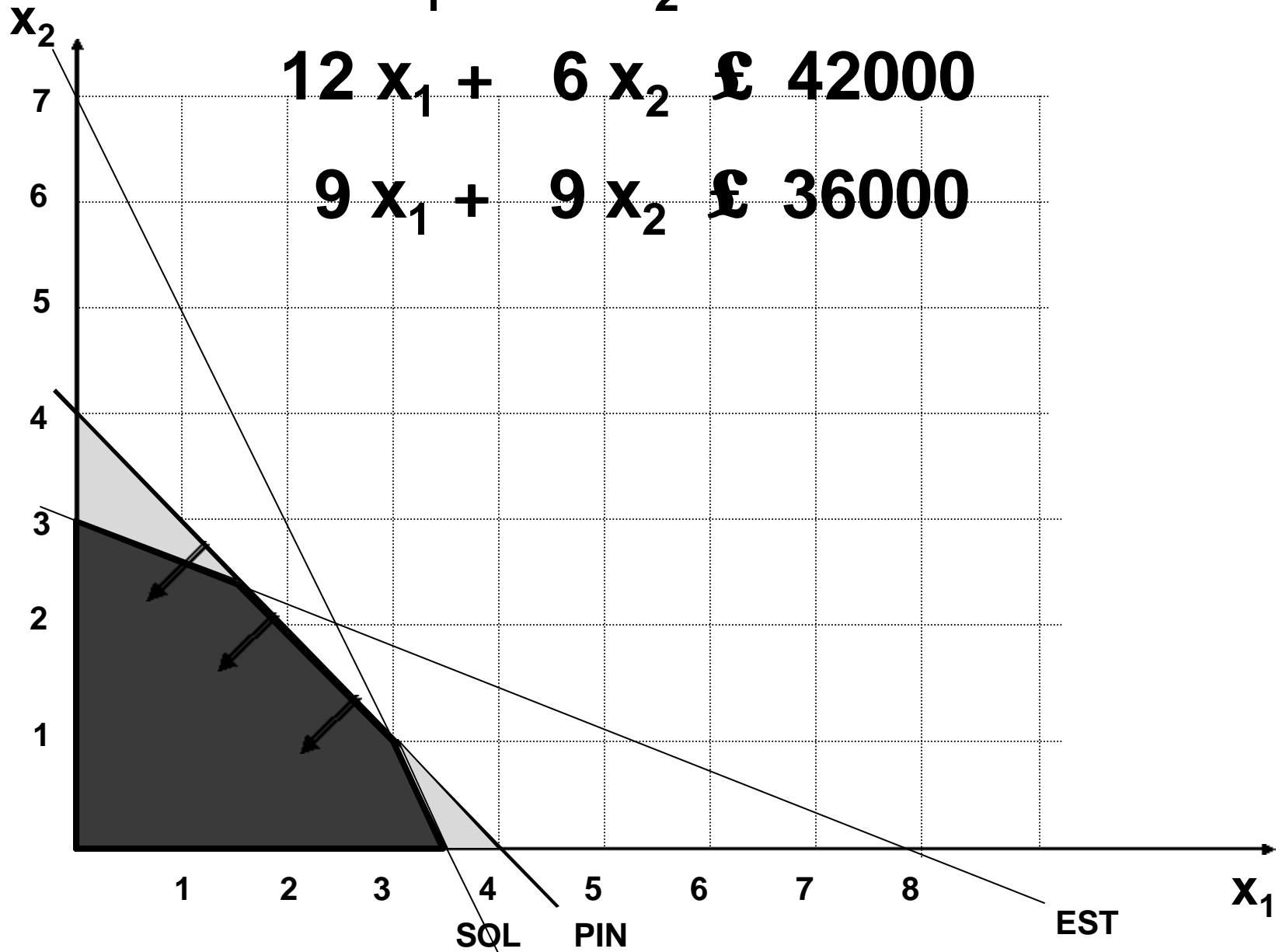
$$9x_1 + 9x_2 \quad \text{£} \quad 36000$$



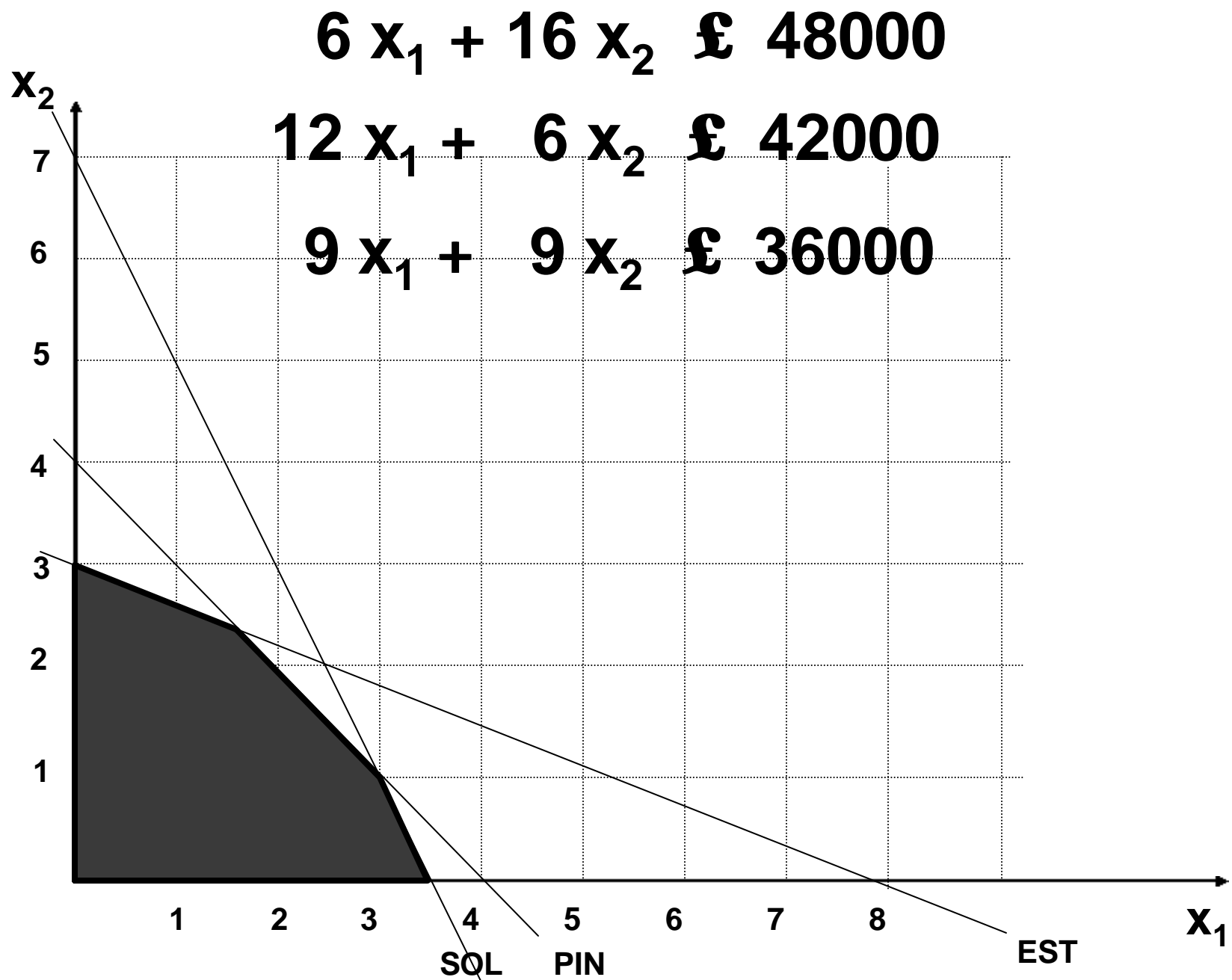
$$6x_1 + 16x_2 \quad \text{£} \quad 48000$$

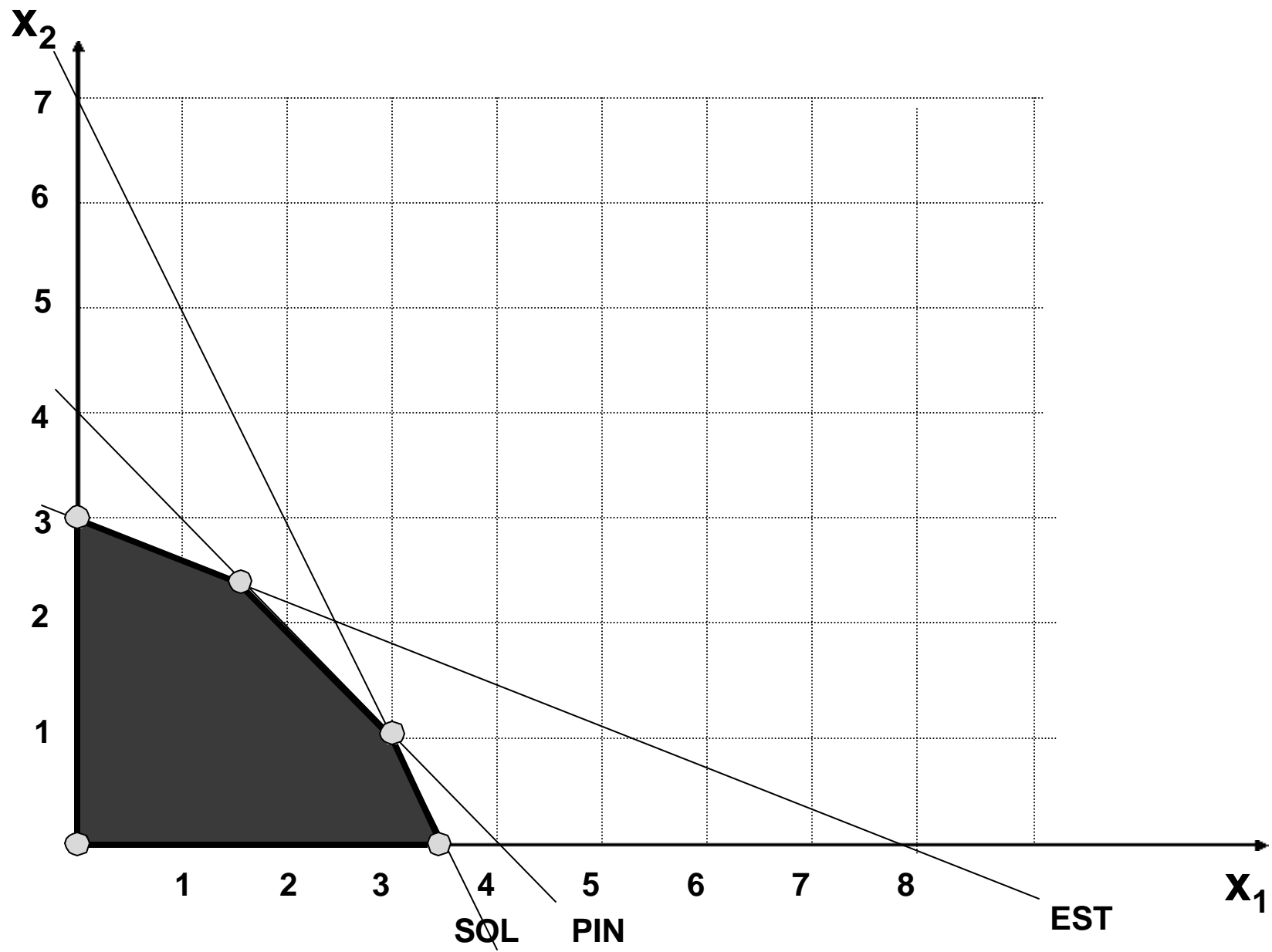
$$12x_1 + 6x_2 \quad \text{£} \quad 42000$$

