Logistic Regression

DAT2

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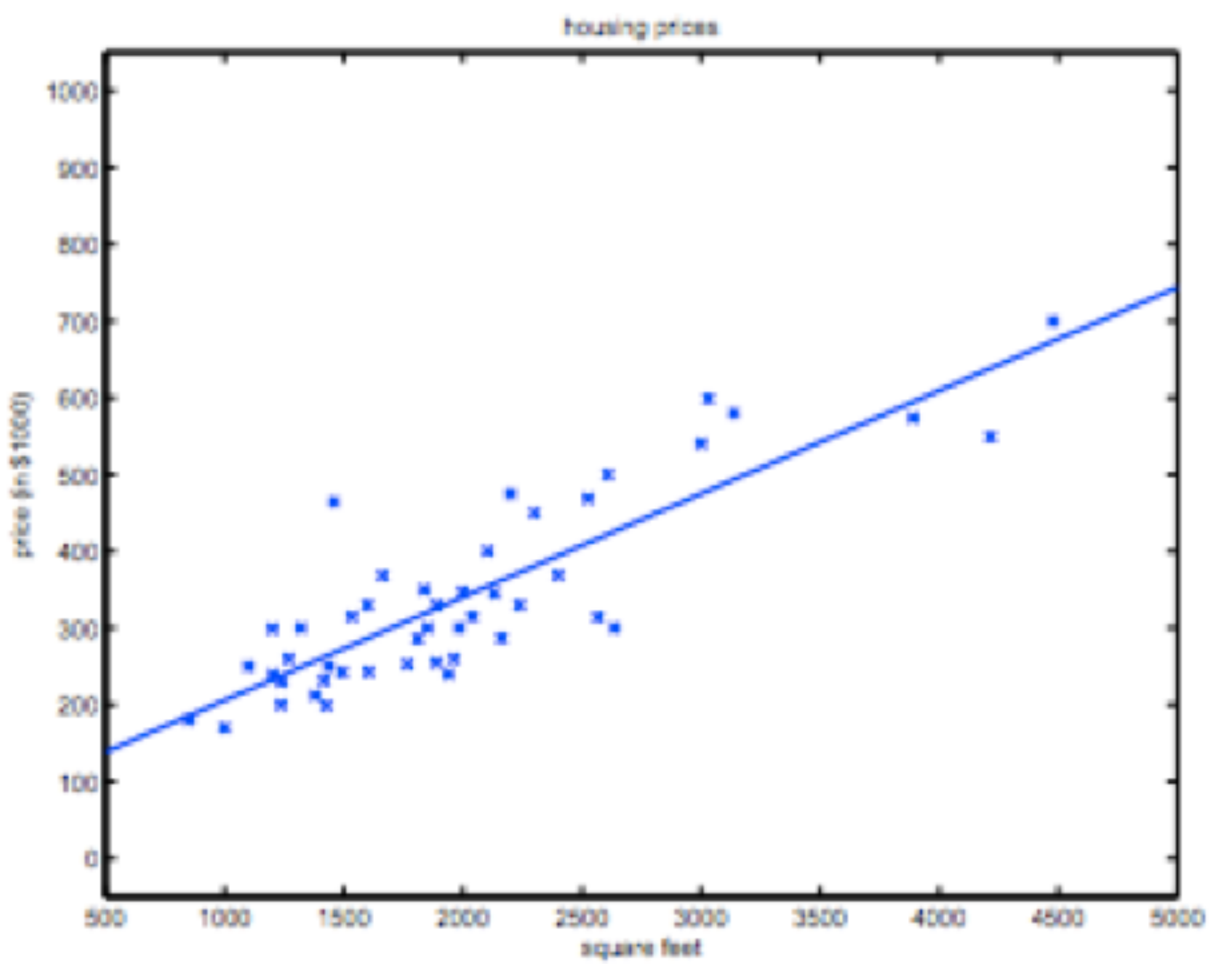
Agenda

- Linear Regression Review
- From Linear to Logistic
- Performance Measures
- GLMs, Exponential Family
- Relationship to Naïve Bayes

Linear Reg Review



$$h(x) = \sum_{i=0}^{n} \theta_i x_i = \theta^T x$$

Squared Error Loss

$$J(\theta) = \frac{1}{2}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$$

Why?