$$9x_1 + 9x_2 \quad \text{£} \quad 36000$$









$x_2$

$$Z = 4x_1 + 3x_2$$

3

2

1

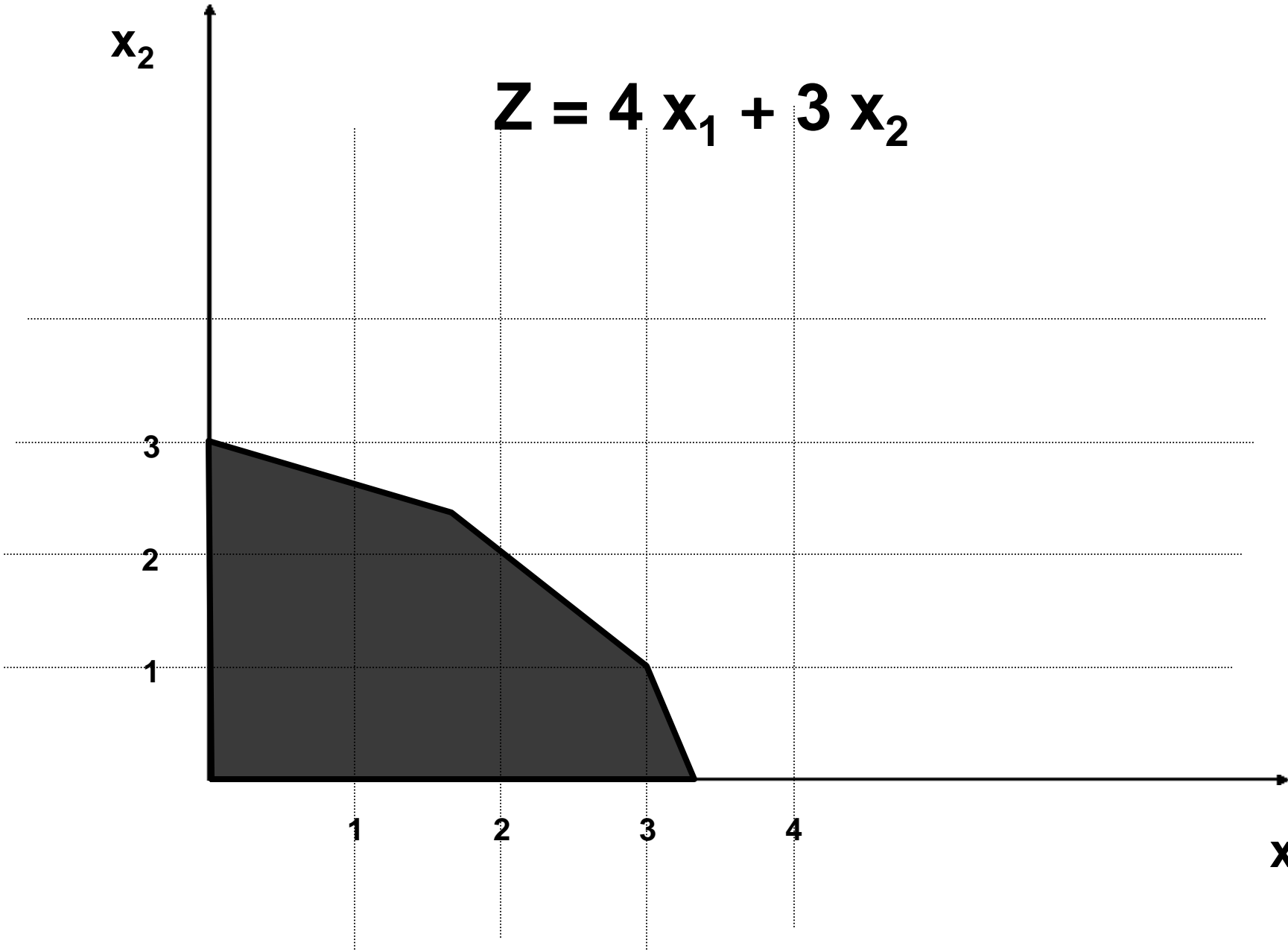
1

2

3

4

$x_1$



$x_2$

$$0 = 4x_1 + 3x_2$$

3

2

1

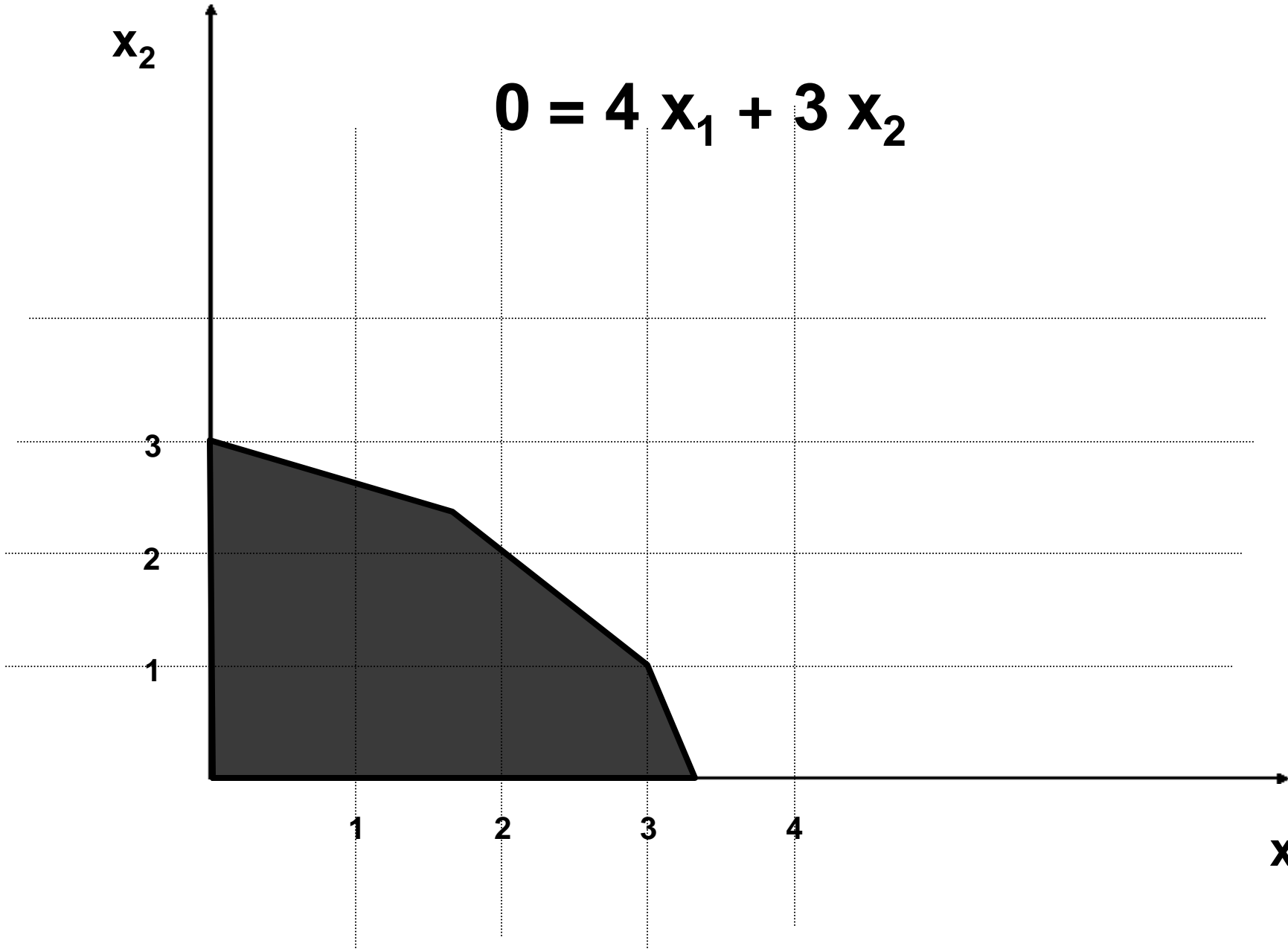
1

2

3

4

$x_1$



$x_2$

$$0 = 4x_1 + 3x_2$$

$$x_1 = -\frac{3}{4}x_2$$

3

2

1

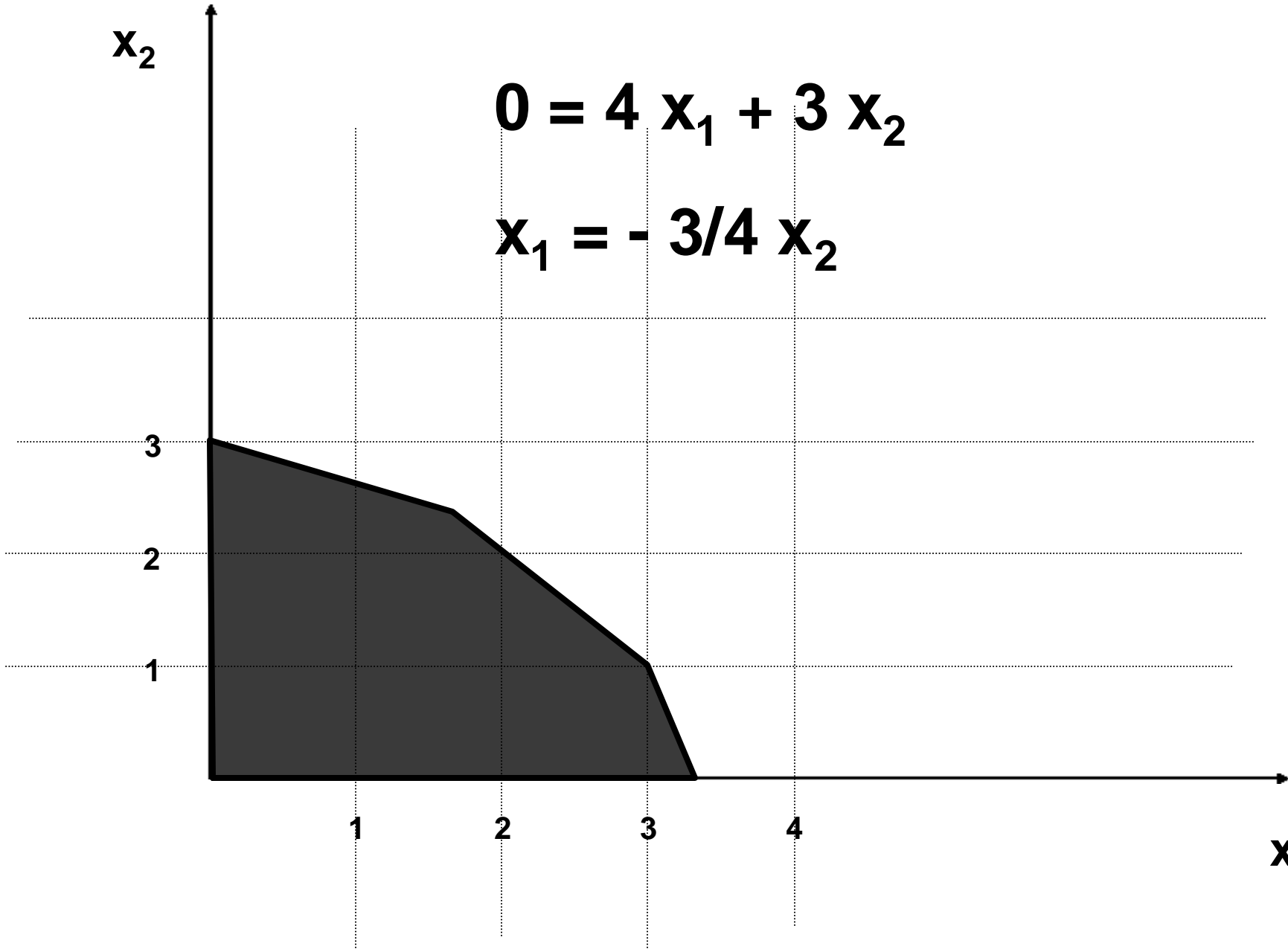
1

2

3

4

$x_1$



$x_2$

$$0 = 4x_1 + 3x_2$$

$$x_1 = -\frac{3}{4}x_2$$

Para  $x_2 = 0$        $x_1 = 0$

3

2

1

1

2

3

4

$x_1$

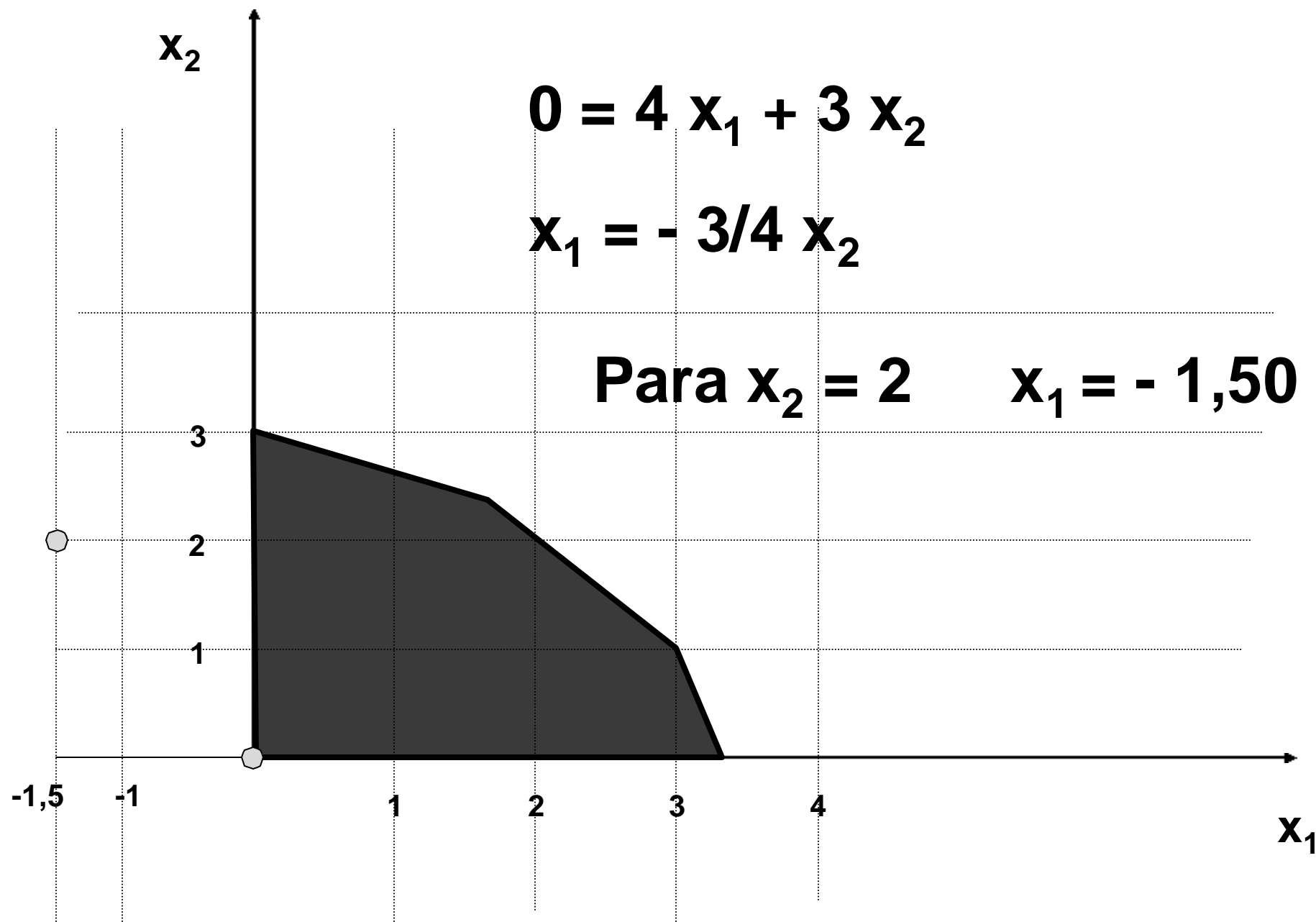


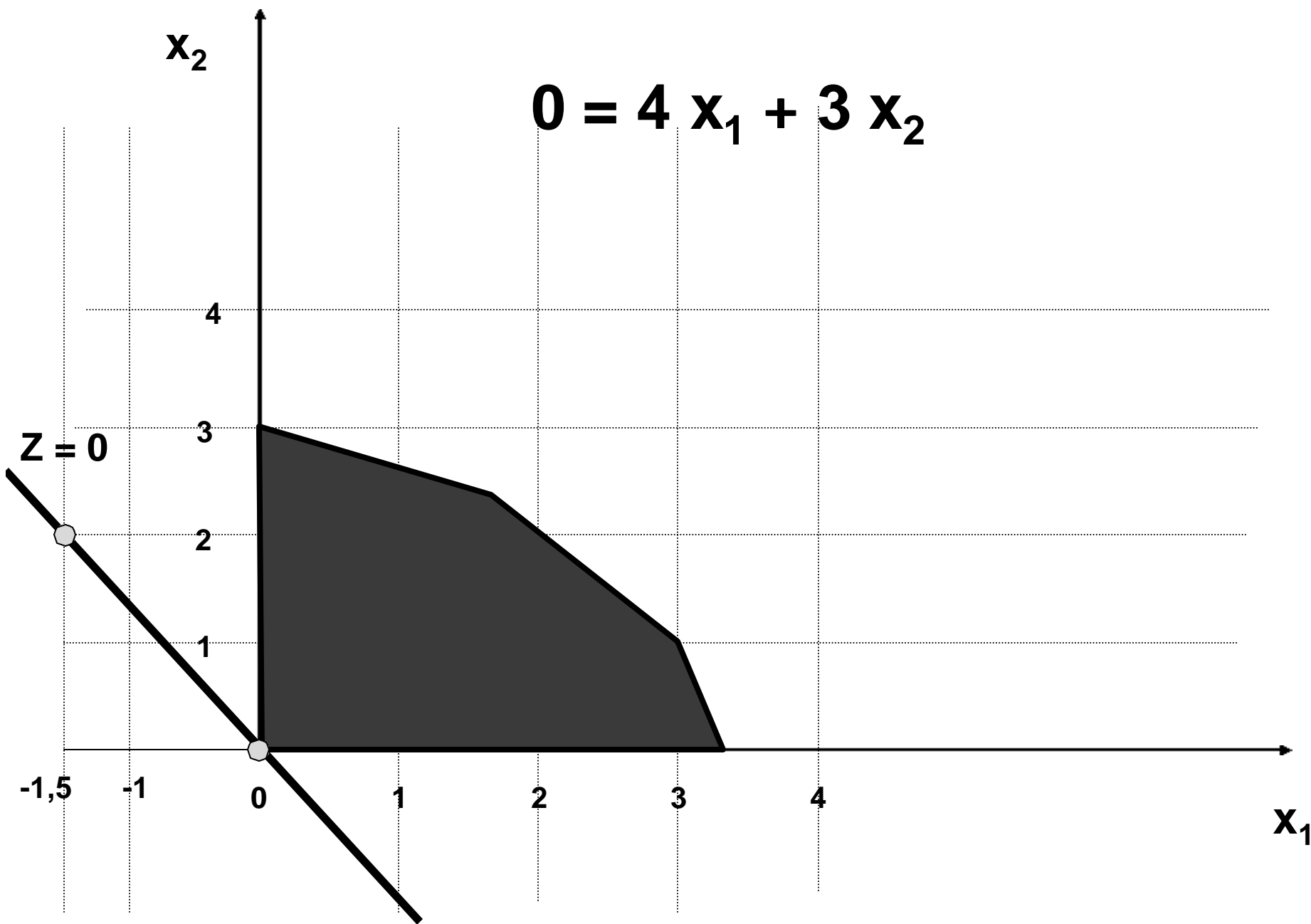
$x_2$

$$0 = 4x_1 + 3x_2$$

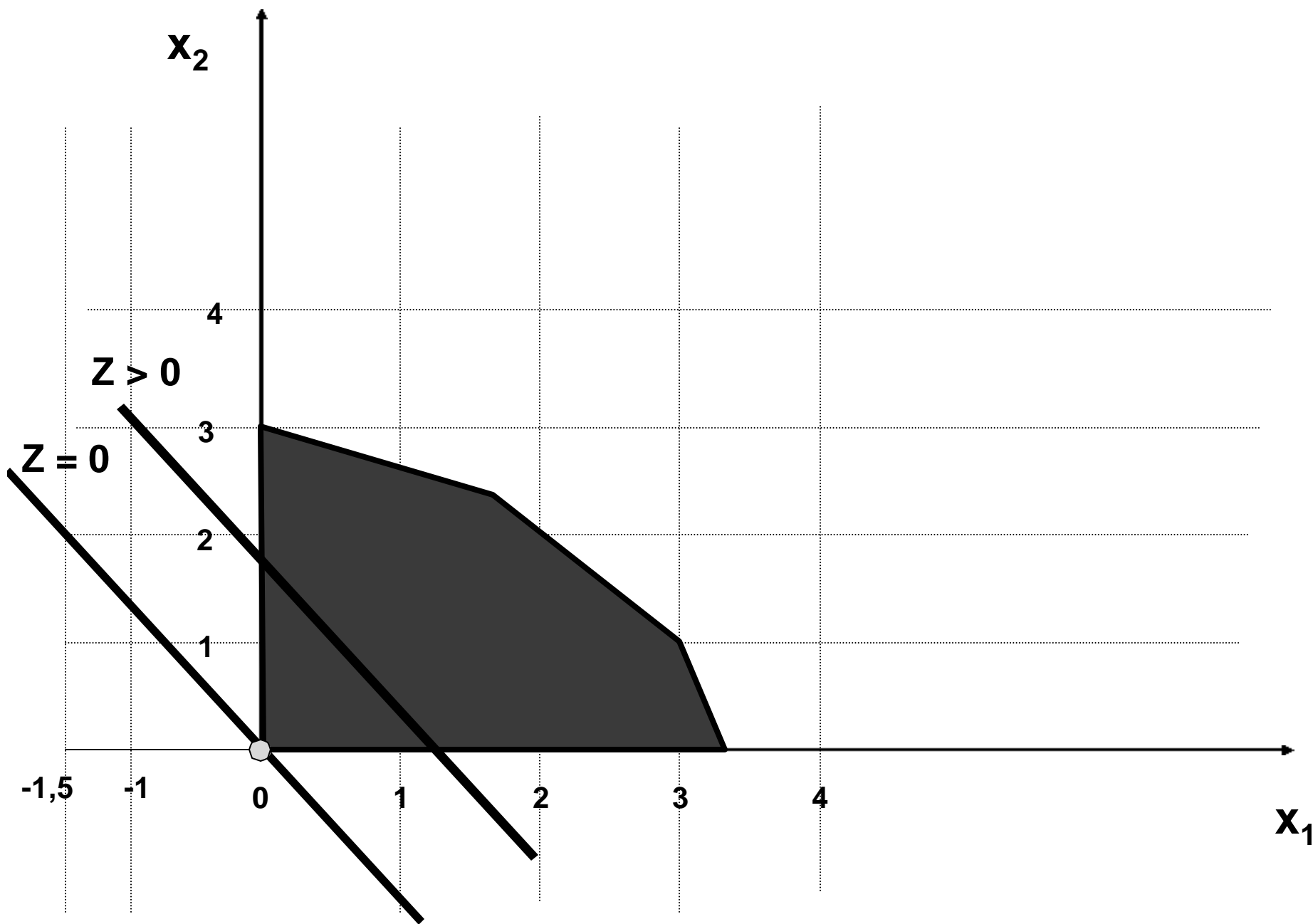
$$x_1 = -\frac{3}{4}x_2$$

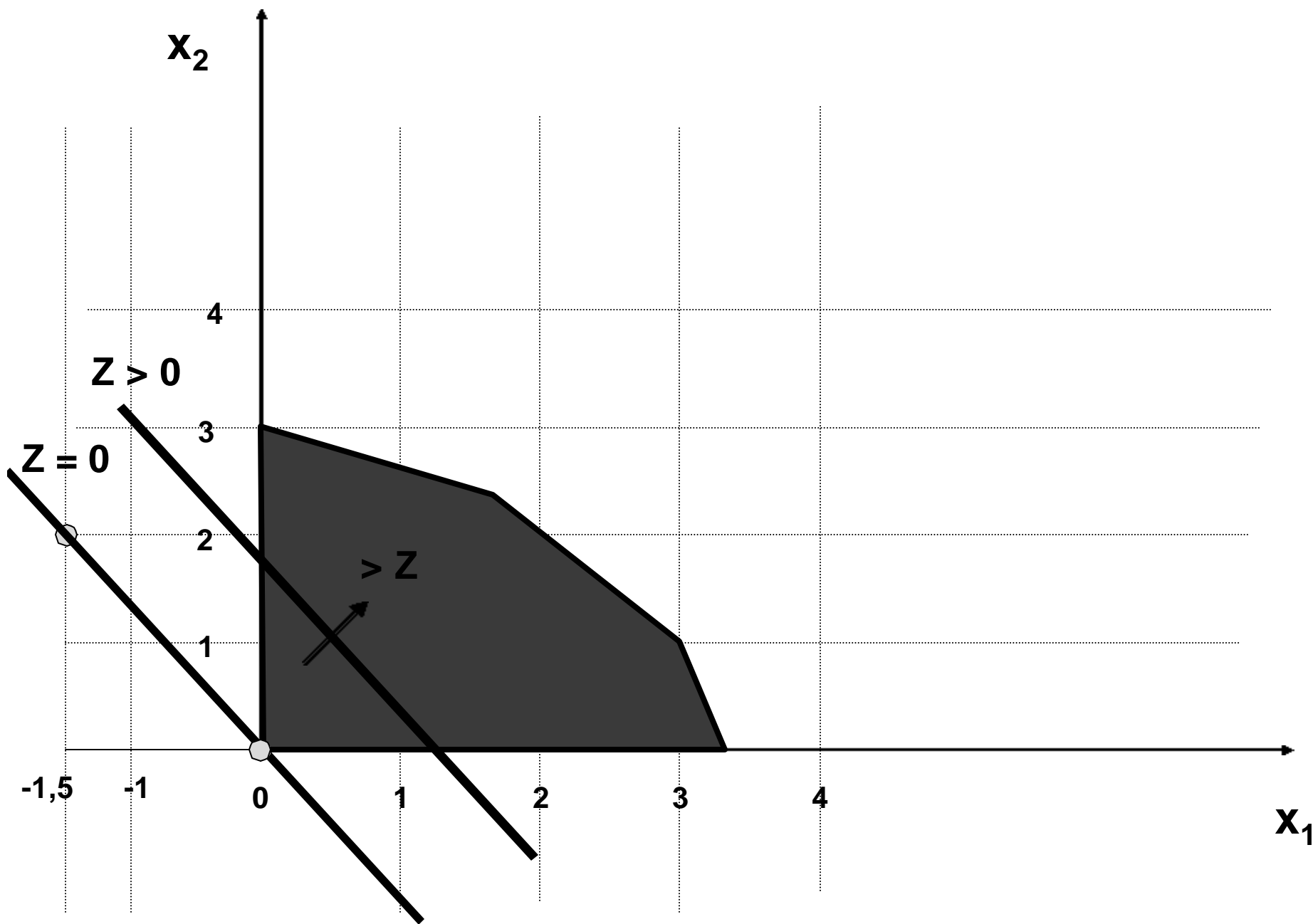
$$\text{Para } x_2 = 2 \quad x_1 = -1,50$$

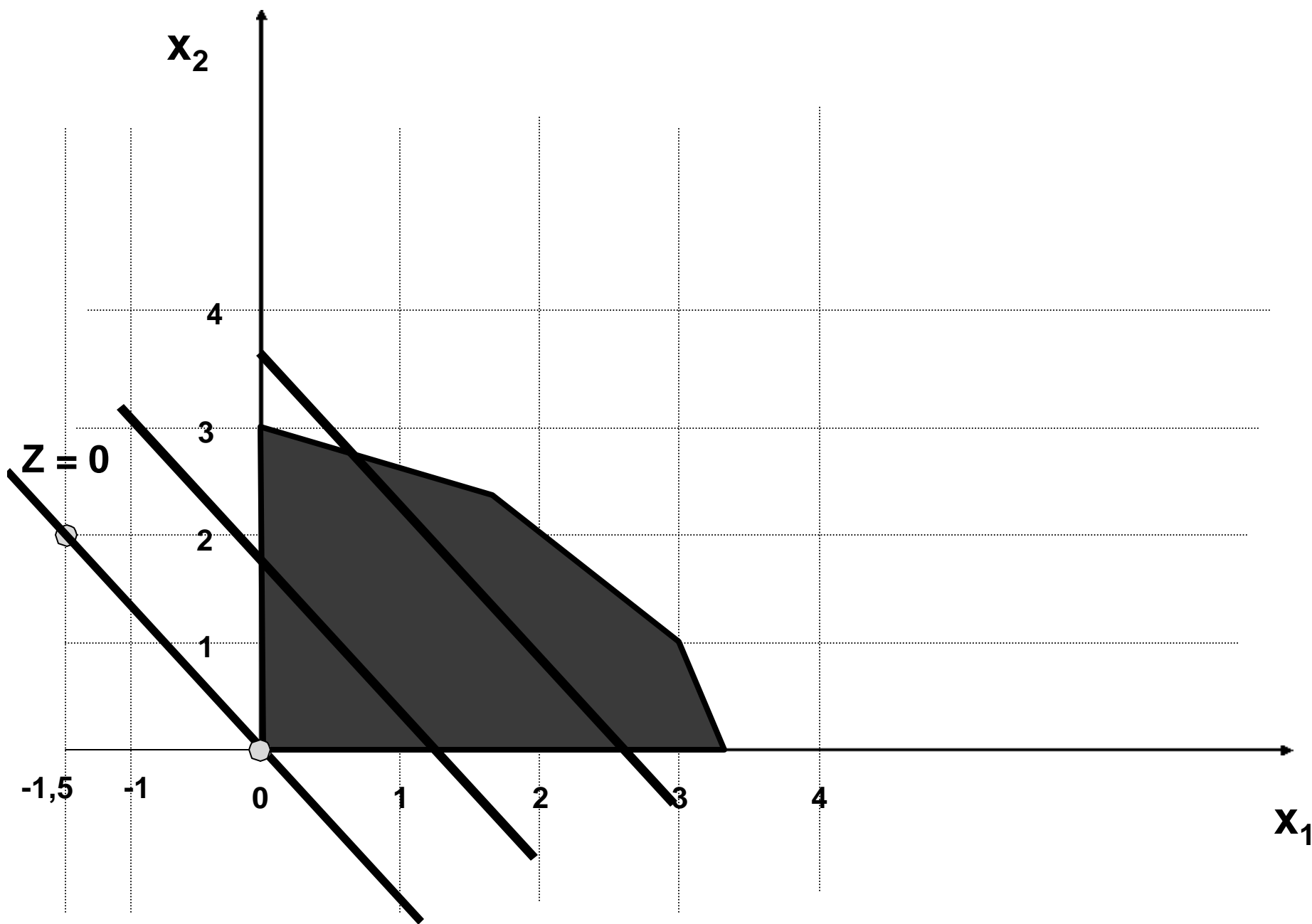


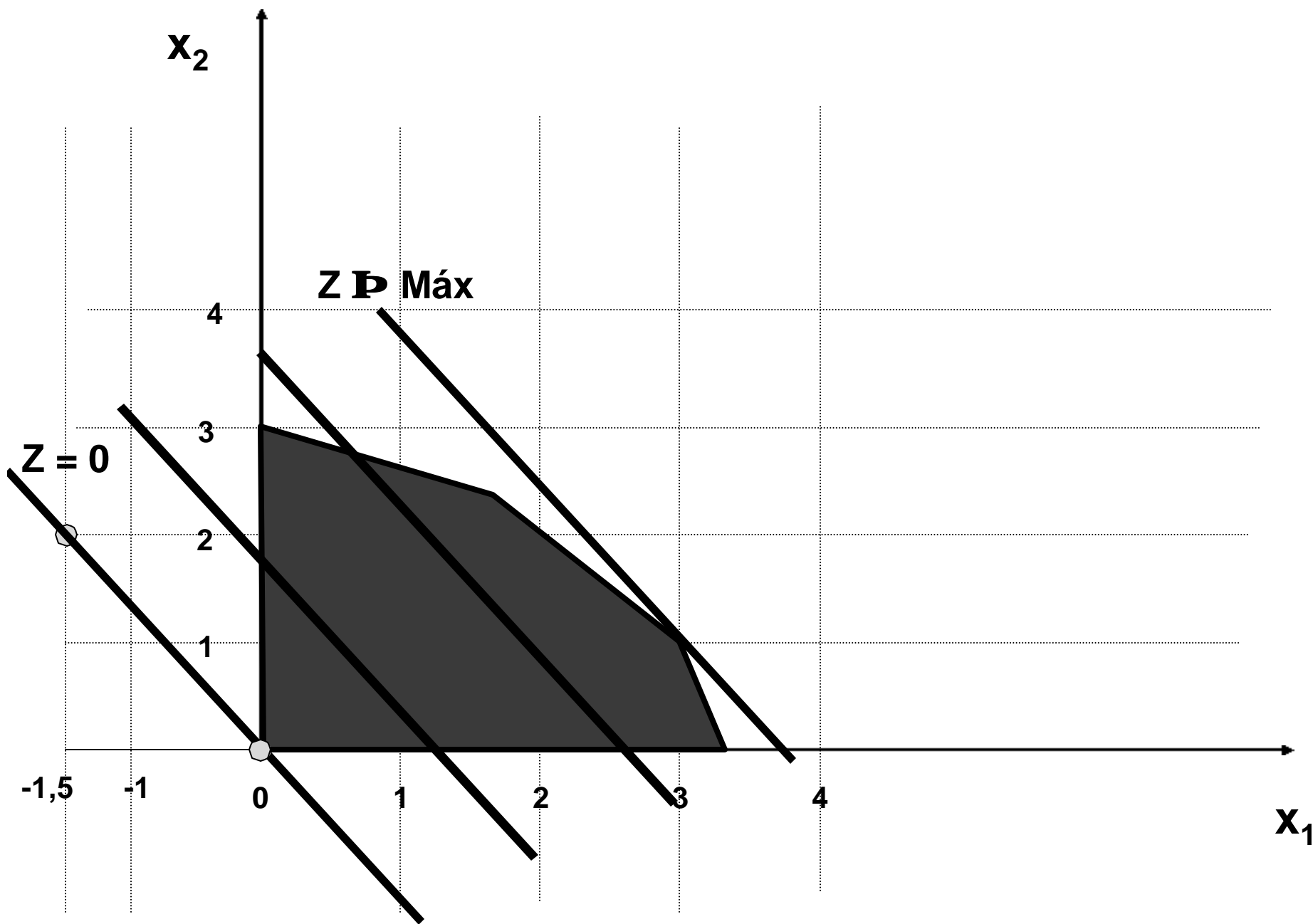


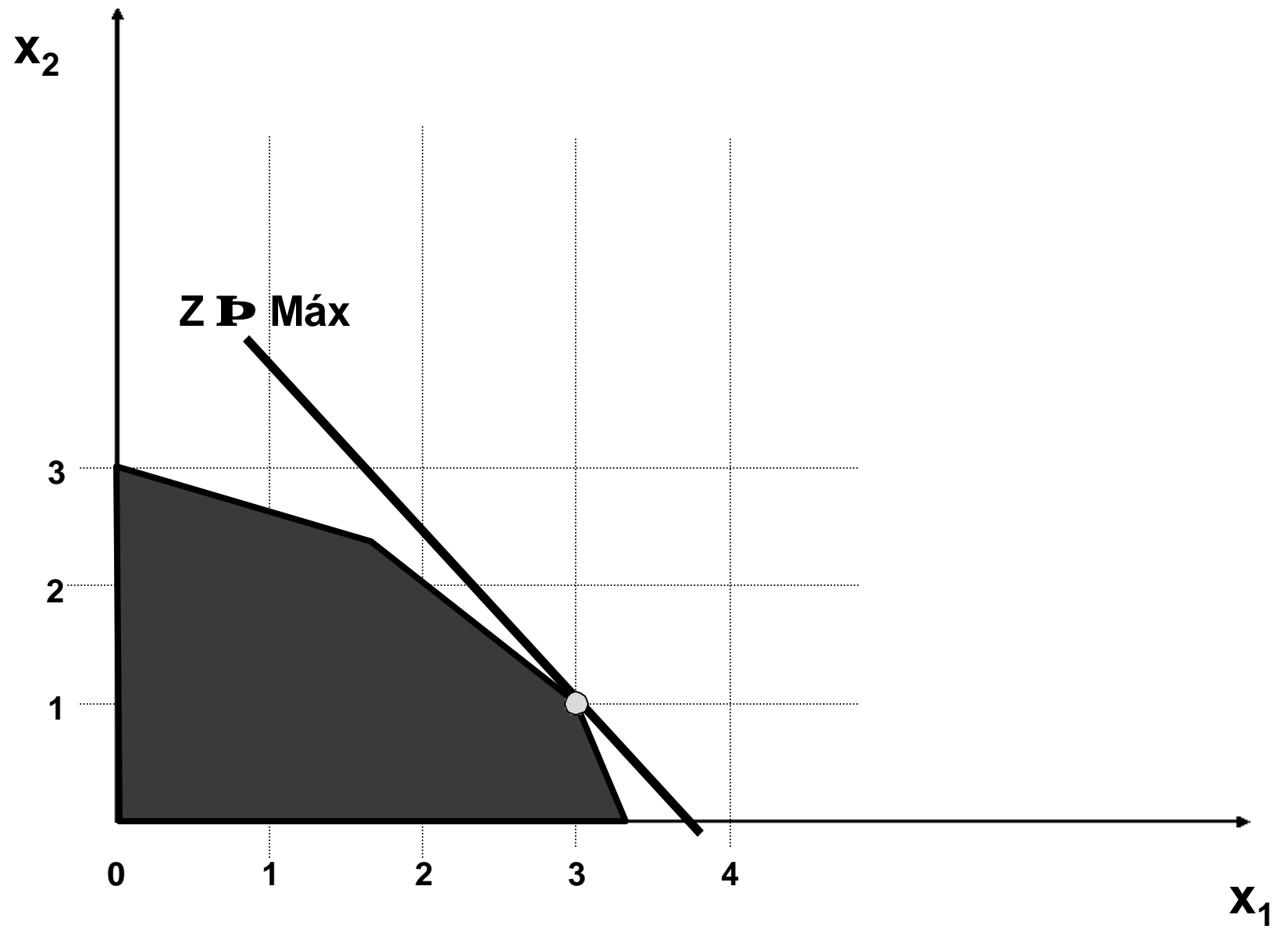


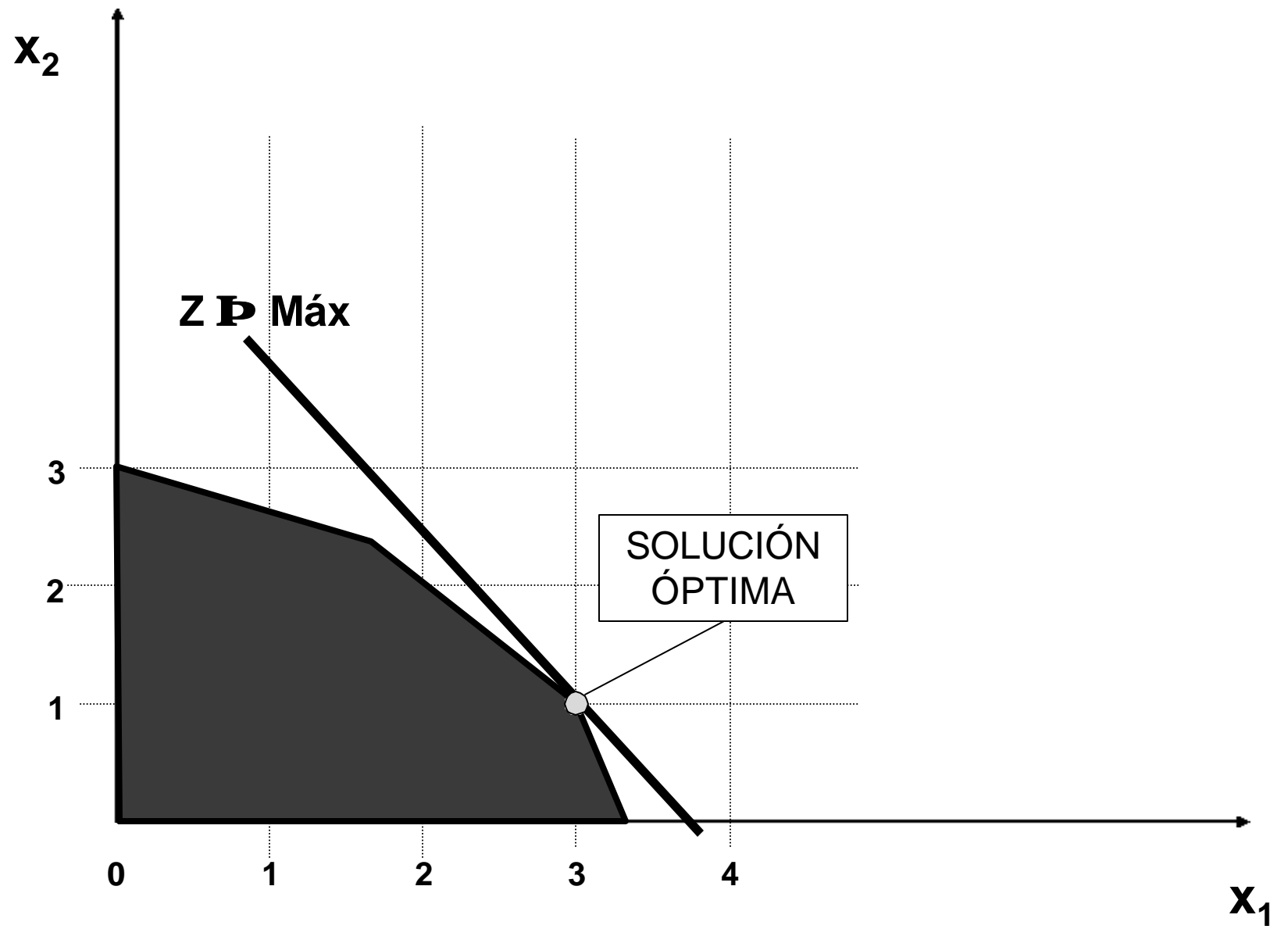


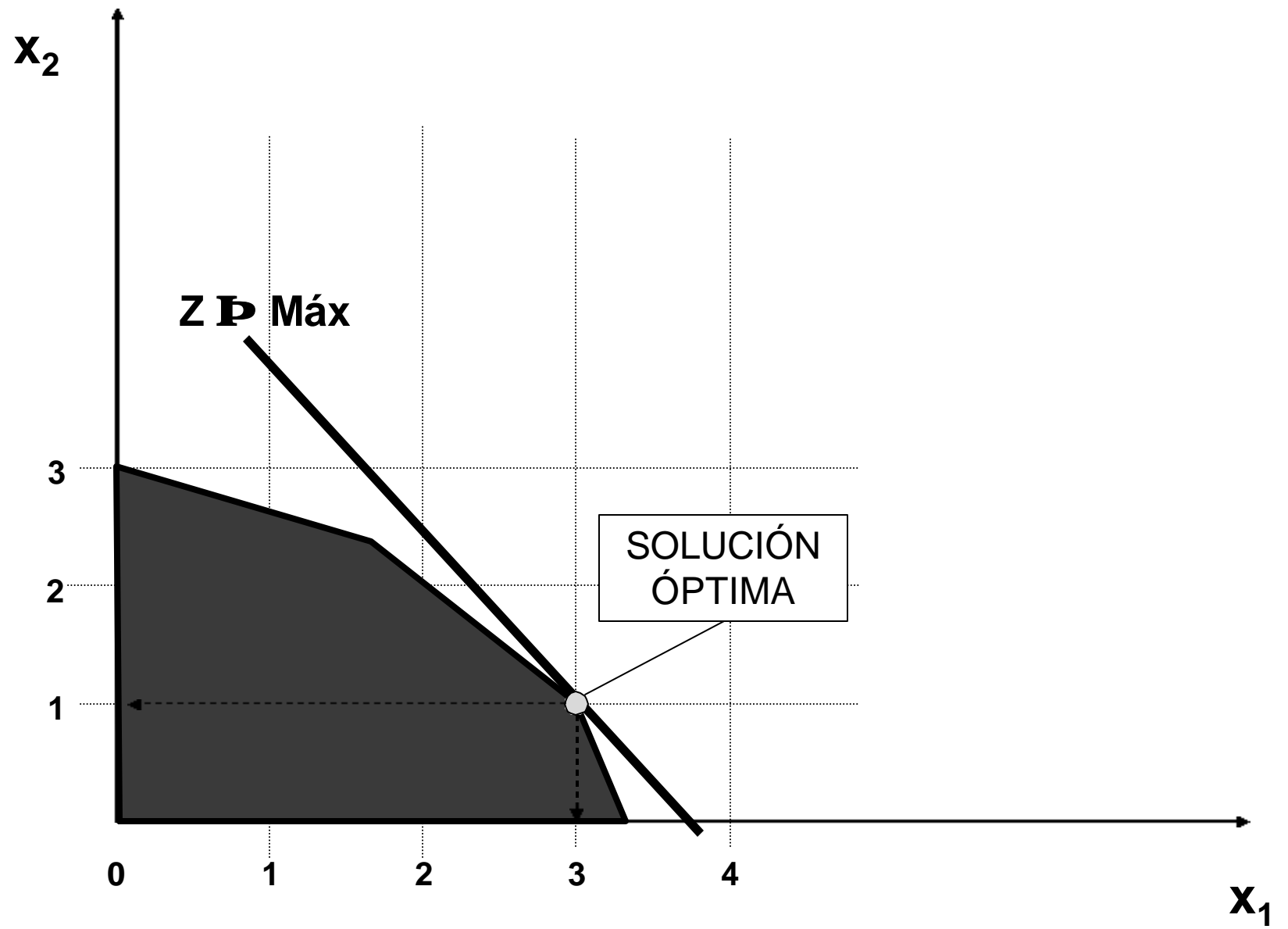


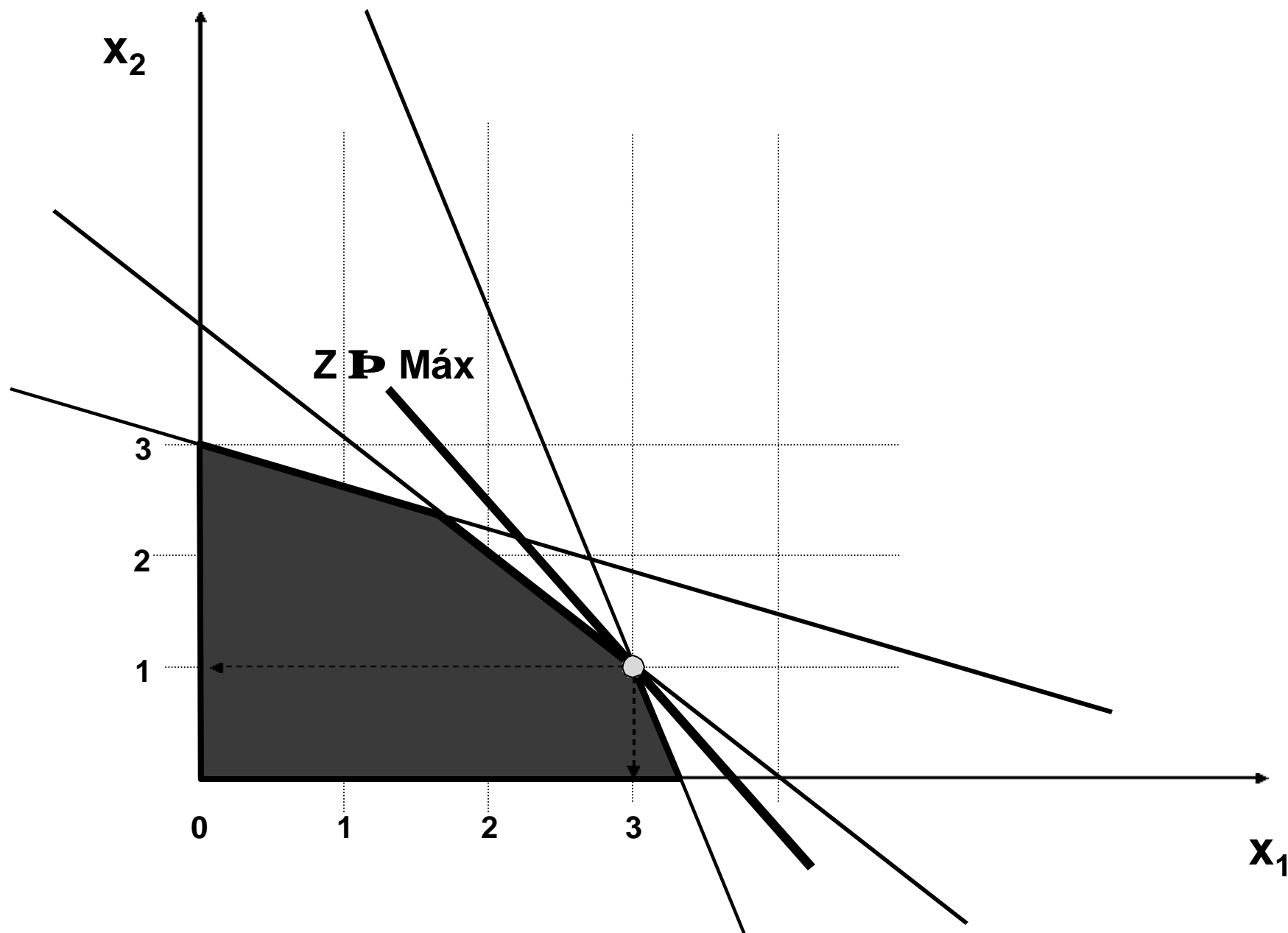




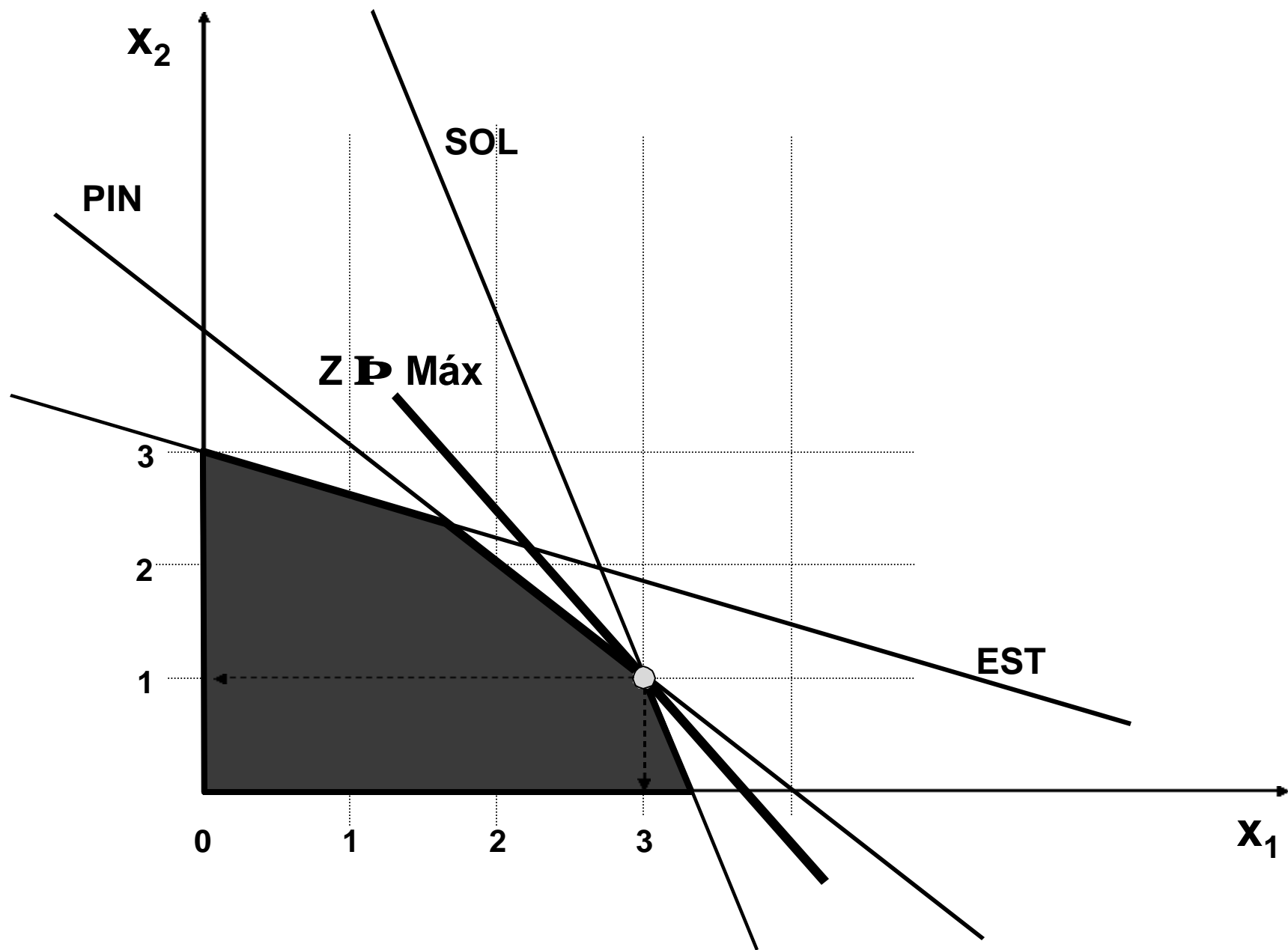


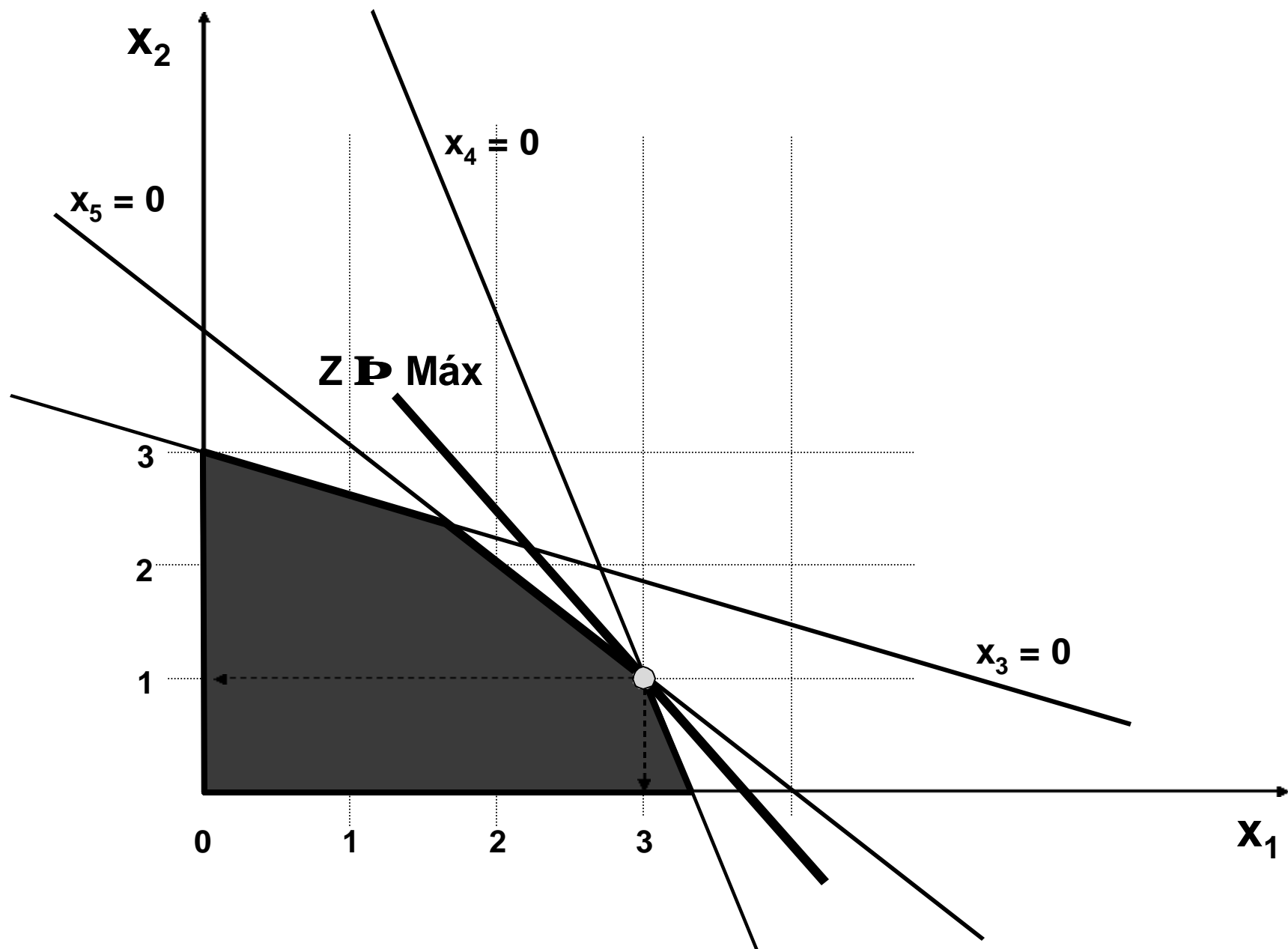


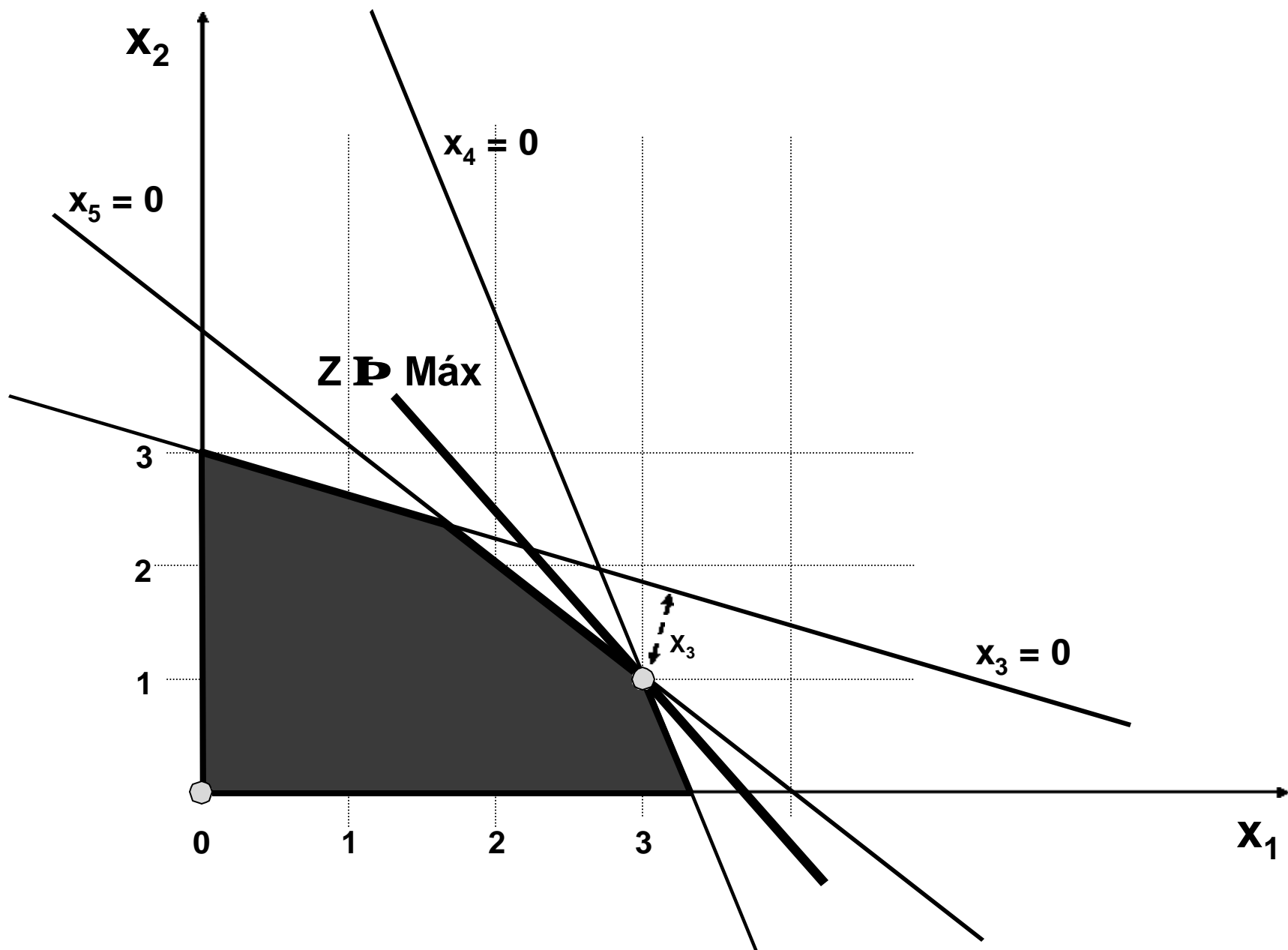


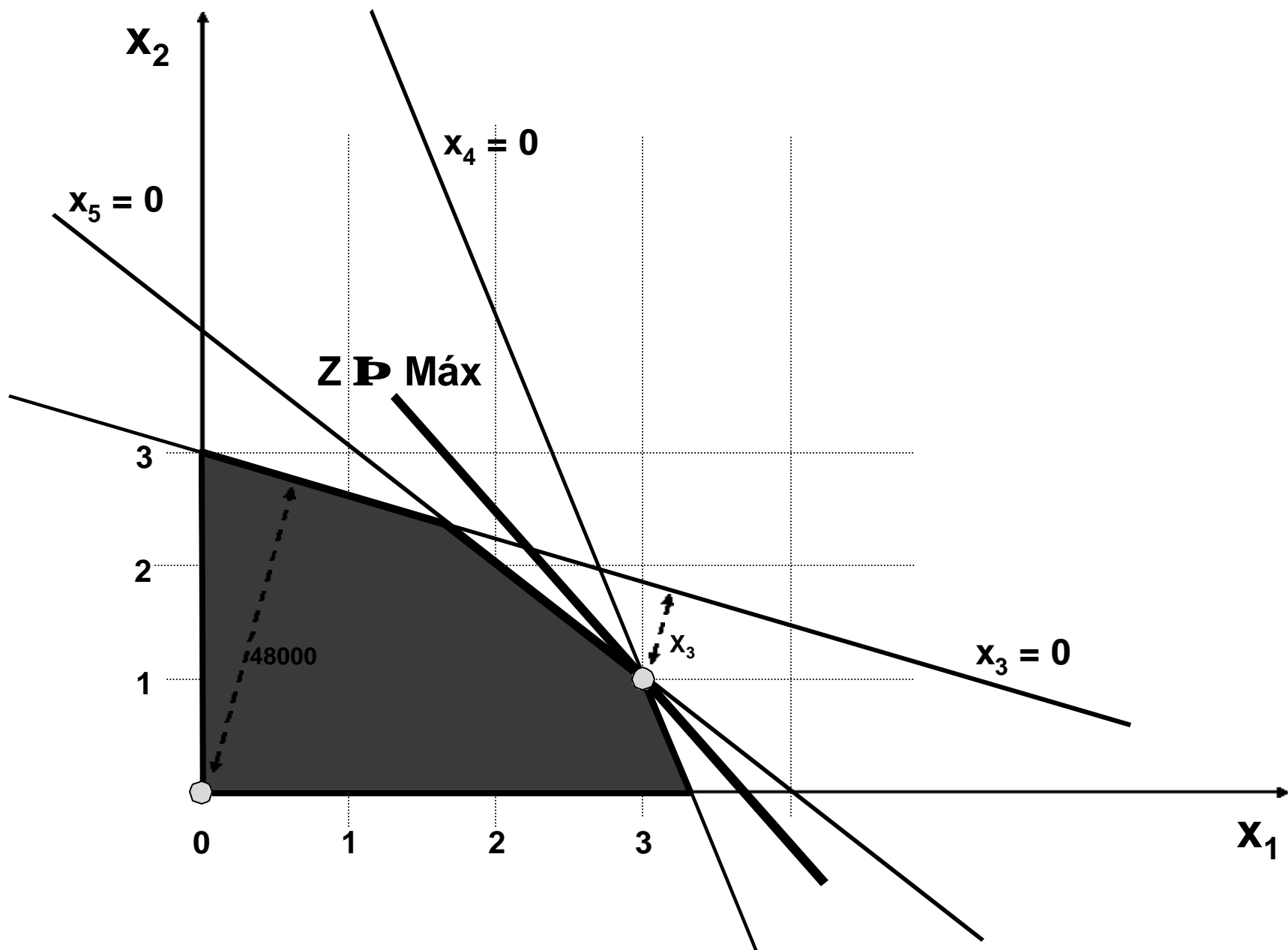












$$\text{MAX: } Z = 4 x_1 + 3 x_2$$

$$\left\{ \begin{array}{l} 6 x_1 + 16 x_2 \leq 48000 \\ 12 x_1 + 6 x_2 \leq 42000 \\ 9 x_1 + 9 x_2 \leq 36000 \end{array} \right.$$

$$x_1, x_2 \geq 0$$

## FORMA MATRICIAL EXTENDIDA

	$X_1$	$X_2$	SIGNO	RHS
Z)	4	3	$\rightarrow$	MAX
EST)	6	16	$\leq$	48000
SOL)	12	6	$\leq$	42000
PIN)	9	9	$\leq$	36000
Var.	NN	NN		

$$\text{MAX: } Z = 4 x_1 + 3 x_2$$

$$\begin{cases} 6 x_1 + 16 x_2 + x_3 & = 48000 \\ 12 x_1 + 6 x_2 + x_4 & = 42000 \\ 9 x_1 + 9 x_2 + x_5 & = 36000 \end{cases}$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