Probabilistic Interpretation

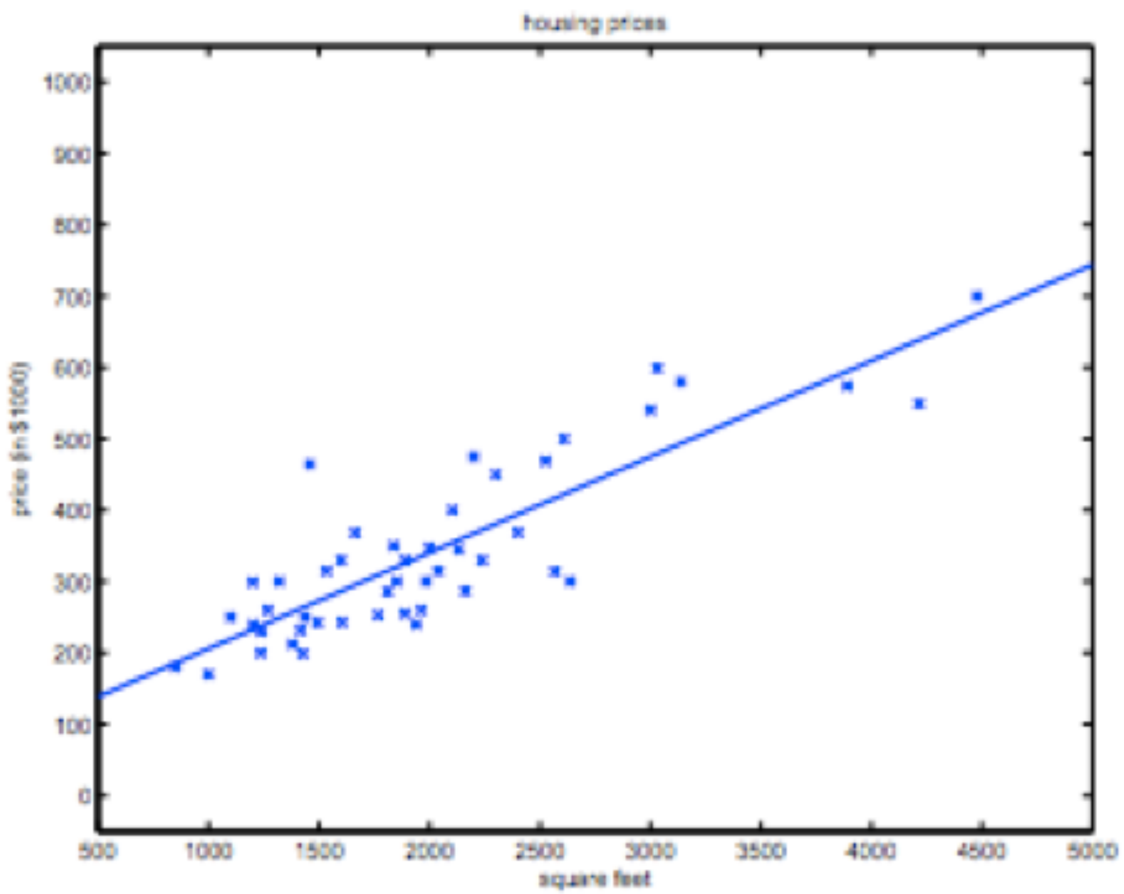
$$y^{(i)} = \theta^T x^{(i)} + \epsilon^{(i)}$$