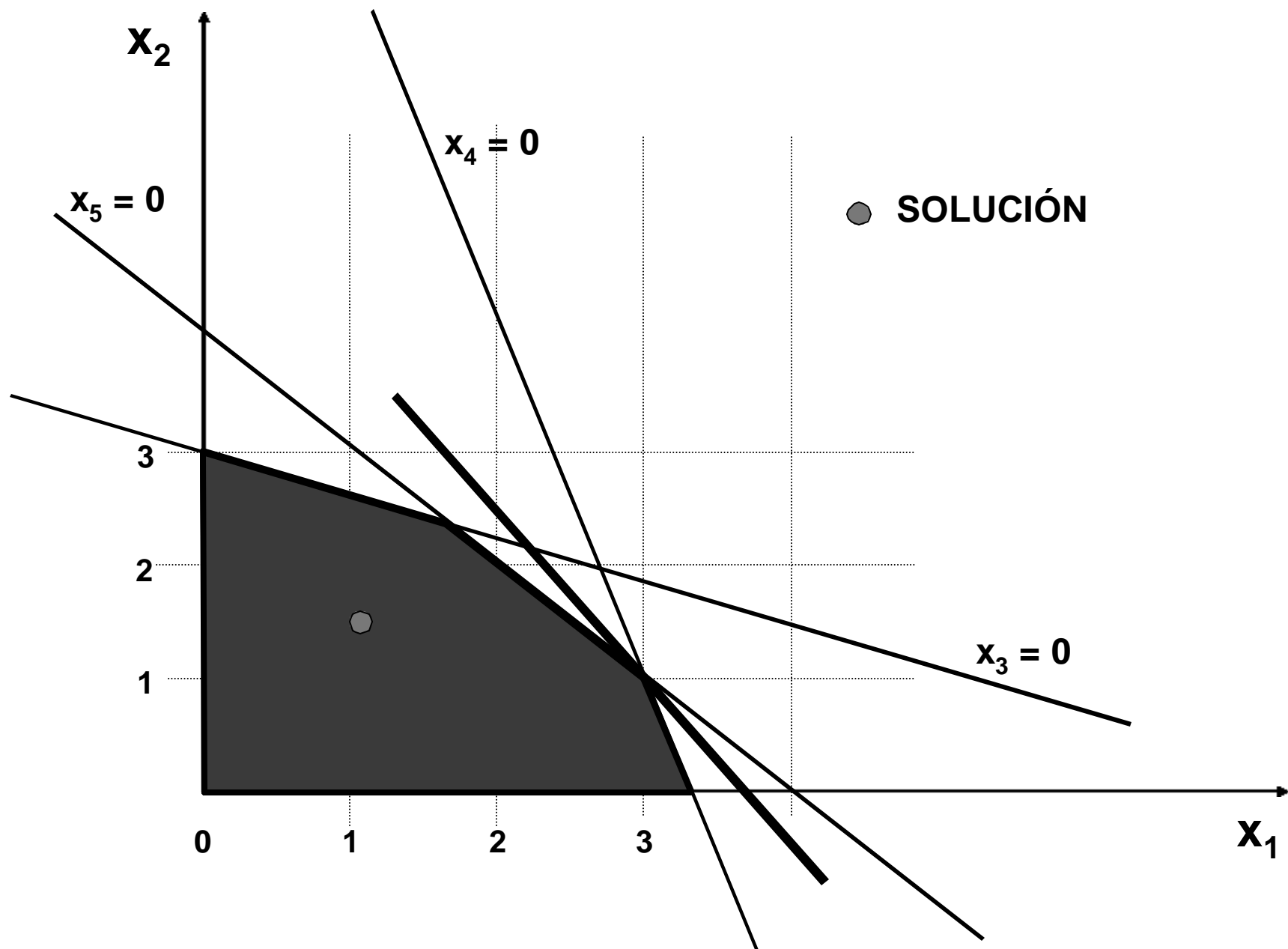
## **SISTEMA DE ECUACIONES LINEALES**

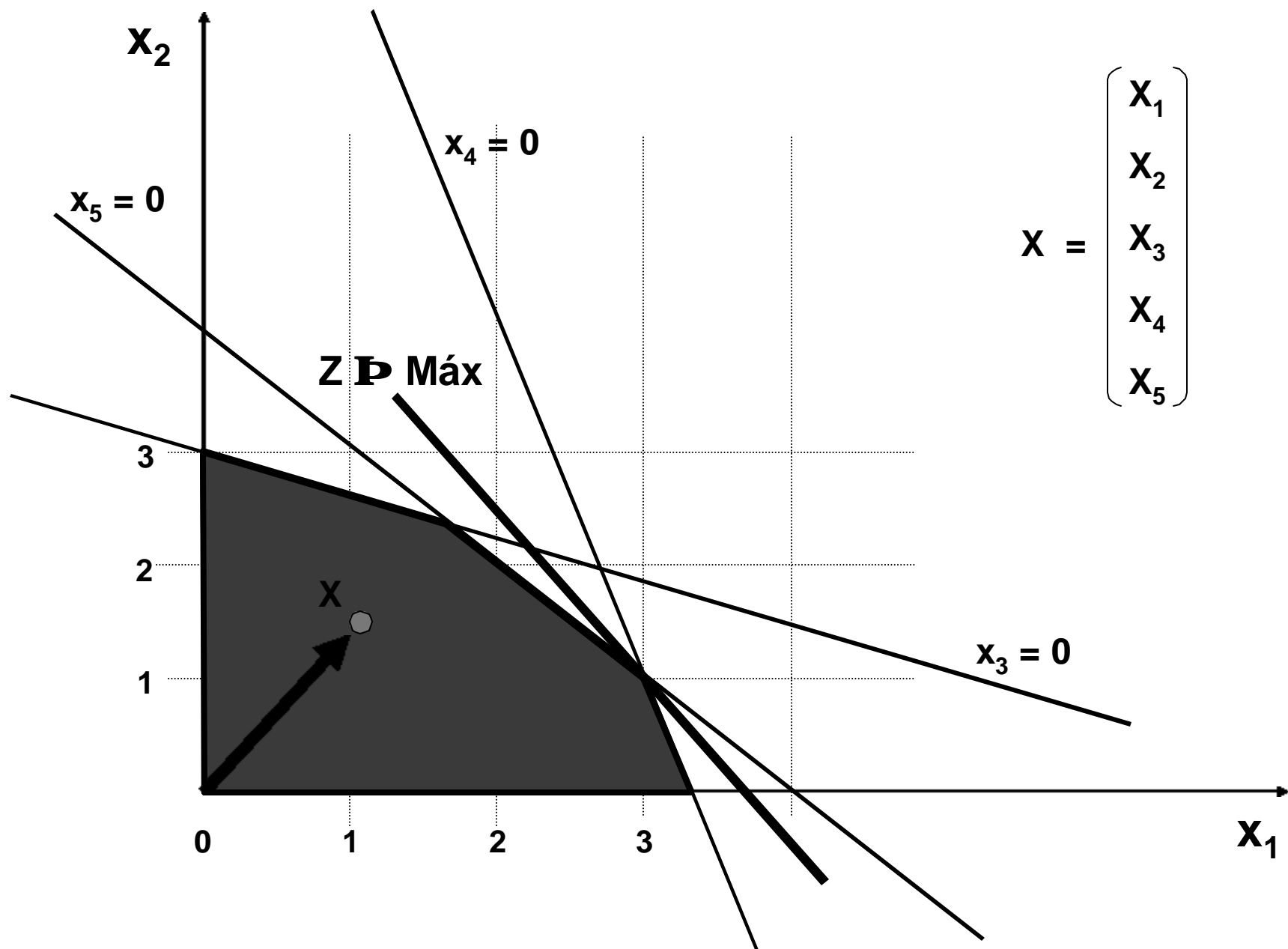
$$\left\{ \begin{array}{lcl} 6 x_1 + 16 x_2 + x_3 & & = 48000 \\ 12 x_1 + 6 x_2 & + x_4 & = 42000 \\ 9 x_1 + 9 x_2 & & + x_5 = 36000 \end{array} \right.$$