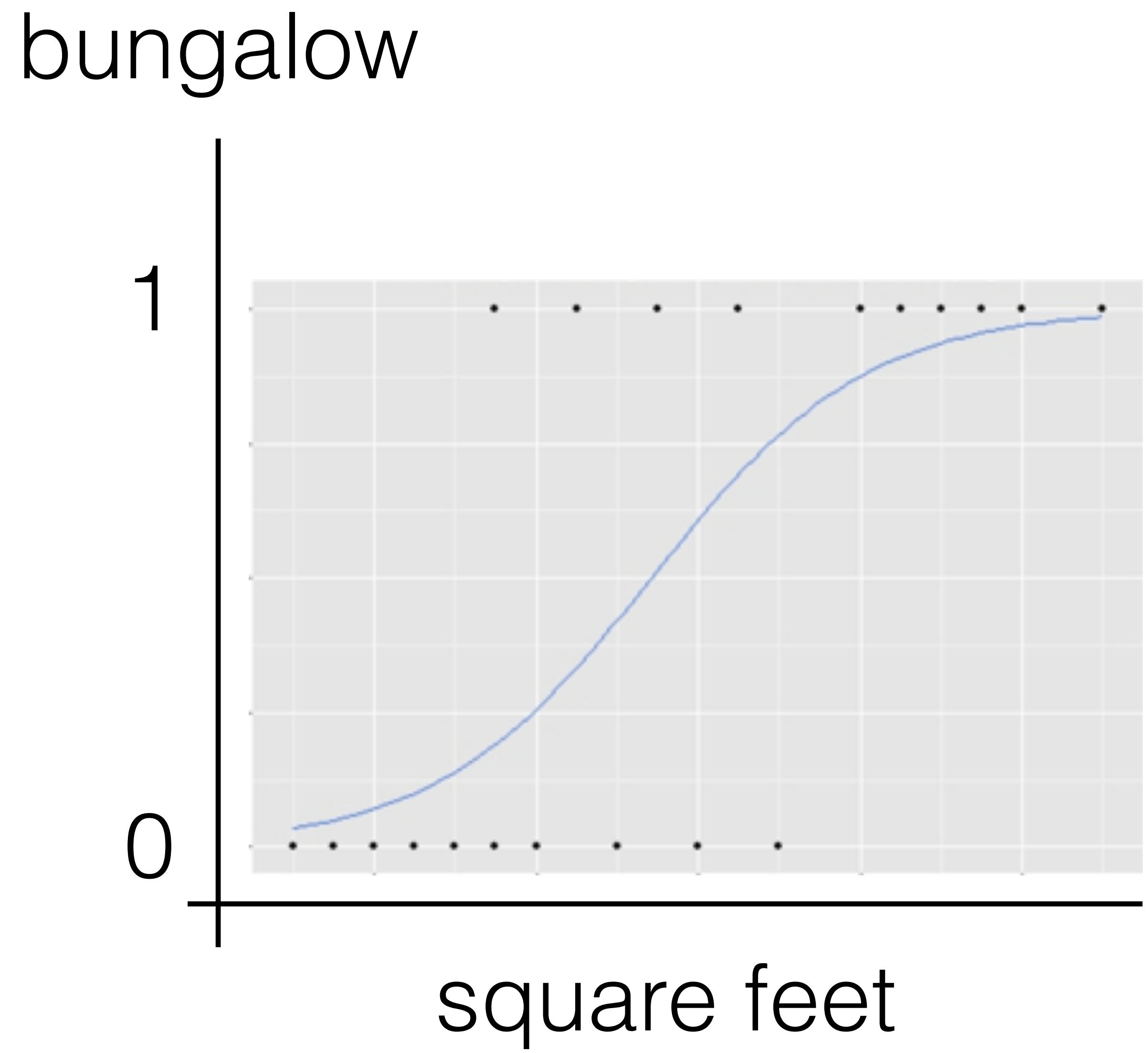
$$p(\epsilon^{(i)}) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\epsilon^{(i)})^2}{2\sigma^2}\right).$$

We can minimise **cost**
or
maximise likelihood

What's the likelihood?