**n = número de variables (5)**

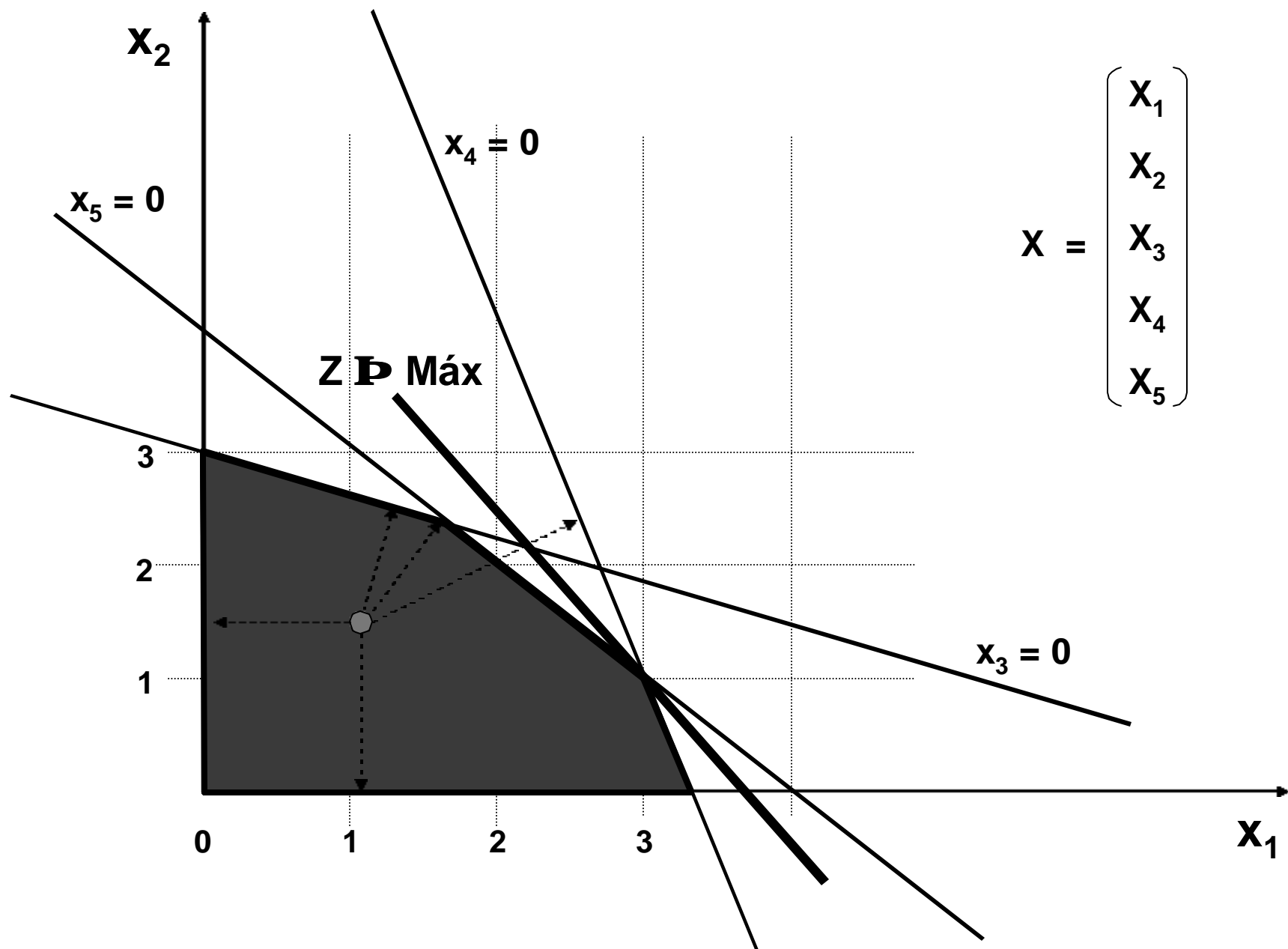
**m = número de restricciones (3)**



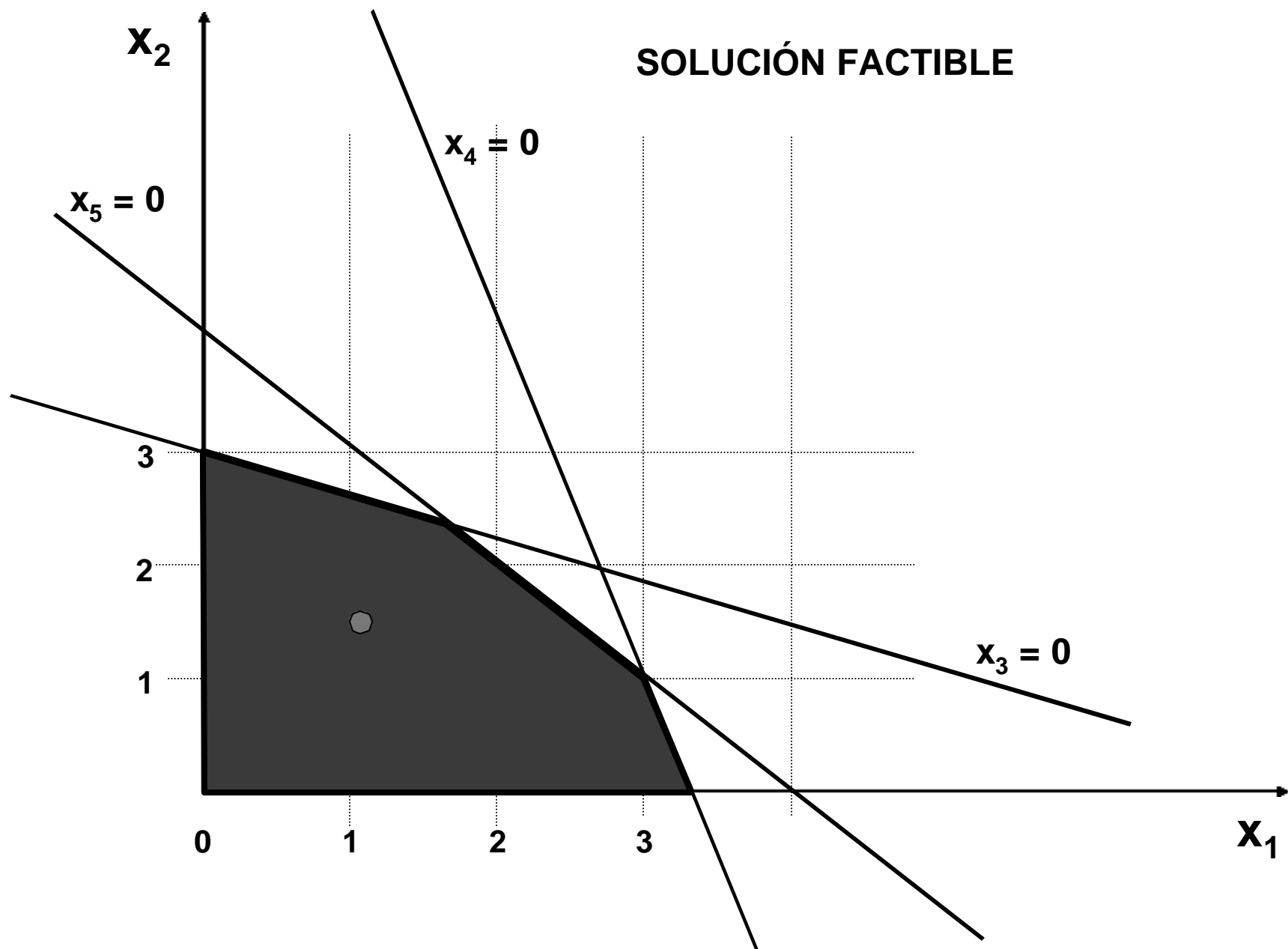


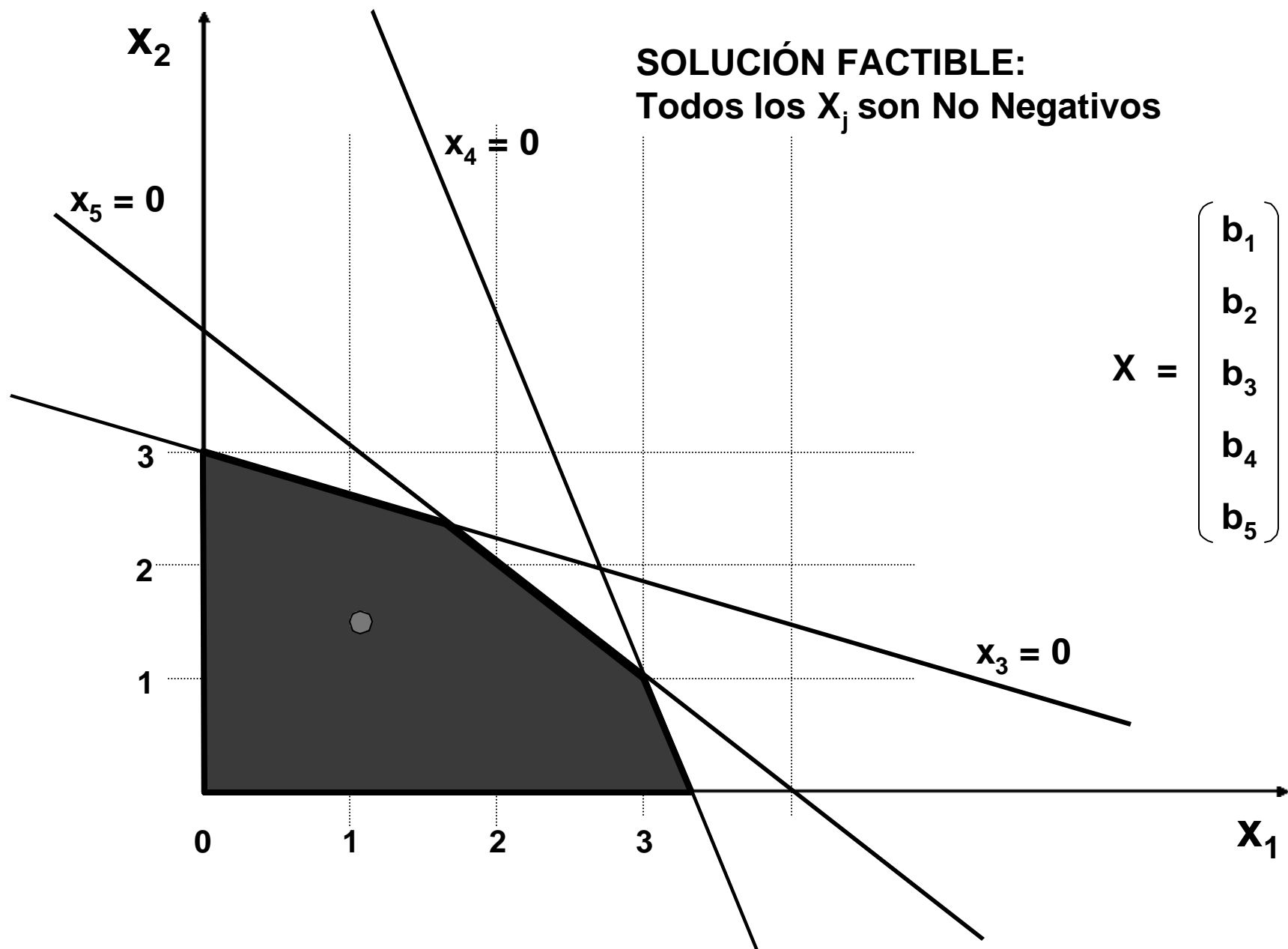


$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$



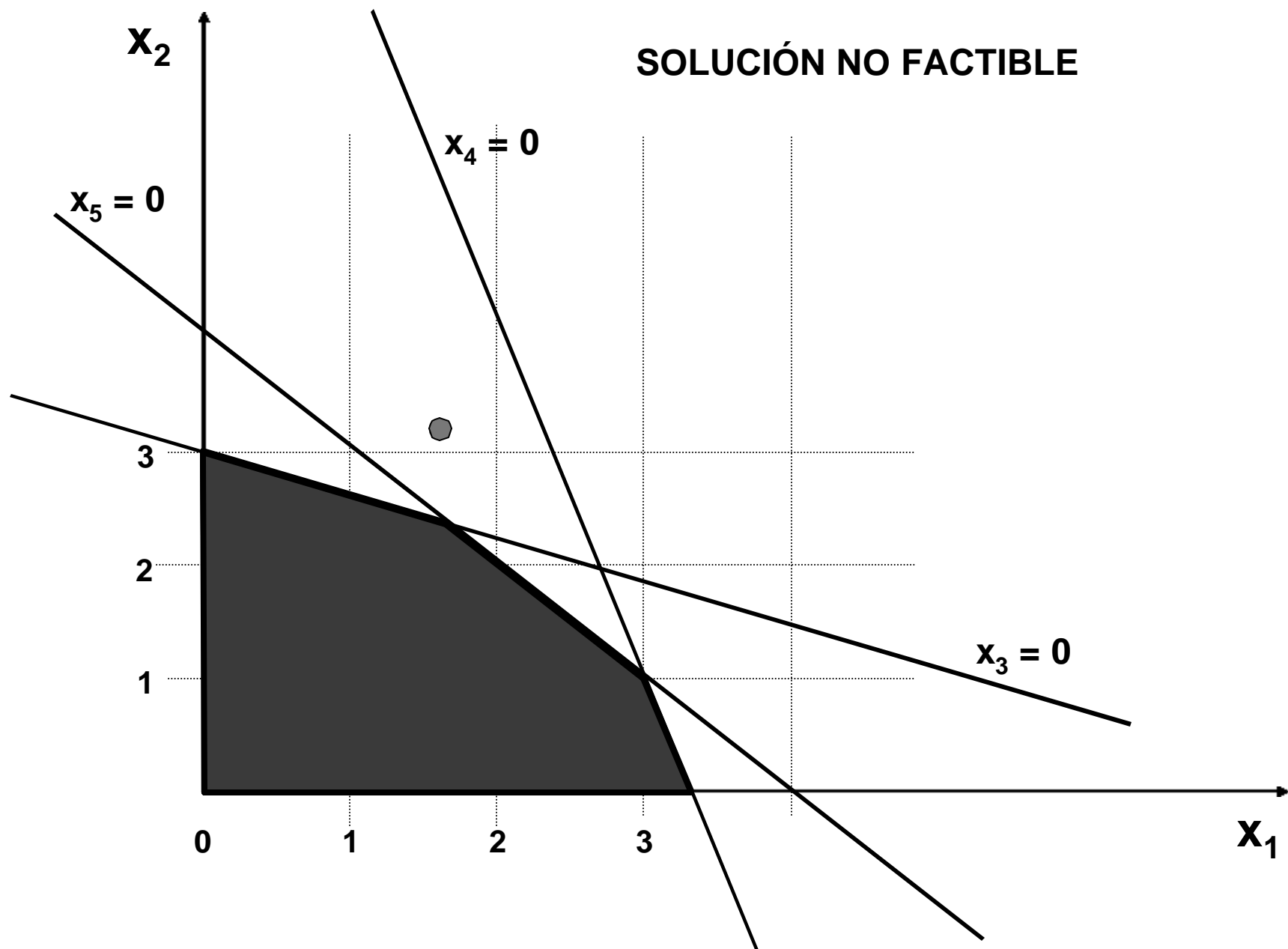
$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$

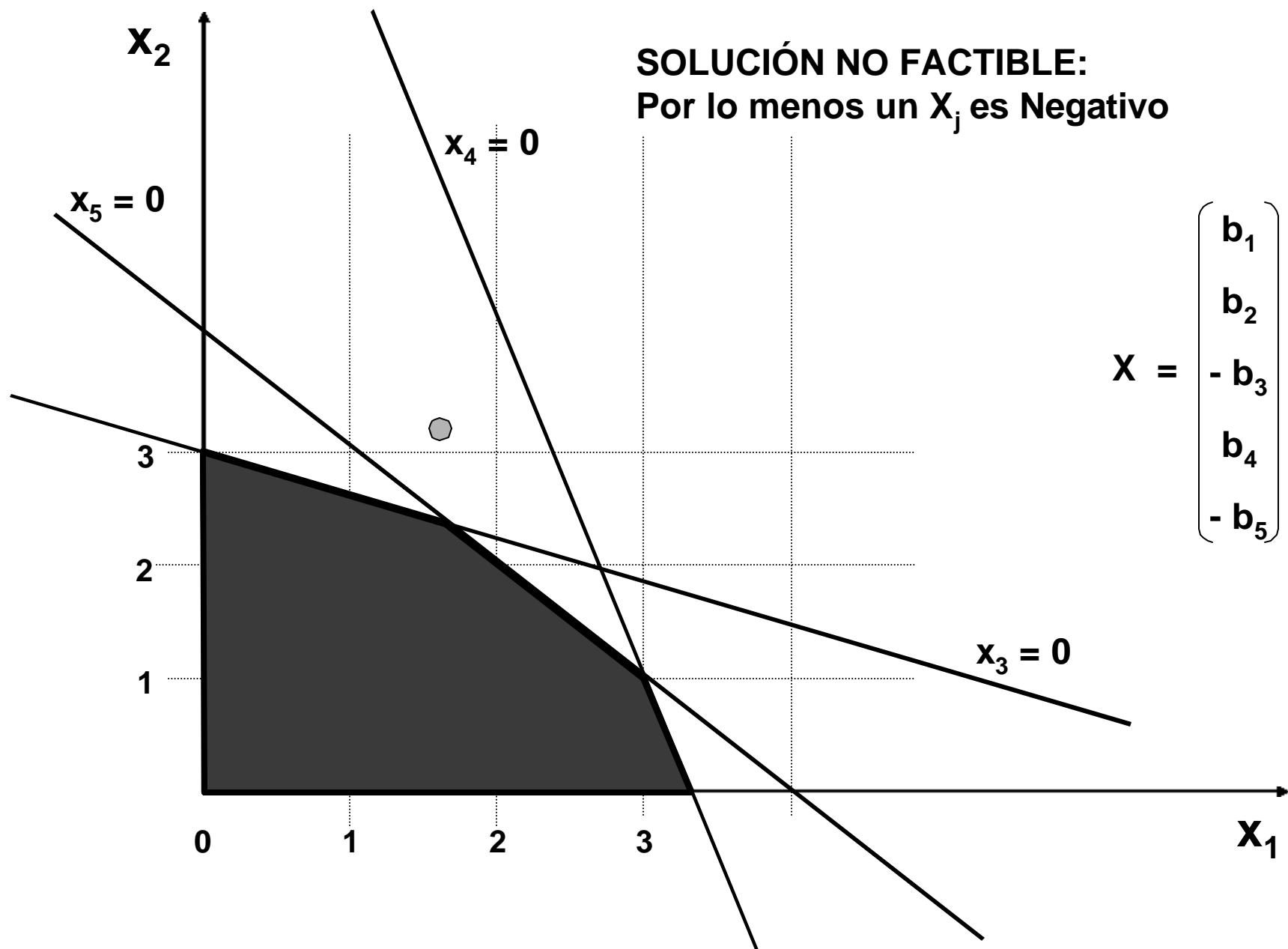




**SOLUCIÓN FACTIBLE:**  
Todos los  $X_j$  son No Negativos

$$X = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}$$



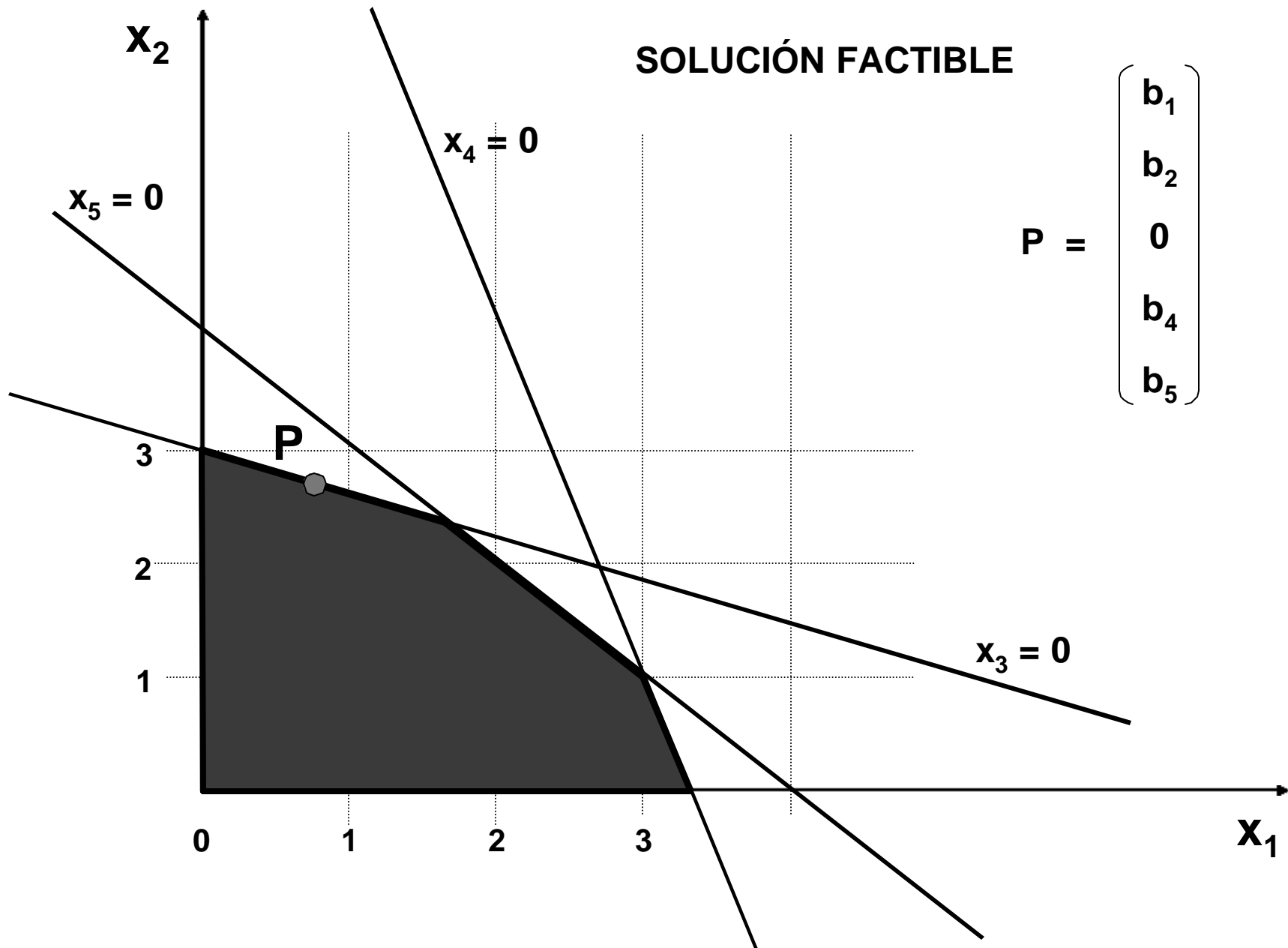


**SOLUCIÓN NO FACTIBLE:**  
Por lo menos un  $X_j$  es Negativo

$$X = \begin{bmatrix} b_1 \\ b_2 \\ -b_3 \\ b_4 \\ -b_5 \end{bmatrix}$$

# SOLUCIÓN FACTIBLE

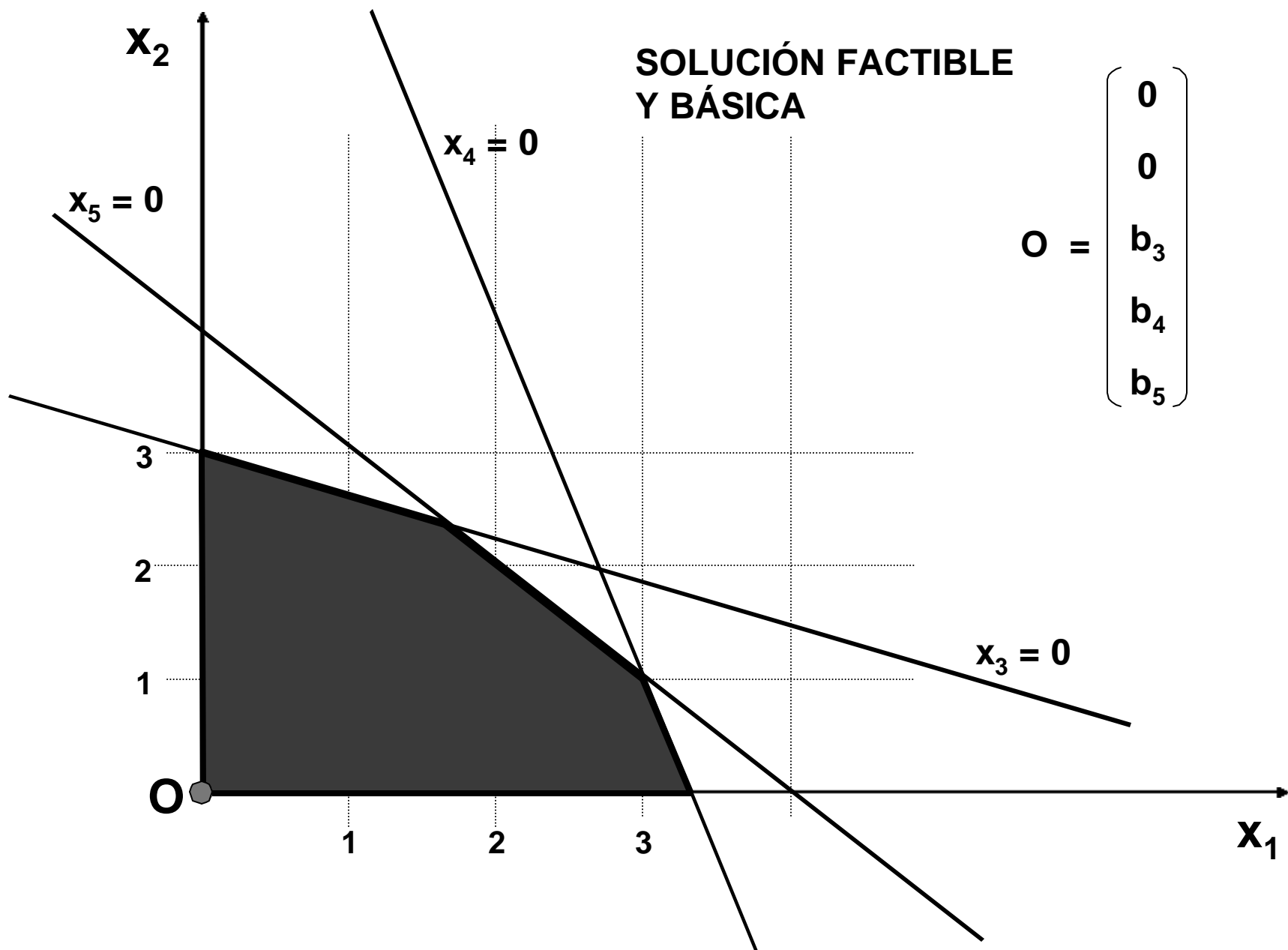
$$P = \begin{bmatrix} b_1 \\ b_2 \\ 0 \\ b_4 \\ b_5 \end{bmatrix}$$