Linear to Logistic

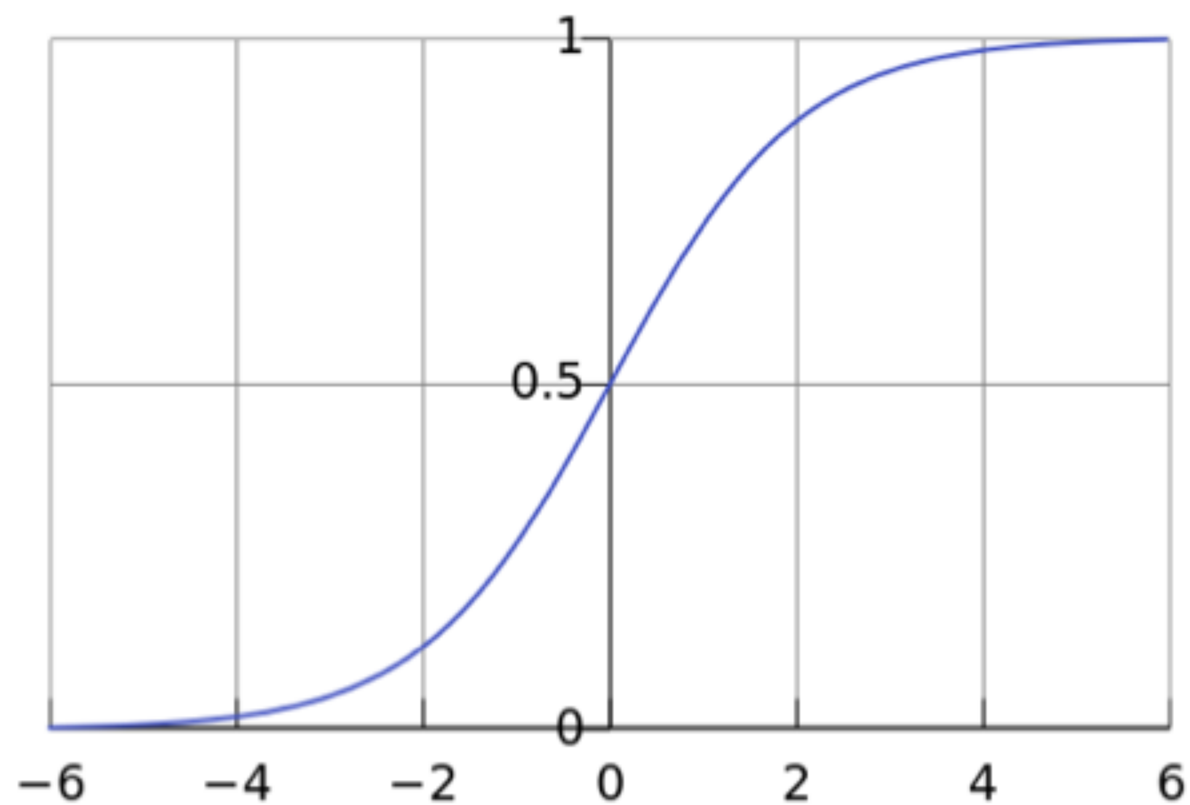




THE LOGISTIC FUNCTION

- The logistic function always returns a value between zero and one.

$$F(t) = \frac{1}{1 + e^{-t}}$$



Problems with just using linear regression to classify?

Classification vs Clustering?

Examples of Classification?

PERFORMANCE MEASURES

Predict whether tumors are malignant or benign:

- ***Accuracy*:** fraction of instances that are classified correctly
  - does not differentiate between malignant tumors that were classified as being benign, and benign tumors that were classified as being malignant.
- In some problems, the costs associated with all types of errors may be the same
- In this problem, failing to identify malignant tumors is likely more severe than failing to identify benign tumors as malignant

Confusion Matrix

	Prediction		
Actual		1	0
	1	**TP**	**FP**
	0	**FN**	**TN**

PERFORMANCE MEASURES

- ***Accuracy*** is the fraction of instances that were classified correctly

$$\text{ACC} = \frac{(\text{TP} + \text{TN})}{(\text{TP} + \text{TN} + \text{FP} + \text{FN})}$$

PERFORMANCE MEASURES

- ***Precision*** is the fraction of the tumors that were predicted to be malignant that are actually malignant.

$$P = TP / (TP + FP)$$

PERFORMANCE MEASURES

- ***Recall*** (or True Positive Rate) is the fraction of malignant tumors that the system identified.

$$R = TP / (TP + FN)$$

PERFORMANCE MEASURES

- ***Fall-out*** *or false positive rate* ***(FPR):***

$$FPR = FP / (FP + TN)$$

Generalised Linear Models (GLMs)

We’ve seen

y | x ~ N(mu, sigma) $\longrightarrow$ linear regression

y | x ~ Bernoulli(phi) $\longrightarrow$ logistic classification

Can we find common ground?