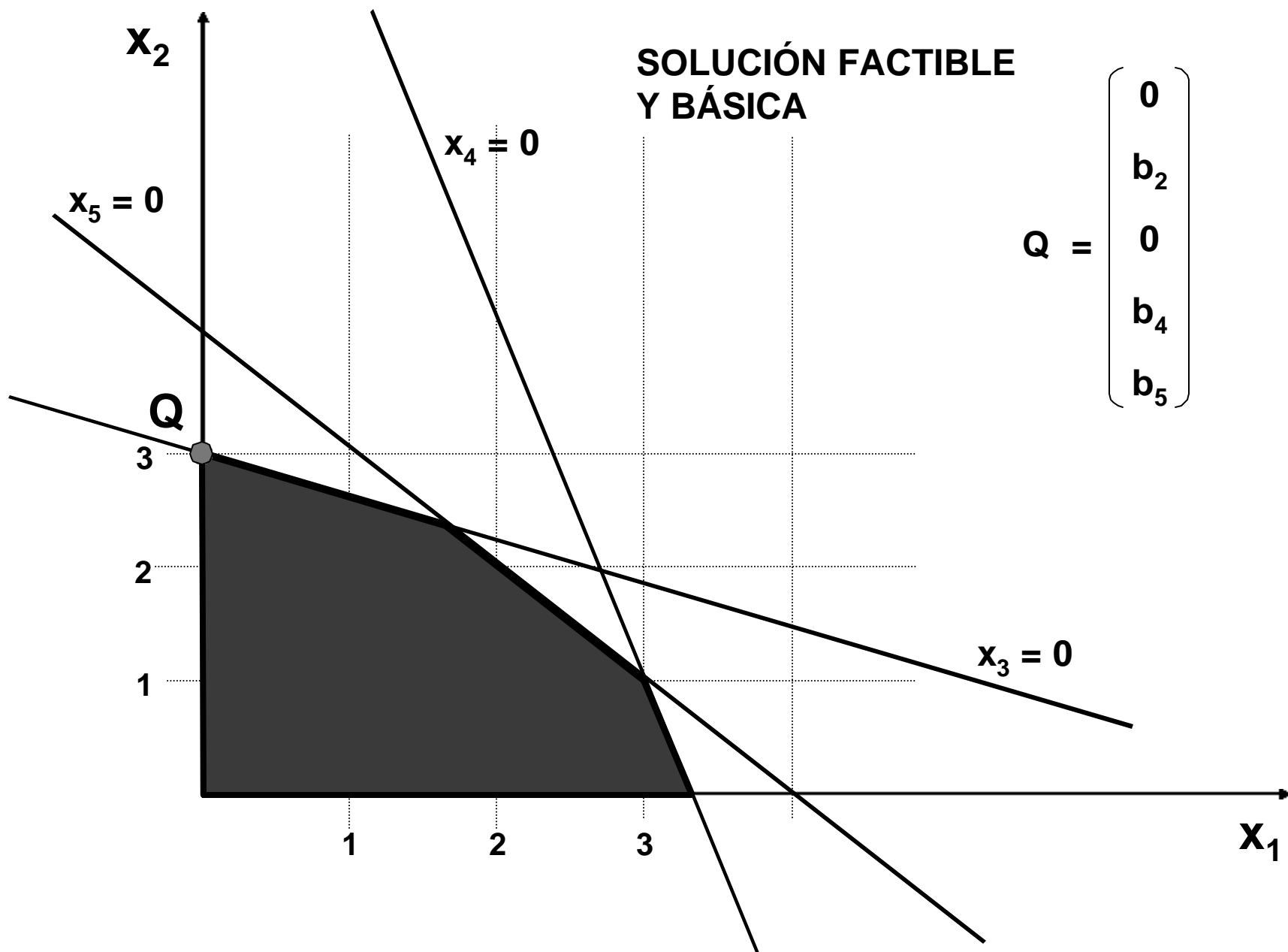
# SOLUCIÓN FACTIBLE Y BÁSICA

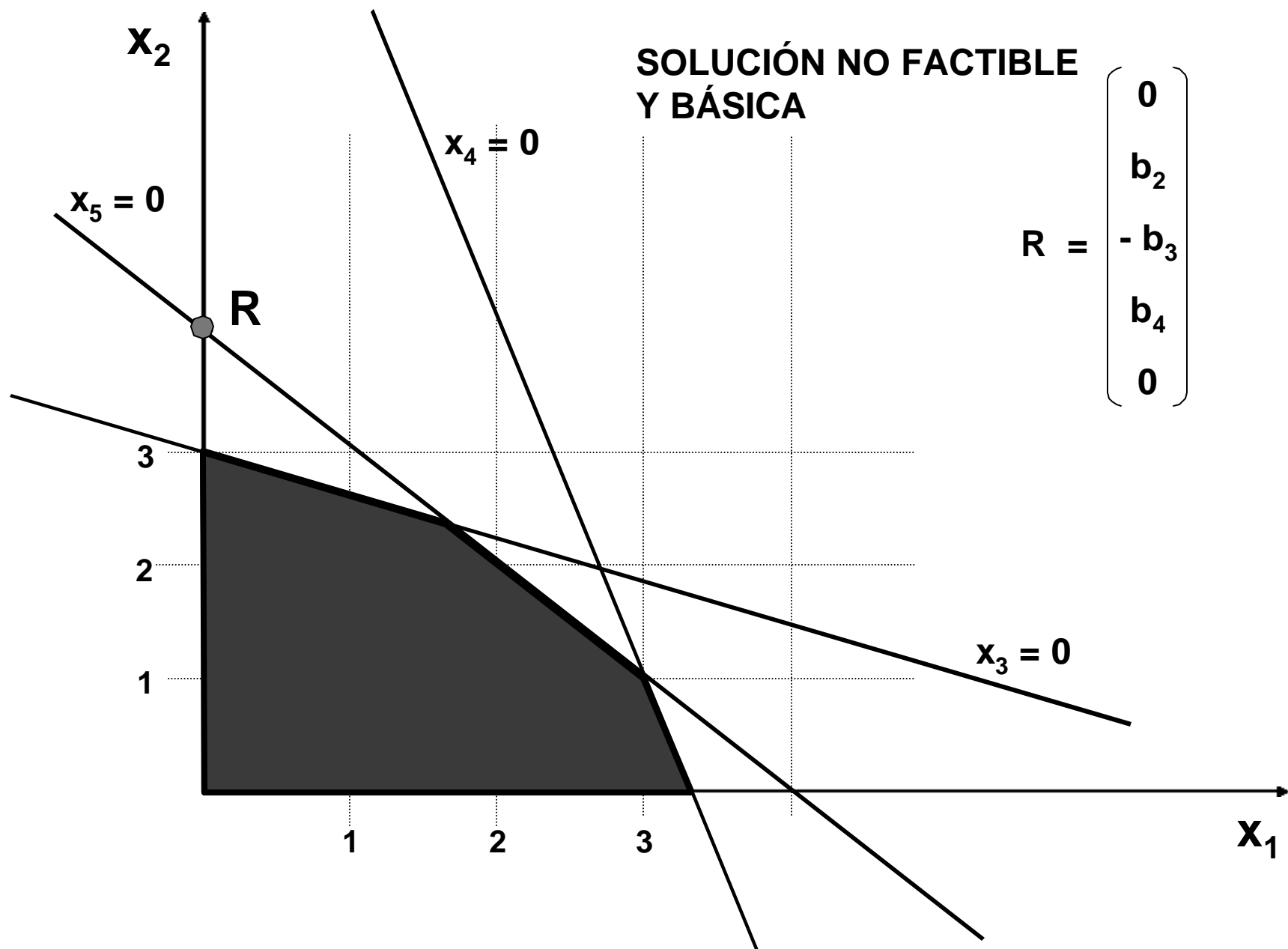
$$O = \begin{bmatrix} 0 \\ 0 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}$$



# SOLUCIÓN FACTIBLE Y BÁSICA

$$Q = \begin{bmatrix} 0 \\ b_2 \\ 0 \\ b_4 \\ b_5 \end{bmatrix}$$

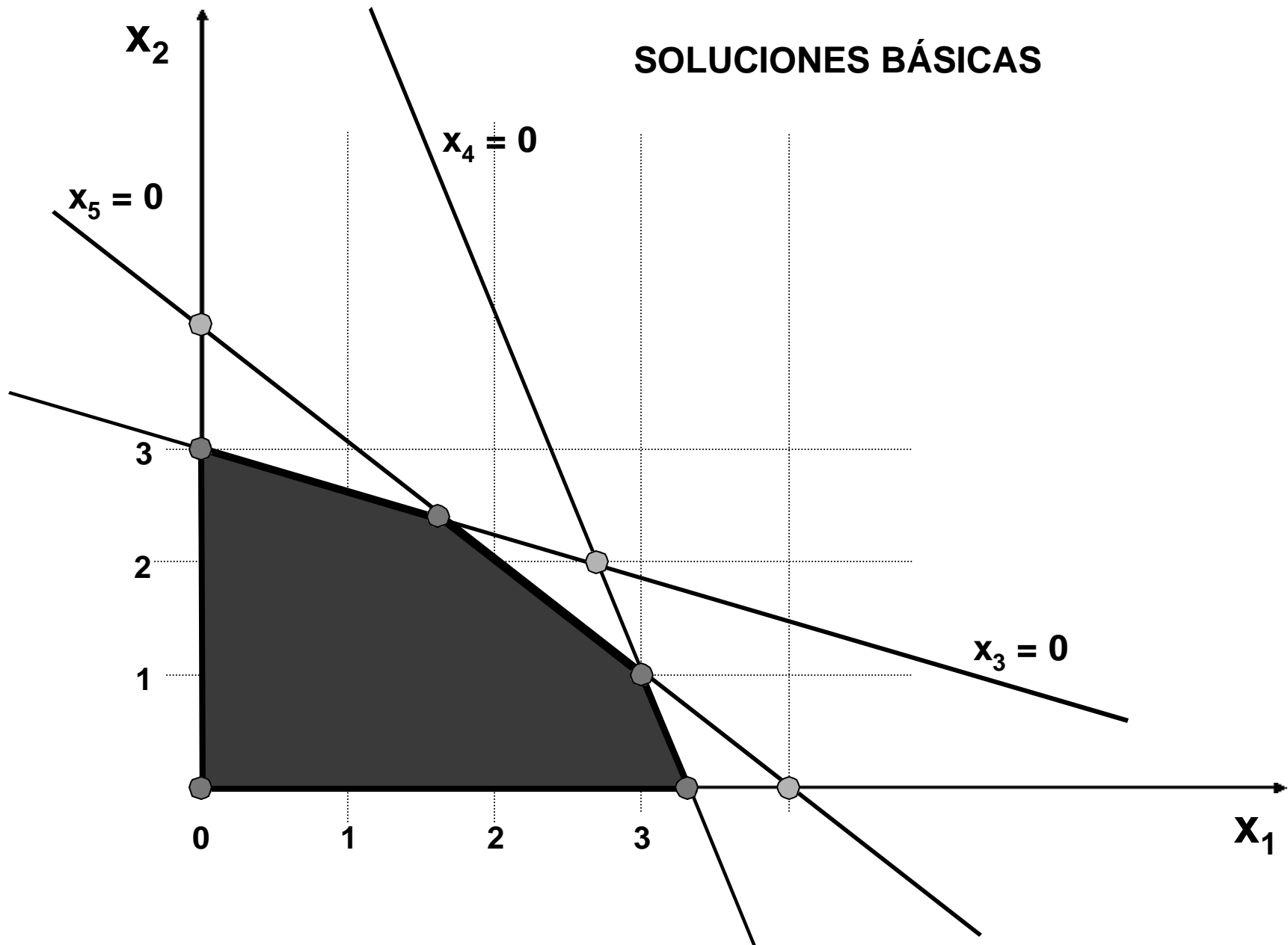




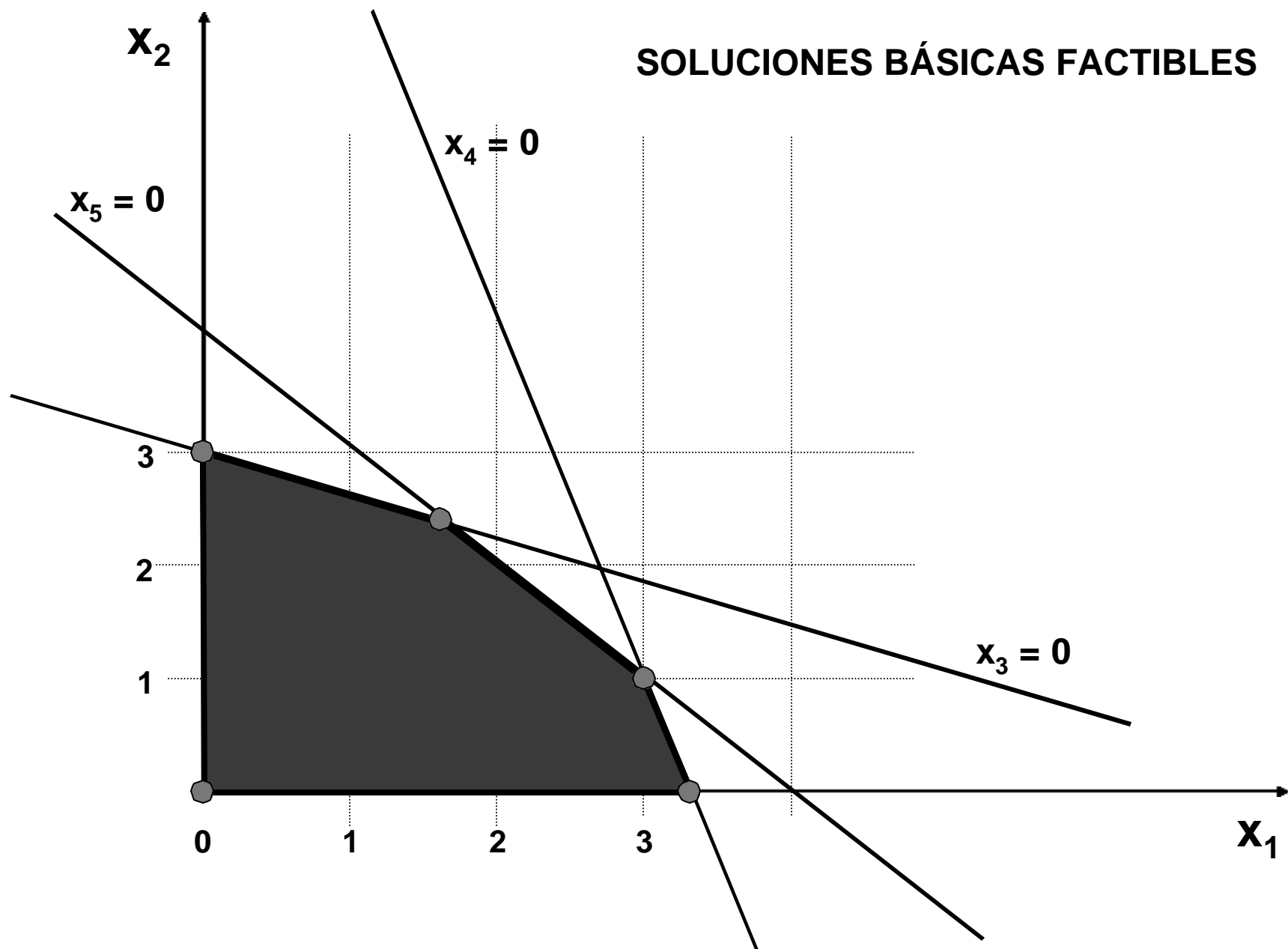
**SOLUCIÓN NO FACTIBLE  
Y BÁSICA**

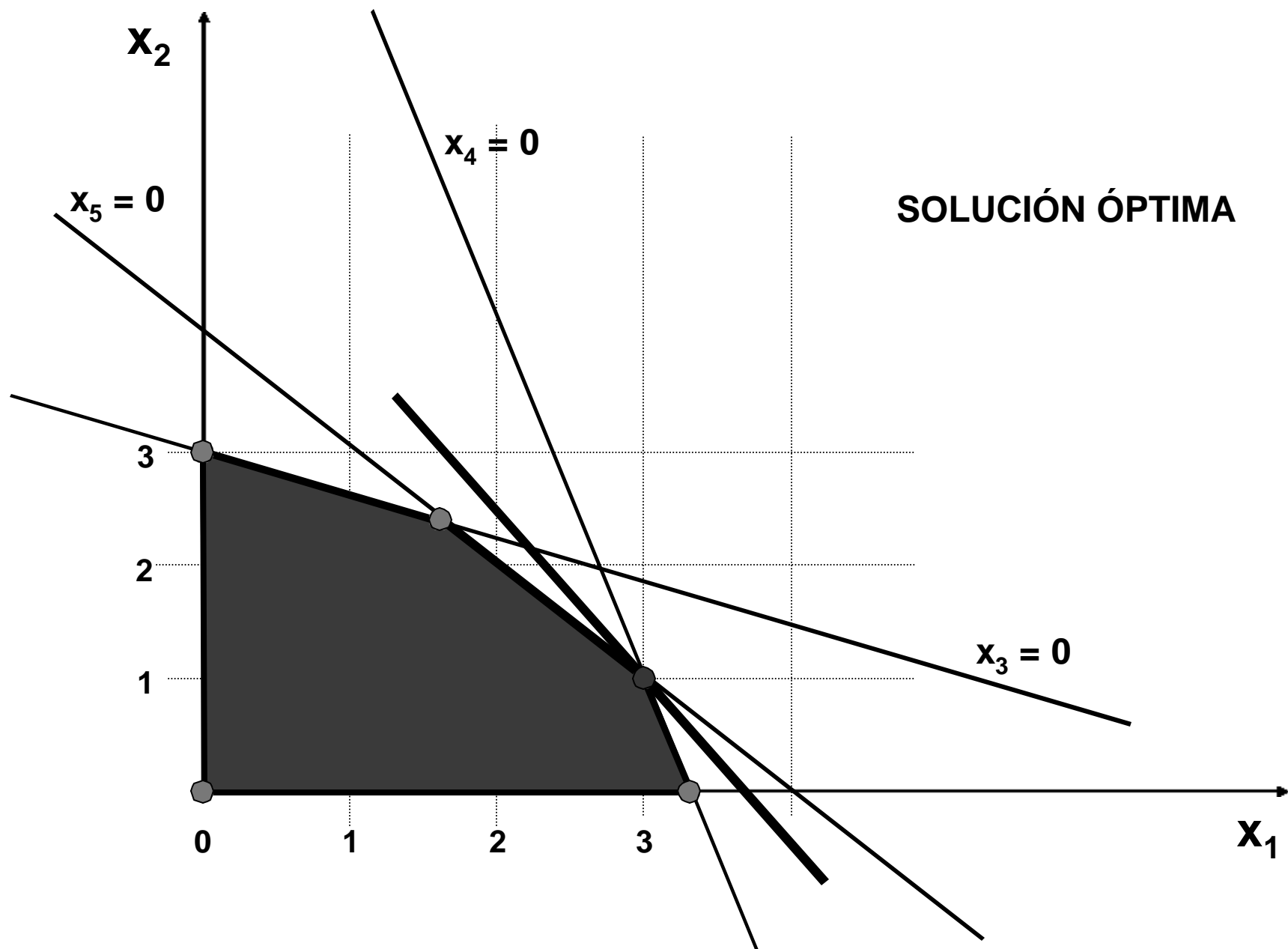
$$R = \begin{bmatrix} 0 \\ b_2 \\ -b_3 \\ b_4 \\ 0 \end{bmatrix}$$

## SOLUCIONES BÁSICAS

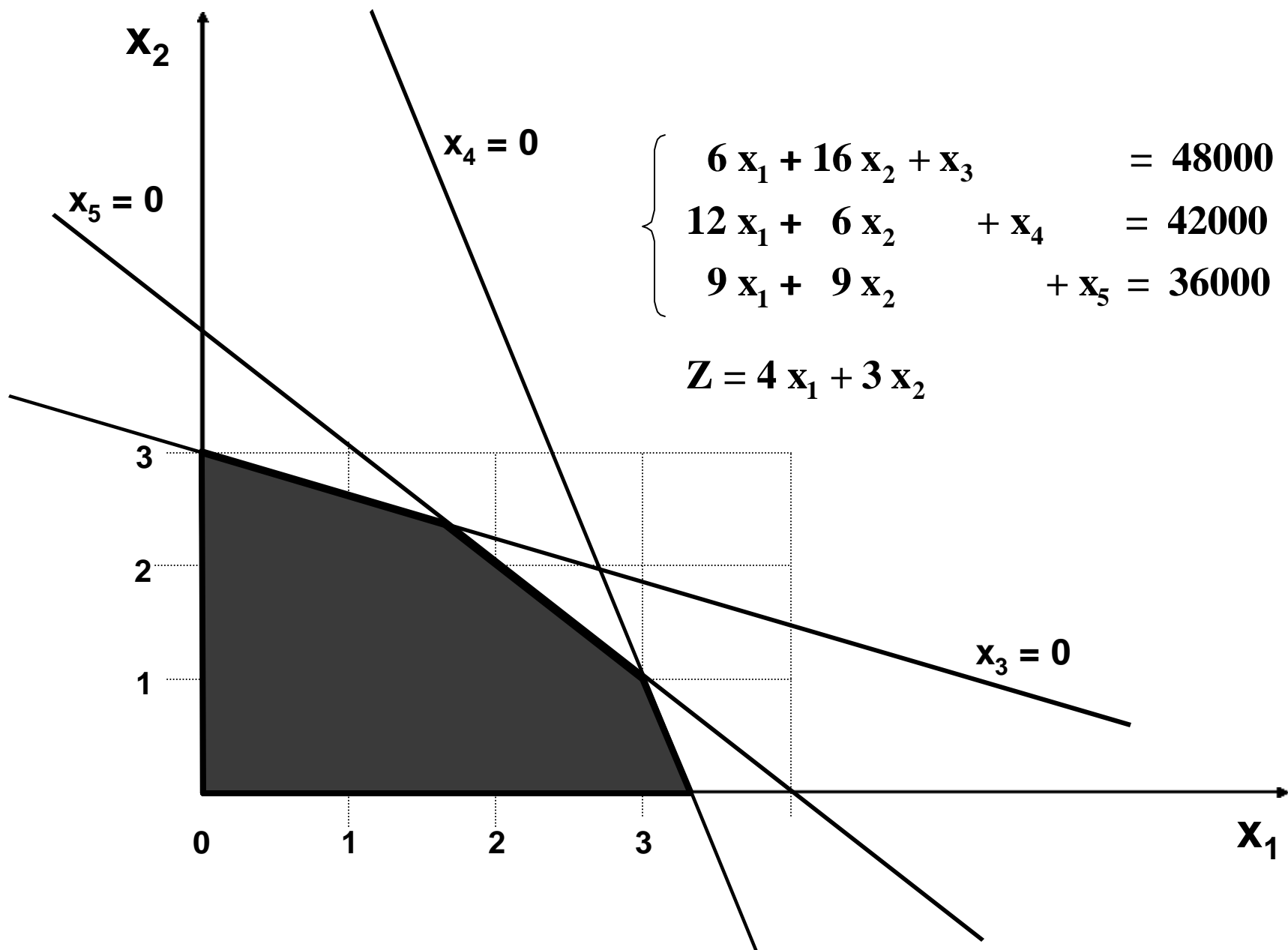


# SOLUCIONES BÁSICAS FACTIBLES





**SOLUCIÓN ÓPTIMA**



$$\begin{cases} 6x_1 + 16x_2 + x_3 & = 48000 \\ 12x_1 + 6x_2 + x_4 & = 42000 \\ 9x_1 + 9x_2 + x_5 & = 36000 \end{cases}$$

$$Z = 4x_1 + 3x_2$$

$$\left\{ \begin{array}{l} 6 \mathbf{x}_1 + 16 \mathbf{x}_2 + \mathbf{x}_3 = 48000 \\ 12 \mathbf{x}_1 + 6 \mathbf{x}_2 + \mathbf{x}_4 = 42000 \\ 9 \mathbf{x}_1 + 9 \mathbf{x}_2 + \mathbf{x}_5 = 36000 \end{array} \right.$$

$$\mathbf{X} = \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \\ \mathbf{x}_5 \end{pmatrix} \quad \mathbf{Z} = 4 \mathbf{x}_1 + 3 \mathbf{x}_2$$



$$\left\{ \begin{array}{lcl} \mathbf{x}_3 & = & 48000 \\ \mathbf{x}_4 & = & 42000 \\ \mathbf{x}_5 & = & 36000 \end{array} \right.$$

$$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \\ 48000 \\ 42000 \\ 36000 \end{pmatrix} \quad \mathbf{Z} = 0$$