The Exponential Family

$$p(y;\eta) = b(y)\exp(\eta^T T(y) - a(\eta))$$

The Exponential Family: Bernoulli

$$p(y;\eta) = b(y)\exp(\eta^T T(y) - a(\eta))$$

$$\begin{aligned}
p(y;\phi) &= \phi^y (1-\phi)^{1-y} \\
&= \exp(y\log\phi + (1-y)\log(1-\phi)) \\
&= \exp\left(\left(\log\left(\frac{\phi}{1-\phi}\right)\right)y + \log(1-\phi)\right)
\end{aligned}$$

What are…

η =

$$\begin{aligned}
T(y) &= \\
a(\eta) &= \\
&= \\
b(y) &=
\end{aligned}$$

The Exponential Family: Bernoulli

$$p(y;\eta) = b(y)\exp(\eta^T T(y) - a(\eta))$$

$$\begin{aligned} p(y;\phi) &= \phi^y(1-\phi)^{1-y} \\ &= \exp(y\log\phi + (1-y)\log(1-\phi)) \\ &= \exp\left(\left(\log\left(\frac{\phi}{1-\phi}\right)\right)y + \log(1-\phi)\right) \end{aligned}$$

What are…

$\eta = \log(\phi/(1-\phi))$.

$$\begin{aligned} T(y) &= y \\ a(\eta) &= -\log(1-\phi) \\ &= \log(1+e^\eta) \\ b(y) &= 1 \end{aligned}$$

The Exponential Family: Normal

$$p(y;\eta) = b(y)\exp(\eta^T T(y) - a(\eta))$$

$$p(y;\mu) = \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}(y-\mu)^2\right)$$

What are…

$$\begin{aligned}
\eta &= \\
T(y) &= \\
a(\eta) &= \\
&= \\
b(y) &=
\end{aligned}$$

The Exponential Family: Normal

$$p(y;\eta) = b(y)\exp(\eta^T T(y) - a(\eta))$$

$$\begin{aligned} p(y;\mu) &= \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}(y-\mu)^2\right) \\ &= \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}y^2\right)\cdot\exp\left(\mu y - \frac{1}{2}\mu^2\right) \end{aligned}$$

What are…

$$\begin{aligned} \eta &= \mu \\ T(y) &= y \\ a(\eta) &= \mu^2/2 \\ &= \eta^2/2 \\ b(y) &= (1/\sqrt{2\pi})\exp(-y^2/2) \end{aligned}$$

Okay, okay…so who cares?

Constructing GLMs

1. Assume y | x; θ ~ ExponentialFamily(η)

2. Given x, we want to predict T(y), usually = y.
We choose h(x) = E[y|x]

3. Further assume η = θ^T.x

So we have a machinery we can crank

Constructing GLMs

Linear Regression

$$\begin{aligned} h_\theta(x) &= E[y|x;\theta] \\ &= \mu \\ &= \eta \\ &= \theta^T x. \end{aligned}$$

Logistic Classification

$$\begin{aligned} h_\theta(x) &= E[y|x;\theta] \\ &= \phi \\ &= 1/(1+e^{-\eta}) \\ &= 1/(1+e^{-\theta^T x}) \end{aligned}$$

Coincidentally, this is how we get softmax regression…

Relationship to Naïve Bayes

Assuming $y \mid x \sim$ some distribution

Assuming $x \mid y \sim$ some distribution

e.g.
text classification
Gaussian Discriminant Analysis (GDA)