{

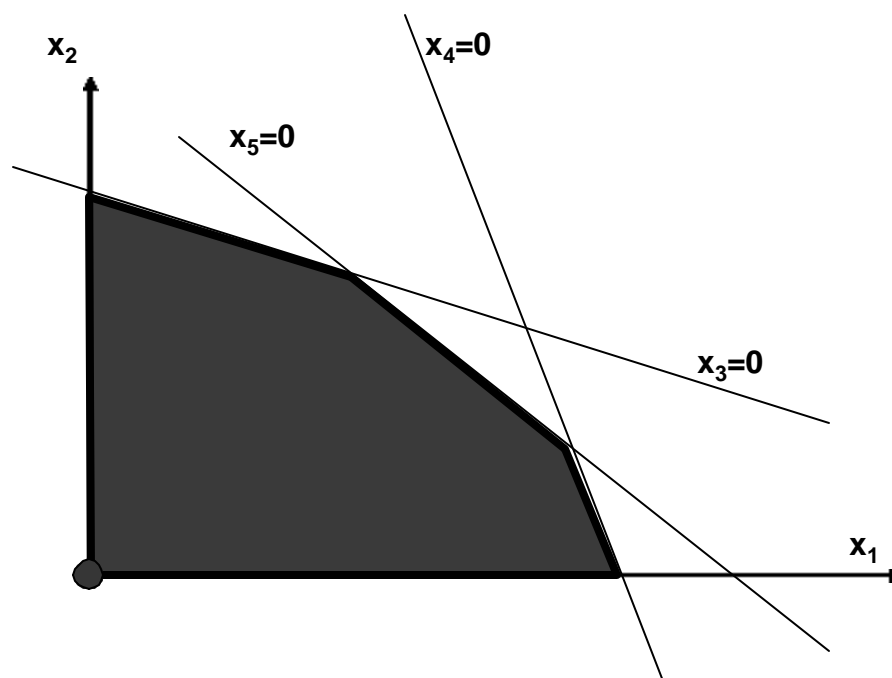
$$\mathbf{x}_3 = 48000$$

$$\mathbf{x}_4 = 42000$$

$$\mathbf{x}_5 = 36000$$

$$\mathbf{X} = \begin{pmatrix} 0 \\ 0 \\ 48000 \\ 42000 \\ 36000 \end{pmatrix}$$

$$\mathbf{Z} = 0$$



$$\left\{ \begin{array}{l} 6 \mathbf{x}_1 + 16 \mathbf{x}_2 + \mathbf{x}_3 = 48000 \\ 12 \mathbf{x}_1 + 6 \mathbf{x}_2 + \mathbf{x}_4 = 42000 \\ 9 \mathbf{x}_1 + 9 \mathbf{x}_2 + \mathbf{x}_5 = 36000 \end{array} \right.$$

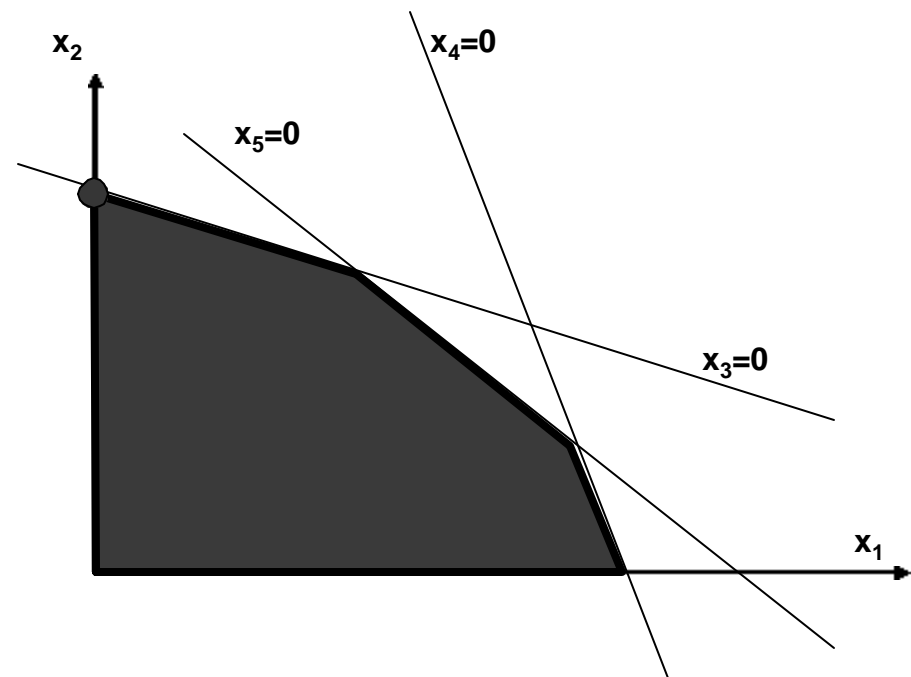
$$\mathbf{X} = \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \\ \mathbf{x}_5 \end{pmatrix} \quad \mathbf{Z} = 4 \mathbf{x}_1 + 3 \mathbf{x}_2$$

$$\left\{ \begin{array}{rcl} 16 \mathbf{x}_2 & & = 48000 \\ 6 \mathbf{x}_2 & + \mathbf{x}_4 & = 42000 \\ 9 \mathbf{x}_2 & & + \mathbf{x}_5 = 36000 \end{array} \right.$$

$$\mathbf{X} = \begin{pmatrix} 0 \\ 3000 \\ 0 \\ 24000 \\ 9000 \end{pmatrix} \quad \mathbf{Z} = 9000$$

$$\left\{ \begin{array}{rcl} 16 x_2 & & = 48000 \\ 6 x_2 & + x_4 & = 42000 \\ 9 x_2 & & + x_5 = 36000 \end{array} \right.$$

$$\mathbf{X} = \begin{pmatrix} 0 \\ 3000 \\ 0 \\ 24000 \\ 9000 \end{pmatrix} \quad \mathbf{Z} = 9000$$



$$\left\{ \begin{array}{l} 6 \mathbf{x}_1 + 16 \mathbf{x}_2 + \mathbf{x}_3 = 48000 \\ 12 \mathbf{x}_1 + 6 \mathbf{x}_2 + \mathbf{x}_4 = 42000 \\ 9 \mathbf{x}_1 + 9 \mathbf{x}_2 + \mathbf{x}_5 = 36000 \end{array} \right.$$

$$\mathbf{X} = \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \\ \mathbf{x}_5 \end{pmatrix} \quad \mathbf{Z} = 4 \mathbf{x}_1 + 3 \mathbf{x}_2$$

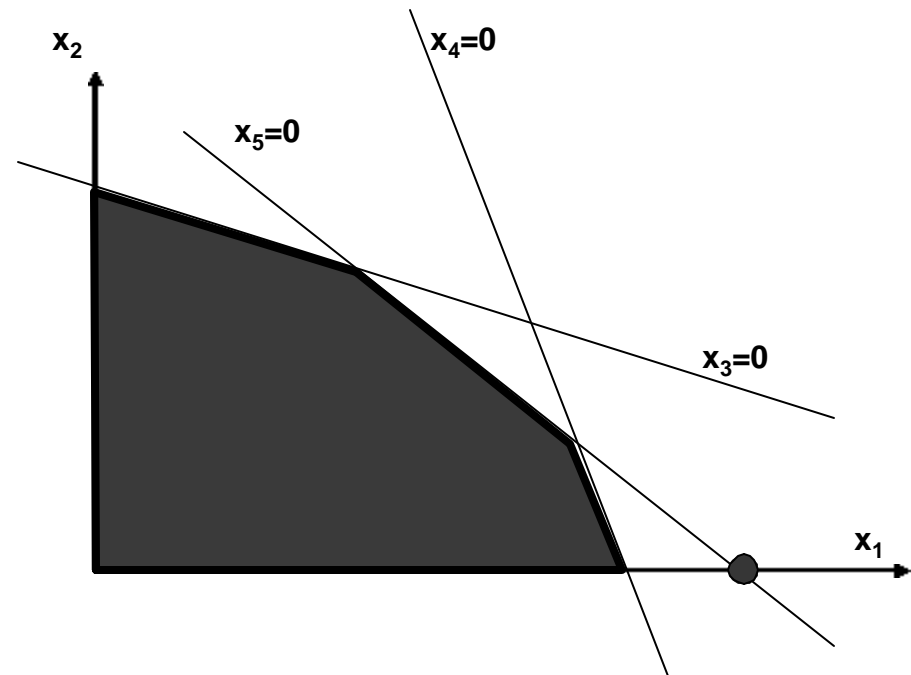
$$\left\{ \begin{array}{rcl} 6 \mathbf{x}_1 & + \mathbf{x}_3 & = 48000 \\ 12 \mathbf{x}_1 & + \mathbf{x}_4 & = 42000 \\ 9 \mathbf{x}_1 & & = 36000 \end{array} \right.$$

$$\mathbf{X} = \begin{pmatrix} 4000 \\ 0 \\ 24000 \\ -6000 \\ 0 \end{pmatrix} \quad \text{NO FACTIBLE}$$

$$\begin{cases} 6x_1 & + x_3 & = 48000 \\ 12x_1 & + x_4 & = 42000 \\ 9x_1 & & = 36000 \end{cases}$$

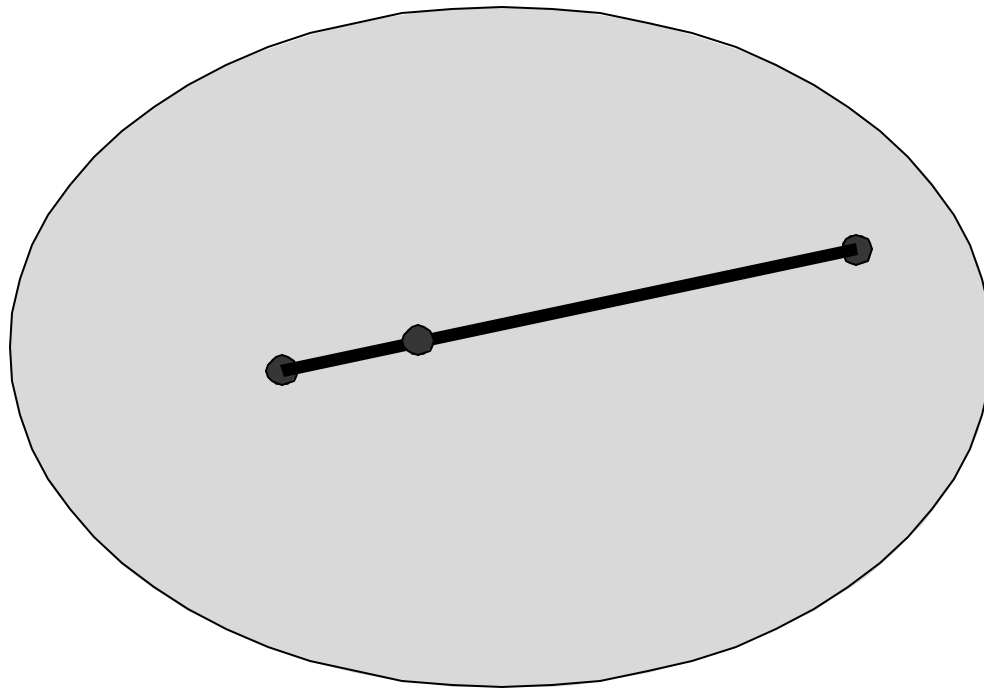
$$X = \begin{pmatrix} 4000 \\ 0 \\ 24000 \\ -6000 \\ 0 \end{pmatrix}$$

NO FACTIBLE

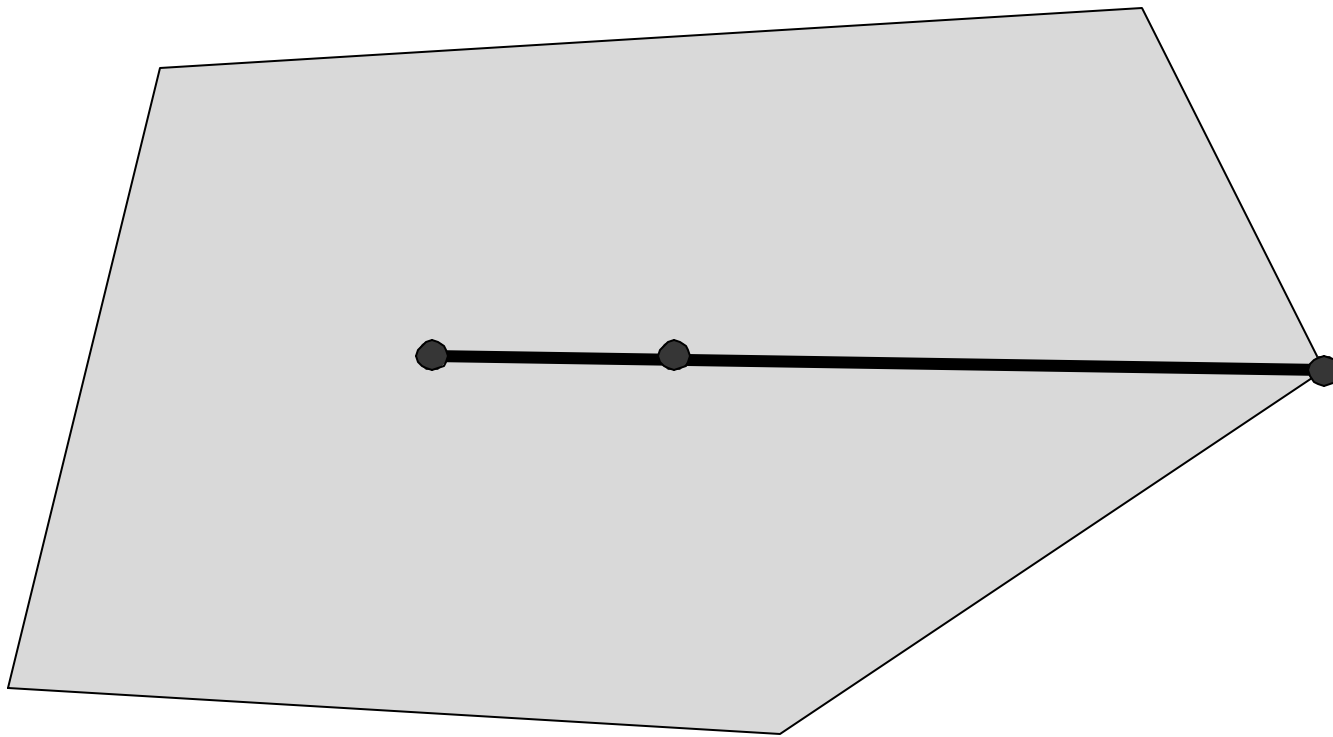




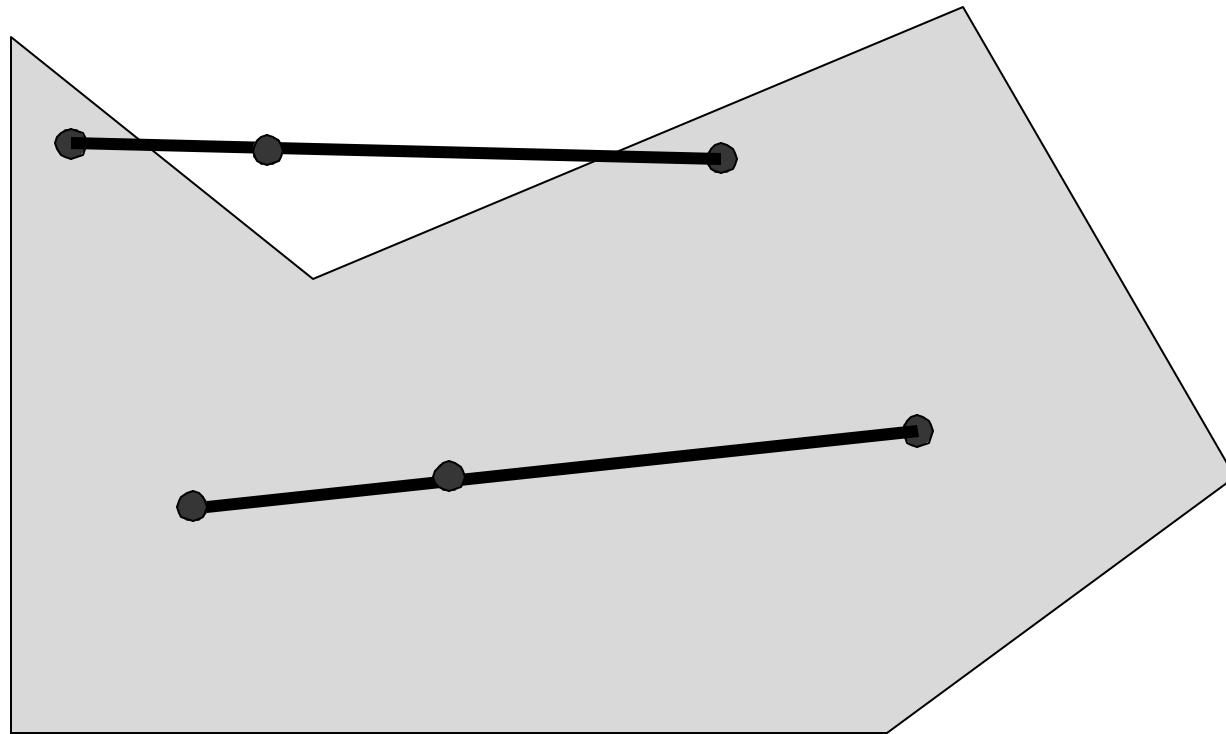
# CONJUNTO CONVEXO



# CONJUNTO CONVEXO



# CONJUNTO NO CONVEXO



# CONJUNTO NO CONVEXO

